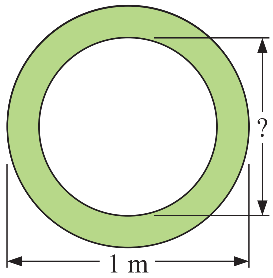
CHALLENGE SET 1

1 Show how to construct a regular hexagon using a compass, ruler and pencil.

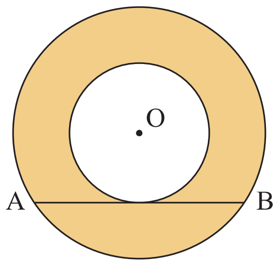
2 A circular target has diameter 1 m. A smaller circle is placed within the target. What should be the diameter of the smaller circle if a complete novice can expect to hit the inner circle as often as he or she hits the outer circle?



3 In a multiple choice test there are 20 questions. A correct answer to a question earns 2 marks and an incorrect answer results in a deduction of 1 mark. Sarah achieved a mark of 19. How many questions did she get right?

4 Find a and b given that $a^b \times b^a = 800$ and that a and b are integers.

5



[AB] is a tangent to the inner circle and is a chord of the larger circle. If [AB] is 6 m long, find the area of the shaded annulus.

6 Show that when a 2-digit number is added to the 2-digit number obtained by reversing the digits of the original number, the result is divisible by 11.
Hint: A number with digit form "ab" can be written algebraically as $10a + b$.

7 Towns X and Y are 90 km apart. Ross runs from X to Y at a uniform speed of 11 km h^{-1}. Peter leaves at a different time and runs from Y to X at a uniform speed of 12 km h^{-1}. When they meet they observe that they have both been running for an exact number of hours. Where did they meet?

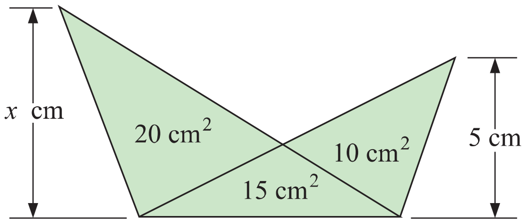
8
- **a** Find $\sqrt{3+2\sqrt{2}} \times \sqrt{3-2\sqrt{2}}$ without using your calculator.
- **b** Check your result using your calculator.
- **c** Consider $X = \sqrt{3+2\sqrt{2}} - \sqrt{3-2\sqrt{2}}$. Find X^2 without using your calculator.
- **d** Find the value of $\sqrt{3+2\sqrt{2}} - \sqrt{3-2\sqrt{2}}$ without using a calculator.

CHALLENGE SET 2

1 To what index must we raise 1.1 in order to obtain an answer of at least 10?

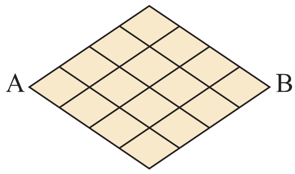
2 What is the last digit of the number 3^{1998}?

3 Find x:



4 4 litres of 8% cordial mix are added to 6 litres of 5% cordial mix. What is the percentage of cordial in the final mixture?

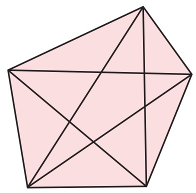
5



If only motion to the right is permitted, how many different paths can be taken from A to B?

6 A pentagon has 5 diagonals.

- **a** How many diagonals has a hexagon?
- **b** How many diagonals has an octagon?
- **c** How many diagonals has an n-gon?

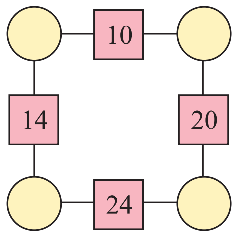


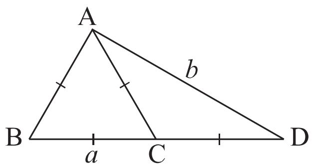
7 A motorcyclist and a cyclist travel from A to B and leave simultaneously from A at 12 noon. The motorcyclist arrives at B after $1\frac{1}{2}$ hours and the cyclist arrives 30 minutes later. At what time was the cyclist twice as far from B as the motorcyclist was?

8 Show that $n^3 - n$ is always divisible by 6 for any integer n.
Hint: Fully factorise $n^3 - n$.

CHALLENGE SET 3

1 Place integers in the circles so that the number in each square is the sum of the circled numbers adjacent to that square.



2 

Show that $b = a\sqrt{3}$.

3 **a** Simplify $(n+1)^2 - n^2$.

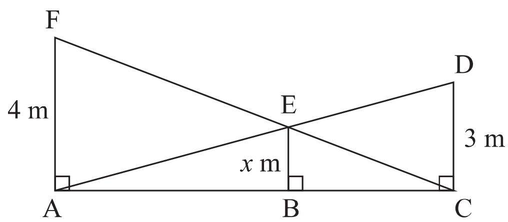
b Two consecutive squares differ by 71. What is the smaller of the two squares?

4 | 3 0 7 D A Q | is a typical Australian alphanumeric registration plate for a motor vehicle.

How many different registration plates could be produced with:

a three numbers followed by three letters

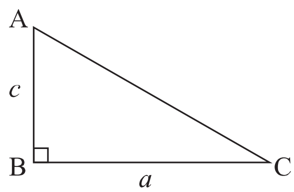
b four letters followed by two numbers?

5 

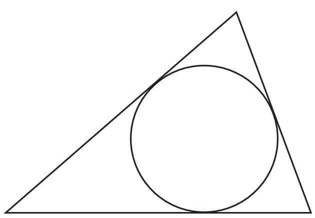
Find x.

Hint: Let AB $= a$ metres and BC $= b$ metres.

6 Show that the shortest distance from B to [AC] is $\dfrac{ac}{\sqrt{a^2+c^2}}$.



Hint: Use the area of ΔABC in two different forms.

7 

A circle is inscribed in a triangle. The triangle has area A and perimeter P. Show that:

a the radius of the circle is given by $r = \dfrac{2A}{P}$

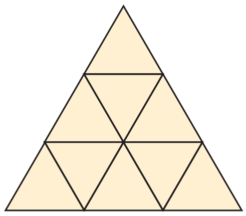
b the ratio of perimeters of the circle and triangle is the same as the ratio of their areas.

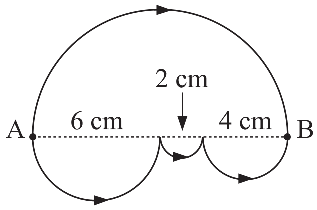
8 Can any two digit number be equal to the product of its digits?

CHALLENGE SET 4

1 When Samuel was 7, Claire was 34. Claire's age is now double Samuel's age. How old will Samuel be in 10 years' time?

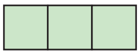
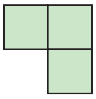
2 How many triangles does the given figure contain?



3 

Which is the shorter path from A to B given that each curve is semi-circular?

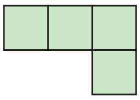
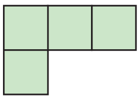
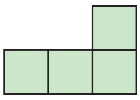
4 When 3 squares are drawn so that sides are shared, there are **two** figures which can be constructed.

These are  and  .

How many different figures are possible if:

a 4 squares are used

b 5 squares are used?

Note:  ,  ,  , etc., are the same.

5 $S_1 = \frac{1}{1\times2}$, $S_2 = \frac{1}{1\times2} + \frac{1}{2\times3}$, $S_3 = \frac{1}{1\times2} + \frac{1}{2\times3} + \frac{1}{3\times4}$.

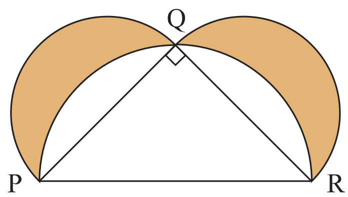
Find the values of S_1, S_2, S_3 and S_4 in fractional form and hence find the value of

$$\frac{1}{1\times2} + \frac{1}{2\times3} + \frac{1}{3\times4} + \frac{1}{4\times5} + \ldots\ldots + \frac{1}{100\times101}.$$

6 Two real numbers differ by 3 and the sum of their squares is 8. Determine their product by considering $(a-b)^2$.

7 Mouldy Oldy claims that if the digits in his age are multiplied, the result would be equal to his age 40 years ago. How old is Mouldy Oldy?

8 PQR is a right angled triangle. 3 semi-circles are constructed on each of its sides as shown. Prove that the sum of the shaded areas is equal to the area of triangle PQR.

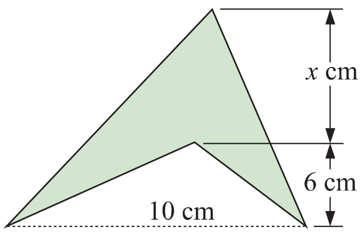


CHALLENGE SET 5

1 To pick all the oranges in an orchard it will take 14 people a total of 10 days. If 3 of the workers are sick and cannot work, and 2 others can work at half the usual rate, how long will it take to pick all of the oranges?

2 A rental car may be hired either for \$80 per day, or at \$45 per day plus 35 cents per kilometre. When would you be better off using the \$80 per day option?

3



The shaded region has area 24 cm^2.
Find x.

4 Find x and y if $\frac{3}{x} + \frac{1}{y} = 9$ and $\frac{5}{x} + \frac{4}{y} = 22$.

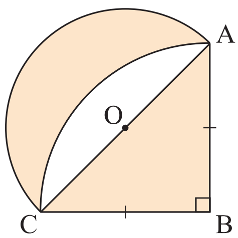
Hint: Let $X = \frac{1}{x}$ and $Y = \frac{1}{y}$.

5 When a positive integer N is divided by 2, the remainder is 1. When N is divided by 3 the remainder is 2, and when N is divided by 4 the remainder is 3.
Show that N could be any integer of the form $12a - 1$ where $a = 1, 2, 3, 4, ...$

6 How many ways can 360 be factorised into the product of 3 factors greater than 1?
For example, $360 = 3 \times 12 \times 10$.

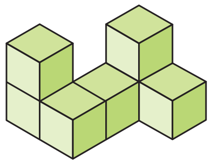
7 Any **even** number can be written in the form $2a$ where a is an integer.
Any **odd** number can be written in the form $2a + 1$ where a is an integer.
If p and q are positive integers and $p^2 + q^2$ is divisible by 4, prove that p and q are both even.
Hint: Consider the 4 possible cases, one of which is when p and q are both even.

8 Show that the area enclosed by the two arcs is equal to the area of the isosceles right angled triangle ABC.



CHALLENGE SET 6

1 7 blocks are glued together as shown. What is the total surface area of the resulting solid given that each cube has sides of 1 cm?



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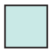
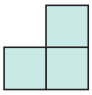
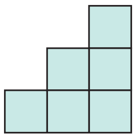


requires 4 toothpicks to make.



requires 10 toothpicks to make.

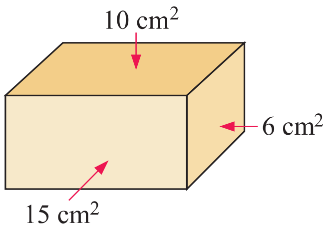


requires 18 toothpicks to make.

How many toothpicks are required to make a structure similar to these but containing 10 squares in the bottom row?

3 Students at a dance are evenly spaced around a circle. One student is numbered 1 and the others are consecutively numbered from 1 around the circle. The student numbered 7 is directly opposite number 32. How many students are in the circle?

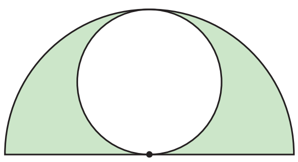
4 Three faces of a rectangular prism have areas of 6 cm^2, 10 cm^2 and 15 cm^2 respectively. Find the volume of the prism.



5 To win a *best of five sets* match at tennis, one possibility is to win in straight sets, i.e., WWW. Another possibility is WLLWW.
How many winning possibilities exist?

6 I wish to buy 100 animals. Cats cost \$5, rabbits \$1 and fish 5 cents each. If I have \$100 and spend every cent buying these animals, how many of each will I have bought?

7



Prove that the shaded region has area equal to that of the circle in the centre.

8 **a** Show that $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$.

b Given that $a+b+c=3$, $ab+bc+ca=2$ and $abc=-2$, find the values of the following:

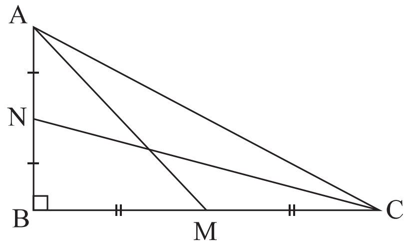
i $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$ **ii** $a^2+b^2+c^2$.

Hint: You should not have to find the actual values of a, b and c.

CHALLENGE SET 7

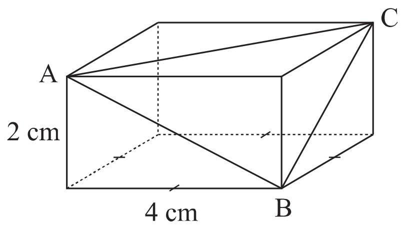
1 If you increase the price of a TV set by 10% each month for 3 successive months then its overall price increase is 33.1%. True or false?

2 In the figure we know that [AM] has length $\sqrt{3}$ cm and [CN] has length $\sqrt{7}$ cm. Find the length of [AC].



3 Find the first time after 3 pm when the hands of a clock are at right angles to each other. Give your answer to the nearest second.

4



For the rectangular prism shown with a square base of 4 cm and height 2 cm, find, correct to 2 decimal places, the size of angle ABC.

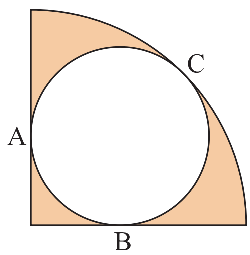
5 When a number is increased by 7, a perfect square results. When the same number is increased by 43, another perfect square results. Show that there are 2 possible original numbers.

6 If $a * b = a + b - 2ab$ for any real numbers a and b, find:

a $7 * (-2)$ **b** $(2 * 3) * (4 * 1)$ **c** m if $5 * m = 4$.

7 [AB] is a line segment and C is a point not on [AB]. Explain how to construct a perpendicular from C to [AB] using a pencil, ruler and compass only.

8



A circle with radius r is inscribed in a quadrant (quarter circle) of radius R, touching it at A, B and C.

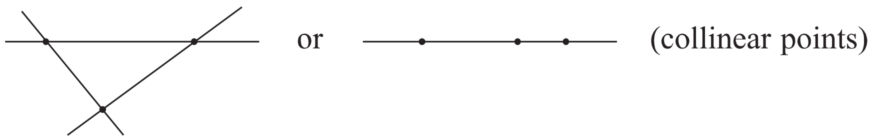
a Find the relationship between R and r.

b Compare the area shaded with that of the small circle.

CHALLENGE SET 8

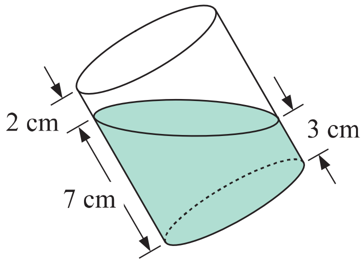
1 $\begin{array}{r} AB \\ +\ B \\ \hline BA \end{array}$ Find integers A and B for which this sum is correct.

2 Through 3 distinct points we can draw either 3 lines or 1 line:



Illustrate **all** possible cases when lines are drawn through 5 distinct points.

3 What fraction of the cylinder is occupied by the liquid?



4 If $x^2 = 1 - 3x$, show that $x^4 = 10 - 33x$.

5 Find a, b and c given that

$$a + b - c = 13$$
$$a - b + c = -3$$
$$a - b - c = 1.$$

6 A rectangular lawn measures 10 m by 6 m. One circuit around the outer edge of the lawn will mow $\frac{1}{4}$ of the lawn. How wide is my mower?

7 Solve the equations:

a $\sqrt{x+2} = x$ **b** $\sqrt{x+13} - \sqrt{7-x} = 2$

8 The factors of 30 are 1, 2, 3, 5, 6, 10, 15 and 30. These can be written in the form:

$$2^0 3^0 5^0 \quad 2^1 3^0 5^0 \quad 2^0 3^1 5^0 \quad 2^0 3^0 5^1$$
$$2^1 3^1 5^0 \quad 2^1 3^0 5^1 \quad 2^0 3^1 5^1 \quad 2^1 3^1 5^1$$

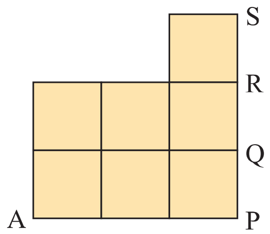
a Noting that $30 = 2 \times 3 \times 5$, explain why the number of factors of 30 is $2 \times 2 \times 2 = 8$.

b How many factors has the number 540, which is $2^2 \times 3^3 \times 5$?

c How many factors has the number $2^a 3^b 5^c$ where a, b and c are non-negative integers?

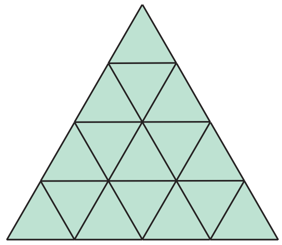
CHALLENGE SET 9

1 What can be deduced about a and b if $(a+b)^2 = (a-b)^2$?

2 

Where should point B be located along [PS] such that [AB] divides the figure into equal areas?

3 How many triangles are there in the given figure?



4 Find x, y and z if
$$x + y + z = 2$$
$$x - y + 2z = -11$$
$$2x + y - z = 15.$$

5 The difference between the cubes of two consecutive integers is 127. Find the integers.

6 By considering the perfect square $(x-1)^2$ where $x > 0$, prove that "the sum of any positive real number and its reciprocal is never less than 2."

7 Find all solutions of the equation $(x^2 + 2x - 3)^2 + 2(x^2 + 2x - 3) - 3 = 0$.
Hint: Let $x^2 + 2x - 3 = m$.

8 Prove that the solution of $2^x = 3$ cannot be a rational number.
Hint: Suppose x is rational, i.e., $x = \dfrac{p}{q}$ where p and q are positive integers.

CHALLENGE SET 10

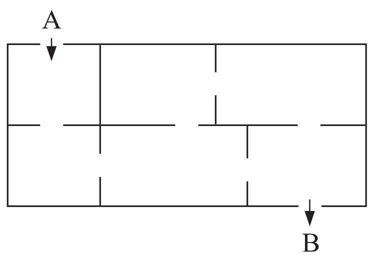
1 One of the greatest mathematicians of all time, **Leonhard Euler**, claimed that $n^2 + n + 41$ is **always** a prime number for $n = 0, 1, 2, 3, 4, 5, ...$

a Find the value of this expression for $n = 0, 1, 2, 3, 4, 5, 6, 7$ and 8.

b By making a suitable substitution, show that Euler's claim was wrong.

2 A log is sawn into 7 pieces in 7 minutes. How long would it have taken to saw the same log into 4 pieces?

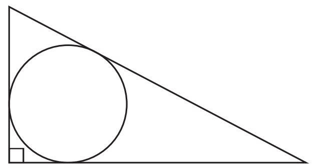
3



Explain why it is impossible to enter this house through door A, pass through every room of the house and through every door exactly once, and then exit through door B.

4 If n is odd, show that $(n+2)^2 - n^2$ is a multiple of 8.

5 Find the radius of the inscribed circle of the right angled triangle with sides 3 cm, 4 cm and 5 cm.



6 Find the possible values of a, b and c given that:

$$a + b = c$$
$$b + c = a$$
$$c + a = b.$$

7 $\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+......}}}}}$ is actually equal to a positive integer. Find it.

Hint: Let the number be x and find x^2.

8 Prove that "a 4-digit integer is divisible by 3 if the sum of its digits is divisible by 3".
Hint: Let the number be $1000a + 100b + 10c + d = 999a + 99b + 9c + ...$