$\boxed{\text{INTEGRALES}}$

M00

$$\int_0^{a^2} \frac{1}{1+x^2}\,dx = 0'22 \longrightarrow \left[\operatorname{arctg} x\right]_0^{a^2} = 0'22 \;;\; \operatorname{arctg} a^2 = 0'22 \Longrightarrow a^2 = \operatorname{tg} 0'22 = 0'2236$$

$$\boxed{a = 0'4729}$$

M01

$$\int t^{1/3}\left(1 - \frac{1}{2t^{5/3}}\right) dt = \int t^{1/3}\,dt - \frac{1}{2}\int t^{-4/3}\,dt = \frac{t^{1/3+1}}{1/3+1} - \frac{1}{2}\,\frac{t^{-4/3+1}}{-\frac{4}{3}+1} = \frac{3}{4}t^{4/3} + \frac{3}{2}t^{-1/3} =$$

$$= \frac{3}{4}\left[t^{4/3} + \frac{2}{t^{1/3}}\right] = \boxed{\frac{3}{4}\,\frac{t^{5/3}+2}{t^{1/3}} + C}$$

N02

$$\int(\theta\cos\theta - \theta)\,d\theta = \int\theta\cos\theta\,d\theta - \int\theta\,d\theta = \theta\operatorname{sen}\theta - \int\operatorname{sen}\theta\,d\theta - \int\theta\,d\theta = \boxed{\theta\operatorname{sen}\theta + \cos x - \frac{\theta^2}{2} + C}$$

$$\boxed{\begin{array}{l} u=\theta \longrightarrow du = d\theta \\ dv = \cos\theta\,d\theta \rightarrow v = \operatorname{sen}\theta \end{array}}$$

M03

$$\int\left(\frac{x}{2-x}\right)^2 dx = -\int\left(\frac{2-t}{t}\right)^2 dt = -\int\frac{4-4t+t^2}{t^2}\,dt = -\int\left(\frac{4}{t^2} - \frac{4}{t} + 1\right) dt =$$

$$\boxed{\begin{array}{l} t = 2-x \longleftrightarrow x = 2-t \\ \qquad\qquad\qquad dx = -dt \end{array}}$$

$$= -4\int t^{-2}\,dt + 4\int\frac{1}{t}\,dt - \int 1\,dt = -4\,\frac{t^{-1}}{-1} + 4\ln|t| - t = \boxed{\frac{4}{t} + 4\ln|t| - t + C}$$

M04

$$\int\frac{\ln x}{\sqrt{x}}\,dx = \int x^{-1/2}\ln x\,dx = 2\sqrt{x}\ln x - 2\int\frac{x^{1/2}}{x}\,dx = 2\sqrt{x}\ln x - 2\int x^{-1/2}\,dx = 2\sqrt{x}\ln x - 2\,\frac{x^{-1/2+1}}{-1/2+1} =$$

$$= \boxed{2\sqrt{x}\ln x - 4\sqrt{x} + C}$$

$$\boxed{\begin{array}{l} u = \ln x \rightarrow du = \frac{1}{x}\,dx \\ dv = x^{-1/2}\,dx \rightarrow v = \frac{x^{-1/2+1}}{-1/2+1} = 2x^{1/2} \end{array}}$$

M04

a)

$$\int_0^m \frac{1}{x^2+4}\,dx = \int\frac{1}{(2t)^2+4}\,2\cdot dt = \int\frac{2}{4t^2+4}\,dt = \frac{2}{4}\int\frac{1}{t^2+1}\,dt = \frac{1}{2}\operatorname{arctg} t =$$

$$\cancel{x^2+4=0}$$
$$x^2 = -4$$
$$x = \pm 2i \longrightarrow \frac{x}{2} = \pm i$$

$$\boxed{\begin{array}{l} t = \frac{x}{2} \longleftrightarrow x = 2t \\ \qquad\qquad dx = 2dt \end{array}}$$

$$= \left[\frac{1}{2}\operatorname{arctg}\frac{x}{2}\right]_0^m = \boxed{\frac{1}{2}\operatorname{arctg}\frac{m}{2}}$$

b)

$$\frac{1}{2}\operatorname{arctg}\frac{m}{2} = 1/3 \Longrightarrow \operatorname{arctg}\frac{m}{2} = \frac{2}{3} \Longrightarrow \frac{m}{2} = \operatorname{tg}\left(\frac{2}{3}\right) \Longrightarrow m = 2\operatorname{tg}\left(\frac{2}{3}\right) = \boxed{1'5737}$$

M04

$$\int x^2e^x\,dx = x^2e^x - \int 2xe^x\,dx = x^2e^x - 2xe^x + 2\int e^x\,dx = \boxed{x^2e^x - 2xe^x + 2e^x + C}$$

$$\boxed{\begin{array}{l} u = x^2 \rightarrow du = 2x\,dx \\ dv = e^x\,dx \rightarrow v = e^x \end{array}} \qquad \boxed{\begin{array}{l} u = 2x \rightarrow du = 2\,dx \\ dv = e^x\,dx \rightarrow v = e^x \end{array}}$$

N04

$$\int\sqrt{1-4x^2}\,dx = \int\sqrt{1-4\left(\frac{\operatorname{sen}\theta}{2}\right)^2}\cdot\frac{1}{2}\cos\theta\,d\theta = \frac{1}{2}\int\sqrt{1-\cancel{4}\,\frac{\operatorname{sen}^2\theta}{\cancel{4}}}\cdot\cos\theta\,d\theta = \frac{1}{2}\int\cos^2\theta\,d\theta =$$

$$\boxed{\begin{array}{l} 2x = \operatorname{sen}\theta \longleftrightarrow \theta = \operatorname{arcsen}(2x) \\ x = \frac{1}{2}\operatorname{sen}\theta \\ dx = \frac{1}{2}\cos\theta\,d\theta \end{array}}$$

$$= \frac{1}{2}\int\frac{1+\cos(2\theta)}{2}\,d\theta = \frac{1}{4}\int 1\,d\theta + \frac{1}{4}\cdot\frac{1}{2}\int 2\cos(2\theta)\,d\theta =$$

$$= \frac{\theta}{4} + \frac{1}{8}\operatorname{sen}(2\theta) = \frac{\theta}{4} + \frac{1}{8}\cdot 2\operatorname{sen}\theta\cos\theta = \boxed{\frac{\operatorname{arcsen}(2x)}{4} + \frac{2x\sqrt{1-4x^2}}{4} + C}$$

M05 $\dfrac{x^2+3x+12}{x(x+2)^2} = \dfrac{A}{x} + \dfrac{B}{x+2} + \dfrac{C}{(x+2)^2} = \dfrac{A(x+2)^2 + Bx(x+2) + Cx}{x(x+2)^2} =$

$= \dfrac{Ax^2+4Ax+4A+Bx^2+2Bx+Cx}{x(x+2)^2} = \dfrac{(A+B)x^2+(4A+2B+C)x+4A}{x(x+2)^2}$

$A+B=1 \longrightarrow B=1-A=1-3=\boxed{-2}$

$4A+2B+C=3 \longrightarrow C=3-4A-2B=3-12+4=\boxed{-5}$

$4A=12 \longrightarrow \boxed{A=3}$

$\displaystyle\int \frac{x^2+3x+12}{x(x+2)^2}dx = \int\left(\frac{3}{x} - \frac{2}{x+2} - \frac{5}{(x+2)^2}\right)dx = 3\ln|x| - 2\ln|x+2| - 5\frac{(x+2)^{-2+1}}{-2+1} =$

$= \boxed{3\ln|x| - 2\ln|x+2| + \dfrac{5}{x+2} + C} = \boxed{\ln\left|\dfrac{x^3}{(x+2)^2}\right| + \dfrac{5}{x+2} + C}$

N05

a) $\dfrac{x^2}{(1+x)(1+x^2)} = \dfrac{a}{1+x} + \dfrac{bx+c}{(1+x^2)} = \dfrac{a(1+x^2)+(bx+c)(1+x)}{(1+x)(1+x^2)} = \dfrac{(a+b)x^2+(b+c)x+(a+c)}{(1+x)(1+x^2)}$

$\left.\begin{array}{l}1=a+b\\0=b+c\\0=a+c\end{array}\right\}\quad \boxed{\begin{array}{l}a=1/2\\b=1/2\\c=-1/2\end{array}}$

b) $\displaystyle\int\frac{x^2}{(1+x)(1+x^2)}dx = \int\frac{1/2}{1+x}dx + \int\frac{\frac{1}{2}x-\frac{1}{2}}{1+x^2}dx = \frac{1}{2}\int\frac{1}{1+x}dx + \frac{1}{2}\cdot\frac{1}{2}\int\frac{2x}{1+x^2}dx - \frac{1}{2}\int\frac{1}{1+x^2}dx =$

$= \boxed{\dfrac{1}{2}\ln|1+x| + \dfrac{1}{4}\ln|1+x^2| - \dfrac{1}{2}\operatorname{arctg} x + C} = \boxed{\dfrac{1}{4}\ln (1+x)^2(1+x^2) - \dfrac{1}{2}\operatorname{arctg} x + C}$

M06 $\displaystyle\int e^{2x}\sin x\,dx = -e^{2x}\cos x + 2\int e^{2x}\cos x\,dx = -e^{2x}\cos x + 2e^{2x}\sin x - 4\int e^{2x}\sin x\,dx$

$\boxed{\begin{array}{l}u=e^{2x} \rightarrow du=2e^{2x}dx\\ dv=\sin x\,dx \rightarrow v=-\cos x\,dx\end{array}}\quad \boxed{\begin{array}{l}u=e^{2x} \rightarrow du=2e^{2x}dx\\ dv=\cos x\,dx \rightarrow v=\sin x\end{array}}$

$\displaystyle 5\cdot\int e^{2x}\sin x\,dx = e^{2x}\cos x + 2e^{2x}\sin x \implies \boxed{\int e^{2x}\sin x\,dx = \frac{e^{2x}\cos x + 2e^{2x}\sin x}{5} + C}$

M07 $\displaystyle\int_0^{\ln 3}\frac{e^x}{e^{2x}+9}dx = \int\frac{3\cancel{t}}{9t^2+9}\cdot\frac{1}{\cancel{t}}dt = \frac{3}{9}\int\frac{1}{t^2+1}dt = \frac{1}{3}\operatorname{arctg} t = \left[\frac{1}{3}\operatorname{arctg}\frac{e^x}{3}\right]_0^{\ln 3} =$

$\boxed{\begin{array}{l}t=\dfrac{e^x}{3} \rightarrow x=\ln 3t\\ \qquad\qquad dx=\dfrac{1}{\cancel{3}t}\cancel{3}\,dt\end{array}}$

$= \dfrac{1}{3}\operatorname{arctg}\dfrac{e^{\ln 3}}{3} = \dfrac{1}{3}\operatorname{arctg}\dfrac{3}{3} = \dfrac{1}{3}\operatorname{arctg} 1 =$

$= \dfrac{1}{3}\cdot\dfrac{\pi}{4} = \boxed{\dfrac{\pi}{12}}$

M07 $f(x) = x\ln x - x$

a) $f'(x) = 1\cdot\ln x + x\cdot\dfrac{1}{x} - \cancel{1} = \ln x + 1 - 1 = \ln x$

b) $\displaystyle\int(\ln x)^2dx = (\ln x)^2 - \int 2\ln x\cdot\frac{1}{\cancel{x}}\cdot\cancel{x}\,dx = (\ln x)^2 - 2\int\ln x\,dx =$ [Aplicando el apartado (a)]

$\boxed{\begin{array}{l}u=(\ln x)^2 \rightarrow du=2\ln x\cdot\frac{1}{x}dx\\ dv=dx \rightarrow v=x\end{array}}$

$= (\ln x)^2 - 2(x\ln x - x) = \boxed{(\ln x)^2 - 2x\ln x + 2x + C}$

N07

$$\int_0^a \arcsin x\,dx = \left[x\arcsin x\right]_0^a - \int_0^a \frac{x}{\sqrt{1-x^2}}\,dx = \left[x\arcsin x\right]_0^a - \int_0^a x\cdot(1-x^2)^{1/2}\,dx =$$

$$\boxed{\begin{array}{ll} u=\arcsin x \rightarrow du=\dfrac{1}{\sqrt{1-x^2}}\,dx \\ dv=dx \rightarrow v=x \end{array}}$$

$$= \left[x\arcsin x\right]_0^a - \frac{1}{-2}\int_0^a -2x\,(1-x^2)^{1/2}\,dx = \left[x\arcsin x\right]_0^a + \left[\frac{1}{2}\,\frac{(1-x^2)^{1/2+1}}{1/2+1}\right]_0^a =$$

$$= \left[x\arcsin x + \frac{\sqrt{(1-x^2)^3}}{3}\right]_0^a = \boxed{a\cdot\arcsin a + \frac{\sqrt{(1-a^2)^3}}{3} - \frac{1}{3}}$$

M08

$$\int_0^{\pi/6} x\sin(2x)\,dx = \left[\frac{x\cos(2x)}{2}\right]_0^{\pi/6} - \frac{1}{2}\int_0^{\pi/6}\cos 2x\,dx = \left[\frac{x\cos(2x)}{2} - \frac{\sin(2x)}{4}\right]_0^{\pi/6} = \frac{\pi/6\,\sin\pi/3}{2} =$$

$$\boxed{\begin{array}{l} u=x \rightarrow du=dx \\ dv=\sin(2x)dx \rightarrow v=\frac{1}{2}\cos(2x) \end{array}}$$

$$= \frac{\pi/6\ \sqrt{3}/2}{2} = \boxed{\frac{\pi\sqrt{3}}{24}}$$

M08

$$\int \frac{tg(\ln y)}{y}\,dy = \int \frac{tg\,t}{\cancel{e^t}}\cdot \cancel{e^t}\,dt = \int tg\,t\,dt = \int \frac{\sin t}{\cos t}\,dt = -\ln|\cos t| =$$

$$\boxed{\begin{array}{l} t=\ln y \rightarrow y=e^t \\ \qquad\qquad dy=e^t\,dt \end{array}}$$

$$= \boxed{-\ln|\cos \ln y| + C}$$

Muestra 08

$$\int_1^e \frac{(\ln x)^3}{x}\,dx = \left[\frac{(\ln x)^4}{4}\right]_1^e = \frac{(\ln e)^4}{4} - \frac{(\ln 1)^4}{4} = \boxed{\frac{1}{4}}$$

Muestra 08

$x^2-4=0$; $x=\pm 2$

		-2		$+2$	
x^2-4	$+$	|	$-$	|	$+$

$$\int_0^4 |x^2-4|\,dx = \int_0^2 |x^2-4|\,dx + \int_2^4 |x^2-4|\,dx = \int_0^2 -(x^2-4)\,dx + \int_2^4 (x^2-4)\,dx =$$

$$= \left[-\frac{x^3}{3}+4x\right]_0^2 + \left[\frac{x^3}{3}-4x\right]_2^4 = -\frac{8}{3}+8+\frac{64}{3}-16-\frac{8}{3}+8 = \boxed{16}$$

Muestra 08

$$\int_1^{\sqrt{3}} \sqrt{4-x^2}\,dx = \int_{\pi/6}^{\pi/3} \sqrt{4-(2\sin\theta)^2}\cdot 2\cos\theta\,d\theta = 2\int_{\pi/6}^{\pi/3}\sqrt{4-4\sin^2\theta}\cdot\cos\theta\,d\theta =$$

$$\boxed{\begin{array}{l} x=2\sin\theta \longleftrightarrow \theta=\arcsin\frac{x}{2} \\ dx=2\cos\theta\,d\theta \end{array}} \qquad \boxed{\begin{array}{l} x=1 \Rightarrow \theta=\arcsin\frac{1}{2}=\frac{\pi}{6} \\ x=\sqrt{3} \Rightarrow \theta=\arcsin\frac{\sqrt{3}}{2}=\frac{\pi}{3} \end{array}}$$

$$= 2\int_{\pi/6}^{\pi/3}\sqrt{4(1-\sin^2\theta)}\cdot\cos\theta\,d\theta = 2\int_{\pi/6}^{\pi/3} 2\cdot\cos\theta\cdot\cos\theta\,d\theta = 4\int_{\pi/6}^{\pi/3}\cos^2\theta\,d\theta = 4\int_{\pi/6}^{\pi/3}\frac{1+\cos(2\theta)}{2}\,d\theta =$$

$$= 2\int_{\pi/6}^{\pi/3} 1\cdot d\theta + \int_{\pi/6}^{\pi/3} 2\cos(2\theta)\,d\theta = \left[2\theta+\sin(2\theta)\right]_{\pi/6}^{\pi/3} = \left(\frac{2\pi}{3}+\frac{1}{2}\right)-\left(\frac{\pi}{3}+\frac{\sqrt{3}}{2}\right) = \boxed{\frac{\pi}{3}+\frac{1-\sqrt{3}}{2}}$$

Muestra 08

$$\int_0^{\pi/4} \frac{\sin x}{\sqrt{\cos x}}\,dx = \int_0^{} \sin x \cdot \cos^{-1/2} x\,dx = -\int_0^{\pi/4} -\sin x \cdot \cos^{-1/2} x\,dx = \left[- \frac{\cos^{-1/2+1} x}{-1/2+1} \right]_0^{\pi/4} =$$

$$= \left[-2\sqrt{\cos x} \right]_0^{\pi/4} = -2\sqrt{\frac{\sqrt{2}}{2}} + 2\sqrt{1} = \boxed{1 - \sqrt[4]{8}}$$

Muestra 06/08

$$f'(x) = 2\sin\left(5x - \frac{\pi}{2}\right)$$

a) $f''(x) = 2\cos\left(5x - \frac{\pi}{2}\right) \cdot 5 = 10\cos\left(5x - \frac{\pi}{2}\right)$

b) $f(x) = \int 2\sin(5x - \pi/2)\,dx = \frac{-2}{5}\int -5\sin\left(5x - \frac{\pi}{2}\right)dx = -\frac{2}{5}\cos(5x - \pi/2) + C$

$$f\left(\frac{\pi}{2}\right) = 1 \Rightarrow 1 = -\frac{2}{5}\cos\left(\frac{5\pi}{2} - \frac{\pi}{2}\right) + C$$

$$1 = -\frac{2}{5}\cos(2\pi) + C$$

$$1 = -\frac{2}{5}\cdot 1 + C \Rightarrow \boxed{C = 7/5} \Rightarrow \boxed{f(x) = -\frac{2}{5}\cos\left(5x - \frac{\pi}{2}\right) + \frac{7}{5}}$$

Muestra 06/08

$$\int \frac{x^3}{(x+2)^2}\,dx = \int \frac{(u-2)^3}{u^2}\,du = \int \frac{u^3 - 6u^2 + 12u - 8}{u^2}\,du = \int \left(u - 6 + \frac{12}{u} - 8u^{-2}\right)du =$$

$$\boxed{\begin{array}{ll} u = x+2 \longleftrightarrow & x = u-2 \\ & dx = du \end{array}}$$

$$= \frac{u^2}{2} - 6u + 12\ln|u| - 8\frac{u^{-1}}{-1} = \boxed{\frac{(x+2)^2}{2} - 6(x+2) + 12\ln|x+2| + \frac{8}{x+2} + C}$$

Nov 08

$$\int_1^e x^2 \ln x\,dx = \left[\frac{x^3 \ln x}{3}\right]_1^e - \int_1^e \frac{x^3}{3}\cdot\frac{1}{x}\,dx = \left[\frac{x^3\ln x}{3}\right]_1^e - \frac{1}{3}\int_1^e x^2\,dx = \left[\frac{x^3\ln x}{3} - \frac{x^3}{9}\right]_1^e =$$

$$\boxed{\begin{array}{l} u = \ln x \to du = \frac{1}{x}dx \\ dv = x^2 dx \to v = \frac{x^3}{3} \end{array}}$$

$$= \left(\frac{e^3 \ln e}{3} - \frac{e^3}{9}\right) - \left(0 - \frac{1}{9}\right) = \boxed{\frac{2e^3+1}{9}}$$

Mo9

$$\frac{3}{x+1} + \frac{2}{x+3} = \frac{3(x+3) + 2(x+1)}{(x+1)(x+3)} = \frac{5x+11}{x^2+4x+3}$$ ✓

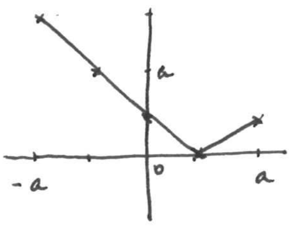
$$\int_0^2 \frac{5x+11}{x^2+4x+3}\,dx = \int_0^2 \left(\frac{3}{x+1} + \frac{2}{x+3}\right)dx = \left[3\ln|x+1| + 2\ln|x+3|\right]_0^2 = 3\ln 3 + 2\ln 5 - 3\ln 1 - 2\ln 3 =$$

$$= \ln 3 + 2\ln 5 = \ln(3\cdot 5^2) = \ln 75 \Rightarrow \boxed{K = 75}$$

Mo9

$$y = \left|x - \frac{a}{2}\right|$$

x	y
$-a$	$3a/2$
$-a/2$	a
0	$a/2$
$a/2$	0
a	$a/2$



$$\int_{-a}^0 \left|x - \frac{a}{2}\right| dx = \int_{-a}^0 \left(-x + \frac{a}{2}\right)dx = \left[-\frac{x^2}{2} + \frac{a}{2}x\right]_{-a}^0 =$$

$$= +\frac{a^2}{2} + \frac{a^2}{2} = a^2$$

$$\int_0^a \left|x - \frac{a}{2}\right| dx = 2\int_0^{a/2}\left(-x + \frac{a}{2}\right)dx = 2\left[\frac{-x^2}{2} + \frac{ax}{2}\right]_0^{a/2} =$$

$$= 2\left(-\frac{a^2}{8} + \frac{a^2}{4}\right) = \frac{a^2}{4}$$

$$a^2 = K\cdot\frac{a^2}{4} \Rightarrow \boxed{K = 4}$$

M09

a) $\displaystyle\int \frac{\sin\theta}{1-\cos\theta}\,d\theta = \boxed{\ln|1-\cos\theta| + C}$

b) $\displaystyle\int_{\pi/2}^{a} \frac{\sin\theta}{1-\cos\theta}\,d\theta = \ln|1-\cos a| - \ln\left|1-\cos\frac{\pi}{2}\right| = \ln|1-\cos a| - \ln 1 = \ln|1-\cos a|$

$\ln|1-\cos a| = \frac{1}{2} \Rightarrow 1-\cos a = e^{1/2} \Rightarrow \cos a = 1-\sqrt{e} \Rightarrow a = \begin{cases} \arccos(1-\sqrt{e}) \pm 2k\pi \\ -\arccos(1-\sqrt{e}) \pm 2k\pi \end{cases} \rightarrow$

$\rightarrow a = \begin{cases} 2'2767 \pm 2k\pi \\ -2'2767 \pm 2k\pi \end{cases} \Rightarrow \boxed{a = 2'2767}$ (porque $\frac{\pi}{2} < a < \pi$)

M11

$I = \displaystyle\int e^{2x}\sin x\,dx = -e^{2x}\cos x + \int 2e^{2x}\cos x\,dx = -e^{2x}\cos x + 2e^{2x}\sin x - 4\int e^{2x}\sin x\,dx = (2\sin x - \cos x)e^{2x} - 4I$

$u = e^{2x}$	$du = 2e^{2x}dx$
$dv = \sin x\,dx$	$v = -\cos x$

$u = 2e^{2x}$	$du = 4e^{2x}dx$
$dv = \cos x\,dx$	$v = \sin x$

$5I = (2\sin x - \cos x)e^{2x}$

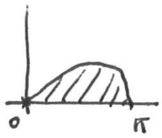
$\boxed{I = \frac{1}{5}(2\sin x - \cos x)e^{2x}}$ ✓

ÁREAS

M00

$y = x\sin(x^2)$

$y = 0 \Rightarrow x\sin(x^2) = 0 \rightarrow \boxed{x=0}$; $\sin(x^2) = 0 \rightarrow \boxed{x = 0 \pm k\pi}$



$\text{Area} = \displaystyle\int_0^{\pi} x\sin(x^2)\,dx = -\frac{1}{2}\int_0^{\pi} -2x\sin(x^2)\,dx = \left[\frac{1}{2}\cos(x^2)\right]_0^{\pi} =$

$= -\frac{1}{2}\cos(\pi^2) + \frac{1}{2}\cos 0 = \boxed{\frac{1}{2} - \frac{1}{2}\cos(\pi^2)}$

M01

a) $\displaystyle\int x\cos(3x)\,dx = \frac{x\sin(3x)}{3} - \frac{1}{3}\int \sin(3x)\,dx = \frac{x\sin(3x)}{3} + \frac{\cos(3x)}{9} + C$ ✓

$u = x \rightarrow du = dx$

$dv = \cos(3x)dx \rightarrow v = \frac{1}{3}\sin(3x)$

b) $y = x\cos(3x)$

$y = 0 \rightarrow x\cos(3x) = 0 \Rightarrow \boxed{x = 0}$

$\cos(3x) = 0 \rightarrow 3x = \frac{\pi}{2} \pm k\pi \rightarrow \boxed{x = \frac{\pi}{6} \pm \frac{k\pi}{3}}$

	0		$\pi/6$		$3\pi/6$		$5\pi/6$		$7\pi/6$
$y = x\cos(3x)$		+		−		+		−	

$\text{Area}_1 = -\displaystyle\int_{\pi/6}^{3\pi/6} x\cos(3x)\,dx = -\left[\frac{x\sin(3x)}{3} + \frac{\cos(3x)}{9}\right]_{\pi/6}^{3\pi/6} = -\left[\frac{\frac{3\pi}{6}\cdot\sin\frac{3\pi}{2}}{3} + \frac{\cos\frac{3\pi}{2}}{9} - \frac{\frac{\pi}{6}\sin\frac{\pi}{2}}{3} - \frac{\cos\frac{\pi}{2}}{9}\right] =$

$= +\frac{\pi}{6} + \frac{\pi}{18} = \boxed{\frac{4\pi}{18}}$

$\text{Area}_2 = \displaystyle\int_{3\pi/6}^{5\pi/6} x\cos(3x)\,dx = \left[\frac{x\sin(3x)}{3} + \frac{\cos(3x)}{9}\right]_{3\pi/6}^{5\pi/6} = \frac{\frac{5\pi}{6}\cdot\sin\frac{5\pi}{2}}{3} + \frac{\cos\frac{5\pi}{2}}{9} - \frac{\frac{3\pi}{6}\cdot\sin\frac{3\pi}{2}}{3} - \frac{\cos 3\pi/2}{9} =$

$= \frac{5\pi}{18} + \frac{\pi}{6} = \boxed{\frac{4\pi}{9}}$

$$Area_3 = -\int_{5\pi/6}^{7\pi/6} x \cos(3x)\,dx = -\left[\frac{x\sin(3x)}{3} + \frac{\cos(3x)}{9}\right]_{5\pi/6}^{7\pi/6} = -\left[\frac{\frac{7\pi}{6}\sin\frac{7\pi}{2}}{3} + \frac{\cos\frac{7\pi}{2}}{9} - \frac{\frac{5\pi}{6}\sin\frac{5\pi}{2}}{3} - \frac{\cos\frac{5\pi}{2}}{9}\right] =$$

$$= \frac{7\pi}{18} + \frac{5\pi}{18} = \boxed{\frac{2\pi}{3}}$$

$\frac{4\pi}{18}, \frac{4\pi}{9}, \frac{2\pi}{3}, \ldots$ $\qquad d = \frac{4\pi}{9} - \frac{4\pi}{18} = \frac{2\pi}{3} - \frac{4\pi}{9} = \ldots = \frac{2\pi}{9} \leftarrow$ diferencia

$$a_n = a_1 + d\cdot(n-1) = \frac{4\pi}{18} + \frac{2\pi}{9}(n-1)$$

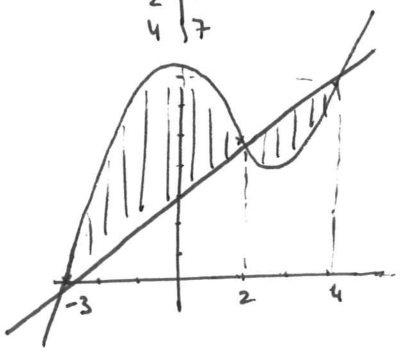
$$S_n = \frac{a_1 + a_n}{2}\cdot n = \frac{\frac{4\pi}{18} + \frac{4\pi}{18} + \frac{2\pi}{9}n - \frac{2\pi}{9}}{2}\cdot n = \frac{\frac{2\pi}{9} + \frac{2\pi}{9}n}{2}\, n = \boxed{\frac{n(n+1)\pi}{9}}$$

Nº4

$$\left.\begin{array}{l} y = x^3 - 3x^2 - 9x + 27 \\ y = x + 3 \end{array}\right\} \quad x^3 - 3x^2 - 9x + 27 = x + 3 \;; \quad x^3 - 3x^2 - 10x + 24 = 0$$

	1	-3	-10	24
2		2	-2	-24
	1	-1	-12	0
-3		-3	12	
	1	-4	0	
4		4		
	1	0		

x	y
-3	0
2	5
4	7



$$Area = \int_{-3}^{2} (x^3 - 3x^2 - 9x + 27) - (x+3)\,dx + \int_{2}^{4} (x+3) - (x^3 - 3x^2 - 9x + 27)\,dx =$$

$$= \int_{-3}^{2} (x^3 - 3x^2 - 10x + 24)\,dx + \int_{2}^{4} (-x^3 + 3x^2 + 10x - 24)\,dx =$$

$$= \left[\frac{x^4}{4} - x^3 - 5x^2 + 24x\right]_{-3}^{2} + \left[-\frac{x^4}{4} + x^3 + 5x^2 - 24x\right]_{2}^{4} =$$

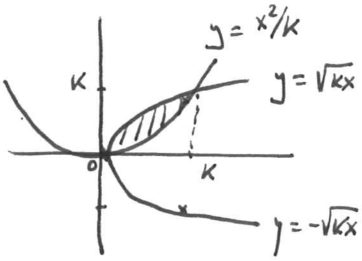
$$= (4 - 8 - 20 + 48) - \left(\frac{81}{4} + 27 - 45 - 72\right) + (-64 + 64 + 80 - 96) - (-4 + 8 + 20 - 48)$$

$$= \boxed{\frac{407}{4}}$$

Nº6

$$\left.\begin{array}{l} y^2 = kx \\ x^2 = ky \rightarrow y = x^2/k \end{array}\right\} \longrightarrow \frac{x^4}{k^2} = kx \;; \quad x^4 = k^3 x \;; \quad x^4 - k^3 x = 0$$

$$x(x^3 - k^3) = 0 \quad \begin{array}{l} \nearrow x = 0 \rightarrow y = 0 \\ \searrow x = k \rightarrow y = k \end{array}$$



$$Area = \int_0^k \left(\sqrt{kx} - \frac{x^2}{k}\right)dx = \sqrt{k}\int_0^k x^{1/2}\,dx - \frac{1}{k}\int_0^k x^2\,dx =$$

$$= \sqrt{k}\left[\frac{x^{3/2}}{3/2}\right]_0^k - \frac{1}{k}\left[\frac{x^3}{3}\right]_0^k = \sqrt{k}\,\frac{k^{3/2}}{3/2} - \frac{1}{k}\,\frac{k^3}{3} =$$

$$= \frac{2k^2}{3} - \frac{k^2}{3} = \frac{k^2}{3}$$

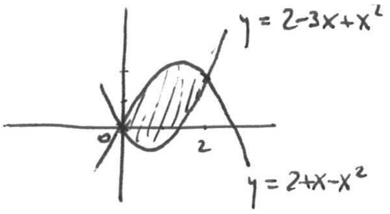
$$\frac{k^2}{3} = 12 \Rightarrow k^2 = 36 \Rightarrow \boxed{k = 6}$$

M08

$$\left.\begin{array}{l} y = 2+x-x^2 \\ y = 2-3x+x^2 \end{array}\right\} \quad 2-3x+x^2 = 2+x-x^2 \; ; \quad 2x^2-4x=0$$

$$2x(x-2)=0 \Longrightarrow \boxed{x = \begin{cases} 0 \\ 2 \end{cases}}$$

x	y
0	2
2	0

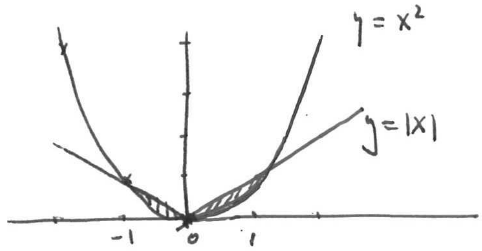


$$\text{Area} = \int_0^2 (2+x-x^2)-(2-3x+x^2)\,dx = \int_0^2 (-2x^2+4x)\,dx =$$

$$= \left[-\frac{2x^3}{3}+2x^2\right]_0^2 = -\frac{16}{3}+8 = \boxed{\frac{8}{3}}$$

Muestra 08

a)



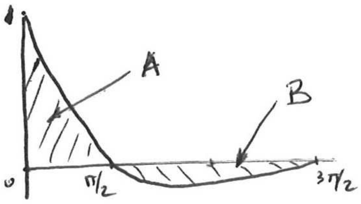
$$\text{Area} = 2\int_0^1 (x-x^2)\,dx = 2\left[\frac{x^2}{2}-\frac{x^3}{3}\right]_0^1 = 2\left(\frac{1}{2}-\frac{1}{3}\right) = \boxed{\frac{1}{3}}$$

M12 P2

$$f(x) = e^{-x}\cos x \quad \left(x \in \left[0, \tfrac{3\pi}{2}\right]\right)$$

a) $f(x)=0 \Rightarrow e^{-x}\cos x = 0$ $\begin{cases} e^{-x}=0 \quad \text{Absurdo} \\ \cos x = 0 \; ; \; \boxed{x = \begin{cases} \pi/2 \\ 3\pi/2 \end{cases}} \end{cases}$

b)



(Hecho con calculadora gráfica)

c) $$A = \int_0^{\pi/2} e^{-x}\cos x\,dx \qquad B = -\int_{\pi/2}^{3\pi/2} e^{-x}\cos x\,dx$$

$$I = \int e^{-x}\cos x\,dx = e^{-x}\sin x + \int e^{-x}\sin x\,dx = e^{-x}\sin x - e^{-x}\cos x - \int e^{-x}\cos x\,dx = e^{-x}(\sin x - \cos x) - I$$

$$\boxed{\begin{array}{l|l} u = e^{-x} & du = -e^{-x}dx \\ dv = \cos x\,dx & v = \sin x \end{array}} \qquad \boxed{\begin{array}{l|l} u = e^{-x} & du = -e^{-x}dx \\ dv = \sin x\,dx & v = -\cos x \end{array}}$$

$$2I = e^{-x}(\sin x - \cos x) \; ; \quad I = \frac{1}{2}e^{-x}(\sin x - \cos x) + K$$

$$A = \frac{1}{2}e^{-x}(\sin x - \cos x)\Big|_0^{\pi/2} = \frac{1}{2}e^{-\pi/2}(\underbrace{\sin \tfrac{\pi}{2}}_{1} - \underbrace{\cos\tfrac{\pi}{2}}_{0}) - \frac{1}{2}\underbrace{e^0}_{1}(\underbrace{\sin 0}_{0} - \underbrace{\cos 0}_{1}) = \frac{1}{2}\left(e^{-\pi/2}+1\right)$$

$$B = -\frac{1}{2}e^{-x}(\sin x - \cos x)\Big|_{\pi/2}^{3\pi/2} = -\frac{1}{2}e^{-3\pi/2}(\underbrace{\sin\tfrac{3\pi}{2}}_{-1} - \underbrace{\cos\tfrac{3\pi}{2}}_{0}) + \frac{1}{2}e^{-\pi/2}(\underbrace{\sin\tfrac{\pi}{2}}_{1} - \underbrace{\cos\tfrac{\pi}{2}}_{0}) = \frac{1}{2}\left(e^{-3\pi/2}+e^{-\pi/2}\right)$$

$$\frac{A}{B} = \frac{\cancel{\tfrac{1}{2}}\left(e^{-\pi/2}+1\right)}{\cancel{\tfrac{1}{2}}\left(e^{-3\pi/2}+e^{-\pi/2}\right)} = \frac{e^{\frac{3\pi}{2}}e^{-\pi/2}+e^{3\pi/2}}{e^{3\pi/2}e^{-3\pi/2}+e^{3\pi/2}e^{-\pi/2}} = \frac{e^{\pi}+e^{3\pi/2}}{1+e^{\pi}} = \frac{e^{\pi}\left(1+e^{\pi/2}\right)}{e^{\pi}+1} \quad ✓$$

multiplicando numerador y denominador por $e^{3\pi/2}$

VOLÚMENES

N06

$$f(x)=\frac{\ln x}{x^3}\ ,\quad x\geq 1$$

a) $f'(x)=\dfrac{\frac{1}{x}\cdot x^3-\ln x\cdot 3x^2}{x^6}=\dfrac{x^2-3x^2\ln x}{x^6}=\boxed{\dfrac{1-3\ln x}{x^4}}$

$f''(x)=\dfrac{-\frac{3}{x}\cdot x^4-(1-3\ln x)\cdot 4x^3}{x^8}=\dfrac{-3x^3-4x^3+12x^3\ln x}{x^8}=\boxed{\dfrac{-7+12\ln x}{x^5}}$

b) $f'(x)=0 \Rightarrow 1-3\ln x=0\ ;\ \ln x=\frac{1}{3}\ ;\ \boxed{x=e^{1/3}}$

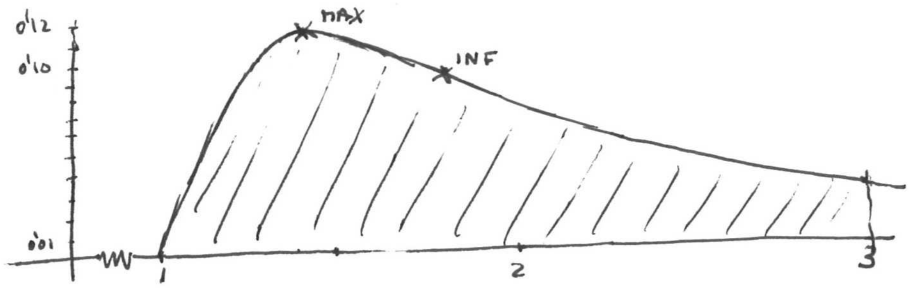
$f''(e^{1/3})=\dfrac{-7+12\ln e^{1/3}}{(e^{1/3})^5}=\dfrac{-7+12\cdot\frac{1}{3}}{e^{5/3}}=\dfrac{-3}{e^{5/3}}<0 \Rightarrow \boxed{\text{Máximo en } x=e^{1/3}}$

$f''(x)=0 \Rightarrow -7+12\ln x=0\ ;\ \ln x=7/12\ ;\ \boxed{x=e^{7/12}}$

$f'''(x)=\dfrac{\frac{12}{x}\cdot x^5-(-7+12\ln x)\cdot 5x^4}{x^{10}}=\dfrac{12x^4+35x^4-60\ln x\cdot x^4}{x^{10}}=\dfrac{47-60\ln x}{x^6}$

$f'''(e^{7/12})=\dfrac{47-60\cdot\ln e^{7/12}}{(e^{7/12})^6}\neq 0 \Rightarrow \boxed{\text{Inflexión en } x=e^{7/12}}$

x	y
$1'40=e^{1/3}$	$\dfrac{\ln e^{1/3}}{(e^{1/3})^3}=0'12$
$1'79=e^{7/12}$	$\dfrac{\ln e^{7/12}}{(e^{7/12})^3}=0'10$
1	0
3	0'04



c) $V=\pi\displaystyle\int_1^3\left(\frac{\ln x}{x^3}\right)^2dx=\pi\int_1^3\frac{\ln^2 x}{x^6}dx=\pi\int_1\ln^2x\cdot x^{-6}dx=-\left[\frac{\pi\ln^2x}{x^5}\right]_1^3+\frac{\pi}{5}\int_1^3\ln x\cdot x^{-6}dx=$

$\boxed{\begin{array}{l} u=\ln^2x \rightarrow du=2\ln x\cdot\frac{1}{x}dx \\ dv=x^{-6}dx\rightarrow v=\frac{x^{-5}}{-5}\end{array}}$

$=\left[-\dfrac{\pi\ln^2x}{x^5}\right]_1^3-\left[\dfrac{\pi\ln x}{25x^5}\right]_1^3+\dfrac{\pi}{25}\displaystyle\int_1^3 x^{-6}dx=\left[-\dfrac{\pi\ln^2x}{x^5}-\dfrac{\pi\ln x}{25x^5}-\dfrac{\pi}{5x^5}\right]_1^3=\boxed{0'6096}$

$\boxed{\begin{array}{l} u=\ln x \rightarrow du=\frac{1}{x}dx \\ dv=x^{-6}dx\rightarrow v=\frac{x^{-5}}{-5}\end{array}}$

$\text{Area}=\displaystyle\int_1^3\frac{\ln x}{x^3}dx=\left[-\frac{\ln x}{2x^2}\right]_1^3+\frac{1}{2}\int_1^3x^{-3}dx=\left[\frac{-\ln x}{2x^2}-\frac{1}{4x^2}\right]_1^3=\left(\frac{-\ln 3}{18}-\frac{1}{36}\right)-\left(0-\frac{1}{4}\right)=$

$\boxed{\begin{array}{l} u=\ln x \rightarrow du=\frac{1}{x}dx \\ dv=x^{-3}dx\rightarrow v=\frac{x^{-2}}{-2}\end{array}}$

$=-\dfrac{\ln 3}{18}-\dfrac{1}{36}+\dfrac{1}{4}=-\dfrac{\ln 3}{18}+\dfrac{2}{9}=\boxed{\dfrac{4-\ln 3}{18}}$ ✓

(M07)

$$V = \pi \int_0^{\pi/4} \sin^2(3x)\,dx = \pi \int_0^{\pi/4} \frac{1-\cos(6x)}{2}\,dx = \frac{\pi}{2}\int_0^{\pi/4} 1\,dx - \frac{\pi}{2}\frac{1}{6}\int_0^{\pi/4} 6\cos(6x)\,dx =$$

$$= \left[\frac{\pi x}{4} - \frac{\pi}{12}\sin(6x)\right]_0^{\pi/4} = \left(\frac{\pi\,\pi/4}{4} - \frac{\pi}{12}\sin\frac{3\pi}{2}\right) - 0 = \boxed{\frac{\pi^2}{16} + \frac{\pi}{12}}$$

(M08)

$$V = \pi \int_1^e \left(\frac{\ln x}{x}\right)^2 dx = \pi \int_1^e \ln^2 x \cdot x^{-2}\,dx = -\left[\frac{\pi \ln^2 x}{x}\right]_1^e + 2\pi \int_1^e \frac{\ln x}{x^2}\,dx = \left[-\frac{\pi \ln^2 x}{x} - \frac{2\pi \ln x}{x}\right]_1^e + 2\pi\int_1^e x^{-2}\,dx =$$

$$\boxed{\begin{array}{l} u = \ln^2 x \rightarrow du = 2\ln x \cdot \frac{1}{x}\,dx \\ dv = x^{-2}dx \rightarrow v = \frac{x^{-1}}{-1} \end{array}} \qquad \boxed{\begin{array}{l} u = \ln x \rightarrow du = \frac{1}{x}\,dx \\ dv = x^{-2}dx \rightarrow v = \frac{x^{-1}}{-1} \end{array}}$$

$$= \left[-\frac{\pi \ln^2 x}{x} - \frac{2\pi \ln x}{x} - \frac{2\pi}{x}\right]_1^e = \left(-\frac{\pi}{e} - \frac{2\pi}{e} - \frac{2\pi}{e}\right) - \left(-0 - 0 - 2\pi\right) = \boxed{2\pi - \frac{5\pi}{e}}$$

(M09)

$$4x^2 + y^2 = 4$$

a) $8x + 2y \cdot \frac{dy}{dx} = 0 \longrightarrow \frac{dy}{dx} = -\frac{8x}{2y}$; $\boxed{\frac{dy}{dx} = \frac{-4x}{y}}$

b) Pendiente de Tangente $= \dfrac{-4 \cdot 2/\sqrt{5}}{2/\sqrt{5}} = \boxed{-4}$

c) $$V = \pi \int_0^1 y^2\,dx = \pi \int_0^1 (4 - 4x^2)\,dx = \pi\left(4x - \frac{4x^3}{3}\right)\Big|_0^1 = \pi\left(4 - \frac{4}{3}\right) = \boxed{\frac{8\pi}{3}}$$

M09 $\quad f(x) = x + 2\cos x \qquad x \in [0, 2\pi]$

a) $\left.\begin{array}{l} y = x + 2\cos x \\ y = x \end{array}\right\}\ \cancel{x} + 2\cos x = \cancel{x}\ ;\ \cos x = 0\ ;\ \boxed{x = \begin{cases} \pi/2 \\ 3\pi/2 \end{cases}} \begin{array}{l} \rightarrow A(\pi/2, \pi/2) \quad \boxed{a=1} \\ \rightarrow C\left(\frac{3\pi}{2}, 3\pi/2\right) \quad \boxed{c=3} \end{array}$

$f'(x) = 1 - 2\sin x$

$f'(x) = 0 \rightarrow \sin x = \frac{1}{2} \Rightarrow x = \begin{cases} \pi/6 \\ 5\pi/6 \rightarrow B(5\pi/6\ ,\ 5\pi/6 - \sqrt{3}) \quad \boxed{b=5} \end{cases}$

b) $f(x)$ tiene un mínimo en $x = \frac{\pi}{6} \longrightarrow y = \frac{\pi}{6} + \sqrt{3} \simeq 2'2556$

" " " máximo en $x = \frac{5\pi}{6} \longrightarrow y = \frac{5\pi}{6} - \sqrt{3} \simeq 0'8859$

Los puntos inicio y fin del dominio: $x = 0 \rightarrow y = 2$

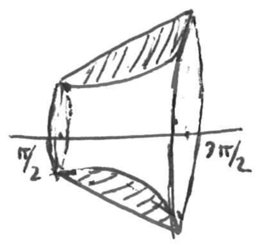
$x = 2\pi \rightarrow y = 2\pi + 2 \simeq 8'28$

El recorrido de $f(x)$ es: $\boxed{\left[\frac{5\pi}{6} - \sqrt{3},\ 2\pi + 2\right]}$

c) $x = \frac{3\pi}{2} \begin{cases} \rightarrow y = 3\pi/2 \\ \rightarrow y' = 1 - 2\sin\frac{3\pi}{2} = 3 \rightarrow \text{pendiente normal} = -1/3 \end{cases}$

$y - \frac{3\pi}{2} = -\frac{1}{3}\left(x - \frac{3\pi}{2}\right) \longrightarrow y = \frac{3\pi}{2} - \frac{x}{3} + \frac{\pi}{2}\ ;\ \boxed{y = -\frac{x}{3} + 2\pi}$

d) $V = \pi \int_{\pi/2}^{3\pi/2} \left[x^2 - (x + 2\cos x)^2\right] dx =$



$= \pi \int_{\pi/2}^{3\pi/2} x^2 - x^2 - 4x\cos x - 4\cos^2 x \ dx =$

$= -4\pi \int_{\pi/2}^{3\pi/2} x\cos x\, dx - 4\pi \int_{\pi/2}^{3\pi/2} \cos^2 x\, dx = \left[-4\pi x \sin x\right]_{\pi/2}^{3\pi/2} - 4\pi \int_{\pi/2}^{3\pi/2} \sin x\, dx - 4\pi \int_{\pi/2}^{3\pi/2} \frac{1 + \cos 2x}{2} dx =$

$\boxed{\begin{array}{l} u = x \rightarrow du = dx \\ dv = \cos x\, dx \rightarrow v = \sin x \end{array}}$

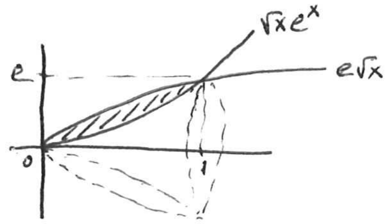
$= \left[-4\pi x \sin x + 4\pi\cos x - 2\pi x - \pi \sin 2x\right]_{\pi/2}^{3\pi/2} =$

$= \left(-4\pi \cdot \frac{3\pi}{2} \cdot (-1) + 4\pi \cdot 0 - 2\pi\frac{3\pi}{2} - \pi \cdot 0\right) - \left(-4\pi\frac{\pi}{2} \cdot 1 + 4\pi \cdot 0 - 2\pi \cdot \frac{\pi}{2} - \pi \cdot 0\right) =$

$= 6\pi^2 - 3\pi^2 + 2\pi^2 + \pi^2 = \boxed{6\pi^2}$

M10
P1

$$\left.\begin{aligned} y &= \sqrt{x}\,e^x \\ y &= e\sqrt{x} \end{aligned}\right\}\ \sqrt{x}\,e^x = e\sqrt{x}\ ;\ \sqrt{x}\cdot(e^x-e)=0 \begin{cases} \sqrt{x}=0 & x=0 \to P(0,0) \\ e^x = e\ ;\ x=1 & \to P(1,e) \end{cases}$$



$$V = \pi\int_0^1 (e\sqrt{x})^2\,dx - \pi\int_0^1 (\sqrt{x}\,e^x)^2\,dx = \pi e^2\int_0^1 x\,dx - \pi\int_0^1 x e^{2x}\,dx =$$

$$\int x e^{2x}\,dx = \tfrac{1}{2}x e^{2x} - \tfrac{1}{2}\int e^{2x}\,dx = \frac{xe^{2x}}{2} - \frac{e^{2x}}{4}$$

$u = x$	$du = dx$
$dv = e^{2x}dx$	$v = \frac{1}{2}e^{2x}$

$$= \pi e^2\left(\frac{x^2}{2}\right)_0^1 - \pi\left(\frac{xe^{2x}}{2} - \frac{e^{2x}}{4}\right)_0^1 =$$

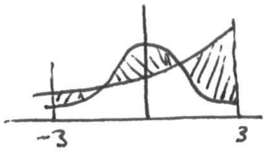
$$= \pi e^2\left(\frac{1}{2}-0\right) - \pi\left(\frac{e^2}{2}-\frac{e^2}{4}\right) + \pi\left(0-\frac{1}{4}\right) =$$

$$= \frac{\pi e^2}{4} - \frac{\pi}{4} = \boxed{\frac{\pi(e^2-1)}{4}}$$

CON CALCULADORA GRÁFICA

N01

$$\text{Area} = \int_{-3}^{3} \left| \frac{2}{1+x^2} - e^{x/3} \right| dx = \boxed{4'4955}$$

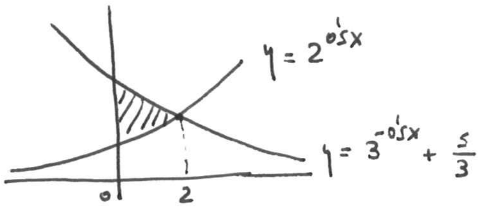


M02

$$y = x^3 - 7x^2 + 14x - 7 \longrightarrow \begin{array}{l} A = 0'7530 \\ B = 2'4450 \\ C = 3'8019 \end{array}$$

$$\text{Area} = \int_{0'7530}^{2'4450} (x^3 - 7x^2 + 14x - 7)\, dx = \boxed{1'7785}$$

M06

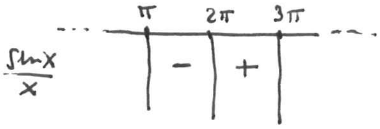


$$\text{Area} = \int_0^2 \left(3^{-0'5x} + \frac{5}{3} - 2^{0'5x} \right) dx = \boxed{1'6616}$$

M01

$$y = \frac{\sin x}{x}, \quad \pi \le x \le 3\pi$$

$$\sin x = 0 \longrightarrow x = \begin{cases} 0 + 2k\pi \\ \pi + 2k\pi \end{cases}$$



$$\text{Area} = -\int_{\pi}^{2\pi} \frac{\sin x}{x}\, dx + \int_{2\pi}^{3\pi} \frac{\sin x}{x}\, dx = 0'4338 + 0'2566 = \boxed{0'6904}$$

También:

$$\text{Area} = \int_{\pi}^{3\pi} \left| \frac{\sin x}{x} \right| dx = \boxed{0'6904}$$

N02

b) $\boxed{x = -3 \quad \text{A. Vertical}}$

$x = 0 \rightarrow y = \boxed{\ln 3 - 2} = -0'9014$

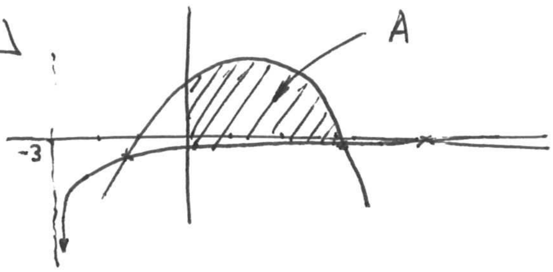
$y = 0 \rightarrow 0 = \ln(x+3) - 2$

$2 = \ln(x+3)$

$e^2 = x + 3$

$x = \boxed{e^2 - 3} = 4'3891$

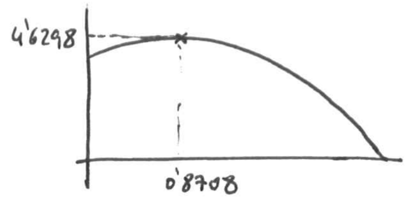
a)



c) $f(x) = g(x) \longrightarrow \boxed{x = \begin{cases} -1'3443 \\ 3'0494 \end{cases}}$

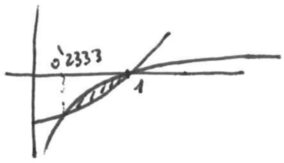
d) $$\text{Area} = \int_0^{3'0494} (4 - (1-x)^2) - (\ln(x+3) - 2)\, dx = \boxed{10'5453}$$

e) Represento $y = [4 - (1-x)^2] - [\ln(x+3) - 2]$ en $0 \le x \le 3'0494$ y busco el máximo:



La màxima distancia es $\boxed{4'6298}$ para $x = 0'8708$.

(M03)



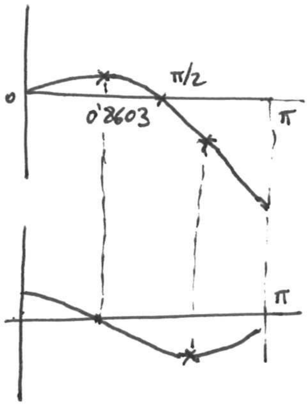
$$\text{Area} = \int_{0'2333}^{1} \ln x - (e^x - e)\,dx = \boxed{0'2014}$$

(M04)

$\boxed{a = 0'8603}$

Para el punto de inflexión represento $f'(x) = \cos x - x\,\text{sen}\,x$ y busco su extremo:

$\boxed{b = 2'2889}$



c) $$\int x\cos x\,dx = x\,\text{sen}\,x - \int \text{sen}\,x\,dx = \boxed{x\,\text{sen}\,x + \cos x + C}$$

$u = x \rightarrow du = dx$

$dv = \cos x\,dx \rightarrow v = \text{sen}\,x$

d) $$\text{Area} = \int_0^{\pi/2} x\cos x\,dx = \left[x\,\text{sen}\,x + \cos x\right]_0^{\pi/2} = \left(\frac{\pi}{2} + 0\right) - (0+1) = \boxed{\frac{\pi}{2} - 1}$$

(M05)

Se cortan en:

$x = -1'9198$

$x = 2'7186$

$$\text{Area} = \int_{-1'9198}^{2'7186} \left[(4-x^2) - (x+1)\cos x\right] dx = \boxed{9'4053}$$

(N08)

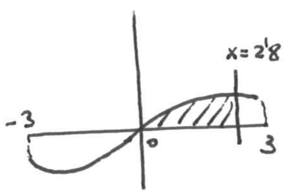
$$f(x) = x\sqrt{9-x^2} + 2\,\text{arc sen}\left(\frac{x}{3}\right)$$

a) $9 - x^2 \geq 0 \rightarrow x \in [-3,3]$

$-1 \leq \frac{x}{3} \leq 1 \rightarrow x \in [-3,3]$

$\Big\}$ $\text{dom} f = [-3,3]$

b) $$V = \pi\int_0^{2'8} f^2(x)\,dx = \boxed{180'5439}$$



c] $f'(x) = \sqrt{9-x^2} + x\frac{-\cancel{2}x}{\cancel{2}\sqrt{9-x^2}} + 2\frac{1/3}{\sqrt{1-(\frac{x}{3})^2}} = \sqrt{9-x^2} - \frac{x^2}{\sqrt{9-x^2}} + \frac{2}{3\sqrt{1-\frac{x^2}{9}}} =$

$= \sqrt{9-x^2} - \frac{x^2}{\sqrt{9-x^2}} + \frac{2}{\cancel{3}\sqrt{\frac{9-x^2}{\cancel{9}}}} = \frac{9-x^2-x^2+2}{\sqrt{9-x^2}} = \boxed{\frac{11-2x^2}{\sqrt{9-x^2}}}$

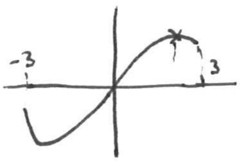
d] $\int_{-p}^{p} \frac{11-2x^2}{\sqrt{9-x^2}}\,dx = \left[x\sqrt{9-x^2} + 2\arcsin\left(\frac{x}{3}\right)\right]_{-p}^{p} = \left[p\sqrt{9-p^2} + 2\arcsin\left(\frac{p}{3}\right)\right] - \left[-p\sqrt{9-p^2} + 2\arcsin\left(\frac{-p}{3}\right)\right] =$

$= p\sqrt{9-p^2} + 2\arcsin\left(\frac{p}{3}\right) + p\sqrt{9-p^2} + 2\arcsin\left(\frac{p}{3}\right) = \boxed{2p\sqrt{9-p^2} + 4\arcsin\left(\frac{p}{3}\right)}$ ✓

e] Represento $2p\sqrt{9-p^2} + 4\arcsin\left(\frac{p}{3}\right)$ y busco su máximo:

Máximo para

$\boxed{p = 2'3452}$



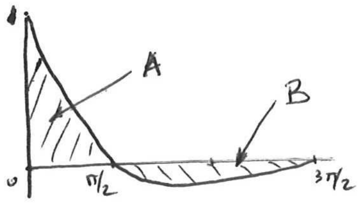
M12
P2

$f(x) = e^{-x}\cos x \quad \left(x \in \left[0, \frac{3\pi}{2}\right]\right)$

a] $f(x)=0 \Rightarrow e^{-x}\cos x = 0$ → $e^{-x}=0$ Absurdo

→ $\cos x = 0$; $\boxed{x = \begin{cases} \pi/2 \\ 3\pi/2 \end{cases}}$

b]



(Hecho con calculadora gráfica)

c] $A = \int_0^{\pi/2} e^{-x}\cos x\,dx \qquad B = -\int_{\pi/2}^{3\pi/2} e^{-x}\cos x\,dx$

$I = \int e^{-x}\cos x\,dx = e^{-x}\sin x + \int e^{-x}\sin x\,dx = e^{-x}\sin x - e^{-x}\cos x - \int e^{-x}\cos x\,dx = e^{-x}(\sin x - \cos x) - I$

$u = e^{-x}$	$du = -e^{-x}dx$
$dv = \cos x\,dx$	$v = \sin x$

$u = e^{-x}$	$du = -e^{-x}dx$
$dv = \sin x\,dx$	$v = -\cos x$

$2I = e^{-x}(\sin x - \cos x)$; $I = \frac{1}{2}e^{-x}(\sin x - \cos x) + K$

$A = \frac{1}{2}e^{-x}(\sin x - \cos x)\Big|_0^{\pi/2} = \frac{1}{2}e^{-\pi/2}(\overset{1}{\sin\frac{\pi}{2}} - \overset{0}{\cos\frac{\pi}{2}}) - \frac{1}{2}\overset{1}{e^{0}}(\overset{0}{\sin 0} - \overset{1}{\cos 0}) = \frac{1}{2}(e^{-\pi/2}+1)$

$B = -\frac{1}{2}e^{-x}(\sin x - \cos x)\Big|_{\pi/2}^{3\pi/2} = -\frac{1}{2}e^{-3\pi/2}(\overset{-1}{\sin\frac{3\pi}{2}} - \overset{0}{\cos\frac{3\pi}{2}}) + \frac{1}{2}e^{-\pi/2}(\overset{1}{\sin\frac{\pi}{2}} - \overset{0}{\cos\frac{\pi}{2}}) = \frac{1}{2}\left(e^{-3\pi/2} + e^{-\pi/2}\right)$

$\frac{A}{B} = \frac{\cancel{\frac{1}{2}}(e^{-\pi/2}+1)}{\cancel{\frac{1}{2}}(e^{-3\pi/2}+e^{-\pi/2})} = \frac{e^{\frac{3\pi}{2}}e^{-\pi/2} + e^{3\pi/2}}{e^{3\pi/2}e^{-3\pi/2} + e^{3\pi/2}e^{-\pi/2}} = \frac{e^{\pi} + e^{3\pi/2}}{1+e^{\pi}} = \frac{e^{\pi}(1+e^{\pi/2})}{e^{\pi}+1}$ ✓

multiplicando numerador y denominador por $e^{3\pi/2}$

DISTRIBUCIONES CONTINUAS DE PROBABILIDAD

M04

a) $\int_{-\infty}^{+\infty} f(x)\,dx=1 \Rightarrow \int_0^2 kx^2\,dx=1$; $\left[\frac{kx^3}{3}\right]_0^2=1$; $\frac{8k}{3}=1 \Rightarrow \boxed{k=3/8}$

b) $E[x]=\int_{-\infty}^{+\infty} x\, f(x)\,dx = \int_0^2 x\cdot kx^2\,dx = \frac{3}{8}\int_0^2 x^3\,dx = \frac{3}{8}\left[\frac{x^4}{4}\right]_0^2=\boxed{\frac{3}{2}}$

m = mediana

$0'5=\int_0^m kx^2\,dx = \frac{3}{8}\left[\frac{x^3}{3}\right]_0^m = \frac{3}{8}\,\frac{m^3}{3}=\frac{m^3}{8} \Rightarrow m^3=4 \to \boxed{m=\sqrt[3]{4}}$

N04

a) $1=\int_0^2 k(2x-x^2)\,dx = k\left[x^2-\frac{x^3}{3}\right]_0^2 = k\cdot\left(4-\frac{8}{3}\right)=\frac{4k}{3} \Rightarrow \boxed{k=3/4}$

b) $P[0'25\le x\le 0'5]=\int_{0'25}^{0'5}k(2x-x^2)\,dx=\frac{3}{4}\left[x^2-\frac{x^3}{3}\right]_{0'25}^{0'5}=\frac{3}{4}\left(0'5^2-\frac{0'5^3}{3}\right)-\frac{3}{4}\left(0'25^2-\frac{0'25^3}{3}\right)=\boxed{0'1133}$

M06

$\mu=\int_4^{10} t\cdot\frac{1}{72}(12t-t^2-20)\,dt=\frac{1}{72}\int_4^{10}(12t^2-t^3-20t)\,dt=\frac{1}{72}\left(4t^3-\frac{t^4}{4}-10t^2\right)_4^{10}=$

$=\frac{1}{72}\left(4\cdot10^3-\frac{10^4}{4}-10\cdot10^2\right)-\frac{1}{72}\left(4\cdot4^3-\frac{4^4}{4}-10\cdot4^2\right)=\boxed{6'5}$

$\sigma^2=\int_4^{10}t^2\cdot\frac{1}{72}(12t-t^2-20)\,dt-6'5^2=\frac{1}{72}\int_4^{10}(12t^3-t^4-20t^2)\,dt-6'5^2=\frac{1}{72}\left[3t^4-\frac{t^5}{5}-\frac{20t^3}{3}\right]_4^{10}-6'5^2=$

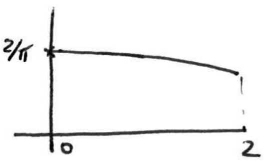
$=\frac{1}{72}\left(3\cdot10^4-\frac{10^5}{5}-\frac{20\cdot10^3}{3}\right)-\frac{1}{72}\left(3\cdot4^4-\frac{4^5}{5}-20\,\frac{4^3}{3}\right)-6'5^2=\boxed{2'15}\to\sigma=\sqrt{2'15}$

b) $P[\mu-\sigma<x<\mu]=\int_{6'5-\sqrt{2'15}}^{6'5}\frac{1}{72}(12t-t^2-20)\,dt=\frac{1}{72}\int_{5'0337}^{6'5}(12t-t^2-20)\,dt=\frac{1}{72}\left[6t^2-\frac{t^3}{3}-20t\right]_{5'0337}^{6'5}=$

$=\frac{1}{72}\left[6\cdot6'5^2-\frac{6'5^3}{3}-20\cdot6'5\right]-\frac{1}{72}\left[6\cdot5'0337^2-\frac{5'0337^3}{3}-20\cdot5'0337\right]=\boxed{0'3211}$

M07

$f(x)=\frac{8}{\pi(x^2+4)}$ $\quad 0\le x\le 2 \longrightarrow$



El máximo valor lo toma en $x=0 \to \boxed{\text{moda}=0}$

$E[x]=\int_0^2 x\,\frac{8}{\pi(x^2+4)}\,dx=\frac{8}{\pi}\int_0^2\frac{x}{x^2+4}\,dx=\frac{4}{\pi}\int_0^2\frac{2x}{x^2+4}\,dx=\left[\frac{4}{\pi}\ln(x^2+4)\right]_0^2=$

$=\frac{4}{\pi}\ln 8-\frac{4}{\pi}\ln 4=\frac{4}{\pi}\ln\left(\frac{8}{4}\right)=\boxed{\frac{4\ln 2}{\pi}}$

TRAYECTORIAS DE MÓVILES

M02

$$v = \frac{1}{2+t^2}$$

a) $\frac{ds}{dt} = v \Rightarrow ds = v\,dt \Rightarrow s = \int v\,dt$

$$s = \int_0^1 \frac{1}{2+t^2}\,dt = \int_0^{1/\sqrt{2}} \frac{1}{2+(u\sqrt{2})^2}\cdot\frac{1}{\sqrt{2}}\,du = \int_0^{1/\sqrt{2}} \frac{1}{2+2u^2}\cdot\sqrt{2}\,du = \int_0^{1/\sqrt{2}} \frac{1}{2(1+u^2)}\cdot\sqrt{2}\,du = \frac{\sqrt{2}}{2}\int_0^{1/\sqrt{2}} \frac{1}{1+u^2}\,du =$$

$$\boxed{\begin{array}{l} t = u\sqrt{2} \longleftrightarrow u = \frac{1}{\sqrt{2}}t \\ dt = \sqrt{2}\,du \\ t=0 \Rightarrow u=0 \\ t=1 \Rightarrow u = 1/\sqrt{2} \end{array}}$$

$$= \frac{\sqrt{2}}{2}\cdot\left[\operatorname{arctg} u\right]_0^{1/\sqrt{2}} = \frac{\sqrt{2}}{2}\left(\operatorname{arctg}\frac{1}{\sqrt{2}} - \operatorname{arctg} 0\right) = \boxed{0'4352}$$

b) $a = \frac{dv}{dt} = \boxed{\frac{-2t}{(2+t^2)^2}}$

M06

$$v_A = -\frac{1}{2}t^2 + 3t + \frac{3}{2}$$
$$v_B = e^{0'2t}$$
$(t \in [0,9])$

a) $v_A' = -t+3$

$v_A' = 0 \Rightarrow t = 3$

$v_A'' = -1$; $v_A''(3) = -1 < 0 \Rightarrow$ $\boxed{\text{Máximo velocidad } v_A \text{ en } t=3}$

b) $a_B = \frac{dv_B}{dt} = 0'2\cdot e^{0'2t}$

$a_B(4) = 0'2\cdot e^{0'2\cdot 4} = \boxed{0'2\cdot e^{0'8}}$

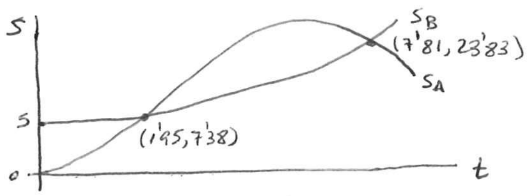
c) $S_A = \int v_A\,dt = \int\left(-\frac{1}{2}t^2 + 3t + \frac{3}{2}\right)dt = \frac{-t^3}{6} + \frac{3t^2}{2} + \frac{3t}{2} + K$

$\left.\begin{array}{l} t=0 \\ S_A = 0 \end{array}\right| \Rightarrow 0 = K$; $\boxed{S_A = \frac{-t^3}{6} + \frac{3t^2}{2} + \frac{3t}{2}}$

$S_B = \int v_B\,dt = \int e^{0'2t} = \frac{1}{0'2}e^{0'2t} + K = 5e^{0'2t} + K$

$\left.\begin{array}{l} t=0 \\ S_B = 5 \end{array}\right| \Rightarrow 5 = 5\cdot e^0 + K$; $K = 0$; $\boxed{S_B = 5\cdot e^{0'2t}}$

d)



$\boxed{\text{Se encontrarán en } t = \begin{cases} 1'95\ s \\ 7'81\ s \end{cases}}$

(Hecho con calculadora gráfica)

M07

$v = 3t^2 - 4t + 2 \longrightarrow s = \int(3t^2 - 4t + 2)\,dt = t^3 - 2t^2 + 2t + K$

$\left.\begin{array}{l} t=0 \\ s=-3 \end{array}\right| \Rightarrow -3 = K$; $\boxed{s = t^3 - 2t^2 + 2t - 3}$

$s = 0 \Rightarrow t^3 - 2t^2 + 2t - 3 = 0 \Rightarrow \boxed{t = 1'81\ s}$

(Resuelto con calculadora gráfica)

M11

$$v = 50(1 - e^{-0'2t})$$

a) $a = \frac{dv}{dt} = +50\cdot 0'2\,e^{-0'2t} = e^{-0'2t}$; $t = 10\,s \Rightarrow \boxed{a = e^{-2}\ m/s^2}$

b) $s = \int_0^{10} 50(1 - e^{-0'2t})\,dt = \left[50t - \frac{50}{-0'2}e^{-0'2t}\right]_0^{10} = \left[50t + 250\,e^{-0'2t}\right]_0^{10} = (500 + 250e^{-2}) - (0 + 250) = 250 + 250e^{-2}$

Altura $= 2000 - (250 + 250e^{-2}) = \boxed{1716\ m}$