M00 a) $\log_2 5 = \dfrac{\log_a 5}{\log_a 2} = \boxed{\dfrac{y}{x}}$

b) $\log_a 20 = \log_a(2^2 \cdot 5) = 2\log_a 2 + \log_a 5 = \boxed{2x+y}$

M00 $(a+b)^{12} = \sum_{r=0}^{12} \binom{12}{r} a^r b^{12-r}$

$\binom{12}{5} a^5 \cdot b^7 = \dfrac{12!}{5!\,7!} a^5 \cdot b^7 = \boxed{792 a^5 b^7}$

M00 $4x^2 + 4kx + 9 = 0$; $(k>0)$

$x = \dfrac{-4k \pm \sqrt{16k^2 - 144}}{8}$

Solución única $\Rightarrow$ $16k^2 = 144$; $k^2 = \dfrac{144}{16}$; $k = \pm\sqrt{\dfrac{144}{16}}$ $\begin{cases} \dfrac{12}{4} = \boxed{3} \\ \cancel{-\dfrac{12}{4} = -3} \end{cases}$ Dice que $k>0$

N00 a) $C = 1000 \cdot (1+0'075)^{10} = 1000 \cdot 1'075^{10} = \boxed{2061,03\ \$}$

b) $C = 1000 \cdot 1'075^{10} + 1000 \cdot 1'075^9 + \cdots + 1000 \cdot 1'075^1 =$ (Suma diez términos de una p.g. con $r = 1'075$)

$= \dfrac{1000 \cdot 1'075^{10} \cdot 1'075 - 1000 \cdot 1'075}{1'075 - 1} = \dfrac{1000 \cdot 1'075 \cdot (1'075^{10} - 1)}{0'075} = \boxed{15208,12\ \$}$

N00 $V = 10000 \cdot 0'933^t$

a) $t=0 \Rightarrow \boxed{V = 10\,000\ l}$

b) $V = 5000 \Rightarrow 5000 = 10000 \cdot 0'933^t$; $0'5 = 0'933^t$;

$t = \log_{0'933} 0'5 = \dfrac{\log 0'5}{\log 0'933} = 9'9949$ min $\simeq$ 9 min 60 seg $\simeq \boxed{10\ \text{min}}$

c) 5% de 10.000 = 500

$500 = 10\,000 \cdot 0'933^t$; $0'05 = 0'933^t$

$t = \log_{0'933} 0'05 = \dfrac{\log 0'05}{\log 0'933} = 43,20$ min $\simeq \boxed{45\ \text{min}}$ ✓

M01 a) $P = 15'2 \cdot (1+0'027)^{-1} = \dfrac{15'2}{1'027} = \boxed{14'8\ \text{millones}}$

b) $P = 15'2 \cdot 1'027^{-5} = \dfrac{15'2}{1'027^5} = \boxed{13'3\ \text{millones}}$

M01 a) $C = 1500 \cdot (1+0'0525)^3 = 1748,87 \simeq \boxed{1749\ f.}$

b) $3000 = 1500 \cdot 1'00525^t$; $2 = 1'00525^t$

$t = \log_{1'00525} 2 = \dfrac{\log 2}{\log 1'00525} = 132,37 \rightarrow \boxed{133\ \text{años}}$

c) $3000 = 1500 \cdot (1+i)^{10}$; $2 = (1+i)^{10}$; $1+i = \sqrt[10]{2}$; $i = \sqrt[10]{2} - 1 = 0'0718 =$

$= \boxed{7'18\ \%}$

N01 $C = 5000 \cdot (1+0'065)^5 = 6850{,}43 \simeq \boxed{6850 \text{ €}}$

N01 a) $N = 5000 \cdot e^{-kt}$

$\left.\begin{matrix} N=2500 \\ t=5 \end{matrix}\right| \Rightarrow 2500 = 5000 \cdot e^{-5k} \; ; \; 0'5 = e^{-5k} \; ; \; -5k = \ln 0'5 \; ; \; k = \frac{\ln 0'5}{-5} = \boxed{0'139}$

b) $N=50$: $50 = 5000 \cdot e^{-0'139t}$; $0'01 = e^{-0'139t}$;

$-0'139t = \ln 0'01$; $t = \frac{\ln 0'01}{-0'193} = 33'13\ldots$ $\boxed{\simeq 33 \text{ años}}$

N01 $(x^3 - 3y^2)^5 = \sum_{r=0}^{5} \binom{5}{r} (x^3)^r (-3y^2)^{5-r} = \sum_{r=0}^{5} \binom{5}{r} x^{3r} \cdot (-3)^{5-r} \cdot y^{10-2r}$

Exponentes iguales $\Rightarrow$ $3r = 10-2r$; $5r = 10$; $r=2$

$r=2 \Rightarrow \binom{5}{2} \cdot (x^3)^2 \cdot (-3y^2)^{5-2} = 10 \cdot x^6 \cdot (-3)^3 \cdot y^6 = \boxed{-270 x^6 y^6}$

N01 $\log_9 81 + \log_9 \left(\frac{1}{9}\right) + \log_9 3 = \log_9 x$

$81 \cdot \frac{1}{9} \cdot 3 = x$; $\boxed{x = 27}$

M02 a) $\left(3x^2 - \frac{1}{x}\right)^9$ Tiene $\boxed{\text{diez}}$ sumandos.

b) $\left(3x^2 - \frac{1}{x}\right)^9 = \sum_{r=0}^{9} \binom{9}{r} \cdot (3x^2)^r \left(\frac{-1}{x}\right)^{9-r} = \sum_{r=0}^{9} \binom{9}{r} \frac{3^r x^{2r} \cdot (-1)^{9-r}}{x^{9-r}}$

$2r = 9-r \Rightarrow r=3$

$r=3 \rightarrow \binom{9}{3} (3x^2)^3 \left(\frac{-1}{x}\right)^6 = 84 \cdot 27 \cdot x^6 \cdot \frac{+1}{\cancel{x^6}} = \boxed{2268}$

M02 $\log_{27} x = 1 - \log_{27}(x - 0'4)$; $\log_{27} x + \log_{27}(x-0'4) = 1$; $\log_{27} x(x-0'4) = 1$;

$x(x-0'4) = 27^1$; $x^2 - 0'4x = 27$; $x^2 - 0'4x - 27 = 0$

$x = \frac{0'4 \pm \sqrt{0'16 + 108}}{2} = \frac{0'4 \pm 10'4}{2}$ $\boxed{5'4}$; $\cancel{-5}$ No existirían los logaritmos

N02 $(2-x)^5 = \sum_{r=0}^{5} \binom{5}{r} 2^r (-x)^{5-r}$

$5-r=3 \Rightarrow r=2 \rightarrow \binom{5}{2} \cdot 2^2 \cdot (-x)^3 = 10 \cdot 4 \cdot (-x^3) = \boxed{-40x^3}$

N02 $3000 = 1000 \cdot \left(1 + \frac{0'15}{12}\right)^t$; $3 = (1 + 0'0125)^t$; $3 = 1'0125^t$;

$t = \log_{1'0125} 3 = \frac{\log 3}{\log 1'0125} = 88{,}44 \rightarrow \boxed{89 \text{ meses}}$

M03 $(5+2x^2)^7 = \sum_{r=0}^{7} \binom{7}{r} 5^r \cdot (2x^2)^{7-r} = \sum_{r=0}^{7} \binom{7}{r} 5^r \cdot 2^{7-r} \cdot x^{14-2r}$

$14 - 2r = 10$; $r=2 \rightarrow \binom{7}{2} 5^2 \cdot (2x^2)^5 = 21 \cdot 25 \cdot 32 \cdot x^{10} = \boxed{16800 x^{10}}$

M03 a) $\log_{\sqrt{5}} x^2 = 2\log_{\sqrt{5}} x = \boxed{2y}$

b) $\log_{\sqrt{5}}\left(\frac{1}{x}\right) = \log_{\sqrt{5}} 1 - \log_{\sqrt{5}} x = -\log_{\sqrt{5}} x = \boxed{-y}$

c) $\log_{\sqrt{25}} x = \dfrac{\log_{\sqrt{5}} x}{\log_{\sqrt{5}} 25} = \dfrac{\log_{\sqrt{5}} x}{\log_{\sqrt{5}} 5^2} = \boxed{\dfrac{y}{2}}$

N03

$Kx^2+3x+1=0$

$x = \dfrac{-3\pm\sqrt{9-4K}}{2K}$

Única solución $\Rightarrow$ $9-4K=0$; $\boxed{K=9/4}$

N03

$m = 4e^{-0'2t}$

a) $t=0 \Rightarrow \boxed{m=4\ \text{kg}}$

b) $1'5 = 4e^{-0'2t}$; $0'375 = e^{-0'2t}$; $-0'2t = \ln 0'375$;

$t = \dfrac{\ln 0'375}{-0'2} = 8'90$ horas $= \boxed{8 \text{ horas } 54 \text{ min}}$

N03

$(2+ax)^4 = 2^4 + 4\cdot 2^3\cdot ax + 6\cdot 2^2\cdot (ax)^2 + 4\cdot 2\cdot (ax)^3 + (ax)^4 =$

$= \boxed{16 + 32ax + 24a^2x^2 + 8a^3x^3 + a^4x^4}$

M04

b) $(1+x^2)^6 = \sum_{r=0}^{6} \binom{6}{r} 1^r (x^2)^{6-r} =$

$= \sum_{r=0}^{6} \binom{6}{r} x^{12-2r}$

$12-2r=8 \Rightarrow r=2$

$\binom{6}{2} x^8 = \boxed{15x^8}$

a)

1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
$\boxed{1\ 6\ 15\ 20\ 15\ 6\ 1}$

M04 a) $e^{\ln x} = \boxed{x}$ b) $e^{\ln x+\ln y} = e^{\ln x}\cdot e^{\ln y} = \boxed{xy}$

c) $\ln (e^{x+y})^2 = 2\ln e^{x+y} = 2(x+y) = \boxed{2x+2y}$

M04 $\log_{10}\left(\dfrac{x}{y^2\sqrt{z}}\right) = \log_{10} x + 2\log_{10} y + \frac{1}{2}\log_{10} z = \boxed{p-2q-\frac{r}{2}}$

M04 $x^2-2kx+1=0$

$x = \dfrac{2k\pm\sqrt{4k^2-4}}{2}$

Dos raíces reales distintas $\Rightarrow$ $4k^2-4>0$; $4k^2>4$; $k^2>1$;

$k^2=1 \rightarrow k=\pm 1$

	-1		$+1$	
$4k^2-4$	+	−	+	

$\boxed{k\in(-\infty,-1)\cup(1,+\infty)}$

M04 $(2+ax)^{10} = \sum_{r=0}^{10} \binom{10}{r} 2^r (ax)^{10-r}$

$10-r=3 \rightarrow r=7 \qquad \binom{10}{7} 2^7 (ax)^3 = 120 \cdot 128 \cdot a^3 x^3 = \underbrace{15360\, a^3 x^3}_{=414720}$

$15360\, a^3 = 414720 \Rightarrow a^3 = 27 \Rightarrow \boxed{a=3}$

M04 (i) domingos de 2004 $\rightarrow n=10 \rightarrow P = 1200000 \cdot 1'025^{10} \simeq \boxed{1536101 \text{ hab}}$

(ii) $\frac{1536101}{1200000} = 1'28 = \boxed{128\%} \longrightarrow \boxed{\text{Creció un } 28\%}$

(iii) $P > 2000000 \Rightarrow 2000000 = 1.200000 \cdot 1'025^n$

$\frac{2000000}{1200000} = 1'025^n$

$1'\hat{6} = 1'025^n \ ; \quad n = \log_{1'025} 1'\hat{6} = 20'687 \Rightarrow n=21 \rightarrow \boxed{2015}$

N04 $(2-3x)^8 = \sum_{r=0}^{8} \binom{8}{r} 2^r (-3x)^{8-r}$

$8-r=3 \Rightarrow r=5 \qquad \binom{8}{5} \cdot 2^5 \cdot (-3x)^3 = 56 \cdot 32 \cdot (-27)\, x^3 = \boxed{48384 x^3}$

N04 $\log \frac{x^2 \sqrt{y}}{z^3} = 2\log x + \frac{1}{2}\log y - 3\log z = \boxed{2a + \frac{b}{2} - 3c}$

N04 a) Sueldo 2º año $= 11000 + 400 = \boxed{11400\ \$}$

" 3er año $= 11400 + 400 = \boxed{11800\ \$}$

$\text{Total} = 11000 + 11400 + 11800 + \cdots + (11000 + 9 \cdot 400) =$

$= 11000 + 11400 + \cdots + 14600 =$ (Suma 10 Términos de una prog. aritm.)

$= \frac{11000 + 14600}{2} \cdot 10 = \boxed{128000\ \$}$

b) Sueldo 2º año $= 10000 + 10000 \times 0'07 = 10000 \cdot 1'07 = \boxed{10.700\ \$}$

Sueldo 3er año $= 10700 + 17000 \cdot 0'07 = 10.700 \cdot 1'07 = \boxed{11449\ \$}$

Sueldo 10º año $= 10.000 \cdot 1'07^9 = \boxed{18384,59\ \$}$

c) Sueldos en el año enésimo: $S_A = 11.000 + (n-1)400$

$S_B = 10.000 \cdot 1'07^{n-1}$

Totales en n años: $\text{Total A} = \frac{11000 + 11000 + (n-1)400}{2} \cdot n = 10800n + 200n^2$

$\text{Total B} = \frac{10000 \cdot 1'07^{n-1} \cdot 1'07 - 10000}{1'07 - 1} = 142857,14 \cdot (1'07^n - 1)$

$\text{Total B} > \text{Total A} \Rightarrow 142.857'14\,(1'07^{n-1} - 1) > 10800n + 200n^2$

Resuelto con calculadora gráfica $\rightarrow n = 6'33 \Rightarrow \boxed{n = 7 \text{ años}}$

Por Tanteo: $\cancel{n=3}$: Total A = 34.200 \$

Total B = 32149 \$

$\cancel{n=4}$: Total A = 46.400 \$

Total B = 44399,43 \$

$\cancel{n=5}$: Total A = 59.000 \$

Total B = 57.507,39 \$

$\cancel{n=6}$: Total A = 72000 \$

Total B = 71532'91 \$

$\boxed{n=7}$ Total A = 85400 \$

Total B = 86540,21 \$

M05 $(x^2-2)^5$

a) Son seis términos

b) $(x^2-2)^5 = \sum_{r=0}^{5} \binom{5}{r} (x^2)^r \cdot (-2)^{5-r}$

$r=2 \rightarrow \binom{5}{2} (x^2)^2 (-2)^3 = 10\, x^4 \cdot (-8) = -80x^4 \rightarrow \boxed{A=-80}$

M05 $9^{2x} = 27^{1-x}$; $(3^2)^{2x} = (3^3)^{1-x}$; $3^{4x} = 3^{3-x}$; $4x = 3-x$; $5x = 3$; $\boxed{x = 3/5}$

M05 a) $\log_3 x - \log_3 (x-5) = \log_3 \frac{x}{x-5} \Rightarrow \boxed{A = \frac{x}{x-5}}$

b) $\log_3 x - \log_3 (x-5) = 1$; $\log_3 \frac{x}{x-5} = 1$; $\frac{x}{x-5} = 3^1$; $x = 3x - 15$; $-2x = -15$; $\boxed{x = 15/2}$

N05 $(3+\sqrt{7})^3 = 3^3 + 3\cdot 3^2 \sqrt{7} + 3 \cdot 3 (\sqrt{7})^2 + (\sqrt{7})^3 = 27 + 27\sqrt{7} + 63 + 7\sqrt{7} =$

$= 90 + 34\sqrt{7} \longrightarrow \boxed{p=90}$ $\boxed{q=34}$

N05

$V = 10000\, e^{-0'3t}$

$V < 1500 \longrightarrow 1500 = 10000\, e^{-0'3t}$; $0'15 = e^{-0'3t}$; $-0'3t = \ln 0'15$;

$t = \frac{\ln 0'15}{-0'3} = 6'32 \rightarrow \boxed{t = 7 \text{ años}}$

Muestra 06/08

a) $\boxed{C = 5000 \cdot 1'063^n}$

b) $C_5 = 5000 \cdot 1'063^5 = \boxed{6786{,}35\ \$}$

c) $\boxed{10000 < 5000 \cdot 1'063^n}$

$2 < 1'063^n$; $n > \log_{1'063} 2 = 11'35 \longrightarrow \boxed{n = 12 \text{ años}}$

M06 a) $\log_c 15 = \log_c (3\cdot 5) = \log_c 3 + \log_c 5 = \boxed{p+q}$

$\log_c 25 = \log_c 5^2 = 2\log_c 5 = \boxed{2q}$

b) $\log_d 6 = \frac{1}{2} \rightarrow 6 = d^{1/2}$; $6 = \sqrt{d}$; $d = 6^2$; $\boxed{d = 36}$

M06

a) $\ln (x+2) = 3$; $x+2 = e^3$; $\boxed{x = e^3 - 2}$

b) $10^{2x} = 500$; $2x = \log 500$; $x = \frac{\log 500}{2} = \boxed{1'35}$

N06

a) $5^{x+1} = 625$; $x+1 = 4$; $\boxed{x=3}$

b) $\log_a (3x+5) = 2$; $3x+5 = a^2$; $3x = a^2 - 5$; $\boxed{x = \frac{a^2-5}{3}}$

N06

a) $\ln a^3 b = 3\ln a + \ln b = \boxed{3p+q}$

b) $\ln \frac{\sqrt{a}}{b} = \frac{1}{2} \ln a - \ln b = \boxed{\frac{p}{2} - q}$

M07

a) $N = 250000 \cdot 1'013 = \boxed{253250 \text{ hab}}$

b) $2002 - 1972 = 30$

$N = 250000 \cdot 1'013^{30} = \boxed{368318 \text{ hab}}$

M07

$$(x+2y)^{10} = \sum_{r=0}^{10} \binom{10}{r} x^r (2y)^{10-r}$$

$$\binom{10}{8} x^8 (2y)^2 = 45x^8 \cdot 4y^2 = 180x^8y^2 \quad ; \quad \boxed{a = 180}$$

M07

a) $\log_a 10 = \log_a (5 \cdot 2) = \log_a 5 + \log_a 2 = \boxed{p+q}$

b) $\log_a 8 = \log_a 2^3 = 3\log_a 2 = \boxed{3q}$

c) $\log_a 2'5 = \log_a \left(\frac{5}{2}\right) = \log_a 5 - \log_a 2 = \boxed{p-q}$

M07

$$(x^3 - 3x)^6 = \sum_{r=0}^{6} \binom{6}{r} (x^3)^r (-3x)^{6-r} = \sum_{r=0}^{6} \binom{6}{r} (-3)^{6-r} \cdot x^{3r+6-r} =$$

$$= \sum_{r=0}^{6} \binom{6}{r} (-3)^{6-r} x^{2r+6}$$

a) Tiene 7 términos

b) $2r+6 = 12 \; ; \; 2r = 6 \; ; \; r = 3 \rightarrow \binom{6}{3}(-3)^3 x^{12} = 20 \cdot (-27)\, x^{12} = \boxed{-540x^{12}}$

M07 a) $(2^x+a)(2^x+b) = (2^x)^2 + a \cdot 2^x + b \cdot 2^x + ab = (2^x)^2 + \underbrace{(a+b)}\, 2^x + \underbrace{ab} =$

$= (2^x)^2 + 2^x - 12$

$a+b = 1 \rightarrow b = 1-a$

$ab = -12 \longrightarrow a(1-a) = -12 \; ; \; a - a^2 = -12 \; ; \; 0 = a^2 - a - 12 \; ;$

$a = \frac{1 \pm \sqrt{1+48}}{2} = \frac{1 \pm 7}{2}$ $\begin{cases} 4 \rightarrow b = -3 \\ -3 \rightarrow b = 4 \end{cases}$ $\Rightarrow (2^x)^2 + 2^x - 12 = (2^x+4)(2^x-3)$

b) $(2^x)^2 + 2^x - 12 = 0 \; ; \; (2^x+4)(2^x-3) = 0$ $\begin{cases} 2^x + 4 = 0 \; ; \; 2^x = -4 \; ; \text{ Sin solución.} \\ 2^x - 3 = 0 \; ; \; 2^x = 3 \; ; \; \boxed{x = \log_2 3} \end{cases}$

Sólo tiene una solución porque las exponenciales toman valores mayores que 0.

N07 a) $\left(e+\frac{1}{e}\right)^4 = e^4 + 4e^3\frac{1}{e} + 6e^2\left(\frac{1}{e}\right)^2 + 4e\left(\frac{1}{e}\right)^3 + \left(\frac{1}{e}\right)^4 =$

$= \boxed{e^4 + 4e^2 + 6 + 4e^{-2} + e^{-4}}$

b) $\left(e+\frac{1}{e}\right)^4 + \left(e-\frac{1}{e}\right)^4 = \left(e^4 + 4e^2 + 6 + 4e^{-2} + e^{-4}\right) + \left(e^4 - 4e^2 + 6 - 4e^{-2} + e^{-4}\right) =$

$= \boxed{2e^4 + 12 + 2e^{-4}}$

Muestra 08

a) $\log_x 49 = 2$; $x^2 = 49$; $x = \begin{cases} \boxed{7} \\ \cancel{-7} \end{cases}$ La base no puede ser negativa

b) $\log_2 8 = x$; $x = \log_2 2^3 = \boxed{3}$

c) $\log_{25} x = -\frac{1}{2}$; $25^{-1/2} = x$; $x = \frac{1}{\sqrt{25}} = \boxed{\frac{1}{5}}$

d) $\log_2 x + \log_2 (x-7) = 3$

$\log_2 x(x-7) = 3$; $x(x-7) = 2^3$; $x^2 - 7x = 8$; $x^2 - 7x - 8 = 0$;

$x = \frac{7 \pm \sqrt{49+32}}{2} = \frac{7 \pm 9}{2} \begin{cases} 8 \\ -1 \end{cases}$

M08

$\left(\frac{2}{3}x - 3\right)^8 = \sum_{r=0}^{8} \binom{8}{r} \left(\frac{2}{3}x\right)^r \cdot (-3)^{8-r}$

$r = 3 \longrightarrow \binom{8}{3} \left(\frac{2}{3}x\right)^3 (-3)^5 = 56 \cdot \frac{8}{27} x^3 \cdot (-243) = \boxed{-4032x^3}$

M08

$T = 280 \cdot 1'12^n$

a) $n = 5 \longrightarrow T = 280 \cdot 1'12^5 = 493,4 \simeq$ $\boxed{493 \text{ taxis}}$

$T = 560 \longrightarrow 560 = 280 \cdot 1'12^n$; $2 = 1'12^n$; $n = \log_{1'12} 2 = 6'1 \longrightarrow \boxed{2007}$

b) $n = 5 \longrightarrow P = \frac{2560000}{10 + 90e^{-0'1 \cdot 5}} \simeq$ $\boxed{39636 \text{ personas}}$

$2 \cdot 25600 = \frac{2560.000}{10 + 90e^{-0'1n}}$; $10 + 90e^{-0'1n} = \frac{2560000}{2 \cdot 25600}$; $10 + 90e^{-0'1n} = 50$;

$90e^{-0'1n} = 40$; $e^{-0'1n} = \frac{4}{9}$; $-0'1n = \ln\frac{4}{9}$; $n = \frac{\ln\frac{4}{9}}{-0'1} = 8'109 \Rightarrow$ NO

c) $n = 0$: $R = \frac{25600}{280} = \boxed{91,43}$

$R < 70 \longrightarrow 70 > \frac{P}{T} = \frac{\frac{2560000}{10 + 90e^{-0'1n}}}{280 \cdot 1'12^n}$; $196\cancel{00} \cdot 1'12^n > \frac{25600\cancel{00}}{1\cancel{0} + 9\cancel{0}e^{-0'1n}}$;

$196 \cdot 1'12^n > \frac{2560}{1 + 9e^{-0'1n}}$

Con calculadora gráfica : $n = 9'31 \longrightarrow$ $\boxed{10 \text{ años}}$

N08

a) $(x-2)^4 = x^4 + 4x^3 \cdot (-2) + 6x^2(-2)^2 + 4x(-2)^3 + (-2)^4 =$

$= \boxed{x^4 - 8x^3 + 24x^2 - 32x + 16}$

b) $(3x+4) \cdot (x-2)^4 = (3x+4)(x^4 - 8x^3 + 24x^2 - 32x + 16)$

$3 \times 24 + 4 \cdot (-8) = \boxed{40}$

M09

a) $\boxed{n = 10}$

b) $\boxed{a = p}$; $\boxed{b = 2q}$

c) $\boxed{\binom{10}{5} p^5 (2q)^5}$

M09

a) $\log_2 32 = \log_2 2^5 = \boxed{5}$

b) $\log_2(32^x/8^y) = x\log_2 32 - y\log_2 8 = 5x-3y \longrightarrow \boxed{p=5}\,;\ \boxed{q=-3}$

M09

a) $kx^2+(k-3)x+1=0$

$$x=\frac{-(k-3)\pm\sqrt{(k-3)^2-4k}}{2k}$$

Soluciones iguales $\Rightarrow (k-3)^2-4k=0\ ;\ k^2-6k+9-4k=0\ ;\ k^2-10k+9=0\ ;$

$$k=\frac{10\pm\sqrt{100-36}}{2}=\frac{10\pm 8}{2}\ \boxed{\begin{matrix}9\\1\end{matrix}}$$

b) $x^2+(k-3)x+k=0$

$$x=\frac{-(k-3)\pm\sqrt{(k-3)^2-4k}}{2k}$$

Soluciones iguales $\Rightarrow (k-3)^2-4k=0 \longrightarrow \boxed{k=\begin{matrix}9\\1\end{matrix}}$

M10

a) $(2+x)^4 = 2^4+4\cdot2^3x+6\cdot2^2x^2+4\cdot2\cdot x^3+x^4=\boxed{16+32x+24x^2+8x^3+x^4}$

b) $(2+x)^4\cdot\left(1+\frac{1}{x^2}\right)=\left(16+32x+24x^2+8x^3+x^4\right)\left(1+\frac{1}{x^2}\right)$

$24+1=\boxed{25}$

M10

$\log_2 x+\log_2(x-3)=3\ ;\ \log_2 x(x-3)=3\ ;\ x(x-3)=2^3\ ;\ x^2-3x=8\ ;\ x^2-3x-8=0$

$$x=\frac{3\pm\sqrt{9+32}}{2}=\begin{cases}\boxed{\dfrac{3+\sqrt{41}}{2}}\\ \cancel{\dfrac{3-\sqrt{41}}{2}}\end{cases}$$

Dicen que $x>2$

M10

$$\left(3x^2-\frac{2}{x}\right)^5=\sum_{r=0}^{5}\binom{5}{r}(3x^2)^r\left(\frac{-2}{x}\right)^{5-r}=\sum_{r=0}^{5}\binom{5}{r}3^r(-2)^{5-r}\cdot\frac{x^{2r}}{x^{5-r}}=\sum_{r=0}^{5}\binom{5}{r}3^r(-2)^{5-r}\cdot x^{3r-5}$$

$3r-5=4\ ;\ 3r=9\ ;\ r=3 \longrightarrow \binom{5}{3}(3x^2)^3\left(\frac{-2}{x}\right)^2=10\cdot27\,x^6\cdot\frac{4}{x^2}=\boxed{1080x^4}$

M11

$$(x+2)^{11}=\sum_{r=0}^{11}\binom{11}{r}x^r2^{11-r}$$

a) Son doce términos.

b) $r=2 \longrightarrow \binom{11}{2}x^2 2^9=\boxed{28160x^2}$

N11

a) $A(0)=\boxed{10\text{ mg/l}}$

b) $A(50)=10\cdot0'5^{0'014\cdot50}=\boxed{6{,}16\text{ mg/l}}$

c) $0'395=10\cdot0'5^{0'014t}\ ;\ 0'0395=0'5^{0'014t}\ ;\ 0'014t=\log_{0'5}0'0395\ ;$

$$t=\frac{\log_{0'5}0'0395}{0'014}=333\text{ min}=5\text{h }33\text{ min}\longrightarrow\boxed{18:33}$$

N11 $(3x^2+2)^9 = \sum_{r=0}^{9} \binom{9}{r}(3x^2)^r(2)^{9-r}$

a) Hay diez términos

b) $r=2 \rightarrow \binom{9}{2}(3x^2)^2 2^7 = 36\cdot 9x^4 \cdot 128 = \boxed{41472x^4}$

M12 $x^2+(k-1)x+1=0$

$x = \dfrac{-(k-1)\pm\sqrt{(k-1)^2-4}}{2}$

Raíces iguales $\Rightarrow (k-1)^2-4=0$; $k^2-2k+1-4=0$; $k^2-2k-3=0$;

$k = \dfrac{2\pm\sqrt{4+12}}{2} = \dfrac{2\pm4}{2}$ $\boxed{3 \quad -1}$

M12 $\left(2x^3+\frac{b}{x}\right)^8 = \sum_{r=0}^{8}\binom{8}{r}(2x^3)^r\left(\frac{b}{x}\right)^{8-r} = \sum_{r=0}^{8}\binom{8}{r}2^r\cdot b^{8-r}x^{4r-8}$

a) $r=7 \rightarrow \binom{8}{7}(2x^3)^7\cdot\frac{b}{x} = 8\cdot128\cdot x^{21}\cdot\frac{b}{x} = \underbrace{1024b}_{=3072}x^{21}$

$1024b = 3072 \rightarrow \boxed{b=3}$

b) $r=2 \rightarrow \binom{8}{2}(2x^3)^2\left(\frac{3}{x}\right)^6 = 28\cdot4\cancel{x^6}\cdot\frac{729}{\cancel{x^6}} = 81648$; $\boxed{k=81648}$

N12 $x^2-3x+k^2=4$; $x^2-3x+(k^2-4)=0$

$x = \dfrac{3\pm\sqrt{9-4(k^2-4)}}{2}$

Dos raíces reales distintas $\Rightarrow 9-4(k^2-4)>0$; $25-4k^2>0$

$25-4k^2=0$; $25=4k^2$; $k=\pm\frac{5}{2}$

		$-5/2$		$5/2$	
$25-4k^2$	$-$		$+$		$-$

$\boxed{k\in(-5/2, 5/2)}$

M12 $\left(1+\frac{2}{3}x\right)^m(3+mx)^2 = 9+84x+\cdots$

$\left[1^m+\binom{m}{1}1^{m-1}\left(\frac{2}{3}x\right)+\cdots\right]\cdot\left[9+6mx+\cdots\right] = 9+84x+\cdots$

$\left[1+\frac{2m}{3}x+\cdots\right]\cdot\left[9+6mx+\cdots\right] = 9+84x+\cdots$

$\left[9+6mx+\frac{18m}{3}x+\cdots\right] = 9+84x+\cdots$

$\left[9+12mx+\cdots\right] = 9+84x+\cdots$; $12m=84$; $\boxed{m=7}$

N12 $(2x+p)^6 = \sum_{r=0}^{6}\binom{6}{r}(2x)^r p^{6-r}$

$r=4 \rightarrow \binom{6}{4}(2x)^4p^2 = 15\cdot16x^4p^2 = \underbrace{240p^2}_{=60}x^4$; $240p^2=60$; $p^2=\frac{1}{4}$; $\boxed{p=\pm\frac{1}{2}}$

M13 a) $\log_3 p^2 = 2\log_3 p = 2\cdot6 = \boxed{12}$

b) $\log_3\left(\frac{p}{q}\right) = \log_3 p - \log_3 q = 6-7 = \boxed{-1}$

c) $\log_3(9p) = \log_3 9 + \log_3 p = 2+6 = \boxed{8}$

M13

a) $\log_2 40 - \log_2 5 = \log_2 \frac{40}{5} = \log_2 8 = \boxed{3}$

b) $8^{\log_2 5} = (2^3)^{\log_2 5} = 2^{3\log_2 5} = \cancel{2}^{\log_2 5^3} = 5^3 = \boxed{125}$

M13

$$\left(\frac{x}{a} + \frac{a^2}{x}\right)^6 = \sum_{r=0}^{6} \binom{6}{r} \left(\frac{x}{a}\right)^r \cdot \left(\frac{a^2}{x}\right)^{6-r} = \sum_{r=0}^{6} \binom{6}{r} \frac{1}{a^r} \cdot a^{12-2r} \cdot x^{2r-6}$$

$2r-6=0$; $r=3$ $\longrightarrow$ $\binom{6}{3}\left(\frac{x}{a}\right)^3\left(\frac{a^2}{x}\right)^3 = 20\,\frac{\cancel{x^3}}{a^3} \cdot \frac{a^6}{\cancel{x^3}} = 20a^3$

$20a^3 = 1280$; $a^3 = 64$; $a = \sqrt[3]{64} = \boxed{4}$

M13

$$(3x-2)^{12} = \sum_{r=0}^{12} \binom{12}{r} (3x)^r \cdot (-2)^{12-r}$$

a) $\boxed{r=5 \rightarrow p=5\ ;\ q=7}$

b) $\binom{12}{5}(3x)^5(-2)^7 = 792 \cdot 243\,x^5 \cdot (-128) = \boxed{-24634368\,x^5}$