# Actividades de Rectas Tangente y Normal en exámenes BI-NS

**Nov 00**

For what values of $m$ is the line $y = mx + 5$ a tangent to the parabola $y = 4 - x^2$ ?

**Mayo 01**

Find the gradient of the tangent to the curve $3x^2 + 4y^2 = 7$ at the point where $x = 1$ and $y > 0$ .

**Nov 01**

The line $y = 16x - 9$ is a tangent to the curve $y = 2x^3 + ax^2 + bx - 9$ at the point $(1, 7)$

Find the values of $a$ and $b$ .

**Nov 01**

Consider the tangent to the curve $y = x^3 + 4x^2 + x - 6$ .

(a) Find the equation of this tangent at the point where $x = -1$ .

(b) Find the coordinates of the point where this tangent meets the curve again.

**Mayo 02**

La ecuación de una curva es $xy^3 + 2x^2y = 3$ . Halle la ecuación de la tangente a esta curva en el punto $(1 , 1)$ .

**Nov 02**

La tangente a la curva $y = f(x)$ en el punto $P(x, y)$ corta al eje de las $x$ en $Q(x-1, 0)$ .
La curva corta al eje de las $y$ en $R(0, 2)$ . Halle la ecuación de la curva.

**Mayo 03**

Una curva tiene la ecuación $x^3y^2 = 8$ . Halle la ecuación de la normal a la curva en el punto $(2, 1)$ .

**Mayo 04**

The point $P(1, p)$, where $p > 0$, lies on the curve $2y^2 - x^3y = 15$

(a) Calculate the value of $p$.

(b) Calculate the gradient of the tangent to the curve at P.

**Mayo 04**

El punto $P(1, p)$, con $p > 0$, pertenece a la curva $2x^2y + 3y^2 = 16$ .

(a) Calcule el valor de $p$.

(b) Calcule la pendiente de la tangente a la curva en P.

**Nov 04**

Halle la ecuación de la normal a la curva $x^3 + y^3 - 9xy = 0$ en el punto (2, 4).

**Mayo 05**

Find the gradient of the curve $2\sin(xy) = 1$ when $y = \frac{1}{2}$ and $\pi < x < 2\pi$

**Mayo 05**

La normal a la curva $y = \frac{k}{x} + \ln x^2$, para $x \neq 0$, $k \in \mathbb{R}$, en el punto $x = 2$, tiene por ecuación $3x + 2y = b$, donde $b \in \mathbb{R}$. Halle el valor **exacto** de $k$.

**Nov 05**

A circle has equation $x^2 + (y-2)^2 = 1$ . The line with equation $y = kx$ , where $k \in \mathbb{R}$ , is a tangent to the circle. Find all possible values of $k$.

**Nov 06**

Let $f$ be the function defined for $x > -\frac{1}{3}$ by $f(x) = \ln(3x+1)$ .

(a) Find $f'(x)$ .

(b) Find the equation of the normal to the curve $y = f(x)$ at the point where $x = 2$
Give your answer in the form $y = ax + b$ where $a, b \in \mathbb{R}$ .

**Nov 06**

Consider the curves $C_1$, $C_2$ with equations

$$C_1: y = x^2 + kx + k \text{ , where } k < 0 \text{ is a constant}$$
$$C_2: y = -x^2 + 2x - 4 \text{ .}$$

Both curves pass through the point P and the tangent at P to one of the curves is also a tangent at P to the other curve.

(a) Find the value of $k$.

(b) Find the coordinates of P.

**Nov 07**

(a) A curve is defined by the implicit equation $2xy + 6x^2 - 3y^2 = 6$.

Show that the tangent at the point A with coordinates $\left(1, \frac{2}{3}\right)$ has gradient $\frac{20}{3}$.

(b) The line $x = 1$ cuts the curve at point A, with coordinates $\left(1, \frac{2}{3}\right)$, and at point B.

Find, in the form $\boldsymbol{r} = \begin{pmatrix} a \\ b \end{pmatrix} + s\begin{pmatrix} c \\ d \end{pmatrix}$

(i) the equation of the tangent at A;

(ii) the equation of the normal at B.

(c) Find the acute angle between the tangent at A and the normal at B.

**Mayo 08**

Find the gradient of the tangent to the curve $x^3y^2 = \cos(\pi y)$ at the point $(-1, 1)$.

**Mayo 08**

A normal to the graph of $y = \arctan(x-1)$, for $x > 0$, has equation $y = -2x + c$, where $c \in \mathbb{R}$.

Find the value of $c$.

**Mayo 08**

Consider the curve with equation $x^2 + xy + y^2 = 3$.

(a) Find in terms of $k$, the gradient of the curve at the point $(-1, k)$.

(b) Given that the tangent to the curve is parallel to the $x$-axis at this point, find the value of $k$.

**Muestra 08**

A curve $C$ is defined implicitly by $xe^y = x^2 + y^2$. Find the equation of the tangent to $C$ at the point $(1, 0)$.

**Muestra 06/08** Find the gradient of the normal to the curve $3x^2y+2xy^2=2$ at the point $(1,-2)$

**Nov 08** Find the equation of the normal to the curve $5xy^2-2x^2=18$ at the point $(1, 2)$

**Mayo 09** Consider the functions $f$ and $g$ defined by $f(x)=2^{\frac{1}{x}}$ and $g(x)=4-2^{\frac{1}{x}}$, $x \neq 0$.

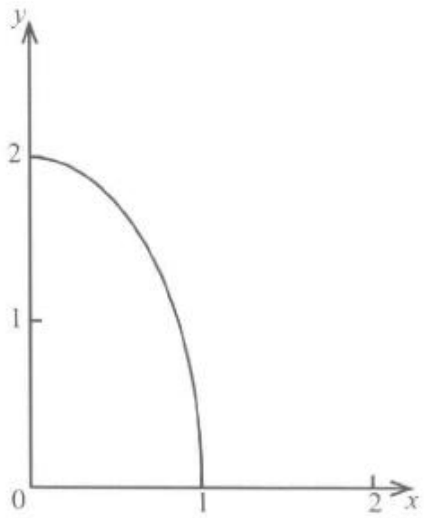
(a) Find the coordinates of P, the point of intersection of the graphs of $f$ and $g$.

(b) Find the equation of the tangent to the graph of $f$ at the point P.

**Mayo 09** (a) Derive $f(x)=\operatorname{arcsen} x+2\sqrt{1-x^2}$, $x \in [-1, 1]$.

(b) Halle las coordenadas del punto perteneciente a la gráfica de $y=f(x)$ en $[-1, 1]$, en que la pendiente de la tangente a la curva es igual a cero.

**Mayo 09** Considere la parte de la curva $4x^2+y^2=4$ que se muestra en la siguiente figura.




(a) Halle una expresión para $\frac{dy}{dx}$ en función de $x$ y de $y$.

(b) Halle la pendiente de la tangente en el punto $\left(\frac{2}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right)$.

**N09 P1#12** A tangent to the graph of $y=\ln x$ passes through the origin.

(a) Sketch the graphs of $y=\ln x$ and the tangent on the same set of axes, and hence find the equation of the tangent.

(b) Use your sketch to explain why $\ln x \leq \frac{x}{e}$ for $x>0$.

(c) Show that $x^e \leq e^x$ for $x>0$.

(d) Determine which is larger, $\pi^e$ or $e^\pi$.

**N09 P2#8**
Find the gradient of the curve $e^{xy}+\ln(y^2)+e^y=1+e$ at the point $(0,1)$.

**M10 TZ2 P1#8**
The normal to the curve $xe^{-y}+e^y=1+x$, at the point $(c,\ln c)$, has a $y$-intercept $c^2+1$.

Determine the value of $c$.

**N10 P2#4**
Halle la ecuación de la recta normal a la curva $x^3y^3-xy=0$ en el punto $(1,1)$.

**N10 P2#10**
La recta $y=m(x-m)$ es tangente a la curva $(1-x)y=1$.

Determine $m$ y las coordenadas del punto donde la tangente toca a la curva.

**M11 TZ1 P1#9**
Show that the points $(0,0)$ and $\left(\sqrt{2\pi},-\sqrt{2\pi}\right)$ on the curve $e^{(x+y)}=\cos(xy)$ have a common tangent.

**M11 TZ2 P1#11**
The curve $C$ has equation $y=\frac{1}{8}(9+8x^2-x^4)$.

(a) Find the coordinates of the points on $C$ at which $\frac{dy}{dx}=0$.

(b) The tangent to $C$ at the point $P(1,2)$ cuts the $x$-axis at the point T. Determine the coordinates of T.

(c) The normal to $C$ at the point P cuts the $y$-axis at the point N. Find the area of triangle PTN.

**M11 TZ2 P2#7**
Consider the functions $f(x)=x^3+1$ and $g(x)=\frac{1}{x^3+1}$. The graphs of $y=f(x)$ and $y=g(x)$ meet at the point $(0,1)$ and one other point, P.

(a) Find the coordinates of P.

(b) Calculate the size of the acute angle between the tangents to the two graphs at the point P.

**M11 TZ2 P2#10**
The point P, with coordinates $(p,q)$, lies on the graph of $x^{\frac{1}{2}}+y^{\frac{1}{2}}=a^{\frac{1}{2}}$, $a>0$. The tangent to the curve at P cuts the axes at $(0,m)$ and $(n,0)$. Show that $m+n=a$.

**M12 TZ1 P1#9**
The curve $C$ has equation $2x^2+y^2=18$. Determine the coordinates of the four points on $C$ at which the normal passes through the point $(1,0)$.

**N12 P1#8**
Consider the curve defined by the equation $x^2+\sin y-xy=0$.

(a) Find the gradient of the tangent to the curve at the point $(\pi,\pi)$.

(b) Hence, show that $\tan\theta=\frac{1}{1+2\pi}$, where $\theta$ is the acute angle between the tangent to the curve at $(\pi,\pi)$ and the line $y=x$.

**M13 TZ1 P1#7** A curve is defined by the equation $8y\ln x - 2x^2 + 4y^2 = 7$. Find the equation of the tangent to the curve at the point where $x = 1$ and $y > 0$.

**M13 TZ2 P1#5** La curva $C$ viene dada por $y = \dfrac{x\cos x}{x + \cos x}$, para $x \geq 0$.

(a) Compruebe que $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\cos^2 x - x^2 \operatorname{sen} x}{(x + \cos x)^2}$, $x \geq 0$.

(b) Halle la ecuación de la recta tangente a $C$ en el punto $\left(\dfrac{\pi}{2}, 0\right)$.

**N13 P2#13a** Una función $f$ viene dada por $f(x) = \dfrac{1}{2}\left(\mathrm{e}^x + \mathrm{e}^{-x}\right)$, $x \in \mathbb{R}$.

(a) (i) Explique por qué no existe la función inversa $f^{-1}$.

(ii) Compruebe que la ecuación de la normal a la curva en el punto P, donde $x = \ln 3$, viene dada por $9x + 12y - 9\ln 3 - 20 = 0$.

(iii) Halle las coordenadas $x$ de los puntos Q y R pertenecientes a la curva, tales que las tangentes en Q y R pasan por $(0, 0)$.

**M13 TZ1 P2#10** Let $f(x) = \dfrac{\mathrm{e}^{2x} + 1}{\mathrm{e}^x - 2}$.

(a) Find the equations of the horizontal and vertical asymptotes of the curve $y = f(x)$.

(b) (i) Find $f'(x)$.

(ii) Show that the curve has exactly one point where its tangent is horizontal.

(iii) Find the coordinates of this point.

(c) Find the equation of $L_1$, the normal to the curve at the point where it crosses the $y$-axis.

The line $L_2$ is parallel to $L_1$ and tangent to the curve $y = f(x)$.

(d) Find the equation of the line $L_2$.

**M14 TZ2 P2#10** Considere la curva definida por la ecuación $\left(x^2 + y^2\right)^2 = 4xy^2$.

(a) Utilice la derivación implícita para hallar una expresión para $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

(b) Halle la ecuación de la recta normal a la curva en el punto $(1, 1)$.

**M15 TZ2 P2#11**

Una curva se define mediante $x^2 - 5xy + y^2 = 7$.

(a) Muestre que $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{5y-2x}{2y-5x}$.

(b) Halle la ecuación de la normal a la curva en el punto $(6, 1)$.

(c) Halle la distancia que hay entre los dos puntos de la curva en los cuales la tangente correspondiente es paralela a la recta $y = x$.

**N15 P1#4**

Consider the curve $y = \frac{1}{1-x}$, $x \in \mathbb{R}$, $x \neq 1$.

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

(b) Determine the equation of the normal to the curve at the point $x = 3$ in the form $ax + by + c = 0$ where $a, b, c \in \mathbb{Z}$.

**N15 P1#7**

A curve is defined by $xy = y^2 + 4$.

(a) Show that there is no point where the tangent to the curve is horizontal.

(b) Find the coordinates of the points where the tangent to the curve is vertical.

**M16 TZ1 P1#10**

Find the $x$-coordinates of all the points on the curve $y = 2x^4 + 6x^3 + \frac{7}{2}x^2 - 5x + \frac{3}{2}$ at which the tangent to the curve is parallel to the tangent at $(-1, 6)$.

**M16 TZ1 P2#11**

Consider the curve, $C$ defined by the equation $y^2 - 2xy = 5 - e^x$. The point $\mathrm{A}$ lies on $C$ and has coordinates $(0, a)$, $a > 0$.

(a) Find the value of $a$.

(b) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2y - e^x}{2(y-x)}$.

(c) Find the equation of the normal to $C$ at the point $\mathrm{A}$.

(d) Find the coordinates of the second point at which the normal found in part (c) intersects $C$.

(e) Given that $v = y^3$, $y > 0$, find $\frac{\mathrm{d}v}{\mathrm{d}x}$ at $x = 0$.

**M16 TZ2 P2#7**

Considere la curva que viene dada por la ecuación $x^3 + y^3 = 4xy$.

(a) Utilice la derivación implícita para mostrar que $\frac{dy}{dx} = \frac{4y - 3x^2}{3y^2 - 4x}$.

La tangente a esta curva es paralela al eje $x$ en el punto donde $x = k$, $k > 0$.

(b) Halle el valor de $k$.

**N16 P1#9**

Una curva viene dada por la ecuación $3x - 2y^2e^{x-1} = 2$.

(a) Halle una expresión para $\frac{dy}{dx}$ en función de $x$ e $y$.

(b) Halle las ecuaciones de las tangentes a esta curva en aquellos puntos donde la curva corta a la recta $x = 1$.