2.- SERIE: LOGARITMOS Y EXPONENCIALES

1 Express in logarithmic notation

a $2^5 = 32$ **b** $10^2 = 100$ **c** $a^b = c$

d $p^3 = q$ **e** $3 = 27^{\frac{1}{3}}$ **f** $\frac{1}{3} = 3^{-1}$

2 Express in index notation

a $\log_2 8 = 3$ **b** $\log_6 36 = 2$ **c** $\log_a b = c$

d $\log_d c^4 = e$ **e** $\log_p 8 = 4$ **f** $q = \log_c 3$

3 Evaluate

a $\log_2 128$ **b** $\log_{10} 1000$ **c** $\log_p p^4$

d $\log_2 16$ **e** $\log_{16} 2$ **f** $\log_e \frac{1}{e^2}$

4 Express in terms of $\log a$, $\log b$ and $\log c$

a $\log \frac{ab^2}{c}$ **b** $\log \sqrt{ab}$ **c** $\log \frac{a^4}{b^2 c}$ **d** $\log a^2 b^3 c^4$

5 Express each of these as a single logarithm.

a $\log(a^2 b) - \log(b^2 a)$ **b** $\frac{1}{2}\log x + 3\log y$

c $\log 5 + 2\log 10 - 3\log 2$ **d** $a\log x + a\log x^2$

6 Simplify

a $\lg 75 + 2\lg 2 - \lg 3$ **b** $\lg 1000\,000$ **c** $\frac{1}{2}\log 64$

d $5\log 3 - \log 81$ **e** $\frac{\log 16}{\log 2}$ **f** $\frac{\log 5}{\log 125}$

7 Solve these.

a $\log_{10}(n^2 - 90n) = 3$ **b** $9^x = 27^{\frac{3}{4}}$ **c** $3^x = 4$

d $5^{2x+1} = 25$ **e** $5^{2x+1} = 8$ **f** $\log_2(y^2 + 7y) = 3$

g $5^{2x} - 6 \times 5^x + 5 = 0$ **h** $4^{2x+1} - 7(4^x) + 3 = 0$ **i** $2^{2x} - 5(2^x) = 14$

j $2^{x^2+x} = 4$

8 Many phenomena – from stock market prices to census data to heat capacities of chemicals – obey Benford's Law. This states that for a set of numerical data, the proportion of numbers starting with the digit D is approximately

$$\log_{10}\left(1 + \frac{1}{D}\right)$$

a Show that Benford's Law predicts that around 30% of numbers will start with a 1, and around 18% with a 2. What proportion of numbers does the law predict will start with a 9?

b Show that $\sum_{D=1}^{9} \log_{10}\left(1 + \frac{1}{D}\right) = 1$.

Test yourself

1 $\log_{64} 4$ is equal to

A 3 **B** -3 **C** -16 **D** $\frac{1}{3}$ **E** $\frac{1}{16}$

2 $4^{3x-1} = 40$. Correct to 3 significant figures, x is equal to

A 3.67 **B** 1.22 **C** 1.10 **D** 0.73 **E** 0.64

3 $\log_{12} 18 + 3\log_{12} 2$ is equal to

A 1.88 **B** 1.31 **C** 1 **D** 2 **E** 4.33

4 Correct to 2 decimal places, $\log_4 5^3$ is equal to

A 1.16 **B** 2.58 **C** 3.48 **D** 0.86 **E** 2.10

5 $\log \dfrac{a\sqrt{b^3}}{c^2}$ is equal to

A $\frac{3}{4}\log a \log b \log c$ **B** $\log(a + \frac{3}{2}b - 2c)$ **C** $\log a + \frac{3}{2}\log b - 2\log c$

D $\frac{1}{2}\log a \log b^3 - \log c^2$ **E** $\frac{3}{2}\log ab - 2\log c$

6 Given that $3^{t+1} = 6^{t-1}$, correct to 3 significant figures, t is equal to

A 3.00 **B** 4.17 **C** -3.82 **D** -6.52 **E** 1.86

7 $\log(a + b)$ is equivalent to

A $(\log a)^b$ **B** $(\log a)(\log b)$ **C** $\log a + \log b$

D $\log a^b$ **E** none of these

8 If $2\log_p q = r$, then

A $(q^p)^2 = r$ **B** $p^{2q} = r$ **C** $q^p = r^2$ **D** $p^r = q^2$ **E** $q^{2r} = p$

9 If $3^{2y} - 2(3^{y+1}) = 27$, then y is equal to

A 2 **B** $\pm\sqrt{28}$ **C** 0 **D** 9 **E** -6

10 $2\log 10 + \log 12 - 2\log 4$ is equal to

A $\log 30$ **B** $\log 75$ **C** $\log 52$ **D** $\log 96$ **E** $\log 60$

2.- SERIE: LOGARITMOS Y EXPONENCIALES (SOLUCIONES)

1 **a** $\log_2 32 = 5$ **b** $\log_{10} 100 = 2$
c $\log_a c = b$ **d** $\log_p q = 3$
e $\log_{27} 3 = \frac{1}{3}$ **f** $\log_3 \frac{1}{3} = -1$

2 **a** $2^3 = 8$ **b** $6^2 = 36$
c $a^c = b$ **d** $d^e = c^4$
e $p^4 = 8$ **f** $c^q = 3$

3 **a** 7 **b** 3 **c** 4
d 4 **e** $\frac{1}{4}$ **f** -2

4 **a** $\log a + 2\log b - \log c$
b $\frac{1}{2}\log a + \frac{1}{2}\log b$
c $4\log a - 2\log b - \log c$
d $2\log a + 3\log b + 4\log c$

5 **a** $\log\left(\frac{a}{b}\right)$ **b** $\log(\sqrt{x}y^3)$
c $\log 62.5$ **d** $\log x^{3a}$

6 **a** 2 **b** 6 **c** $\log 8$
d $\log 3$ **e** 4 **f** $\frac{1}{3}$

7 **a** $n = -10$ or $n = 100$
b $x = \frac{9}{8}$
c $x = 1.26$
d $x = \frac{1}{2}$
e $x = 0.146$
f $y = -8$ or $y = 1$
g $x = 0$ or $x = 1$
h $x = 0$ or $x = -0.208$
i $x = 2.81$
j $x = -2$ or $x = 1$

8 **a** 4.6%

Test Yourself

1 D **2** B **3** D **4** C **5** C
6 B **7** E **8** D **9** A **10** B