Actividades variadas de Cálculo Diferencial en exámenes BI

Actividades a realizar con Calculadora Gráfica

Mayo 04

Sea $f(x) = x\cos x$, para $0 \le x \le \pi$. La curva de $f(x)$ tiene un máximo local en $x = a$ y un punto de inflexión en $x = b$.

(a) Dibuje aproximadamente la gráfica de $f(x)$ indicando las posiciones aproximadas de a y b.

(b) Halle el valor de

(i) a;

(ii) b.

Mayo 05

Consider the functions $f(x) = e^{2x}$ and $g(x) = \sin\dfrac{\pi x}{2}$

(a) Find the period of the function $f \circ g$.

(b) Find the intervals for which $(f \circ g(x)) > 4$.

Mayo 05

Resuelva la ecuación $\left| e^{2x} - \dfrac{1}{x+2} \right| = 2$

Mayo 05

(a) The function g is defined by $g(x) = \dfrac{e^x}{\sqrt{x}}$, for $0 < x \le 3$.

(i) Sketch the graph of g.

(ii) Find $g'(x)$.

(iii) Write down an expression representing the gradient of the normal to the curve at any point.

(b) Let P be the point (x, y) on the graph of g, and Q the point $(1, 0)$.

(i) Find the gradient of (PQ) in terms of x.

(ii) Given that the line (PQ) is a normal to the graph of g at the point P, find the minimum distance from the point Q to the graph of g.

Nov 07

Determine the values of x that satisfy the following inequalities

(a) $\dfrac{|x| + 2}{|x| - 3} < 4$;

(b) $\dfrac{xe^x}{(x^2 - 1)} \ge 1$.

Mayo 08

A system of equations is given by

$$\cos x + \cos y = 1.2$$
$$\sin x + \sin y = 1.4.$$

(a) For each equation express y in terms of x.

(b) **Hence** solve the system for $0 < x < \pi$, $0 < y < \pi$.

Nov 08

(a) Sketch the curve $y = |\ln x| - |\cos x| - 0.1$, $0 < x < 4$ showing clearly the coordinates of the points of intersection with the x-axis and the coordinates of any local maxima and minima.

(b) Find the values of x for which $|\ln x| > |\cos x| + 0.1$, $0 < x < 4$.

Actividades de Rectas Tangente y Normal a una curva

Mayo 05

La normal a la curva $y=\frac{k}{x}+\ln x^2$, para $x\neq 0$, $k\in\mathbb{R}$, en el punto $x=2$, tiene por ecuación $3x+2y=b$, donde $b\in\mathbb{R}$. Halle el valor **exacto** de k.

Nov 05

A circle has equation $x^2+(y-2)^2=1$. The line with equation $y=kx$, where $k\in\mathbb{R}$, is a tangent to the circle. Find all possible values of k.

Nov 06

Consider the curves C_1, C_2 with equations

$$C_1: y=x^2+kx+k \text{ , where } k<0 \text{ is a constant}$$
$$C_2: y=-x^2+2x-4 \text{ .}$$

Both curves pass through the point P and the tangent at P to one of the curves is also a tangent at P to the other curve.

(a) Find the value of k.

(b) Find the coordinates of P.

Nov 07

(a) A curve is defined by the implicit equation $2xy+6x^2-3y^2=6$.

Show that the tangent at the point A with coordinates $\left(1, \frac{2}{3}\right)$ has gradient $\frac{20}{3}$.

(b) The line $x=1$ cuts the curve at point A, with coordinates $\left(1, \frac{2}{3}\right)$, and at point B.

Find, in the form $\boldsymbol{r}=\begin{pmatrix}a\\b\end{pmatrix}+s\begin{pmatrix}c\\d\end{pmatrix}$

(i) the equation of the tangent at A;

(ii) the equation of the normal at B.

(c) Find the acute angle between the tangent at A and the normal at B.

Mayo 08

Find the gradient of the tangent to the curve $x^3y^2=\cos(\pi y)$ at the point $(-1, 1)$.

Muestra 08

A curve C is defined implicitly by $xe^y=x^2+y^2$. Find the equation of the tangent to C at the point $(1, 0)$.

Nov 08

Find the equation of the normal to the curve $5xy^2-2x^2=18$ at the point $(1, 2)$

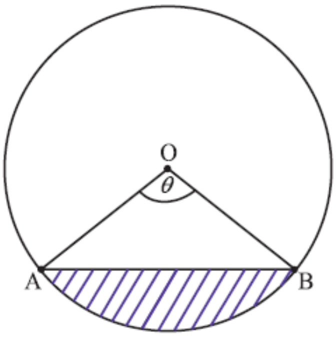
Actividades de Tasas de Variación

Nov 02

Se bombea aire dentro de una pelota esférica la cual se expande $8\ \text{cm}^3$ por segundo $(8\ \text{cm}^3\text{s}^{-1})$. Halle la tasa **exacta** de incremento del radio de la pelota cuando el radio es 2 cm.

Nov 05

The following diagram shows the points A and B on the circumference of a circle, centre O, and radius 4 cm, where $A\hat{O}B = \theta$. Points A and B are moving on the circumference so that θ is increasing at a constant rate.



Given that the rate of change of the length of the minor arc AB is numerically equal to the rate of change of the area of the shaded segment, find the acute value of θ.

Nov 06

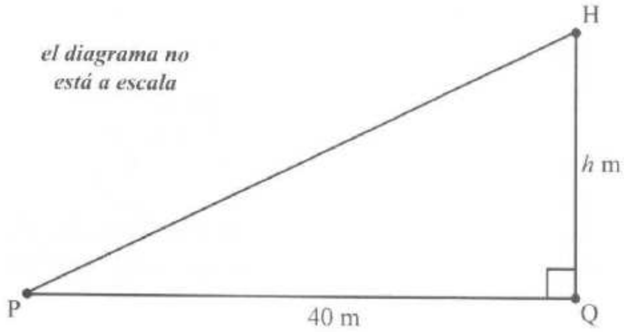
The radius and height of a cylinder are both equal to x cm. The curved surface area of the cylinder is increasing at a constant rate of $10\ \text{cm}^2/\text{sec}$. When $x = 2$, find the rate of change of

(a) the radius of the cylinder,

(b) the volume of the cylinder.

Mayo 09

Un helicóptero H se mueve verticalmente hacia arriba, con una velocidad de $10\ \text{m s}^{-1}$. El helicóptero está a una altura de h m, justo encima del punto Q, que se encuentra a nivel del suelo. El helicóptero es observado desde el punto P, que también se encuentra a nivel del suelo, siendo $PQ = 40$ m. Esta información se representa en el siguiente diagrama.



Para $h = 30$,

(a) compruebe que la razón de cambio de $H\hat{P}Q$ es igual a 0,16 radianes por segundo;

(b) halle la razón de cambio de PH.

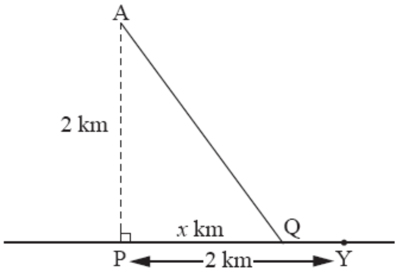
Nov11 P2#9

A stalactite has the shape of a circular cone. Its height is 200 mm and is increasing at a rate of 3 mm per century. Its base radius is 40 mm and is decreasing at a rate of 0.5 mm per century. Determine if its volume is increasing or decreasing, and the rate at which the volume is changing.

Actividades de Optimización

Mayo 08

André wants to get from point A located in the sea to point Y located on a straight stretch of beach. P is the point on the beach nearest to A such that $AP = 2$ km and $PY = 2$ km. He does this by swimming in a straight line to a point Q located on the beach and then running to Y.



When André swims he covers 1 km in $5\sqrt{5}$ minutes. When he runs he covers 1 km in 5 minutes.

(a) If $PQ = x$ km, $0 \le x \le 2$, find an expression for the time T minutes taken by André to reach point Y.

(b) Show that $\dfrac{dT}{dx} = \dfrac{5\sqrt{5}\,x}{\sqrt{x^2+4}} - 5$.

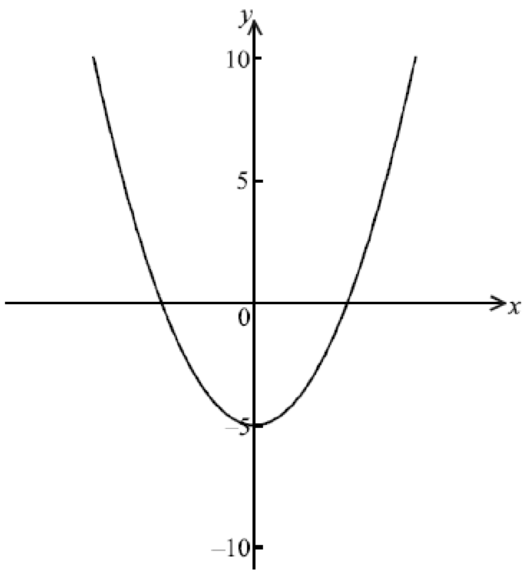
(c) (i) Solve $\dfrac{dT}{dx} = 0$.

(ii) Use the value of x found in **part (c) (i)** to determine the time, T minutes, taken for André to reach point Y.

(iii) Show that $\dfrac{d^2T}{dx^2} = \dfrac{20\sqrt{5}}{(x^2+4)^{\frac{3}{2}}}$ and **hence** show that the time found in **part (c) (ii)** is a minimum.

Muestra 08

The curve $y = x^2 - 5$ is shown below.



A point P on the curve has x-coordinate equal to a.

(a) Show that the distance OP is $\sqrt{a^4 - 9a^2 + 25}$.

(b) Find the values of a for which the curve is closest to the origin.

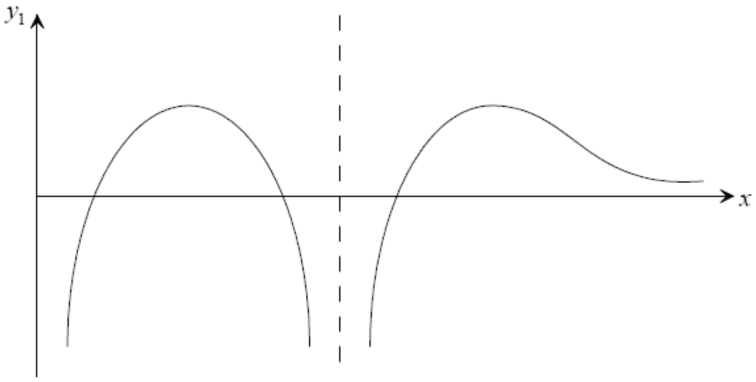
Actividades de Funciones Derivables

Mayo 01

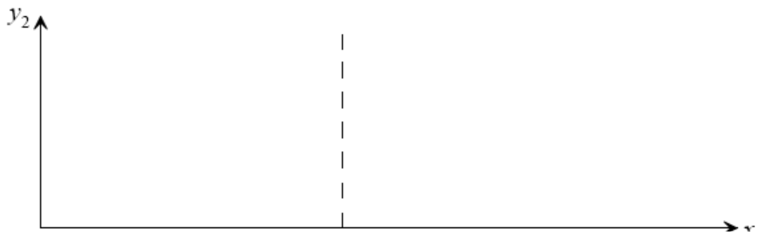
Using mathematical induction, prove that $\frac{d^n}{dx^n}(\cos x) = \cos\left(x + \frac{n\pi}{2}\right)$, for all positive integer values of n.

Mayo 03

El diagrama a continuación muestra la curva $y_1 = f(x)$.



En los ejes a continuación, trace la gráfica de $y_2 = |f'(x)|$.



Mayo 05

La función f está definida por $f(x) = e^{px}(x+1)$, con $p \in \mathbb{R}$.

(a) (i) Compruebe que $f'(x) = e^{px}\left(p(x+1)+1\right)$.

(ii) Sea $f^{(n)}(x)$ la derivada $f(x)$ respecto de x, n veces. Demuestre por inducción matemática que

$$f^{(n)}(x) = p^{n-1}e^{px}\left(p(x+1)+n\right),\ n \in \mathbb{Z}^+.$$

Nov 05

(a) Escriba el término en x^r del desarrollo de $(x+h)^n$, donde $0 \le r \le n$, $n \in \mathbb{Z}^+$.

(b) A partir de lo anterior, derive x^n, $n \in \mathbb{Z}^+$, aplicando la definición de derivada.

(c) Comenzando del resultado $x^n \times x^{-n} = 1$, deduzca la derivada de x^{-n}, $n \in \mathbb{Z}^+$.

Nov 06

The function f is defined by $f(x) = \frac{\ln x}{x^3}$, $x \ge 1$.

(a) Find $f'(x)$ and $f''(x)$, simplifying your answers.

(b) (i) Find the **exact** value of the x-coordinate of the maximum point and justify that this is a maximum.

(ii) Solve $f''(x) = 0$, and show that at this value of x, there is a point of inflexion on the graph of f.

(iii) Sketch the graph of f, indicating the maximum point and the point of inflexion.

Mayo 07

The following table shows the values of two functions f and g and their first derivatives when $x=1$ and $x=0$.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	4	1	-4	5
1	-2	3	-1	2

(a) Find the derivative of $\dfrac{3f(x)}{g(x)-1}$ when $x=0$.

(b) Find the derivative of $f\left(g(x)+2x\right)$ when $x=1$.

Mayo 07

La función f se define como: $f(x)=\dfrac{2x}{x^2+6}$ para $x\geq b$, y donde $b\in\mathbb{R}$.

(a) Compruebe que $f'(x)=\dfrac{12-2x^2}{(x^2+6)^2}$.

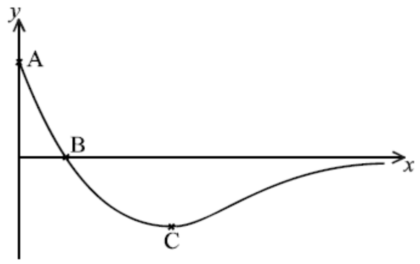
(b) A partir de lo anterior, halle el menor valor **exacto** de b para el cual existe la función inversa f^{-1}. Justifique su respuesta.

Muestra 08

(a) Find the root of the equation $e^{2-2x}=2e^{-x}$ giving the answer as a logarithm.

(b) The curve $y=e^{2-2x}-2e^{-x}$ has a minimum point. Find the coordinates of this minimum.

(c) The curve $y=e^{2-2x}-2e^{-x}$ is shown below.



Write down the coordinates of the points A, B and C.

(d) Hence state the set of values of k for which the equation $e^{2-2x}-2e^{-x}=k$ has two distinct positive roots.

Nov 08 P1#11

(a) Dibuje aproximadamente la curva $f(x)=\text{sen}\,2x$, $0\leq x\leq\pi$.

(b) A partir de lo anterior, y en un diagrama aparte, dibuje aproximadamente la gráfica de $g(x)=\text{cosec}\,2x$, $0\leq x\leq\pi$, indicando claramente las coordenadas de todos los máximos o mínimos locales, así como las ecuaciones de todas las asíntotas.

(c) Compruebe que $\text{tg}\,x+\text{cotg}\,x\equiv 2\,\text{cosec}\,2x$.

(d) A partir de lo anterior o de cualquier otro modo, halle las coordenadas de los máximos y mínimos locales de la gráfica de $y=\text{tg}\,2x+\text{cotg}\,2x$, $0\leq x\leq\dfrac{\pi}{2}$.

(e) Halle la solución de la ecuación $\text{cosec}\,2x=1,5\,\text{tg}\,x-0,5$, $0\leq x\leq\dfrac{\pi}{2}$.