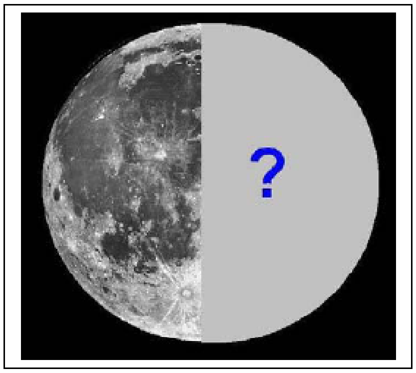
The Moon's Density - What's inside?



The Moon has a mass of 7.4×10^{22} kilograms and a radius of 1,737 kilometers. Seismic data from the Apollo seismometers also shows that there is a boundary inside the Moon at a radius of about 400 kilometers where the rock density or composition changes. Astronomers can use this information to create a model of the Moon's interior.

In 1973 a rocket was send to a very distant Solar System (B) to collect some kind of information of the planets from that Solar System B. The obtained information is shown in the following table:

Student	Radius		Mass		boundary at	
Moon	1737	km	7,4E+22	kg	400	km
Planet 1	5000	km	4,1972E+24	kg	1000	km
Planet 2	5000	km	5,9264E+24	kg	1000	km
Planet 3	1000	km	1,0415E+22	kg	200	km
Planet 4	7655	km	9,3082E+24	kg	1500	km
Planet 5	9843	km	1,0888E+19	kg	1800	km
Planet 6	11700	km	9,316E+25	kg	2000	km
Planet 7	10600	km	4,0446E+25	kg	2100	km
Planet 8	2124	km	7,2304E+22	kg	500	km
Planet 9	9877	km	2,426E+25	kg	2400	km
Planet 10	3400	km	7,8845E+23	kg	600	km
Planet 11	9034	km	3,3028E+25	kg	800	km
Planet 12	14680	km	3,2689E+25	kg	4800	km
Planet 13	24660	km	4,7584E+26	kg	6000	km
Planet 14	4850	km	1,6809E+24	kg	1200	km

Problem 1

a)

- What is the average density of the Moon in grams per cubic centimeter (g/cm^3) ? (Assume the Moon is a perfect sphere.)

b) Find the density of your planet in Solar System B in g/cm^3

Problem 2

- What is the volume, in cubic centimeters, of A) the Moon's interior out to a radius of 400 km? and B) The remaining volume out to the surface?

You can make a simple model of a planet's interior by thinking of it as an inner sphere (the core) with a radius of R(core), surrounded by a spherical shell (the mantle) that extends from R(core) to the planet's surface, R(surface). We know the total mass of the planet, and its radius, R(surface). The challenge is to come up with densities for the core and mantle and R(core) that give the total mass that is observed.

C) Find the volume of your planet's core in cm^3

D) Find the volume of your planet's shell in cm^3

Problem 3

- From this information, what is the total mass of the planet model in terms of the densities of the two rock types (D1 and D2) and the radius of the core and mantle regions R(core) and R(surface)?

So, find the formula for the total mass, using densities and different radius (or volumes).

Problem 4 (don't do it, just check how it's done)

- The densities of various rock types are given in the table below.

Type	Density
I - Iron+Nickle mixture (Earth's core)	15.0 gm/cc
E - Earth's mantle rock (compressed)	4.5 gm/cc
B - Basalts	2.9 gm/cc
G - Granite	2.7 gm/cc
S - Sandstone	2.5 gm/cc

A) How many possible lunar models are there? B) List them using the code letters in the above table, C) If denser rocks are typically found deep inside a planet, which possibilities survive? D) Find combinations of the above rock types for the core and mantle regions of the lunar interior model, that give approximately the correct lunar mass of 7.4×10^{25} grams. [Hint: use an *Excel* spread sheet to make the calculations faster as you change the parameters.] E) If Apollo rock samples give an average surface density of 3.0 gm/cc, which models give the best estimates for the Moon's interior structure?

Answer to problem 4:

- A) There are 5 types of rock for 2 lunar regions so the number of unique models is 5 x 5 = 25 possible models. B) The possibilities are: II, IE, IB, IG, IS, EE, EI, EB, EG, ES, BI, BE, BB, BG, BS, GI ,GE, GB, GG, GS, SI, SE, SB, SG, SS. C) The ones that are physically reasonable are: IE, IB, IG, IS, EB, EG, ES, BG, BS, GS. The models, II, EE, BB, GG and SS are eliminated because the core must be denser than the mantle. D) Each possibility in your answer to Part C has to be evaluated by using the equation you derived in Problem 3. This can be done very efficiently by using an Excel spreadsheet. The possible answers are as follows:

Model Code	Mass (in units of 10^{25} grams)
I E	10.2
I B	6.7
E B	6.4
I G	6.3
E G	6.0
B G	6.0
I S	5.8
E S	5.5
B S	5.5
G S	5.5

E) The models that have rocks with a density near 3.0 gm/cc as the mantle top layer are the more consistent with the density of surface rocks, so these would be IB and EB which have mass estimates of 6.7 x 10^{25} and 6.4 x 10^{25} grams respectively. These are both very close to the actual moon mass of 7.4 x 10^{25} grams (e.g. 7.4 x 10^{22} kilograms) so it is likely that the moon has an outer mantle consisting of basaltic rock, similar to Earth's mantle rock (4.5 gm/cc) and a core consisting of a denser iron/nickel mixture (15 gm/cc).

Problem 5

Do similar calculations with the data taken from your planet in the first table, but we know that in the Solar System B the possible existing rock types are only four:

Type	Density g/cm^3
I – Iron + Nickle mixture	15.0
E- Earth`s mantle rocks	4.5
B- Basalts	2.9
G- Liquid Gas (compressed)	1.2

A) How many possible planet models are there?
B) List them using the code letters in the above table.
C) If denser rocks are typically found deep inside a planet, which possibilities survive?
D) Find combinations of the above rock types for the core and mantle regions of your planet interior model that give approximately the correct planet mass shown in the first table. Use an excel spread sheet to make the calculations faster as you change the parameters.
E) Which models give the best estimates for your planet's interior structure?

TO REMEMBER:

The whole project must be written on a maximum of 6 sheets, and must include:

1. A title
2. Name and present course
3. Date and Term
4. At the end of your written project you must include a bibliography or references to books, articles, web links, etc. that you have used (e.g. books

where you have found Solar System's data, or web links where you found them, etc.) In general, all books and web links you have used in the making of **any** part of your project.

5. If you used mathematical references from an outside source, they must be included as well.
6. Your project can be handwritten (It is not compulsory to do it on a computer).

Project deadline: December 9th.