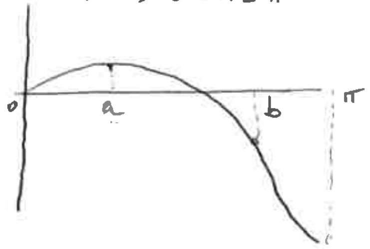
Con Calculadora Gráfica

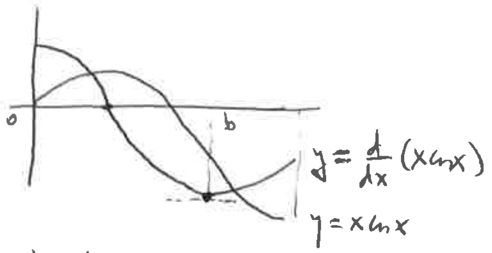
M04

a) $f(x) = x\cos x \;,\; 0 \le x \le \pi$



b) Utilizando G-Solv + MAX $\rightarrow$ $\boxed{a = 0'8603}$

Representando también $\frac{d}{dx} f(x)$ vemos:



f(x) tendrá una inflexión en el valor de x para el que su derivada tiene recta tangente horizontal -concretamente un mínimo-

Utilizando G-Solv + MIN $\rightarrow$ $\boxed{b = 2'8894}$

M05

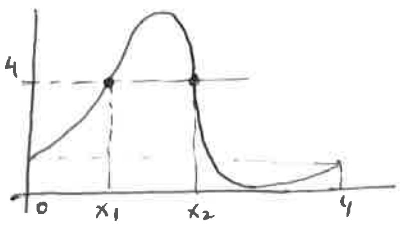
$f(x) = e^{2x}$

$g(x) = \operatorname{sen} \frac{\pi x}{2}$

a) $(f \circ g)(x) = f(g(x)) = f\left(\operatorname{sen} \frac{\pi x}{2}\right) = e^{2 \operatorname{sen} \frac{\pi x}{2}}$

$\frac{\pi x}{2} = 2\pi \Rightarrow x = 4$ $\quad\boxed{\text{Periodo} = 4}$

b) Representando $(f \circ g)(x)$ en todo un periodo:



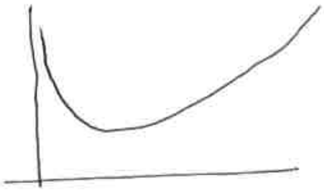
Utilizando G-Solv + X-Calc + Y = 4 $\rightarrow$ $x_1 = 0'4876$, $x_2 = 1'5124$ $\Rightarrow$ $\boxed{(0'4876,\ 1'5124)}$

M05

Gráfico $y = \left| e^{2x} - \frac{1}{x+2} \right|$

G-Solv + X-Calc + Y = 2 $\rightarrow$ $\boxed{x = \begin{cases} -2'50 \\ -1'51 \\ 0'44 \end{cases}}$

M05 a) Gráfico $g(x)=\dfrac{e^x}{\sqrt{x}}$, $0<x\le 3$



$$g'(x)=\frac{e^x\sqrt{x}-e^x\frac{1}{2\sqrt{x}}}{x}=$$

$$=\frac{2xe^x-e^x}{2x\sqrt{x}}=\frac{(2x-1)e^x}{2x\sqrt{x}}$$

$P(a,g(a)) \rightarrow m=\dfrac{(2a-1)e^a}{2a\sqrt{a}}=$ pendiente recta tangente $\rightarrow \boxed{m'=\dfrac{-2a\sqrt{a}}{(2a-1)e^a}}$ pendiente recta normal

b) $P\left(x,\dfrac{e^x}{\sqrt{x}}\right)$, $Q(1,0)$ | $\vec{PQ}=\left(1-x,-\dfrac{e^x}{\sqrt{x}}\right) \rightarrow$ pendiente $(PQ)=\dfrac{-\frac{e^x}{\sqrt{x}}}{1-x}=\dfrac{-e^x}{\sqrt{x}(1-x)}$

$\vec{PQ}$ normal a $g(x)$ en $P \Rightarrow \dfrac{-2x\sqrt{x}}{(2x-1)e^x}=\dfrac{-e^x}{\sqrt{x}(1-x)} \rightarrow$

$\rightarrow \dfrac{2x^2(1-x)}{2x-1}=(e^x)^2 \Rightarrow \boxed{x=0'5454}$ (Represento ambas gráficas y busqué el punto intersección)

$$|\vec{PQ}|=\sqrt{(1-x)^2+\left(-\frac{e^x}{\sqrt{x}}\right)^2}=\sqrt{(1-x)^2+\frac{e^{2x}}{x}}$$

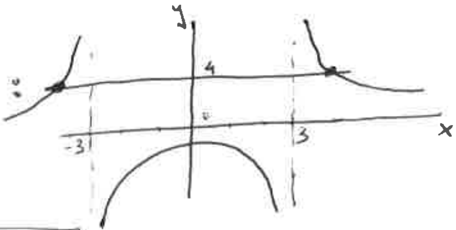
Mínima distancia $=|\vec{PQ}|=\sqrt{(1-0'5454)^2+\dfrac{e^{2\cdot 0'5454}}{0'5454}}=\boxed{2'38}$

También $|\vec{PQ}|=\sqrt{(1-x)^2+\dfrac{e^{2x}}{x}}$

Represento y busco el mínimo: $x=1'545428 \rightarrow y=2'38$ ✓

N07 a) $\dfrac{|x|+2}{|x|-3}<4$

Represento $y=\dfrac{|x|+2}{|x|-3}$ conjuntamente con $y=4$

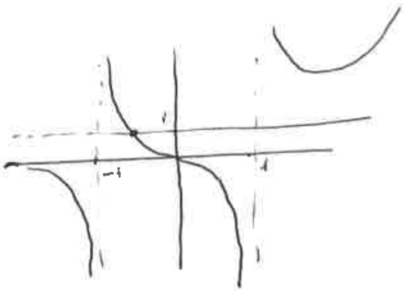


que tienen intersección en $x=-4'\hat{6}$, $x=4'\hat{6}$

Observando la gráfica: $\boxed{x\in(-\infty,-4'\hat{6})\cup(-3,3)\cup(4'\hat{6},+\infty)}$

b) $\dfrac{xe^x}{(x^2-1)}\ge 1$

Represento $y=\dfrac{xe^x}{x^2-1}$ conjuntamente con $y=1$:



que tienen intersección en $x=-0'800$

Observando la gráfica:

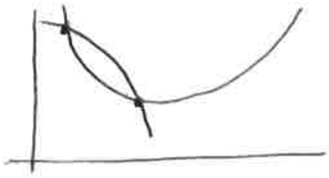
$\boxed{x\in(-1,-0'8]\cup(1,+\infty)}$

M08

$$\left.\begin{array}{l}\cos x + \cos y = 1'2 \\ \operatorname{sen} x + \operatorname{sen} y = 1'4\end{array}\right\} \longrightarrow \begin{array}{ll} y = \arccos(1'2 - \cos x) & \text{que toma valores en } [0, \pi] \\ y = \operatorname{arcsen}(1'4 - \operatorname{sen} x) & \text{que '' '' '' } [-\frac{\pi}{2}, \frac{\pi}{2}]\end{array}$$

$\arccos(1'2 - \cos x) = \operatorname{arcsen}(1'4 - \operatorname{sen} x)$

Represento ambas funciones:



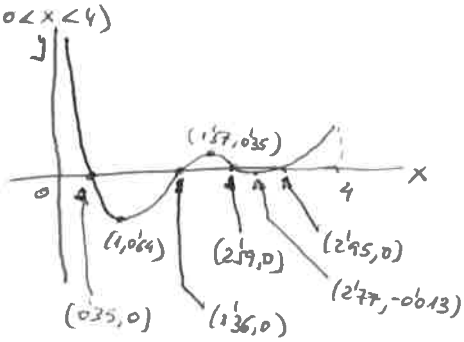
que tienen intersección en:

$$\boxed{\begin{array}{l} x = 0'4645 \longrightarrow y = 1'2599 \\ x = 1'2599 \longrightarrow y = 0'4645 \end{array}}$$

N08

$y = |\ln x| - |\cos x| - 0'1 \quad (0 < x < 4)$

a) Representando la función y utilizando G-SOLVE + ROOT + MAX + MIN se obtienen los 7 puntos



b) $|\ln x| > |\cos x| + 0'1 \Rightarrow |\ln x| - |\cos x| - 0'1 > 0 \Rightarrow \boxed{x \in (0, 0'35) \cup (1'36, 2'59) \cup (2'95, 4)}$

Recta Tangente y Recta Normal

M05

$$y=\frac{K}{x}+\ln x^2 \longrightarrow y'=\frac{-K}{x^2}+\frac{1}{x^2}\cdot 2x=\frac{2x-K}{x^2}$$

$$x=2 \longrightarrow y=\frac{K}{2}+\ln 4$$

$$\longrightarrow m=\frac{2\cdot 2-K}{2^2}=\frac{4-K}{4} \longrightarrow m'=-\frac{4}{4-K}$$

$$3x+2y=b \Rightarrow y=\frac{b-3x}{2} \Rightarrow m'=-\frac{3}{2}$$

$$\Rightarrow -\frac{3}{2}=-\frac{4}{4-K}\ ; \quad 12-3K=8\ ; \quad \boxed{K=\frac{4}{3}}$$

N05

$$x^2+(y-2)^2=1$$

$$2x+2(y-2)y'=0 \longrightarrow y'=\frac{-2x}{2(y-2)}=\frac{-x}{y-2}$$

$$P(x_0,y_0) \longrightarrow y-y_0=\frac{-x_0}{y_0-2}(x-x_0)\ ; \quad \left.\begin{array}{l} y=\dfrac{-x_0}{y_0-2}x+\left(y_0+\dfrac{x_0^2}{y_0-2}\right) \\ y=Kx \end{array}\right\} \Rightarrow$$

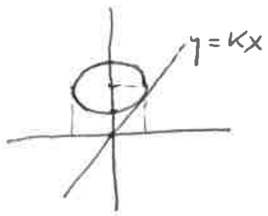
$$\Rightarrow \left| \begin{array}{l} y_0+\dfrac{x_0^2}{y_0-2}=0\ ; \\ K=-\dfrac{x_0}{y_0-2} \end{array}\right.$$

$$\frac{x_0^2}{y_0-2}=-y_0\ ; \quad \left.\begin{array}{l} x_0^2=-y_0^2+2y_0 \\ x_0^2+(y_0-2)^2=1 \end{array}\right\} \quad 1-(y_0-2)^2=-y_0^2+2y_0$$

$$1-\cancel{y_0^2}+4y_0-4=-\cancel{y_0^2}+2y_0 \Rightarrow 2y_0=3 \Rightarrow \boxed{y_0=\frac{3}{2}} \rightarrow$$

$$\rightarrow x_0^2=1-(y_0-2)^2\ ; \quad x_0^2=1-\left(\frac{3}{2}-2\right)^2\ ; \quad x_0^2=1-\frac{1}{4}\ ; \quad x_0^2=\frac{3}{4}\ ; \quad \boxed{x_0=\pm\frac{\sqrt{3}}{2}}$$

$$K=-\frac{\pm\sqrt{3}/2}{\frac{3}{2}-2}=\frac{\mp\sqrt{3}/2}{-1/2}=\boxed{\pm\sqrt{3}}$$



Otro procedimiento:

$$\left.\begin{array}{l} x^2+(y-2)^2=1 \\ y=Kx \end{array}\right\}$$ Debe tener sólo 1 solución

$$x^2+(Kx-2)^2=1$$

$$x^2+K^2x^2-4Kx+4=1$$

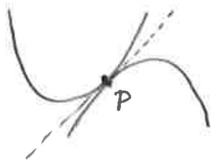
$$(1+K^2)x^2-4Kx+3=0$$

$$x=\frac{4K\pm\sqrt{16K^2-12(1+K^2)}}{2(1+K^2)} \Longrightarrow 16K^2-12(1+K^2)=0$$

$$4K^2=12$$

$$K^2=3\ ; \quad \boxed{K=\pm\sqrt{3}}$$

N06



$C_1: \; y = x^2+Kx+K \longrightarrow y' = 2x+K$

$C_2: \; y = -x^2+2x-4 \longrightarrow y' = -2x+2$

$x=a \rightarrow y = a^2+Ka+K = -a^2+2a-4 \Rightarrow 2a^2+(K-2)a+(K+4)=0$

$\quad \hookrightarrow y' = 2a+K = -2a+2 \Rightarrow K = -4a+2$

$\rightarrow 2a^2 + (-4a+\cancel{2}-\cancel{2})a + (-4a+2+4) = 0$

$2a^2-4a^2-4a+6=0$

$-2a^2-4a$

$a^2+2a-3=0 \rightarrow a = \begin{cases} \boxed{1} \Rightarrow K = -4\cdot1+2 = \boxed{-2} \; ; \; x=1 \rightarrow y = -1^2+2\cdot1-4 = \boxed{-3} \\ -3 \Rightarrow \cancel{K = -4\cdot(-3)+2 = 14} \quad \text{porque dicen que } K<0 \end{cases}$

Otro procedimiento:

$\left.\begin{array}{l} y = x^2+Kx+K \\ y = -x^2+2x-4 \end{array}\right\} \rightarrow$ Deben tener 1 único punto de corte.

$x^2+Kx+K = -x^2+2x-4$

$2x^2+(K-2)x+(K+4)=0$

$x = \dfrac{-(K-2) \pm \sqrt{(K-2)^2-8(K+4)}}{4} \Longrightarrow (K-2)^2-8(K+4)=0$

$K^2-4K+4-8K-32=0$

$K^2-12K-28=0$

$K = \begin{cases} -2 \\ \cancel{14} \end{cases} \Rightarrow x = \dfrac{-(K-2)\pm 0}{4} = \dfrac{-(-2-2)}{4} = \boxed{1}$

$\Downarrow$

$\boxed{y=-3}$

N07

$2xy+6x^2-3y^2=6$

$2y+2xy'+12x-6yy'=0 \; ; \; y'(2x-6y) = -2y-12x \; ; \; y' = \dfrac{-2y-12x}{2x-6y} = \dfrac{-y-6x}{x-3y}$

a) $A(1,\frac{2}{3}) \rightarrow y' = \dfrac{-\frac{2}{3}-6}{1-2} = \dfrac{-20/3}{-1} = \dfrac{20}{3}$ ✓

b) $A(1,\frac{2}{3}) \rightarrow y-\frac{2}{3} = \frac{20}{3}(x-1) \rightarrow \dfrac{x-1}{3} = \dfrac{y-2/3}{20} \Longrightarrow \left.\begin{array}{l} x = 1+3s \\ y = \frac{2}{3}+20s \end{array}\right\} \; \boxed{\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 2/3 \end{pmatrix} + s\begin{pmatrix} 3 \\ 20 \end{pmatrix}}$

$\left.\begin{array}{l} 2xy+6x^2-3y^2=6 \\ x=1 \end{array}\right\} \; 2y+6-3y^2=6 \; ; \; 0 = 3y^2-2y \; ; \; 0 = y(3y-2) \begin{cases} y=0 \\ y=2/3 \end{cases} \; \boxed{B(1,0)}$

$B(1,0) \rightarrow m = \dfrac{-0-6}{1-0} = -6 \Rightarrow m' = \dfrac{1}{6} \Rightarrow y-0 = \dfrac{1}{6}(x-1)$

$\dfrac{x-1}{6} = \dfrac{y}{1} \Rightarrow \left.\begin{array}{l} x = 1+6s \\ y = s \end{array}\right\}$

$\boxed{\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + s\begin{pmatrix} 6 \\ 1 \end{pmatrix}}$

Tasas de Variación

NO2

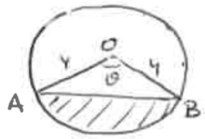
$$\frac{dV}{dt} = 8 \ cm^3/s$$

$$V = \frac{4}{3}\pi R^3$$

$$\frac{dV}{dt} = \frac{4}{\cancel{3}}\pi \cdot \cancel{3}R^2 \cdot \frac{dR}{dt} \Rightarrow \frac{dR}{dt} = \frac{1}{4R^2}\frac{dV}{dt}$$

$$R = 2cm \Rightarrow \frac{dR}{dt} = \frac{1}{4\cdot 2^2}\cdot 8 = \boxed{0'5 \ cm/s}$$

NO5



$$\begin{array}{ccc} 2\pi & \text{———} & 2\pi R \\ \theta & \text{———} & \text{Arco} \end{array} \Rightarrow \text{Arco} = \theta \cdot R$$

$$\text{Arco}AB = \theta \cdot 4 = 4\theta$$

$$\text{Area } \overset{\triangle}{OAB} = \frac{4\cdot 4\cdot \sin\theta}{2} = 8\sin\theta$$

$$\begin{array}{ccc} 2\pi & \text{———} & \pi R^2 \\ \theta & \text{———} & \text{sector} \end{array} \Rightarrow \text{Sector} = \frac{\theta R^2}{2}$$

$$\text{Area Sector } OAB = \frac{\theta\cdot 4^2}{2} = 8\theta$$

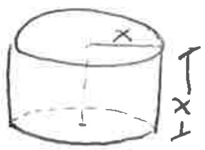
$$\text{Area Sombreada} = 8\theta - 8\sin\theta$$

$$\text{Tasa Variación Arco } AB = 4\cdot\frac{d\theta}{dt}$$

$$\text{Tasa Variación Area Sombreada} = 8\frac{d\theta}{dt} - 8\cos\theta\frac{d\theta}{dt} \rightarrow$$

$$\rightarrow 4\frac{d\theta}{dt} = 8\frac{d\theta}{dt} - 8\cos\theta\frac{d\theta}{dt} \Rightarrow 8\cos\theta = 4 \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \begin{cases}\boxed{\pi/3} \\ \cancel{5\pi/3}\end{cases}$$

NO6



$$\text{Area Lateral} = 2\pi x\cdot x = 2\pi x^2$$

$$\frac{dA}{dt} = 4\pi x\frac{dx}{dt}$$

$$\left.\begin{array}{l}\frac{dA}{dt} = 10 \ cm^2/s \\ x = 2\end{array}\right| \Rightarrow 10 = 4\pi\cdot 2\frac{dx}{dt} \Rightarrow \frac{dx}{dt} = \boxed{\frac{5}{4\pi} \ cm/s}$$

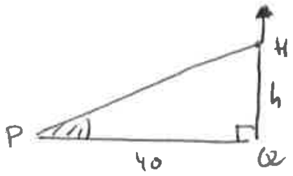
Tasa de variación del radio.

$$V = \pi x^2\cdot x = \pi x^3$$

$$\frac{dV}{dt} = 3\pi x^2\frac{dx}{dt}$$

$$\left.\begin{array}{l}\frac{dx}{dt} = \frac{5}{4\pi} \\ x = 2\end{array}\right| \Rightarrow \frac{dV}{dt} = 3\pi\cdot 2^2\cdot\frac{5}{4\pi} = \boxed{15 \ cm^3/s}$$

M09



$$\operatorname{tg} H\hat{P}Q = \frac{h}{40} \longrightarrow H\hat{P}Q = \operatorname{arcTg}\frac{h}{40}$$

$$\frac{d H\hat{P}Q}{dt} = \frac{1}{1+\left(\frac{h}{40}\right)^2} \cdot \frac{1}{40} \cdot \frac{dh}{dt}$$

a)

$\frac{dh}{dt} = 10$ m/s
$h = 30$ m

$$\longrightarrow \frac{d H\hat{P}Q}{dt} = \frac{1}{1+\left(\frac{30}{40}\right)^2} \cdot \frac{1}{40} \cdot 10 = \frac{16}{25}\cdot\frac{1}{4} = \frac{4}{25} = \boxed{0'16 \text{ rad/s}}$$

b)

$$\overline{PH} = \sqrt{h^2+40^2}$$

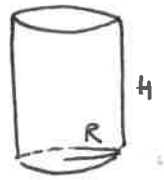
$$\frac{dPH}{dt} = \frac{1}{2\sqrt{h^2+40^2}} \cdot 2h \cdot \frac{dh}{dt}$$

$\frac{dh}{dt} = 10$ m/s
$h = 30$ m

$$\Longrightarrow \frac{dPH}{dt} = \frac{30}{\sqrt{30^2+40^2}} \cdot 10 = \frac{300}{50} = \boxed{6 \text{ m/s}}$$

N11

$$V = \pi R^2 H$$



$$\frac{dV}{dt} = 2\pi R H \frac{dR}{dt} + \pi R^2 \frac{dH}{dt} =$$

$$= 2\pi R H \cdot (-0'5) + \pi R^2 \cdot 3 =$$

$$= -\pi R H + 3\pi R^2 = \pi R(3R - H)$$

- Si $R < \frac{H}{3}$ el volumen estará disminuyendo
- Si $R > \frac{H}{3}$ " " " aumentando.

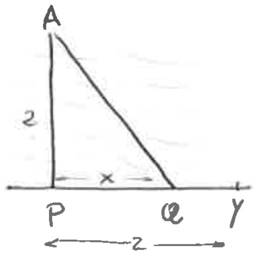
$H = 200$ mm
$R = 40$ mm

$$\frac{dV}{dt} = \pi \cdot 40 \cdot (3\cdot 40 - 200) = -10053{,}1 \text{ mm}^3/\text{sg}$$

El volumen esté disminuyendo a un ritmo de $\boxed{10053 \text{ mm}^3/\text{sg}}$

Optimización

M08



$\overline{AQ} = \sqrt{x^2+2^2} = \sqrt{x^2+4}$

Tiempo nadando $= 5\sqrt{5}\sqrt{x^2+4}$ min

$\overline{QY} = 2-x$

Tiempo corriendo $= 5(2-x)$ min

a) $\boxed{T = 5\sqrt{5}\sqrt{x^2+4} + 5(2-x)}$

b) $\dfrac{dT}{dx} = 5\sqrt{5}\,\dfrac{1}{\cancel{2}\sqrt{x^2+4}}\cdot\cancel{2}x + 5\cdot(-1) = \boxed{\dfrac{5\sqrt{5}\,x}{\sqrt{x^2+4}} - 5}$ ✓

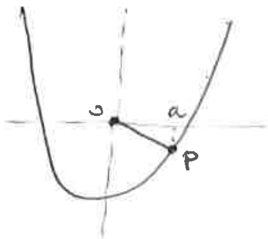
c) $\dfrac{dT}{dx} = 0 \Rightarrow \dfrac{\cancel{5}\sqrt{5}\,x}{\sqrt{x^2+4}} = \cancel{5}$; $\sqrt{5}\,x = \sqrt{x^2+4}$; $5x^2 = x^2+4$; $4x^2 = 4$; $x^2 = 1$; $\boxed{x=1}$

$x=1 \Rightarrow T = 5\sqrt{5}\sqrt{1^2+4} + 5\cdot(2-1) = 25+5 = \boxed{30 \text{ min}}$

$\dfrac{d^2T}{dx^2} = \dfrac{5\sqrt{5}\sqrt{x^2+4} - 5\sqrt{5}\,x\,\dfrac{1}{\cancel{2}\sqrt{x^2+4}}\cdot\cancel{2}x}{x^2+4} = \dfrac{5\sqrt{5}\sqrt{x^2+4} - \dfrac{5\sqrt{5}\,x^2}{\sqrt{x^2+4}}}{x^2+4}$

$x=1 \Rightarrow \dfrac{d^2T}{dx^2} = \dfrac{5\sqrt{5}\sqrt{5} - \dfrac{5\sqrt{5}}{\sqrt{5}}}{5} = \dfrac{20}{5} = 4 > 0 \Rightarrow$ $\boxed{\text{Tiempo mínimo en } x=1}$

Muestra 08



a) $P(a, a^2-5) \rightarrow \overline{OP} = \sqrt{a^2+(a^2-5)^2} = \sqrt{a^2+a^4-10a^2+25} = \sqrt{a^4-9a^2+25}$ ✓

b) $\dfrac{d\overline{OP}}{da} = \dfrac{1}{2\sqrt{a^4-9a^2+25}}\cdot(4a^3-18a) = \dfrac{2a^3-9a}{\sqrt{a^4-9a^2+25}}$

$\dfrac{d\overline{OP}}{da} = 0 \rightarrow 2a^3-9a = 0$; $a(2a^2-9) = 0 \rightarrow \cancel{a=0}$; $\rightarrow a^2 = \dfrac{9}{2}$; $\boxed{a = \pm\dfrac{3}{\sqrt{2}}}$

Vemos que se trataría de un máximo relativo

Derivadas

M01 $\frac{d^n}{dx^n}(\cos x) = \cos\left(x + \frac{n\pi}{2}\right) \quad n = 1, 2, 3, \ldots$

$\boxed{n=1}$ $\frac{d}{dx}\cos x = -\operatorname{sen} x$

$\cos\left(x + \frac{1\cdot\pi}{2}\right) = \cos x \cos\frac{\pi}{2} - \operatorname{sen} x \cdot \operatorname{sen}\frac{\pi}{2} = \cos x \cdot 0 - \operatorname{sen} x \cdot 1 = -\operatorname{sen} x$ ✓

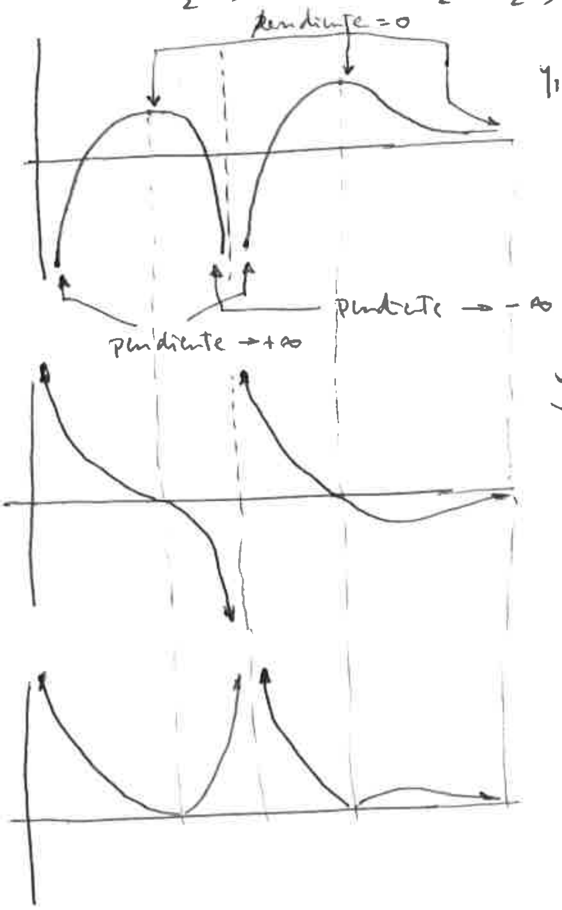
$\boxed{n=K}$ Suponiendo cierto que $\frac{d^K}{dx^K}(\cos x) = \cos\left(x + \frac{K\pi}{2}\right)$

demostraremos que $\frac{d^{K+1}}{dx^{K+1}}(\cos x) = \cos\left(x + \frac{(K+1)\pi}{2}\right)$

$\frac{d^{K+1}}{dx^{K+1}}(\cos x) = \frac{d}{dx}\left(\frac{d^K}{dx^K}(\cos x)\right) = \frac{d}{dx}\cos\left(x + \frac{K\pi}{2}\right) = -\operatorname{sen}\left(x + \frac{K\pi}{2}\right)$ ✓

$\cos\left(x + \frac{(K+1)\pi}{2}\right) = \cos\left(x + \frac{K\pi}{2} + \frac{\pi}{2}\right) = \cos\left(x + \frac{K\pi}{2}\right)\underbrace{\cos\frac{\pi}{2}}_{=0} - \operatorname{sen}\left(x + \frac{K\pi}{2}\right)\underbrace{\operatorname{sen}\frac{\pi}{2}}_{=1} = -\operatorname{sen}\left(x + \frac{K\pi}{2}\right)$

M03



M05 $f(x) = e^{px}(x+1)$

a) $f'(x) = p\cdot e^{px}(x+1) + e^{px}\cdot 1 = e^{px}(px + p + 1) = e^{px}\left((p+1)x + 1\right)$ ✓

$f^{n)}(x) = p^{n-1}e^{px}\left(p(x+1) + n\right)$

$\boxed{n=1}$ $p^{1-1}\cdot e^{px}\left(p(x+1)+1\right) = p^0\cdot e^{px}\left(p(x+1)+1\right) = e^{px}\left(p(x+1)+1\right) = f'(x)$ ✓

$\boxed{n=K}$ Suponiendo cierto que $f^{K)}(x) = p^{K-1}e^{px}\left(p(x+1) + K\right)$

demostraremos que $f^{K+1)}(x) = p^K e^{px}\left(p(x+1) + K + 1\right)$

$f^{K+1)}(x) = \left[f^{K)}(x)\right]' = \left[p^{K-1}e^{px}\left(p(x+1)+K\right)\right]' = p^{K-1}\cdot e^{px}\cdot p\left(p(x+1)+K\right) +$

$+ p^{K-1}e^{px}\cdot p = p^K\cdot e^{px}\left[p(x+1) + K + 1\right]$ ✓

N05

a) $(x+h)^n = \sum_{r=0}^{n} \binom{n}{r} x^r h^{n-r}$

b) $(x^n)' = \lim_{h\to 0} \dfrac{(x+h)^n - x^n}{h} = \lim_{h\to 0} \dfrac{\cancel{x^n} + \binom{n}{1}x^{n-1}\cdot h + \binom{n}{2}x^{n-2}h^2 + \dots + h^n - \cancel{x^n}}{h} =$

$= \lim_{h\to 0} \left[\binom{n}{1}x^{n-1} + \binom{n}{2}x^{n-2}\cdot h + \dots + h^{n-1} \right] = \binom{n}{1}x^{n-1} + 0 = n\cdot x^{n-1}$ ✓

c) $x^{-n} = \dfrac{1}{x^n}$

$(x^{-n})' = \left(\dfrac{1}{x^n}\right)' = \dfrac{0 - 1\cdot (x^n)'}{(x^n)^2} = \dfrac{-n\cdot x^{n-1}}{x^{2n}} = -n\cdot x^{n-1-2n} = -n\cdot x^{-n-1}$ ✓

N06

$f(x) = \dfrac{\ln x}{x^3}$. $x \geq 1 \longrightarrow$ El dominio 'completo' de $f(x)$ sería $(0,+\infty)$, pero nos indican: $\operatorname{dom} f = [1,+\infty)$

a) $f'(x) = \dfrac{\frac{1}{x}x^3 - \ln x \cdot 3x^2}{x^6} = \dfrac{x^2(1-3\ln x)}{x^6} = \boxed{\dfrac{1-3\ln x}{x^4}}$

$f''(x) = \dfrac{-3\frac{1}{x}\cdot x^4 - (1-3\ln x)\cdot 4x^3}{x^8} = \dfrac{x^3(-3-4+12\ln x)}{x^8} = \boxed{\dfrac{12\ln x - 7}{x^5}}$

b) $f'(x) = 0 \Rightarrow \dfrac{1-3\ln x}{x^4} = 0$; $\ln x = \frac{1}{3}$; $x = e^{1/3}$

$f''(e^{1/3}) = \dfrac{12\cdot \ln e^{1/3} - 7}{(e^{1/3})^5} = \dfrac{12\cdot\frac{1}{3} - 7}{e^{5/3}} = \dfrac{-3}{e^{5/3}} < 0$

$\Rightarrow$ Máximo relativo en $x = e^{1/3}$

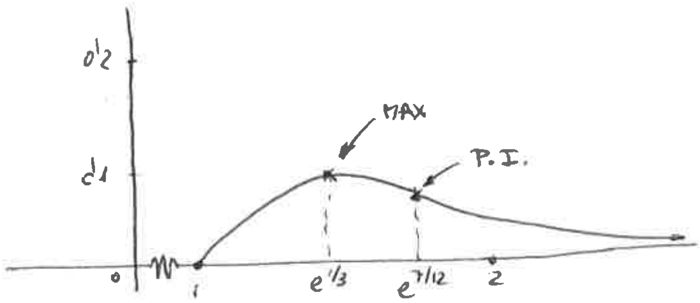
$f''(x) = 0 \Rightarrow \dfrac{12\ln x - 7}{x^5} = 0$; $\ln x = \frac{7}{12}$; $x = e^{7/12}$

$f'''(x) = \dfrac{12\cdot\frac{1}{x}\cdot x^5 - (12\ln x - 7)\cdot 5x^4}{x^{10}} = \dfrac{x^4(12 - 60\ln x + 35)}{x^{10}} = \dfrac{47 - 60\ln x}{x^6}$

$f'''(e^{7/12}) = \dfrac{47 - 60\ln e^{7/12}}{(e^{7/12})^6} = \dfrac{47 - 60\cdot\frac{7}{12}}{e^{7/2}} = \dfrac{12}{e^{7/2}} \neq 0$

$\Rightarrow$ Punto inflexión en $x = e^{7/12}$ ✓

	x	y
	1	0
1'40 =	$e^{1/3}$	0'12
1'79 =	$e^{7/12}$	0'10
	$+\infty$	0



$\lim_{x\to+\infty} \dfrac{\ln x}{x^3} = \dfrac{+\infty}{+\infty} = \lim_{x\to+\infty} \dfrac{1/x}{3x^2} = \lim_{x\to+\infty} \dfrac{1}{3x^3} = \dfrac{1}{+\infty} = 0$

M07

a) $\left[\dfrac{3f(x)}{g(x)-1}\right]' = \dfrac{3f'(x)\cdot(g(x)-1) - 3f(x)\cdot g'(x)}{(g(x)-1)^2}$

En $x=0$: $\left[\dfrac{3f(x)}{g(x)-1}\right]' = \dfrac{3f'(0)\cdot(g(0)-1) - 3f(0)\,g'(0)}{(g(0)-1)^2} = \dfrac{1\cdot(-4-1) - 3\cdot 4\cdot 5}{(-4-1)^2} = \dfrac{-65}{25} = \boxed{\dfrac{-13}{5}}$

b) $\left[f(g(x)+2x)\right]' = f'(g(x)+2x)\cdot(g'(x)+2)$

En $x=1$: $\left[f(g(x)+2x)\right]' = f'(g(1)+2\cdot 1)\,(g'(1)+2) = f'(-1+2)\cdot(2+2) =$

$= f'(1)\cdot 4 = 3\cdot 4 = \boxed{12}$

M07

$f(x) = \dfrac{2x}{x^2+6}$; $x \geq b$; $x^2+6=0 \rightarrow x^2=-6 \Rightarrow$ El dominio completo es $\mathbb{R}$; nos proponen $\text{dom} f = [b, +\infty)$

a) $f'(x) = \dfrac{2(x^2+6) - 2x\cdot 2x}{(x^2+6)^2} = \dfrac{12-2x^2}{(x^2+6)^2}$ ✓

b) $f'(x)=0 \rightarrow 12 = 2x^2$; $x = \pm\sqrt{6}$

	$-\infty$ a $-\sqrt{6}$	$-\sqrt{6}$ a $\sqrt{6}$	$\sqrt{6}$ a $+\infty$
$f'(x)$	$-$	$+$	$-$
$f(x)$	↘	↗	↘

MIN en $-\sqrt{6}$, MAX en $\sqrt{6}$

Por lo tanto $\boxed{b=\sqrt{6}}$, ya que $f(x)$ tomaría a la izquierda de $\sqrt{6}$ los mismos valores de y que a su derecha.

Muestra 08

a) $e^{2-2x} = 2e^{-x}$; $e^2\cdot e^{-2x} = 2e^{-x}$; $t = e^{-x} \rightarrow e^2\cdot t^2 = 2t$;

$e^2t^2 - 2t = 0$; $t(e^2t-2)=0 \rightarrow t=0 \Rightarrow e^{-x}=0$ ✗ ; $\rightarrow e^2t-2=0$; $t=\frac{2}{e^2} \Rightarrow e^{-x}=\frac{2}{e^2}$;

$e^2e^{-x}=2$; $e^{2-x}=2$; $2-x=\ln 2$; $x = \boxed{2-\ln 2} = \boxed{\ln\frac{e^2}{2}}$

b) $y = e^{2-2x} - 2e^{-x}$

$y' = -2e^{2-2x} + 2e^{-x}$

$y'=0 \rightarrow -2e^{2-2x} + 2e^{-x} = 0 \Rightarrow (t=e^{-x}) \Rightarrow -e^2t^2+t=0$; $t(-e^2t+1)=0 \rightarrow t=0$ ✗ ; $t=\frac{1}{e^2} \Rightarrow$

$\Rightarrow e^{-x} = \frac{1}{e^2} \Rightarrow \boxed{x=2} \rightarrow y = e^{-2} - 2e^{-2} = \boxed{-e^{-2}}$

c) $x=0 \rightarrow y = e^2-2$ $\boxed{A(0, e^2-2)}$

$y=0 \rightarrow e^{2-2x} = 2e^{-x} \Rightarrow \boxed{x = \ln\frac{e^2}{2}}$ (Apartado a) $\boxed{B\left(\ln\frac{e^2}{2}, 0\right)}$

Mínimo: $\boxed{C(2, -e^{-2})}$

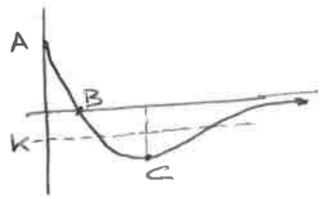
d) $k < -e^{-2} \rightarrow$ Sin raíces

$k > e^2-2 \rightarrow$ Sin raíces

$k \in [0, e^2-2] \rightarrow$ Una raíz

$k = -e^{-2} \rightarrow$ Una raíz

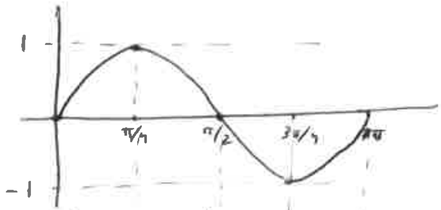
$\boxed{k \in (-e^{-2}, 0) \rightarrow \text{Dos raíces distintas}}$



N08

a) $f(x) = \operatorname{sen} 2x \,, \quad 0 \le x \le \pi$

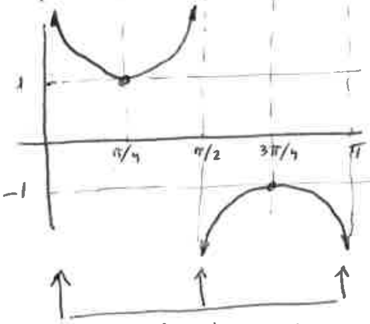
x	$y = f(x)$
0	0
$\pi/4$	1
$\pi/2$	0
$3\pi/4$	-1
π	0



$y = \operatorname{sen} 2x$

b) $g(x) = \operatorname{cosec} 2x = \dfrac{1}{\operatorname{sen} 2x}$

x	$y = g(x)$
0^+	$+\infty$
$\pi/4$	1
$\pi/2^-$	$+\infty$
$\pi/2^+$	$-\infty$
$3\pi/4$	-1
π^-	$-\infty$



$y = \operatorname{cosec} 2x$

Asintotas: $y=0, \; y=\pi/2, \; y=\pi$

Mínimo Relativo en $(\pi/4, 1)$
Máximo Relativo en $(3\pi/4, -1)$

c) $\operatorname{tg} x + \operatorname{ctg} x = \dfrac{\operatorname{sen} x}{\cos x} + \dfrac{\cos x}{\operatorname{sen} x} = \dfrac{\operatorname{sen}^2 x + \cos^2 x}{\cos x \operatorname{sen} x} = \dfrac{1}{\operatorname{sen} x \cos x} = \dfrac{2}{\operatorname{sen} 2x} = 2 \operatorname{cosec} 2x$ ✓

d) $y = \operatorname{tg} 2x + \operatorname{ctg} 2x \,, \quad 0 \le x \le \pi/2$

Cambiando la variable : $2x = t$

$y = \operatorname{tg} t + \operatorname{ctg} t \,, \; 0 \le t \le \pi \longrightarrow y = 2 \operatorname{cosec} 2t \,, \; 0 \le t \le \pi$

Tendré un mínimo relativo en $t = \pi/4$ y un máximo relativo en $t = 3\pi/4$

Por lo tanto :

$t = \pi/4 \longrightarrow 2x = \pi/4 \Rightarrow x = \pi/8 \longrightarrow y = \operatorname{tg} \frac{2\pi}{8} + \operatorname{ctg} \frac{2\pi}{8} = \operatorname{tg} \pi/4 + \operatorname{ctg} \pi/4 = 1+1 = 2$

$t = 3\pi/4 \longrightarrow 2x = 3\pi/4 \Rightarrow x = 3\pi/8 \longrightarrow y = \operatorname{tg} \frac{2\cdot 3\pi}{8} + \operatorname{ctg} \frac{2\cdot 3\pi}{8} = \operatorname{tg} \frac{3\pi}{4} + \operatorname{ctg} \frac{3\pi}{4} = -1-1 = -2$

Mínimo Relativo en el punto $(\pi/8, 2)$
Máximo Relativo en el punto $(3\pi/8, -2)$

e) $\operatorname{cosec} 2x = 1'5 \operatorname{tg} x - 0'5 \,, \quad 0 \le x \le \pi/2$

$2 \operatorname{cosec} 2x = 3 \operatorname{tg} x - 1$

$\operatorname{tg} x + \operatorname{ctg} x = 3 \operatorname{tg} x - 1$

$\operatorname{ctg} x = 2 \operatorname{tg} x - 1$

$\operatorname{tg} x = t \qquad \dfrac{1}{t} = 2t - 1 \; ; \quad 1 = 2t^2 - t \; ; \quad 0 = 2t^2 - t - 1$

$t = \dfrac{1 \pm \sqrt{1+8}}{4} = \dfrac{1 \pm 3}{4} \begin{cases} -\frac{1}{2} \Rightarrow \text{No es posible} \\ 1 \Rightarrow \boxed{x = \pi/4} \end{cases}$

[En $x \in [0, \pi/2]$, $\operatorname{tg} x$ toma valores positivos]