Geometría Analítica del Espacio en exámenes BI (extracto)

Muestra 08

Consider the points $A(1,\ 2,\ 1)$, $B(0,\ -1,\ 2)$, $C(1,\ 0,\ 2)$ and $D(2,\ -1,\ -6)$.

(a) Find the vectors $\vec{AB}$ and $\vec{BC}$.

(b) Calculate $\vec{AB}\times\vec{BC}$.

(c) Hence, or otherwise find the area of triangle ABC.

(d) Find the Cartesian equation of the plane P containing the points A, B and C.

(e) Find a set of parametric equations for the line L through the point D and perpendicular to the plane P.

(f) Find the point of intersection E, of the line L and the plane P.

(g) Find the distance from the point D to the plane P.

(h) Find a unit vector which is perpendicular to the plane P.

(i) The point F is a reflection of D in the plane P. Find the coordinates of F.

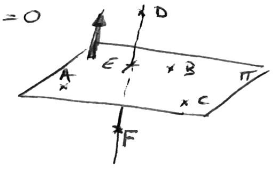
a) $\vec{AB} = (-1,-3,+1)$
$\vec{BC} = (+1,1,0)$

b) $\vec{AB}\times\vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & -3 & +1 \\ +1 & 1 & 0 \end{vmatrix} = \boxed{-\vec{i}+\vec{j}+2\vec{k}} = (-1,1,2)$

c) Area $\widehat{ABC} = \dfrac{|\vec{AB}\times\vec{AC}|}{2} = \dfrac{\sqrt{1+1+4}}{2} = \boxed{\dfrac{\sqrt{6}}{2}}$

d) $\vec{AB}\times\vec{AC} = (-1,1,2) \longrightarrow -1\cdot(x-1)+1\cdot(y-2)+2(z-1)=0$

$P \equiv \boxed{-x+y+2z-3=0}$



e) $\boxed{\left.\begin{array}{l} x=2-r \\ y=-1+r \\ z=-6+2r \end{array}\right\} \equiv L}$

f) $-(2-r)+(-1+r)+2(-6+2r)-3=0$

$-2+r-1+r-12+4r-3=0$

$6r=18$

$r=3 \rightarrow x=-1,\ y=2,\ z=0$ $\boxed{E(-1,2,0)}$

g) $d = d\left(D(2,-1,-6)\ ;\ -x+y+2z-3=0\right) = \dfrac{|-2-1-12-3|}{\sqrt{1+1+4}} = \boxed{\dfrac{18}{\sqrt{6}}} = \dfrac{18\sqrt{6}}{6} = 3\sqrt{6}$

También: $d = |\vec{DE}| = |(-3,3,6)| = \sqrt{9+9+36} = \sqrt{54} = \boxed{3\sqrt{6}}$ ✓

h) $|(-1,1,2)| = \sqrt{1+1+4} = \sqrt{6}$

$\boxed{\vec{u} = \dfrac{1}{\sqrt{6}}(-1,1,2)}$ ó $\boxed{\dfrac{1}{\sqrt{6}}(1,-1,-2)}$

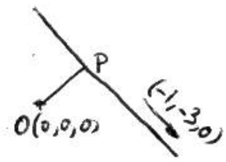
i) $F = D + 2\vec{DE} = (2,-1,-6) + 2(-3,3,6) = \boxed{(-4,5,6)}$

Muestra 06 = 08

The line L is given by the parametric equations $x = 1 - \lambda$, $y = 2 - 3\lambda$, $z = 2$. Find the coordinates of the point on L which is nearest to the origin.

$\vec{OP}\ (1-\lambda,\ 2-3\lambda,\ 2)$

$\vec{OP} \perp (-1,-3,0) \Rightarrow$



$\Rightarrow -1(1-\lambda) - 3(2-3\lambda) + 0 \cdot 2 = 0$

$-1+\lambda-6+9\lambda = 0 \quad ; \quad 10\lambda = 7 \quad ; \quad \lambda = \frac{7}{10}$

$\lambda = \frac{7}{10} \Rightarrow P = \left(1-\frac{7}{10},\ 2-\frac{21}{10},\ 2\right) = \boxed{\left(\frac{3}{10},\ \frac{1}{10},\ 2\right)}$

Mayo 09 P2 TZ1 #7

Consider the planes defined by the equations $x + y + 2z = 2$, $2x - y + 3z = 2$ and $5x - y + az = 5$ where a is a real number.

(a) If $a = 4$ find the coordinates of the point of intersection of the three planes.

(b) (i) Find the value of a for which the planes do not meet at a unique point.

(ii) For this value of a show that the three planes do not have any common point.

a) $\left.\begin{array}{l} x+y+2z=2 \\ 2x-y+3z=2 \\ 5x-y+4z=5 \end{array}\right\}$

$$x = \frac{\begin{vmatrix} 2 & 1 & 2 \\ 2 & -1 & 3 \\ 5 & -1 & 4 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 2 \\ 2 & -1 & 3 \\ 5 & -1 & 4 \end{vmatrix}} = \frac{-8-4+15+10-8+6}{-4-4+15+10-8+3} = \boxed{\frac{11}{12}}$$

$$y = \frac{\begin{vmatrix} 1 & 2 & 2 \\ 2 & 2 & 3 \\ 5 & 5 & 4 \end{vmatrix}}{12} = \frac{8+\cancel{20}+30-\cancel{20}-16-15}{12} = \boxed{\frac{7}{12}}$$

$$z = \frac{\begin{vmatrix} 1 & 1 & 2 \\ 2 & -1 & 2 \\ 5 & -1 & 5 \end{vmatrix}}{12} = \frac{-5-4+10+\cancel{10}-\cancel{10}+2}{12} = \boxed{\frac{3}{12}}$$

b) $A = \begin{pmatrix} 1 & 1 & 2 \\ 2 & -1 & 3 \\ 5 & -1 & a \end{pmatrix}$ $\quad \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} = -3 \neq 0 \Rightarrow r(A) \geq 2$

$|A| = \begin{vmatrix} 1 & 1 & 2 \\ 2 & -1 & 3 \\ 5 & -1 & a \end{vmatrix} = -a-4+15+10-2a+3 = 24-3a \longrightarrow a=8$

• Si $\boxed{a=8} \Rightarrow r(A)=2 \Rightarrow$ El sistema no puede ser Compatible Determ.

$a=8 \rightarrow A^* = \left(\begin{array}{ccc|c} 1 & 1 & 2 & 2 \\ 2 & -1 & 3 & 2 \\ 5 & -1 & 8 & 5 \end{array}\right)$ $\quad \begin{vmatrix} 1 & 1 & 2 \\ 2 & -1 & 2 \\ 5 & -1 & 5 \end{vmatrix} = -5-4+10+\cancel{10}-\cancel{10}+2 = 3 \neq 0 \Rightarrow r(A^*)=3$

$\left.\begin{array}{l} r(A)=2 \\ r(A^*)=3 \end{array}\right| \Rightarrow$ Sistema Incompatible ✓

Mayo 09
P2 TZ1
#11

The position vector at time t of a point P is given by

$$\vec{OP} = (1+t)\boldsymbol{i} + (2-2t)\boldsymbol{j} + (3t-1)\boldsymbol{k},\ t \geq 0.$$

(a) Find the coordinates of P when $t = 0$.

(b) Show that P moves along the line L with Cartesian equations

$$x - 1 = \frac{y-2}{-2} = \frac{z+1}{3}.$$

(c) (i) Find the value of t when P lies on the plane with equation $2x + y + z = 6$.

(ii) State the coordinates of P at this time.

(iii) Hence find the total distance travelled by P before it meets the plane.

The position vector at time t of another point, Q, is given by

$$\vec{OQ} = \begin{pmatrix} t^2 \\ 1-t \\ 1-t^2 \end{pmatrix},\ t \geq 0.$$

(d) (i) Find the value of t for which the distance from Q to the origin is minimum.

(ii) Find the coordinates of Q at this time.

(e) Let $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ be the position vectors of Q at times $t = 0$, $t = 1$ and $t = 2$ respectively.

(i) Show that the equation $\boldsymbol{a} - \boldsymbol{b} = k(\boldsymbol{b} - \boldsymbol{c})$ has no solution for k.

(ii) Hence show that the path of Q is not a straight line.

a) $P_0 = \vec{i} + 2\vec{j} - \vec{k} = \boxed{(1,2,-1)}$

b) $(x,y,z) = (1+t)\vec{i} + (2-2t)\vec{j} + (3t-1)\vec{k} =$

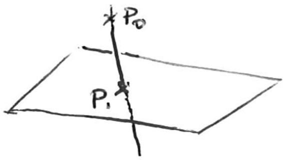
$= (\vec{i} + 2\vec{j} - \vec{k}) + t(\vec{i} - 2\vec{j} - \vec{k}) \Rightarrow$

$\Rightarrow \left.\begin{array}{l} x = 1+t \\ y = 2-2t \\ z = 3t-1 \end{array}\right\} \longrightarrow \boxed{\frac{x-1}{1} = \frac{y-2}{-2} = \frac{z+1}{3}} = t$ ✓

c) $2(1+t) + (2-2t) + (3t-1) = 6$

$2+\cancel{2t}+2-\cancel{2t}+3t-1=6 \;;\; 3t=3 \Rightarrow \boxed{t=1}$

$t=1 \rightarrow x=2,\; y=0,\; z=2$ $\boxed{P_1(2,0,2)}$



$t=0 \rightarrow P_0=(1,2,-1)$

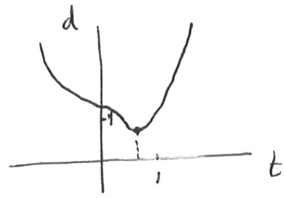
$t=1 \rightarrow P_1=(2,0,2)$

$d=|\overrightarrow{P_0P_1}| = |(1,-2,3)| = \sqrt{1+4+9} = \boxed{\sqrt{14}}$

d) $\overrightarrow{OQ}=(t^2,\,1-t,\,1-t^2)$

$d=|\overrightarrow{OQ}|=\sqrt{t^4+(1-t)^2+(1-t^2)^2}$

Representando con calculadora gráfica, localizamos el mínimo en $\boxed{t=0'7607} \longrightarrow Q=\boxed{(0'579,\,0'239,\,0'421)}$



e) $t=0 \rightarrow \vec{a}=(0,1,1)$

$t=1 \rightarrow \vec{b}=(1,0,0)$

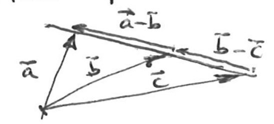
$t=2 \rightarrow \vec{c}=(4,-1,-3)$

$\vec{a}-\vec{b}=(-1,1,1)$

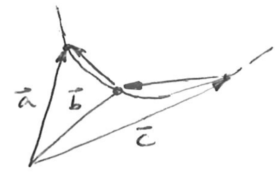
$\vec{b}-\vec{c}=(-3,1,3)$

$\vec{a}-\vec{b}=K\cdot(\vec{b}-\vec{c}) \Rightarrow$ $-1=-3K \rightarrow K=1/3$; $1=K \rightarrow K=1$; $1=3K \rightarrow K=1/3$ No tiene solución.

Por lo tanto la trayectoria no es recta, ya que de serlo la trayectoria y los vectores $\vec{a}-\vec{b}$ y $\vec{b}-\vec{c}$ estarían sobre ella y serían paralelos:

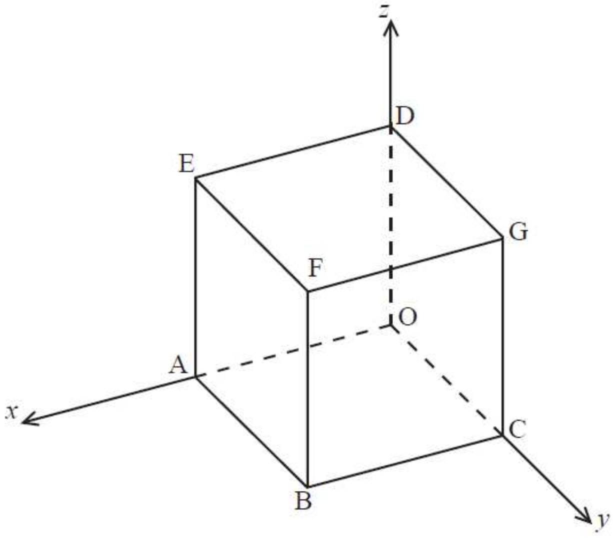


Será entonces algo así:



Nov 10
P2 #12

La siguente figura muestra un cubo OABCDEFG.



Sea O el origen, (OA) el eje x, (OC) el eje y y (OD) el eje z.
Sean M, N y P los puntos medios de [FG], [DG] y [CG], respectivamente.
Las coordenadas de F son $(2, 2, 2)$.

(a) Halle los vectores de posición $\overrightarrow{OM}$, $\overrightarrow{ON}$ y $\overrightarrow{OP}$ en función de sus componentes.

(b) Halle $\overrightarrow{MP} \times \overrightarrow{MN}$.

(c) **A partir de lo anterior**,

(i) calcule el área del triángulo MNP;

(ii) compruebe que la recta (AG) es perpendicular al plano MNP;

(iii) halle la ecuación del plano MNP.

(d) Determine las coordenadas del punto donde la recta (AG) corta al plano MNP.

a) $M = \frac{F+G}{2} = \frac{(2,4,4)}{2} = \boxed{(1,2,2)}$

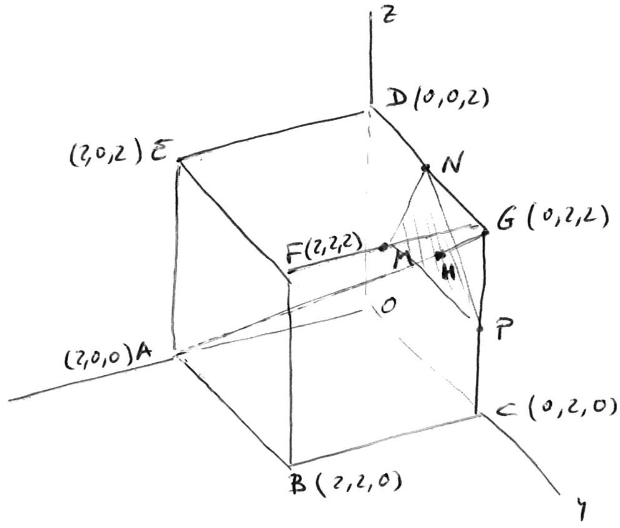
$N = \frac{D+G}{2} = \frac{(0,2,4)}{2} = \boxed{(0,1,2)}$

$P = \frac{C+G}{2} = \frac{(0,4,2)}{2} = \boxed{(0,2,1)}$

$\overrightarrow{MP} = (-1,0,-1)$

$\overrightarrow{MN} = (-1,-1,0)$

b) $\overrightarrow{MP} \times \overrightarrow{MN} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 0 & -1 \\ -1 & -1 & 0 \end{vmatrix} = \boxed{(-1, 1, 1)}$



c) Area $\widehat{MNP} = \dfrac{|\overrightarrow{MP} \times \overrightarrow{MN}|}{2} = \dfrac{\sqrt{1+1+1}}{2} = \boxed{\dfrac{\sqrt{3}}{2}}$

$\overrightarrow{AG} = (-2,2,2)$

$\overrightarrow{AG} \parallel \overrightarrow{MP} \times \overrightarrow{MN} \Rightarrow$ $\boxed{\overrightarrow{AG} \text{ es perpendicular al plano } MNP}$

$-1\cdot(x-1) + 1\cdot(y-2) + 1(z-2) = 0$

$\boxed{-x+y+z-3=0} \equiv \widehat{MNP}$

d) $\overrightarrow{AG}: \left.\begin{array}{l} x = 2-2r \\ y = 2r \\ z = 2r \end{array}\right\}$

$-(2-2r) + 2r + 2r - 3 = 0$

$-2+2r+2r+2r-3=0$

$6r = 5$

$r = \dfrac{5}{6}$ $\rightarrow$ $x = 2 - \dfrac{10}{6} = \dfrac{1}{3}$, $y = \dfrac{10}{6} = \dfrac{5}{3}$, $z = \dfrac{10}{6} = \dfrac{5}{3}$

$\boxed{H\left(\frac{1}{3}, \frac{5}{3}, \frac{5}{3}\right)}$

Mayo 11
P1 TZ1
#11

The points $A(1, 2, 1)$, $B(-3, 1, 4)$, $C(5, -1, 2)$ and $D(5, 3, 7)$ are the vertices of a tetrahedron.

(a) Find the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

(b) Find the Cartesian equation of the plane Π that contains the face ABC.

(c) Find the vector equation of the line that passes through D and is perpendicular to Π. Hence, or otherwise, calculate the shortest distance to D from Π.

(d) (i) Calculate the area of the triangle ABC.

(ii) Calculate the volume of the tetrahedron ABCD.

(e) Determine which of the vertices B or D is closer to its opposite face.

a) $\vec{AB} = (-4,-1,3)$

$\vec{AC} = (4,-3,1)$

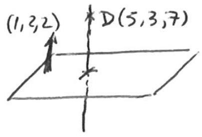
b) $\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -4 & -1 & 3 \\ 4 & -3 & 1 \end{vmatrix} = (8,16,16) \parallel (1,2,2)$

$x+2y+2z+D=0$

$1+4+2+D=0 \rightarrow D=-7$

$ABC \equiv \boxed{x+2y+2z-7=0}$

c) $\boxed{\vec{r} = 5\vec{i}+3\vec{j}+7\vec{k}+r(\vec{i}+2\vec{j}+2\vec{k})}$



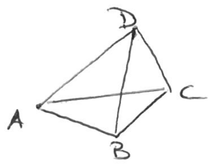
$d = d(D(5,3,7);\ x+2y+2z-7=0) =$

$= \dfrac{|5+6+14-7|}{\sqrt{1+4+4}} = \dfrac{18}{3} = \boxed{6}$

d) $\text{Área } \widehat{ABC} = \dfrac{|\vec{AB}\times\vec{AC}|}{2} = \dfrac{\sqrt{8^2+16^2+16^2}}{2} = \dfrac{24}{2} = \boxed{12}$

$\vec{AD} = (4,1,6)$

$\text{Volumen Tetraedro } ABCD = \dfrac{1}{6}\begin{vmatrix} 4 & 1 & 6 \\ -4 & -1 & 3 \\ 4 & -3 & 1 \end{vmatrix} = \dfrac{144}{6} = \boxed{24}$



e) $d(D, \text{plano } ABC) = 6$ (Apartado c)

$\vec{AC} = (4,-3,1)$

$\vec{AD} = (4,1,6)$

$\begin{vmatrix} x-1 & y-2 & z-1 \\ 4 & -3 & 1 \\ 4 & 1 & 6 \end{vmatrix} = 0$

$-19(x-1)-20(y-2)+16(z-1)=0$

$\text{Plano } ACD \equiv \boxed{-19x-20y+16z+43=0}$

$d(B, \text{plano } ACD) = \dfrac{|-19\cdot(-3)-20\cdot 1+16\cdot 4+43|}{\sqrt{(-19)^2+(-20)^2+16^2}} = \dfrac{144}{3\sqrt{113}} = \dfrac{48}{\sqrt{113}} \simeq 4'52$

$\boxed{\text{B está más cercano que D}}$ de su respectiva cara opuesta.

También:

$\text{Volumen Tetraedro} = 24 = \dfrac{\text{Área } \widehat{ABC}\cdot h_D}{3} = \dfrac{\text{Área } \widehat{ACD}\cdot h_B}{3}$

$\vec{AC}\times\vec{AD} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & -3 & 1 \\ 4 & 1 & 6 \end{vmatrix} = (-19,-20,16)$

$\text{Área } \widehat{ACD} = \dfrac{|(-19,-20,16)|}{2} = \dfrac{\sqrt{113}}{2}$

$24 = \dfrac{12\cdot h_D}{3} = \dfrac{\frac{\sqrt{113}}{2}\cdot h_B}{3}$

$h_D = \dfrac{24\times 3}{12} = 6$ ✓

$h_B = \dfrac{24\times 3\times 2}{3\sqrt{113}} = \dfrac{48}{\sqrt{113}}$ ✓

Nov 11
P2 #13

Dos planos Π_1 y Π_2 tienen por ecuación $2x+y+z=1$ y $3x+y-z=2$, respectivamente.

(a) Halle la ecuación vectorial de L, la recta de intersección de Π_1 y Π_2.

(b) Compruebe que el plano Π_3, que es perpendicular a Π_1 y contiene a L, tiene por ecuación $x-2z=1$.

(c) El punto P tiene coordenadas $(-2, 4, 1)$, el punto Q pertenece a Π_3 y PQ es perpendicular a Π_2. Halle las coordenadas de Q.

a)

$$\left.\begin{array}{l}\Pi_1 \equiv 2x+y+z=1 \\ \Pi_2 \equiv 3x+y-z=2\end{array}\right\} \longrightarrow \left.\begin{array}{l}2x+y=1-z \\ 3x+y=2+z\end{array}\right\}$$

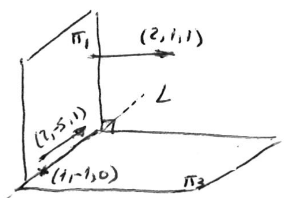
$z \in \mathbb{R}$

$$x = \frac{\begin{vmatrix}1-z & 1 \\ 2+z & 1\end{vmatrix}}{\begin{vmatrix}2 & 1 \\ 3 & 1\end{vmatrix}} = \frac{1-z-2-z}{2-3} = \frac{-1-2z}{-1} = 1+2z$$

$$y = \frac{\begin{vmatrix}2 & 1-z \\ 3 & 2+z\end{vmatrix}}{-1} = \frac{4+2z-3+3z}{-1} = -1-5z$$

$$L \equiv \boxed{\left.\begin{array}{l}x=1+2r \\ y=-1-5r \\ z=r\end{array}\right\}}$$

b)



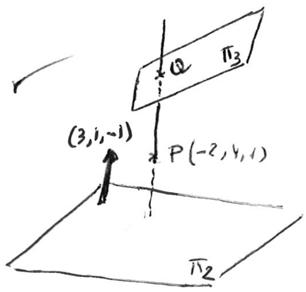
Como 'L' está contenida en Π_1, si fuera que también esté contenida en Π_3, será su intersección

$$\begin{vmatrix}x-1 & y+1 & z \\ 2 & 1 & 1 \\ 2 & -5 & 1\end{vmatrix} = 0$$

$$6(x-1)+0(y+1)-12z=0$$

$$6x-6-12z=0 \longrightarrow \Pi_3 \equiv \boxed{x-2z=1}$$

c)



recta PQ

$$\left.\begin{array}{l}x=-2+3r \\ y=4+r \\ z=1-r\end{array}\right\}$$

Punto Q

$$\left.\begin{array}{l}\Pi_3 \equiv x-2z=1 \\ PQ \equiv \left\{\begin{array}{l}x=-2+3r \\ y=4+r \\ z=1-r\end{array}\right.\end{array}\right\} \longrightarrow \begin{array}{l}-2+3r-2(1-r)=1 \\ -2+3r-2+2r=1 \\ 5r=5\end{array}$$

$$r=1 \longrightarrow \begin{cases}x=-2+3=1 \\ y=4+1=5 \\ z=1-1=0\end{cases} \qquad \boxed{Q(1,5,0)}$$

Mayo 12
P2 TZ1
#13

The coordinates of points A, B and C are given as $(5, -2, 5)$, $(5, 4, -1)$ and $(-1, -2, -1)$ respectively.

(a) Show that $AB = AC$ and that $B\hat{A}C = 60°$.

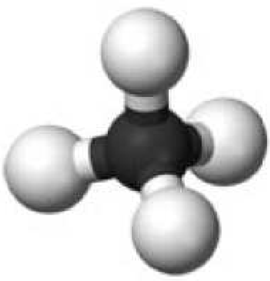
(b) Find the Cartesian equation of Π, the plane passing through A, B, and C.

(c) (i) Find the Cartesian equation of Π_1, the plane perpendicular to (AB) passing through the midpoint of [AB].

(ii) Find the Cartesian equation of Π_2, the plane perpendicular to (AC) passing through the midpoint of [AC].

(d) Find the vector equation of L, the line of intersection of Π_1 and Π_2, and show that it is perpendicular to Π.

A methane molecule consists of a carbon atom with four hydrogen atoms symmetrically placed around it in three dimensions.



The positions of the centres of three of the hydrogen atoms are A, B and C as given. The position of the centre of the fourth hydrogen atom is D.

(e) Using the fact that $AB = AD$, show that the coordinates of one of the possible positions of the fourth hydrogen atom is $(-1, 4, 5)$.

(f) Letting D be $(-1, 4, 5)$, show that the coordinates of G, the position of the centre of the carbon atom, are $(2, 1, 2)$. Hence calculate $D\hat{G}A$, the bonding angle of carbon.

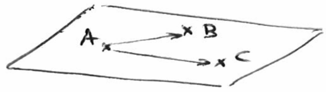
a) $\vec{AB} = (0, 6, -6) \rightarrow AB = \sqrt{0+36+36} = 6\sqrt{2}$ ✓

$\vec{AC} = (-6, 0, -6) \rightarrow AC = \sqrt{36+0+36} = 6\sqrt{2}$

$\cos B\hat{A}C = \dfrac{\vec{AB}\cdot\vec{AC}}{|\vec{AB}|\,|\vec{AC}|} = \dfrac{0+0+36}{6\sqrt{2}\cdot 6\sqrt{2}} = \dfrac{\cancel{36}}{\cancel{36}\cdot 2} = \dfrac{1}{2} \Rightarrow B\hat{A}C = 60°$ ✓

b)

$$\begin{vmatrix} x-5 & y+2 & z-5 \\ 0 & 6 & -6 \\ -6 & 0 & -6 \end{vmatrix} = 0$$

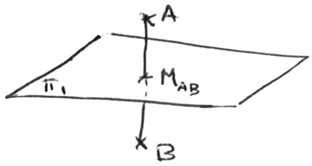


$$-36(x-5) + 36(y+2) + 36(z-5) = 0$$

$$-(x-5) + (y+2) + (z-5) = 0 \longrightarrow \boxed{-x+y+z+2=0} \equiv \pi$$

c)

$$M_{AB} = \frac{A+B}{2} = (5,1,2)$$

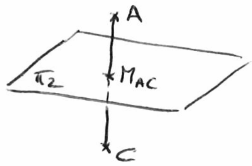


$$\vec{AB} = (0,6,-6)$$

$$0\cdot(x-5) + 6(y-1) - 6(z-2) = 0$$

$$(y-1) - (z-2) = 0 \longrightarrow \boxed{y-z+1=0} \equiv \pi_1$$

$$M_{AC} = \frac{A+C}{2} = (2,-2,2)$$



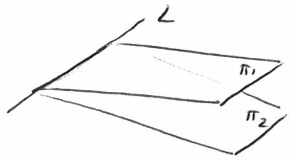
$$\vec{AC} = (-6,0,-6)$$

$$-6(x-2) + 0\cdot(y+2) - 6(z-2) = 0$$

$$+(x-2) + (z-2) = 0 \longrightarrow \boxed{x+z-4=0} \equiv \pi_2$$

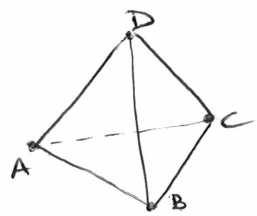
d)

$$\left.\begin{array}{l} y-z+1=0 \\ x+z-4=0 \end{array}\right\} \longrightarrow \left.\begin{array}{l} y=-1+z \\ x=4-z \end{array}\right\} \quad \boxed{\left.\begin{array}{l} x=4-r \\ y=-1+r \\ z=r \end{array}\right\} \equiv L}$$



$$L \equiv \vec{r} = 4\vec{i} - \vec{j} + r(-\vec{i}+\vec{j}+\vec{k})$$

$$\left.\begin{array}{l} \pi \equiv -x+y+z+2=0 \\ L \parallel -\vec{i}+\vec{j}+\vec{k} \end{array}\right| \Rightarrow L \perp \pi \quad ✓$$



e)

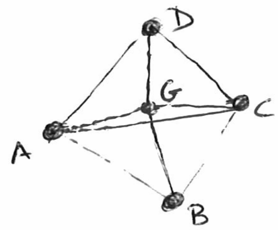
$$D \in L \Rightarrow D = (4-r, -1+r, r)$$

$$\vec{AD} = (-1-r, 1+r, -5+r)$$

$$|\vec{AD}| = |\vec{AB}| \Rightarrow \sqrt{(-1-r)^2 + (1+r)^2 + (-5+r)^2} = 6\sqrt{2};$$

$$1+2r+r^2+1+2r+r^2+25-10r+r^2 = 72; \quad 3r^2-6r-45=0; \quad r = \begin{cases} 5 \Rightarrow D(-1,4,5) \ ✓ \\ -3 \Rightarrow D=(7,-4,-3) \end{cases}$$

f) G será equidistante de A, B, C y D:



$$\vec{AG} = (-3,3,-3) \longrightarrow |\vec{AG}| = \sqrt{27}$$

$$\vec{BG} = (-3,3,3) \longrightarrow |\vec{BG}| = \sqrt{27}$$

$$\vec{CG} = (3,3,3) \longrightarrow |\vec{CG}| = \sqrt{27}$$

$$\vec{DG} = (3,-3,-3) \longrightarrow |\vec{DG}| = \sqrt{27} \quad ✓$$

__También__: G será el centro de masas (baricentro) del tetraedro ABCD:

$$G = \frac{A+B+C+D}{4} = \left(\frac{5+5-1-1}{4}, \frac{-2+4-2+4}{4}, \frac{5-1-1+5}{4}\right) = (2,1,2) \quad ✓$$

$$\cos D\hat{G}A = \frac{\vec{GD}\cdot\vec{GA}}{GD\cdot GA} = \frac{(-3,3,3)\cdot(3,-3,3)}{\sqrt{27}\cdot\sqrt{27}} = \frac{-9}{27} = -\frac{1}{3} \rightarrow \boxed{D\hat{G}A \simeq 109'47^\circ}$$