AREAS DE FIGURAS PLANAS

Calcular la superficie comprendida entre las funciones:

1. $f(x) = x^4 - 5x^2 + 4$; $x = -3$; $x = 3$ y el eje X

2. $f(x) = x^3 - 4x$ y el eje OX .

3 $y^2 = 4x$ y $x = 3$

4. $x = y^2$ e $y - x + 2 = 0$.

5. $f(x) = \text{sen } x$ y $g(x) = \cos x$ en $[0, 2\pi]$

6. $f(x) = x^2 - x$ y $g(x) = 3x - x^2$

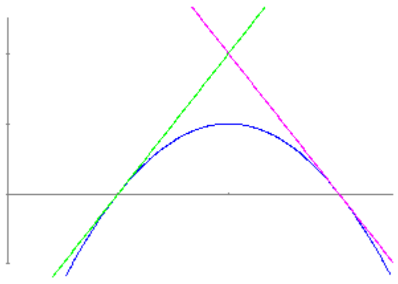
7. $f(x) = 2x - x^2$ y $g(x) = -x$.

8. $f(x) = |x|$ y $g(x) = |x^2 - 2|$

9. $y = x^2$ e $y^2 = x$

10. $x^2 + y^2 = 1$ y $f(x) = x^2 - 2x + 1$. Dibujar el recinto, 1 unidad = 2 cm.

11. Hallar el área del triángulo mixtilíneo formado por $y^2 = 6x$ y sus rectas tangentes en los puntos de abscisa $x = 6$.



12. La figura muestra una parte de la curva de ecuación $y = (x - 1)(3 - x)$ junto con las tangentes a la curva en sus puntos de intersección con el eje ***X***
Halla el área del triángulo curvilíneo encerrado entre la curva y sus tangentes.

13. $f(x) = |x^2 - 1| + 1$ y $g(x) = 2$

14. Hallar m para que el área comprendida entre $y = x^3$ e $y = mx$ sea $8u^2$.

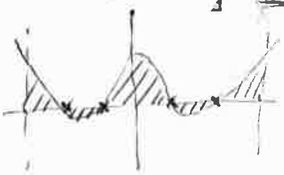
15. Sea $f : x \rightarrow \frac{senx}{x}$, $\pi \leq x \leq 3\pi$. Halle el área encerrada por la gráfica de f y el eje OX (Usar calculadora gráfica)

① $y = x^4 - 5x^2 + 4$

$y = 0 \Longrightarrow x^4 - 5x^2 + 4 = 0 \Longrightarrow x^2 = \frac{5 \pm \sqrt{25-16}}{2} = \frac{5 \pm 3}{2} \begin{cases} \nearrow 4 \Longrightarrow x = \pm 2 \\ \searrow 1 \Longrightarrow x = \pm 1 \end{cases}$

$\begin{array}{ccccccccc} + & & - & & + & & - & & + \\ & -2 & & -1 & & 1 & & 2 & \end{array}$

signo $x^4 - 5x^2 + 4$



$$\text{Area} = \int_{-3}^{3} |x^4 - 5x^2 + 4|\,dx = \int_{-3}^{-2} f(x)\,dx - \int_{-2}^{-1} f(x)\,dx + \int_{-1}^{1} f(x)\,dx - \int_{1}^{2} f(x)\,dx + \int_{2}^{3} f(x)\,dx =$$

$$= 2\int_0^1 f(x)\,dx - 2\int_1^2 f(x)\,dx + 2\int_2^3 f(x)\,dx =$$

$$= 2\left(\frac{x^5}{5} - \frac{5x^3}{3} + 4x\right)\Big|_0^1 - 2\left(\frac{x^5}{5} - \frac{5x^3}{3} + 4x\right)\Big|_1^2 + 2\left(\frac{x^5}{5} - \frac{5x^3}{3} + 4x\right)\Big|_2^3 =$$

$$= 2\left(\frac{1}{5} - \frac{5}{3} + 4\right) - 2\left(\frac{32}{5} - \frac{40}{3} + 8\right) + 2\left(\frac{1}{5} - \frac{5}{3} + 4\right) + 2\left(\frac{243}{5} - \frac{135}{3} + 12\right) - 2\left(\frac{32}{5} - \frac{40}{3} + \dots\right.$$

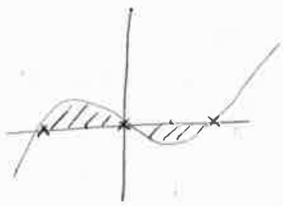
$$= 4\left(\frac{1}{5} - \frac{5}{3} + 4\right) - 4\left(\frac{32}{5} - \frac{40}{3} + 8\right) + 2\left(\frac{243}{5} - \frac{135}{3} + 12\right) =$$

$$= \frac{4 - 128 + 486}{5} + \frac{160 - 20 - 270}{3} + 16 - 32 + 24 = \frac{362}{5} - \frac{130}{3} + 8 =$$

$$= \frac{1086 - 650 + 120}{15} = \boxed{\frac{556}{15}\ \text{us}}$$

② $y = x^3 - 4x$

$y = 0 \Longrightarrow x(x^2 - 4) = 0 \Longrightarrow \begin{array}{l} x = -2 \\ x = 0 \\ x = 2 \end{array}$



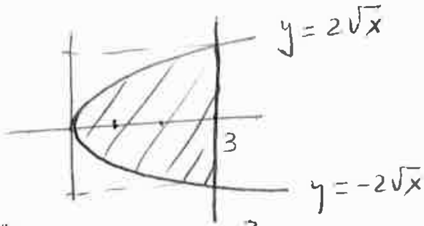
$\begin{array}{ccccccc} - & & + & & - & & + \\ & -2 & & 0 & & 2 & \end{array}$

signo $y = x^3 - 4x$

$$\text{Area} = \int_{-2}^{2} |x^3 - 4x|\,dx = \int_{-2}^{0} (x^3 - 4x)\,dx - \int_0^2 (x^3 - 4x)\,dx = -2\int_0^2 (x^3 - 4x)\,dx =$$

$$= -2\left(\frac{x^4}{4} - 2x^2\right)\Big|_0^2 = -2\left(\frac{16}{4} - 8\right) = \boxed{8\ \text{us}}$$

③ $\left.\begin{array}{l} y^2 = 4x \\ x = 3 \end{array}\right\} \Longrightarrow y^2 = 12 \Longrightarrow y = \pm\sqrt{12}$

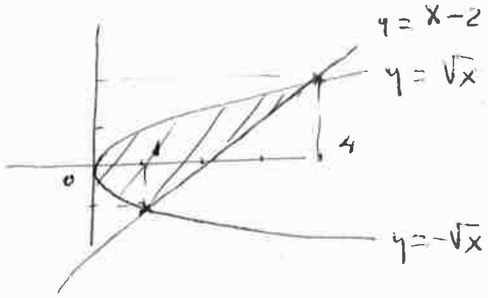


$$\text{Area} = 2\int_0^3 2\sqrt{x}\,dx = 4\int_0^3 x^{1/2}\,dx = 4\,\frac{x^{3/2}}{3/2}\Big|_0^3 = \frac{8}{3}\sqrt{x^3}\Big|_0^3 =$$

$$= \frac{8}{3}\sqrt{27} = \boxed{8\sqrt{3}\ \text{us}}$$

(4) $\left.\begin{array}{l} x = y^2 \\ y - x + 2 = 0 \end{array}\right\} \quad y - y^2 + 2 = 0 \longrightarrow y^2 - y - 2 = 0$

$$y = \frac{1 \pm \sqrt{1+8}}{2} = \frac{1 \pm 3}{2} \begin{array}{l} \nearrow 2 \Rightarrow x = 4 \\ \searrow -1 \Rightarrow x = 1 \end{array}$$

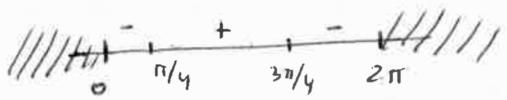


$$\text{Area} = 2\int_0^1 \sqrt{x}\, dx + \int_1^4 \left[\sqrt{x} - (x-2)\right] dx =$$

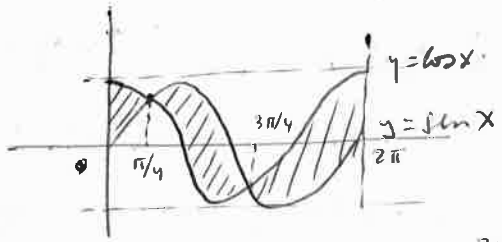
$$= 2\,\frac{x^{3/2}}{3/2}\Bigg|_0^1 + \left(\frac{x^{3/2}}{3/2} - \frac{x^2}{2} + 2x\right)\Bigg|_1^4 =$$

$$= \frac{4}{3} + \frac{16}{3} - 8 + 8 - \frac{2}{3} + \frac{1}{2} - 2 = \boxed{\frac{9}{2}\ \text{us}}$$

(5) $\left.\begin{array}{l} f(x) = \operatorname{sen} x \\ g(x) = \cos x \end{array}\right\} \Rightarrow \operatorname{sen} x = \cos x \Rightarrow \operatorname{tg} x = 1 \Rightarrow \begin{array}{l} x = \cancel{45^\circ} \\ x = \cancel{225^\circ} \end{array} \quad \begin{array}{l} x = \pi/4 \\ x = 3\pi/4 \end{array}$



Signo $\operatorname{sen} x - \cos x$

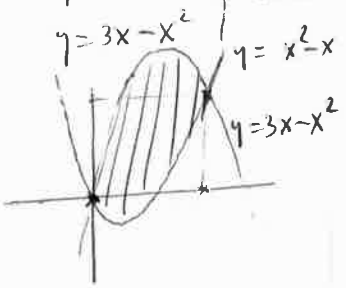


$$\text{Area} = \int_0^{2\pi} |\operatorname{sen} x - \cos x|\, dx = \int_0^{\pi/4} (\cos x - \operatorname{sen} x)\, dx + \int_{\pi/4}^{3\pi/4} (\operatorname{sen} x - \cos x)\, dx + \int_{3\pi/4}^{2\pi} (\cos x - \operatorname{sen} x)\, dx =$$

$$= (\operatorname{sen} x + \cos x)\Big|_0^{\pi/4} + (-\cos x - \operatorname{sen} x)\Big|_{\pi/4}^{3\pi/4} + (\operatorname{sen} x + \cos x)\Big|_{3\pi/4}^{2\pi} =$$

$$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - \cancel{1} + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} + \cancel{\frac{\sqrt{2}}{2}} + \cancel{\frac{\sqrt{2}}{2}} + \cancel{1} - \cancel{\frac{\sqrt{2}}{2}} - \cancel{\frac{\sqrt{2}}{2}} = \boxed{2\sqrt{2}\ \text{us}}$$

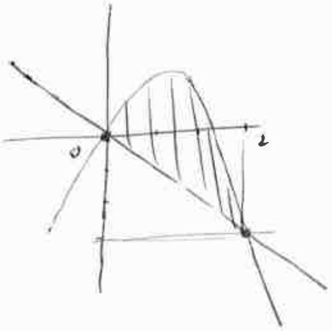
(6) $\left.\begin{array}{l} y = x^2 - x \\ y = 3x - x^2 \end{array}\right\} \Longrightarrow x^2 - x = 3x - x^2 \Longrightarrow 2x^2 = 4x \Longrightarrow 2x(x-2) = 0 \begin{array}{l} \nearrow x = 0 \\ \searrow x = 2 \end{array}$



$$\text{Area} = \int_0^2 (3x - x^2) - (x^2 - x)\, dx = \int_0^2 (4x - 2x^2)\, dx =$$

$$= \left(2x^2 - \frac{2}{3}x^3\right)\Bigg|_0^2 = 8 - \frac{16}{3} = \boxed{\frac{8}{3}\ \text{us}}$$

⑦ $\left.\begin{array}{l} y=2x-x^2 \\ y=-x \end{array}\right\} \Longrightarrow 2x-x^2=-x \Longrightarrow 0=x^2-3x \Longrightarrow 0=x(x-3) \begin{array}{l} x=0 \\ x=3 \end{array}$

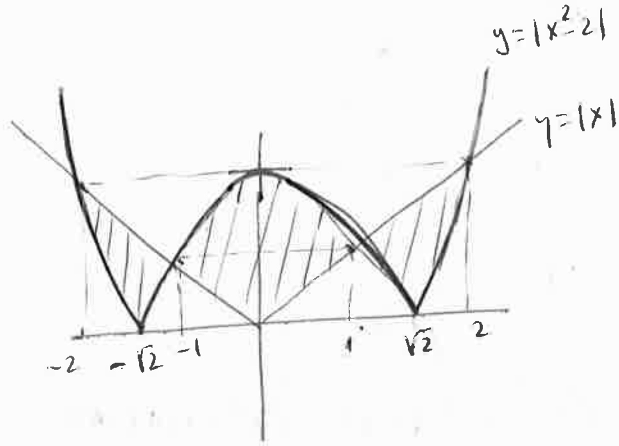


$$\text{Area} = \int_0^2 \left[(2x-x^2)-(-x)\right] dx =$$

$$= \int_0^2 (3x-x^2)\,dx = \left(\frac{3x^2}{2}-\frac{x^3}{3}\right)\Big|_0^2 = 6-\frac{8}{3} = \boxed{\frac{10}{3}\ \text{us}}$$

⑧ $y=|x| = \begin{cases} -x & \text{si } x\le 0 \\ x & \text{si } x>0 \end{cases}$

$y=|x^2-2| = \begin{cases} x^2-2 & \text{si } x\le -\sqrt{2} \\ 2-x^2 & \text{si } -\sqrt{2}\le x\le \sqrt{2} \\ x^2-2 & \text{si } x\ge\sqrt{2} \end{cases}$



$|x|=|x^2-2|$

$x=x^2-2$

$x^2-x-2=0$

$x=\frac{1\pm\sqrt{1+8}}{2}=\frac{1\pm3}{2}= \begin{array}{l} 2 \\ -1 \end{array}$

$-x=x^2-2 \longrightarrow 0=x^2+x-2$

$x=\frac{-1\pm\sqrt{1+8}}{2}=\frac{-1\pm3}{2} \begin{array}{l} 1 \\ -2 \end{array}$

$$\text{Area}=\int_{-2}^{2}\left||x|-|x^2-2|\right|dx = 2\int_0^2 \left||x|-|x^2-2|\right|dx = 2\int_0^1\left[(2-x^2)-x\right]dx +$$

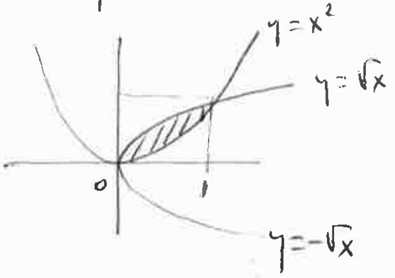
$$+2\int_1^{\sqrt{2}}\left[x-(2-x^2)\right]dx + 2\int_{\sqrt{2}}^{2}\left[x-(x^2-2)\right]dx =$$

$$= 2\left(2x-\frac{x^3}{3}-\frac{x^2}{2}\right)\Big|_0^1 + 2\left(\frac{x^2}{2}-2x+\frac{x^3}{3}\right)\Big|_1^{\sqrt{2}} + 2\left(\frac{x^2}{2}-\frac{x^3}{3}+2x\right)\Big|_{\sqrt{2}}^{2} =$$

$$= 2\left(2-\frac{1}{3}-\frac{1}{2}\right)+2\left(1-2\sqrt{2}+\frac{2\sqrt{2}}{3}\right)-2\left(\frac{1}{2}-2+\frac{1}{3}\right)+2\left(2-\frac{8}{3}+4\right)-2\left(1-\frac{2\sqrt{2}}{3}+2\sqrt{2}\right)$$

$$= 2\left(10-\frac{10}{3}-\frac{8\sqrt{2}}{3}-1\right) = 2\ \frac{27-10-8\sqrt{2}}{3} = \boxed{\frac{34-16\sqrt{2}}{3}\ \text{us}}$$

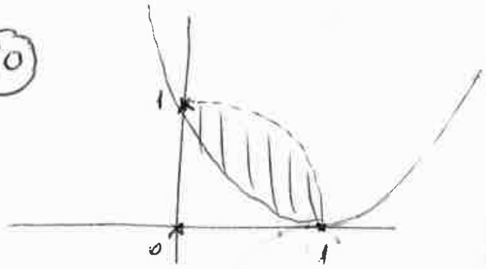
(9) $\left.\begin{array}{l} y=x^2 \\ y^2=x \end{array}\right\} \Rightarrow y=(y^2)^2 \Rightarrow y=y^4 \Rightarrow y(1-y^3)=0 \Rightarrow \begin{cases} y=0 \Rightarrow x=0 \\ y=1 \Rightarrow x=1 \end{cases}$



$$\text{Area} = \int_0^1 (\sqrt{x} - x^2)\,dx = \left(\frac{x^{3/2}}{3/2} - \frac{x^3}{3}\right)\Big|_0^1 =$$

$$= \frac{2}{3} - \frac{1}{3} = \boxed{\frac{1}{3}\ u^2}$$

(10)



$\left.\begin{array}{l} x^2+y^2=1 \\ y=x^2-2x+1 \end{array}\right\} \rightarrow y=\pm\sqrt{1-x^2}$

$\pm\sqrt{1-x^2} = x^2-2x+1$

$1-x^2 = x^4+4x^2+1-4x^3+2x^2-4x$

$0 = x^4-4x^3+7x^2-4x$

$0 = x(x^3-4x^2+7x-4) \rightarrow \boxed{x=0}$

	1	-4	7	-4
$\boxed{1}$		1	-3	4
	1	-3	4	0

$x^2-3x+4=0$

$x = \frac{3\pm\sqrt{9-16}}{2}$ ✗

$$\text{Area} = \int_0^1 \sqrt{1-x^2} - (x^2-2x+1)\,dx$$

$$\int_0^1 \sqrt{1-x^2}\,dx = \int_0^{\pi/2} \cos^2 t\,dt = \int_0^{\pi/2} \frac{1+\cos(2t)}{2}\,dt = \frac{t}{2} + \frac{\sin(2t)}{4}\Big|_0^{\pi/2} = \frac{\pi}{4}$$

$x = \sin t$

$dx = \cos t\,dt$

$$\int_0^1 (x^2-2x+1)\,dx = \left(\frac{x^3}{3} - x^2 + x\right)\Big|_0^1 = \frac{1}{3} - 1 + 1 = \frac{1}{3}$$

$$\text{Area} = \boxed{\frac{\pi}{4} - \frac{1}{3}}$$

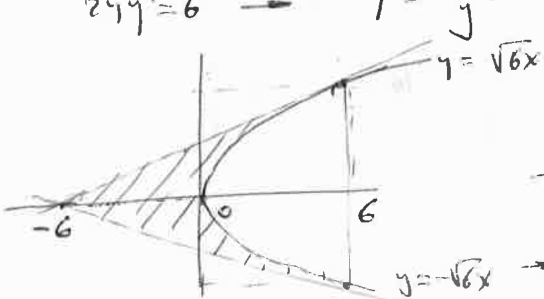
(11) $y^2 = 6x$

$2yy' = 6 \rightarrow y' = \frac{3}{y}$

$\begin{cases} x=6 \Rightarrow y^2=36 \Rightarrow y=\pm 6 \\ P(6,6) \Rightarrow \text{pend} = \frac{3}{6} = \frac{1}{2} \\ P(6,-6) \Rightarrow \text{pend} = \frac{3}{-6} = -\frac{1}{2} \end{cases}$

$\boxed{y-6 = \frac{1}{2}(x-6)}$ →

$\boxed{y+6 = -\frac{1}{2}(x-6)}$ →



→ $\boxed{y = \frac{x+6}{2}}$

→ $\boxed{y = \frac{-x-6}{2}}$

$$\text{Area} = 2\int_{-6}^{0} \frac{x+6}{2}\,dx + 2\int_{0}^{6}\left(\frac{x+6}{2} - \sqrt{6x}\right)dx = \left(\frac{x^2}{2}+6x\right)\Big|_{-6}^{0} + \left(\frac{x^2}{2}+6x - 2\sqrt{6}\,\frac{x^{3/2}}{3/2}\right)\Big|_{0}^{6} =$$

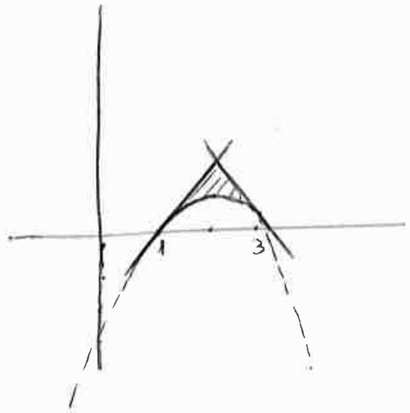
$$= 0 - \left(\frac{36}{2} - 36\right) + \left(\frac{36}{2} + 36 - \frac{4\sqrt{6}}{3}\sqrt{6^3}\right) - 0 = \boxed{24 \text{ us}}$$

(12)

$y = (x-1)(3-x)$

$y = -x^2 + 4x - 3$

$y' = -2x + 4$



$x = 1 \begin{cases} y = 0 \\ y' = 2 \end{cases} \qquad y - 0 = 2(x-1) \longrightarrow \boxed{y = 2x - 2}$

$x = 3 \begin{cases} y = 0 \\ y' = -2 \end{cases} \qquad y - 0 = -2(x-3) \longrightarrow \boxed{y = 6 - 2x}$

$\left.\begin{array}{l} y = 2x-2 \\ y = 6-2x \end{array}\right\} \longrightarrow 2x - 2 = 6 - 2x \; ; \quad 4x = 8 \longrightarrow \boxed{x = 2 \rightarrow y = 2}$

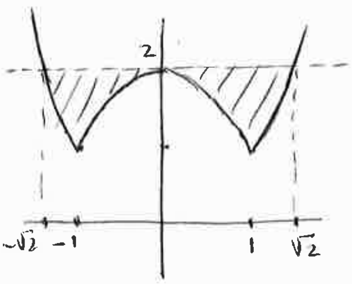
$$\text{Area} = \int_1^2 \left[(2x-2) - (-x^2+4x-3)\right]dx + \int_2^3 \left[(6-2x) - (-x^2+4x-3)\right]dx =$$

$$= \int_1^2 (x^2 - 2x + 1)\,dx + \int_2^3 (x^2 - 6x + 9)\,dx = \left(\frac{x^3}{3} - x^2 + x\right)\Big|_1^2 + \left(\frac{x^3}{3} - 3x^2 + 9x\right)\Big|_2^3$$

$$= \left(\frac{7}{3} - 2\right) + \left(\frac{19}{3} - 6\right) = \frac{1}{3} + \frac{1}{3} = \boxed{\frac{2}{3}}$$

Salen idénticos los valores de área en ambos recintos. Era previsible por la simetría del dibujo. Se podía haber hecho con una sola integral: $\text{Area} = 2\int_1^2 (x^2 - 2x + 1)\,dx$

(13) $f(x) = |x^2 - 1| + 1 = \begin{cases} x^2 - 1 + 1 & \text{si } x < -1 \\ 1 - x^2 + 1 & \text{si } -1 \le x \le 1 \\ x^2 - 1 + 1 & \text{si } x > 1 \end{cases} = \begin{cases} x^2 & \text{si } x < -1 \\ 2 - x^2 & \text{si } -1 \le x \le 1 \\ x^2 & \text{si } x > 1 \end{cases}$



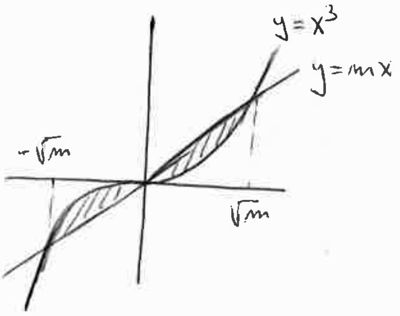
$y = 2 \Rightarrow \begin{cases} x^2 = 2 \longrightarrow & x = -\sqrt{2} \\ 2 - x^2 = 2 \longrightarrow & x = 0 \\ x^2 = 2 \longrightarrow & x = 2 \end{cases}$

$$\text{Area} = 2\left(\int_0^1 \left[2 - (2 - x^2)\right]dx + \int_1^{\sqrt{2}} \left[2 - x^2\right]dx\right) =$$

$$= 2\int_0^1 x^2\,dx + 2\int_1^{\sqrt{2}} (2 - x^2)\,dx = \frac{2x^3}{3}\Big|_0^1 + 2\left(2x - \frac{x^3}{3}\right)\Big|_1^{\sqrt{2}} =$$

$$= \frac{2}{3} + \left(4\sqrt{2} - \frac{4\sqrt{2}}{3}\right) - \left(4 - \frac{2}{3}\right) = \frac{2 + 12\sqrt{2} - 4\sqrt{2} - 12 + 2}{3} = \boxed{\frac{8(\sqrt{2}-1)}{3}}$$

(14) $\left.\begin{array}{l} y=x^3 \\ y=mx \end{array}\right\} \rightarrow x^3=mx \Rightarrow x^3-mx=0 \rightarrow x(x^2-m)=0 \rightarrow \boxed{\begin{array}{l} x=0 \\ x=\sqrt{m} \\ x=-\sqrt{m} \end{array}}$ m debe ser positivo



$$\text{Area} = 2\int_0^{\sqrt{m}} (mx-x^3)\,dx = \left(2m\frac{x^2}{2} - 2\frac{x^4}{4}\right)\Bigg|_0^{\sqrt{m}} =$$

$$= \left(mx^2 - \frac{x^4}{2}\right)\Bigg|_0^{\sqrt{m}} = m^2 - \frac{m^2}{2} = \frac{m^2}{2}$$

$$\frac{m^2}{2} = 8 \Rightarrow m^2 = 16 \Rightarrow \boxed{m=4}$$

(15) $f(x) = \dfrac{\sin x}{x}$

π ⊖ 2π ⊕ 3π

Signo $\dfrac{\sin x}{x}$

$$\text{Area} = -\int_{\pi}^{2\pi} \frac{\sin x}{x}\,dx + \int_{2\pi}^{3\pi} \frac{\sin x}{x}\,dx = +0'43379 + 0'25661 = \boxed{0'69040}$$

También se puede utilizar la calculadora gráfica con la función en valor absoluto, sin estudiar previamente su signo ni, en consecuencia, separar en dos integrales.

$$\text{Area} = \int_{\pi}^{3\pi} \left|\frac{\sin x}{x}\right| dx = \boxed{0'69040}$$