# Ecuaciones Diferenciales en exámenes BI - NS

**Mayo 01**

Find the general solution of the differential equation $\frac{dx}{dt} = kx(5 - x)$, where $0 < x < 5$, and $k$ is a constant.

**Mayo 02**

La función $y = f(x)$ satisface la ecuación diferencial

$$2x^2 \frac{dy}{dx} = x^2 + y^2 \qquad (x > 0)$$

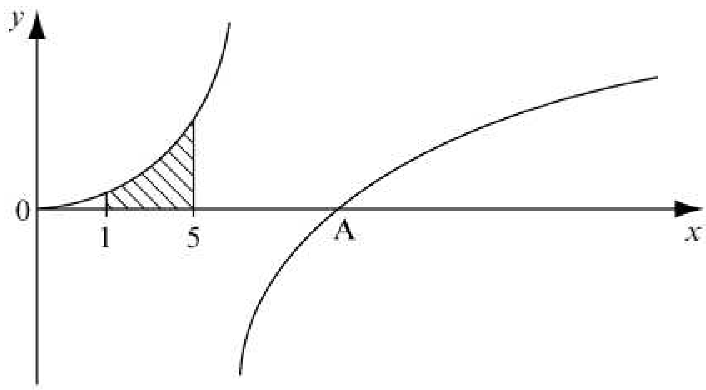
(a) (i) Sustituyendo $y = vx$, demuestre que

$$2x \frac{dv}{dx} = (v - 1)^2 .$$

(ii) A partir de aquí demuestre que la solución de la ecuación diferencial original es $y = x - \frac{2x}{(\ln x + c)}$, donde $c$ es una constante arbitraria.

(iii) Halle el valor de $c$, sabiendo que $y = 2$ cuando $x = 1$.

(b) A continuación aparece la gráfica de $y = f(x)$. La gráfica corta el eje de las $x$ en A.




(i) Escriba la ecuación de la asíntota vertical.

(ii) Halle el valor **exacto** de la coordenada $x$ del punto A.

(iii) Halle el área de la región sombreada.

**Nov 02**

Sea la ecuación diferencial $\frac{dy}{dx} = \frac{3y^2 + x^2}{2xy}$, para $x > 0$.

(a) Sustituya $y = vx$ para mostrar que $v + x\frac{dv}{dx} = \frac{3v^2 + 1}{2v}$.

(b) Partiendo de aquí halle la solución de la ecuación diferencial, sabiendo que $y = 2$ para $x = 1$.

**Nov 02**

La tangente a la curva $y = f(x)$ en el punto $\mathrm{P}(x, y)$ corta al eje de las $x$ en $\mathrm{Q}(x-1, 0)$. La curva corta al eje de las $y$ en $\mathrm{R}(0, 2)$. Halle la ecuación de la curva.

**Mayo 04**

Sabiendo que $\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{e}^x - 2x$, y que cuando $x = 0$, $y = 3$, halle la expresión de $y$ en función de $x$.

**Mayo 04**

Given that $\frac{\mathrm{d}y}{\mathrm{d}x} = 2x - \sin x$ and $y = 2$ when $x = 0$, find an expression for $y$ in terms of $x$.

**Nov 04**

Considere la ecuación diferencial $\frac{\mathrm{d}y}{\mathrm{d}\theta} = \frac{y}{(\mathrm{e}^{2\theta}+1)}$.

(a) Utilice la sustitución $x = \mathrm{e}^\theta$ para comprobar que

$$\int \frac{\mathrm{d}y}{y} = \int \frac{\mathrm{d}x}{x(x^2+1)}.$$

(b) Halle $\int \frac{\mathrm{d}x}{x(x^2+1)}$.

(c) A partir de lo anterior, halle $y$ en función de $\theta$, si $y = \sqrt{2}$ cuando $\theta = 0$.

**Muestra 06/08**

Solve the differential equation $x\frac{\mathrm{d}y}{\mathrm{d}x} - y^2 = 1$, given that $y = 0$ when $x = 2$. Give your answer in the form $y = f(x)$.

**Nov 06**

Solve the differential equation

$$(x+2)^2 \frac{\mathrm{d}y}{\mathrm{d}x} = 4xy \quad (x > -2)$$

given that $y = 1$ when $x = -1$.

**Nov 06**

Let $y = \cos\theta + \mathrm{i}\sin\theta$.

(a) Show that $\frac{\mathrm{d}y}{\mathrm{d}\theta} = \mathrm{i}y$.
[You may assume that for the purposes of differentiation and integration, i may be treated in the same way as a real constant.]

(b) **Hence** show, using integration, that $y = \mathrm{e}^{\mathrm{i}\theta}$.

(c) Use this result to deduce de Moivre's theorem.

**Mayo 07**

Resuelva la ecuación diferencial $\frac{\mathrm{d}y}{\mathrm{d}x} = 2xy^2$, sabiendo que $y = 1$ cuando $x = 0$.
Exprese la respuesta en la forma $y = f(x)$.

**Mayo 07**

Solve the differential equation $(x^2+1)\frac{\mathrm{d}y}{\mathrm{d}x} - xy = 0$ where $x > 0$, $y > 0$, given that $y = 1$ when $x = 1$.

**Nov 07**

Solve the differential equation $\dfrac{dy}{dx} = \dfrac{1+y^2}{1+x^2}$, given that $y = \sqrt{3}$ when $x = \dfrac{\sqrt{3}}{3}$

Give your answer in the form $y = \dfrac{ax+\sqrt{a}}{a - x\sqrt{a}}$ where $a \in \mathbb{Z}^+$.

**Muestra 08**

The acceleration of a body is given in terms of the displacement $s$ metres as $a = \dfrac{2s}{s^2+1}$.

(a) Give a formula for the velocity as a function of the displacement given that when $s = 1$ metre, $v = 2\ \text{ms}^{-1}$.

(b) Hence find the velocity when the body has travelled 5 metres.

**Mayo 08**

A particle moves in a straight line in a positive direction from a fixed point O.

The velocity $v$ m s$^{-1}$, at time $t$ seconds, where $t \geq 0$, satisfies the differential equation

$$\frac{dv}{dt} = \frac{-v(1+v^2)}{50}.$$

The particle starts from O with an initial velocity of 10 m s$^{-1}$.

(a) (i) Express as a definite integral, the time taken for the particle's velocity to decrease from 10 m s$^{-1}$ to 5 m s$^{-1}$.

(ii) **Hence** calculate the time taken for the particle's velocity to decrease from 10 m s$^{-1}$ to 5 m s$^{-1}$.

(b) (i) Show that, when $v > 0$, the motion of this particle can also be described by the differential equation $\dfrac{dv}{dx} = \dfrac{-(1+v^2)}{50}$ where $x$ metres is the displacement from O.

(ii) Given that $v = 10$ when $x = 0$, solve the differential equation expressing $x$ in terms of $v$.

(iii) **Hence** show that $v = \dfrac{10 - \tan\dfrac{x}{50}}{1 + 10\tan\dfrac{x}{50}}$.

**Mayo 08**

A gourmet chef is renowned for her spherical shaped soufflé. Once it is put in the oven, its volume increases at a rate proportional to its radius.

(a) Show that the radius $r$ cm of the soufflé, at time $t$ minutes after it has been put in the oven, satisfies the differential equation $\frac{dr}{dt} = \frac{k}{r}$, where $k$ is a constant.

(b) Given that the radius of the soufflé is 8 cm when it goes in the oven, and 12 cm when it's cooked 30 minutes later, find, to the nearest cm, its radius after 15 minutes in the oven.

**Nov 08**

The population of mosquitoes in a specific area around a lake is controlled by pesticide. The rate of decrease of the number of mosquitoes is proportional to the number of mosquitoes at any time $t$. Given that the population decreases from 500 000 to 400 000 in a five year period, find the time it takes in years for the population of mosquitoes to decrease by half.

**Mayo 09**

Let $f$ be a function with domain $\mathbb{R}$ that satisfies the conditions,

$$f(x+y) = f(x)f(y), \text{ for all } x \text{ and } y \text{ and } f(0) \neq 0.$$

(a) Show that $f(0) = 1$.

(b) Prove that $f(x) \neq 0$, for all $x \in \mathbb{R}$.

(c) Assuming that $f'(x)$ exists for all $x \in \mathbb{R}$, use the definition of derivative to show that $f(x)$ satisfies the differential equation $f'(x) = k\,f(x)$, where $k = f'(0)$.

(d) Solve the differential equation to find an expression for $f(x)$.

**Mayo 09**

(a) Solve the differential equation $\frac{\cos^2 x}{e^y} - e^{e^y}\frac{dy}{dx} = 0$, given that $y = 0$ when $x = \pi$.

(b) Find the value of $y$ when $x = \frac{\pi}{2}$.

**Mayo 09**

La aceleración, en $\mathrm{m\,s^{-2}}$, de una partícula que se mueve en línea recta en el instante $t$ segundos, con $t \geq 0$, viene dada por la fórmula $a = -\frac{1}{2}v$. Para $t = 0$, la velocidad es igual a 40 $\mathrm{m\,s^{-1}}$.

Halle una expresión para $v$ en función de $t$.

**Nov 09**
**P1#9**

A certain population can be modelled by the differential equation $\frac{dy}{dt} = ky\cos kt$, where $y$ is the population at time $t$ hours and $k$ is a positive constant.

(a) Given that $y = y_0$ when $t = 0$, express $y$ in terms of $k$, $t$ and $y_0$.

(b) Find the ratio of the minimum size of the population to the maximum size of the population.

**Mayo 10**
**TZ2**
**P1#14**

Throughout this question $x$ satisfies $0 \le x < \frac{\pi}{2}$.

(a) Solve the differential equation $\sec^2 x \frac{dy}{dx} = -y^2$, where $y = 1$ when $x = 0$.

Give your answer in the form $y = f(x)$.

(b) (i) Prove that $1 \le \sec x \le 1 + \tan x$.

(ii) Deduce that $\frac{\pi}{4} \le \int_0^{\frac{\pi}{4}} \sec x \, dx \le \frac{\pi}{4} + \frac{1}{2}\ln 2$.

**Mayo 10**
**TZ1**
**P2#14**

A body is moving through a liquid so that its acceleration can be expressed as

$$\left(-\frac{v^2}{200} - 32\right) \text{m s}^{-2},$$

where $v$ m s$^{-1}$ is the velocity of the body at time $t$ seconds.

The initial velocity of the body was known to be $40$ m s$^{-1}$.

(a) Show that the time taken, $T$ seconds, for the body to slow to $V$ m s$^{-1}$ is given by

$$T = 200\int_V^{40} \frac{1}{v^2 + 80^2} \, dv.$$

(b) (i) Explain why acceleration can be expressed as $v\frac{dv}{ds}$, where $s$ is displacement, in metres, of the body at time $t$ seconds.

(ii) **Hence** find a similar integral to that shown in part (a) for the distance, $S$ metres, travelled as the body slows to $V$ m s$^{-1}$.

(c) **Hence**, using parts (a) and (b), find the distance travelled and the time taken until the body momentarily comes to rest.

**Mayo 10 TZ2 P2#14**

The functions $f$, $g$ and $h$ are defined by

$$f(x)=1+e^x, \text{ for } x\in\mathbb{R},$$
$$g(x)=\frac{1}{x}, \text{ for } x\in\mathbb{R}/\{0\},$$
$$h(x)=\sec x, \text{ for } x\in\mathbb{R}/\left\{\frac{2n+1}{2}\pi,\ n\in\mathbb{Z}\right\}.$$

(a) Determine the range of the composite function $g\circ f$.

(b) Determine the inverse of the function $g\circ f$, clearly stating the domain.

(c) (i) Show that the function $y=(f\circ g\circ h)(x)$ satisfies the differential equation

$$\frac{dy}{dx}=(1-y)\sin x.$$

(ii) Hence, or otherwise, find $\int y\sin x\,dx$, as a function of $x$.

(iii) You are given that the domain of $y=(f\circ g\circ h)(x)$ can be extended to the whole real axis. That part of the graph of $y=(f\circ g\circ h)(x)$, between its maximum at $x=0$ and its first minimum for positive $x$, is rotated by $2\pi$ about the $y$-axis. Calculate the volume of the solid generated.

**Nov 10 P1#8**

Halle $y$ en función de $x$, sabiendo que $(1+x^3)\frac{dy}{dx}=2x^2\,\text{tg}\,y$ y que para $x=0$, $y=\frac{\pi}{2}$.

**Mayo 11 TZ1 P2#14**

An open glass is created by rotating the curve $y=x^2$, defined in the domain $x\in[0,10]$, $2\pi$ radians about the $y$-axis. Units on the coordinate axes are defined to be in centimetres.

(a) When the glass contains water to a height $h$ cm, find the volume $V$ of water in terms of $h$.

(b) If the water in the glass evaporates at the rate of 3 $cm^3$ per hour for each $cm^2$ of exposed surface area of the water, show that,

$$\frac{dV}{dt}=-3\sqrt{2\pi V}, \text{ where } t \text{ is measured in hours.}$$

(c) If the glass is filled completely, how long will it take for all the water to evaporate?

**Mayo 11 TZ2 P2#13**

Solve the differential equation $\frac{dy}{dx} = \sqrt{1-y^2}\, e^{2x} \sin x$, given that $y = 0$ when $x = 0$, writing your answer in the form $y = f(x)$.

(i) Sketch the graph of $y = f(x)$, found in part (b), for $0 \le x \le 1.5$. Determine the coordinates of the point P, the first positive intercept on the $x$-axis, and mark it on your sketch.

(ii) The region bounded by the graph of $y = f(x)$ and the $x$-axis, between the origin and P, is rotated $360^\circ$ about the $x$-axis to form a solid of revolution. Calculate the volume of this solid.

**Nov 11 P1#13**

The curve C with equation $y = f(x)$ satisfies the differential equation

$$\frac{dy}{dx} = \frac{y}{\ln y}(x+2),\ y > 1,$$

and $y = e$ when $x = 2$.

(a) Find the equation of the tangent to C at the point $(2, e)$.

(b) Find $f(x)$.

(c) Determine the largest possible domain of $f$.

(d) Show that the equation $f(x) = f'(x)$ has no solution.

**Mayo 12 TZ2 P2#12**

Una partícula se mueve en línea recta a una velocidad de $v$ metros por segundo. En un instante cualquiera, $t$ segundos, $0 \le t < \frac{3\pi}{4}$, la velocidad viene dada por la ecuación diferencial $\frac{dv}{dt} + v^2 + 1 = 0$. También se sabe que $v = 1$ cuando $t = 0$.

(a) Halle una expresión para $v$ en función de $t$.

(b) Dibuje aproximadamente la gráfica de $v$ en función de $t$, mostrando claramente las coordenadas de todos los puntos de corte con los ejes y las ecuaciones de todas las asíntotas.

(c) (i) Escriba el tiempo $T$ para el cual la velocidad es igual a cero.

(ii) Halle la distancia recorrida en el intervalo $[0, T]$.

(d) Halle una expresión para el desplazamiento $s$ en función de $t$, sabiendo que $s = 0$ cuando $t = 0$.

(e) A partir de lo anterior, o de cualquier otro modo, compruebe que $s = \frac{1}{2}\ln\frac{2}{1+v^2}$.

**Mayo 13**
**TZ2**
**P2#12**

Considere la ecuación diferencial $y\dfrac{dy}{dx} = \cos 2x$.

(a) (i) Compruebe que la función $y = \cos x + \operatorname{sen} x$ satisface la ecuación diferencial.

(ii) Halle la solución general de la ecuación diferencial. Exprese la solución de la forma $y = f(x)$, incluyendo en la respuesta una constante de integración.

(iii) ¿Para qué valor de la constante de integración coincide su solución con la función dada en el apartado (i)?

(b) Otra solución de la ecuación diferencial, para la cual $y = 2$ cuando $x = \dfrac{\pi}{4}$, define una curva $C$.

(i) Determine la ecuación de $C$, expresando la respuesta de la forma $y = g(x)$, e indique el recorrido de la función $g$.

Una región $R$ del plano $xy$ está delimitada por $C$, el eje $x$ y las rectas verticales $x = 0$ y $x = \dfrac{\pi}{2}$.

(ii) Halle el área de $R$.

(iii) Halle el volumen que se genera cuando la parte de $R$ que se encuentra por encima de la recta $y = 1$ se hace girar $2\pi$ radianes en torno al eje $x$.