1.- SERIE: Números, potencias, raíces

1 Simplify as far as possible, given that $x = \sqrt{5}$, $y = \sqrt{20}$

a x^2 **b** y^2 **c** $x + y$ **d** $2x - 3y$

e $\frac{y}{x}$ **f** x^3 **g** $\sqrt{xy}$

2 Express these in terms of the simplest possible surds.

a $\sqrt{80}$ **b** $\sqrt{32}$ **c** $\sqrt{72}$ **d** $\sqrt{180} + \sqrt{125}$

3 Rationalise the denominator of these fractions.

a $\frac{1}{\sqrt{3}}$ **b** $\frac{1}{\sqrt{2}}$ **c** $\frac{4}{\sqrt{7}}$ **d** $\frac{1}{4 - \sqrt{10}}$

e $\frac{2}{\sqrt{6} + 2}$ **f** $\frac{3}{2\sqrt{6}}$ **g** $\frac{3}{\sqrt{6} - \sqrt{5}}$

4 A square has area $6\,\text{cm}^2$.
Find its perimeter.

5 A rectangle has sides $(4 - \sqrt{7})\,\text{cm}$ and $(3 + 2\sqrt{7})\,\text{cm}$.
Find its perimeter and its area.

6 A rectangle has area $(6 - \sqrt{3})\,\text{cm}^2$ and the length of one of its sides is $(2 + \sqrt{3})\,\text{cm}$.
Find the length of the other side.

7 The sides of a rectangle are in the ratio 2:3. The diagonal is of length 26 cm.
Find the perimeter.

8 A cube has volume $10\,\text{cm}^3$.
Find the sum of the lengths of the sides, and the total surface area.

9 Given that $72 = 2^x \times 3^y$, find x and y, given that $x, y \in \mathbb{Z}$.

10 Simplify

a $3^{\frac{1}{4}} \times 3^{\frac{3}{4}}$ **b** $\sqrt{64} \times \sqrt[3]{64} \times \sqrt[6]{64}$ **c** $7^{\frac{1}{2}} \times 7^{\frac{1}{3}} \times 7^{\frac{1}{6}}$

d $(0.2)^4 \times 5^4$ **e** $(2.5)^3 \times 4^3$ **f** $\frac{6^{\frac{1}{4}} \times 36^{\frac{1}{8}}}{\sqrt{6}}$

11 Work out these values, giving each answer in index form where possible.

a $7^3 \times 7^2 \times 7^4$ **b** $3^4 \times 3^5 \times 3^2$ **c** $4^3 + 4^2$

d $5^2 \times 5^4 \times 5$ **e** $2^{10} - 2^5$ **f** $\dfrac{2^{10}}{2^5}$

g $\dfrac{8^3}{8}$ **h** $6^3 + 6$ **i** $\dfrac{7^6}{7^3}$

j $(7^2)^2$ **k** $(5^3)^3$

12 Simplify these, where possible.

a $a^4 \times a^3 \times a$ **b** $b^5 + b^3$ **c** $2c^3 + 3c^3$

d $d^3 \times d^4 \times d^2$ **e** $e^2 + e^3$ **f** $3f^2 \times 2f^3$

g $4g^3 \times 5g^2$ **h** $(h^4)^3$ **i** $(i^3)^4$

j $(3j)^2$ **k** $(2k^3)^2$ **l** $(7l^6)^2$

m $(m^2n)^3$ **n** $(p^2q^4)^3$ **o** $(3rs)^2$

p $v^2w \times vw^2$ **q** $\dfrac{x^4}{x^3}$ **r** $\dfrac{y^6}{y^2}$

s $\dfrac{z^6}{z}$

13 Simplify these, where possible.

a $a^4 - a^3$ **b** $3b^6 - 3b^3$ **c** $7c^5 - 7c^3$

d $\dfrac{3d^6}{3d^2}$ **e** $\dfrac{10e^3}{2e^2}$ **f** $\dfrac{49f^4}{7f^2}$

g $\sqrt{g^6}$ **h** $\sqrt[3]{h^{12}}$ **i** $\sqrt{25i^4}$

j $9j^5 - 8j^5$ **k** $3k^3 + 4k^4$ **l** $\sqrt[4]{l^8m^{20}}$

m $\sqrt{4n^2p^2}$ **n** $\sqrt{q^4}$ **o** $r^7 + r^7$

p $3s^4 + 4s^4$ **q** $15t^9 - 15t^9$ **r** $u^2 - u$

Test yourself

1 $(3\sqrt{5})^2$ is equal to

A 45 **B** 15 **C** $9\sqrt{5}$ **D** $\sqrt{\sqrt{45}}$ **E** $6\sqrt{5}$

2 $(a^6)^{-2}$ is equivalent to

A a^4 **B** $\frac{1}{a^{12}}$ **C** a^3 **D** $\frac{1}{a^3}$ **E** $-\sqrt{a^6}$

3 The expression $4\sqrt{63} - 5\sqrt{28}$ is equal to

A $-\sqrt{35}$ **B** $2\sqrt{7}$ **C** $16\sqrt{7}$

D $\sqrt{308}$ **E** none of these

4 $7 \times 10^{100} + 8 \times 10^{102}$ is equal to

A 1.5×10^{102} **B** 5.6×10^{101} **C** 7.08×10^{100}

D 8.07×10^{102} **E** 1.5×10^{203}

5 $\frac{(2a^2b)^3}{(ab)^5}$ is equivalent to

A $\frac{6}{a^3b^4}$ **B** $\frac{8a}{b^2}$ **C** $\frac{8}{b^2}$ **D** $(2a)^{\frac{3}{5}}$ **E** $8a^{-9}b^{-12}$

6 $\sqrt{12}\sqrt{15}\sqrt{20}$ is equal to

A 60 **B** $60\sqrt{15}$ **C** $30\sqrt{2}$ **D** $60\sqrt{12}$ **E** $\sqrt{\sqrt{60}}$

7 $\left(\frac{8}{27}\right)^{\frac{2}{3}}$ is equal to

A $\frac{64}{19683}$ **B** $\frac{64}{729}$ **C** $21\frac{1}{3}$ **D** $\frac{4}{9}$ **E** $\frac{2\sqrt{2}}{3}$

8 When two surds are multiplied, the result is

A always a surd

B never a perfect square

C never rational

D sometimes rational

E either irrational or prime

9 $\frac{\sqrt{6}}{2+\sqrt{3}}$ is equal to

A $\sqrt{2}+\frac{\sqrt{6}}{2}$ **B** $2\sqrt{6}-3\sqrt{2}$ **C** $\frac{12+3\sqrt{2}}{7}$ **D** $3\sqrt{2}-2\sqrt{6}$ **E** $\sqrt{12}-\sqrt{6}$

10 $\left(\frac{a^4b}{c^2}\right)^{-\frac{1}{2}}$ is equivalent to

A $-\frac{a^2\sqrt{b}}{c}$ **B** $\frac{a^{\frac{7}{2}}b^{-\frac{1}{2}}}{c^{\frac{3}{2}}}$ **C** $\frac{c}{a^4b}$ **D** $\frac{c\sqrt{b}}{a^2b}$

E none of these

1.- SERIE: Números, potencias, raíces (SOLUCIONES)

1 **a** 5 **b** 20 **c** $3\sqrt{5}$
d $-4\sqrt{5}$ **e** 2 **f** $5\sqrt{5}$
g $\sqrt{10}$

2 **a** $4\sqrt{5}$ **b** $4\sqrt{2}$ **c** $6\sqrt{2}$
d $11\sqrt{5}$

3 **a** $\frac{1}{3}\sqrt{3}$ **b** $\frac{1}{2}\sqrt{2}$ **c** $\frac{4}{7}\sqrt{7}$
d $\frac{4+\sqrt{10}}{6}$ **e** $\sqrt{6}-2$ **f** $\frac{1}{4}\sqrt{6}$
g $3(\sqrt{6}+\sqrt{5})$

4 $4\sqrt{6}\,\text{cm}$

5 $(14+2\sqrt{7})\,\text{cm};\ (5\sqrt{7}-2)\,\text{cm}^2$

6 $(15-8\sqrt{3})\,\text{cm}$

7 $20\sqrt{13}\,\text{cm}$

8 $12\sqrt[3]{10}\,\text{cm};\ (6\times 10^{\frac{2}{3}})\,\text{cm}^2$

9 $x=3,\ y=2$

10 **a** 3 **b** 64 **c** 7
d 1 **e** 1000 **f** 1

11 **a** 7^9 **b** 3^{11} **c** 80
d 5^7 **e** 992 **f** 2^5
g 8^2 **h** 222 **i** 7^3
j 7^4 **k** 5^9

12 **a** a^8 **c** $5c^3$ **d** d^9
f $6f^5$ **g** $20g^5$ **h** h^{12}
i i^{12} **j** $9j^2$ **k** $4k^6$
l $49l^{12}$ **m** m^6n^3 **n** p^6q^{12}
o $9r^2s^2$ **p** v^3w^3 **q** x
r y^4 **s** z^5

13 **d** d^4 **e** $5e$ **f** $7f^2$
g g^3 **h** h^4 **i** $5i^2$
j j^5 **l** l^2m^5 **m** $2np$
n q^2 **o** $2r^7$ **p** $7s^4$
q 0

Test Yourself (p. 21)

1 A **2** B **3** B **4** D **5** B
6 A **7** D **8** D **9** B **10** D