N02

$V = \frac{4}{3}\pi R^3$

$\frac{dV}{dt} = \frac{4}{3}\pi \cdot 3R^2 \frac{dR}{dt} \rightarrow \frac{dR}{dt} = \frac{1}{4\pi R^2}\frac{dV}{dt} = \frac{1}{4\pi R^2}\cdot 8 = \frac{2}{\pi R^2}$

$R = 2\ cm \rightarrow \frac{dR}{dt} = \frac{2}{\pi \cdot 2^2} = \boxed{0'159\ cm/sg}$

N03

$Tg\theta = \frac{3}{x}\ ;\ x = \frac{3}{Tg\theta}$

$\frac{dx}{dt} = \frac{-3\sec^2\theta}{Tg^2\theta}\frac{d\theta}{dt} = \frac{-3}{\sin^2\theta}\frac{d\theta}{dt} = \frac{-3}{\sin^2\theta}\cdot\frac{1}{60} = \frac{-1}{20\sin^2\theta}$

$\theta = \frac{\pi}{3} \rightarrow \frac{dx}{dt} = \frac{-1}{20\cdot\sin^2\frac{\theta}{3}} = \frac{-1}{20\left(\frac{\sqrt{3}}{2}\right)^2} = -\frac{1}{15}\ Km/sg =$

$= -\frac{1}{15}\frac{Km}{sg}\cdot\frac{3600\,sg}{1\,h} = -240\ Km/h$

La velocidad a la que se aleja es de $\boxed{240\ Km/h}$

M04

Area Sector $= \pi R^2 \cdot \frac{\theta}{2\pi} = \frac{R^2\theta}{2}$

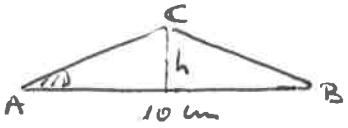
$\frac{dA}{dt} = \frac{R^2}{2}\frac{d\theta}{dt} + \frac{\theta}{2}2R\frac{dR}{dt} = \frac{R^2}{2}\frac{d\theta}{dt} + \theta R\frac{dR}{dt} = \frac{R^2}{2}\cdot 0'1 + \theta R\cdot 2 =$

$= \frac{R^2}{20} + 2\theta R$

$\left.\begin{matrix} R=3 \\ \theta=\pi/4 \end{matrix}\right| \rightarrow \frac{dA}{dt} = \frac{3^2}{20} + 2\cdot\frac{\pi}{4}\cdot 3 = \boxed{5'16\ cm^2/sg}$

M04

$tg\,\hat{A} = \frac{h}{5}\ ;\ \hat{A} = arcTg\frac{h}{5}$

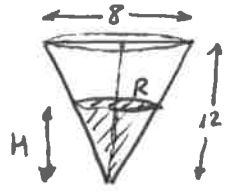


$\frac{d\hat{A}}{dt} = \frac{1}{1+\left(\frac{h}{5}\right)^2}\cdot\frac{1}{5}\cdot\frac{dh}{dt} =$

$= \frac{1}{1+\frac{h^2}{25}}\cdot\frac{1}{5}\cdot 2 = \frac{10}{25+h^2}$

Triángulo Equilátero $\Rightarrow h = \sqrt{10^2-5^2} = \sqrt{75} \rightarrow \frac{d\hat{A}}{dt} = \frac{10}{25+(\sqrt{75})^2} = \boxed{0'1\ rad/sg}$

M05



$V = \frac{\pi R^2 H}{3} \rightarrow V = \frac{\pi\left(\frac{H}{3}\right)^2 H}{3} = \frac{\pi H^3}{27}$

$\frac{4}{12} = \frac{R}{H} \rightarrow R = \frac{H}{3}$

$\frac{dV}{dt} = \frac{\pi}{27}\cdot 3H^2\frac{dH}{dt} \rightarrow \frac{dH}{dt} = \frac{9}{\pi H^2}\frac{dV}{dt} = \frac{9}{\pi H^2}\cdot 10 = \frac{90}{\pi H^2}$

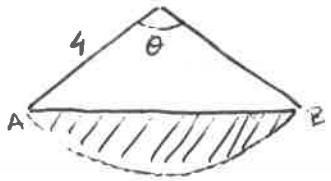
$H = 6\ m \rightarrow \frac{dH}{dt} = \frac{90}{\pi\cdot 6^2} = \boxed{0'796\ m/min}$

N05

$\text{Arco } AB = 2\pi R \frac{\theta}{2\pi} = R\theta = 4\theta$

$\text{Área sombreada} = \text{Área Sector} - \text{Área Triángulo} = \pi R^2 \frac{\theta}{2\pi} - \frac{4^2 \operatorname{sen}\theta}{2} = \frac{R^2\theta}{2} - 8\operatorname{sen}\theta =$

$= 8\theta - 8\operatorname{sen}\theta$



$\frac{d\text{Arco}}{dt} = 4\frac{d\theta}{dt}$

$\frac{d\text{Área}}{dt} = (8 - 8\cos\theta)\frac{d\theta}{dt}$

$\frac{d\text{Arco}}{dt} = \frac{d\text{Área}}{dt} \Rightarrow 4\frac{d\theta}{dt} = (8-8\cos\theta)\frac{d\theta}{dt}$; $8\cos\theta = 4$; $\cos\theta = \frac{1}{2}$; $\boxed{\theta = \frac{\pi}{3}\ \text{rad}}$

N06

$\text{Área Lateral} = 2\pi x \cdot x = 2\pi x^2$

$\frac{d\text{Área Lateral}}{dt} = 2\pi \cdot 2x \frac{dx}{dt}$;

$\frac{dx}{dt} = \frac{1}{4\pi x}\frac{d\text{Área Lateral}}{dt} = \frac{1}{4\pi x} \cdot 10 = \frac{5}{2\pi x}$

a) $x = 2 \rightarrow \frac{dx}{dt} = \frac{5}{4\pi} = \boxed{0'398\ \text{cm/sg}}$

b) $V = \pi x^2 \cdot x = \pi x^3$

$\frac{dV}{dt} = 3\pi x^2 \frac{dx}{dt}$

$x = 2 \rightarrow \frac{dV}{dt} = 2\pi \cdot 3^2 \cdot 0'398 = \boxed{22'5\ \text{cm}^3/\text{sg}}$

N07

$V = \frac{4}{3}\pi R^3 + \pi R^2 H$

$\frac{dV}{dt} = \frac{4}{3}\pi 3R^2 \frac{dR}{dt} + \pi \cdot 2RH\frac{dR}{dt} + \pi R^2 \frac{dH}{dt}$

$\frac{dH}{dt} = \frac{1}{\pi R^2}\left(\frac{dV}{dt} - 4\pi R^2 \frac{dR}{dt} - 2\pi RH\frac{dR}{dt}\right) =$

$= \frac{1}{\pi R^2}\left(20\pi - 4\pi R^2 \cdot 2 - 2\pi RH \cdot 2\right) = \frac{20 - 8R^2 - 4RH}{R^2}$

$R = 3,\ V = 81\pi \rightarrow 81\pi = \frac{4}{3}\pi \cdot 3^3 + \pi \cdot 3^2 \cdot H$

$81 = 36 + 9H$; $H = 5$

$R = 3,\ H = 5 \rightarrow \frac{dH}{dt} = \frac{20 - 8\cdot 3^2 - 4\cdot 3\cdot 5}{3^2} = \boxed{8\ \text{cm/min}}$

Otra forma: $H = \frac{V - \frac{4}{3}\pi R^3}{\pi R^2} = \frac{V}{\pi R^2} - \frac{4R}{3} = \frac{1}{\pi}\frac{V}{R^2} - \frac{4R}{3}$

$\frac{dH}{dt} = \frac{1}{\pi}\frac{R^2\frac{dV}{dt} - V\cdot 2R\frac{dR}{dt}}{R^4} - \frac{4}{3}\frac{dR}{dt} = \frac{R\frac{dV}{dt} - 2V\frac{dR}{dt}}{\pi R^3} - \frac{4}{3}\frac{dR}{dt}$

$R = 3,\ V = 81\pi \rightarrow \frac{dH}{dt} = \frac{3\cdot 20\pi - 2\cdot 81\pi\cdot 2}{\pi 3^3} - \frac{4}{3}\cdot 2 = \frac{32}{3} - \frac{8}{3} = 8\ \text{cm/min}$ ✓

M07

$z = \sqrt{x^2+y^2}$

$$\frac{dz}{dt} = \frac{1}{2\sqrt{x^2+y^2}} \cdot \left(2x\frac{dx}{dt} + 2y\frac{dy}{dt}\right) = \frac{1}{\sqrt{x^2+y^2}}\left(x\frac{dx}{dt} + y\frac{dy}{dt}\right) =$$

$$= \frac{1}{\sqrt{x^2+y^2}}\left(x\cdot(-60) + y\cdot 70\right) = \frac{70y-60x}{\sqrt{x^2+y^2}}$$

$$\left.\begin{matrix} x = 0'8 \\ y = 0'6 \end{matrix}\right| \rightarrow \frac{dz}{dt} = \frac{70\cdot 0'6 - 60\cdot 0'8}{\sqrt{0'8^2+0'6^2}} = -90 \text{ Km/h}$$

Los coches están separados por una distancia que disminuye a una tasa de 90 Km/h

M09

$$tg\,\hat{P} = \frac{h}{40} \; ; \; \hat{P} = \text{arc}tg\,\frac{h}{40}$$

$$\frac{d\hat{P}}{dt} = \frac{1}{1+\left(\frac{h}{40}\right)^2}\cdot\frac{1}{40}\cdot\frac{dh}{dt} = \frac{40}{1600+h^2}\cdot 10 = \frac{400}{1600+h^2}$$

a) $h=30 \rightarrow \frac{d\hat{P}}{dt} = \frac{400}{1600+30^2} = 0'16$ rad/sg ✓

b) $\overline{PH} = \sqrt{h^2+40^2} = \sqrt{h^2+1600}$

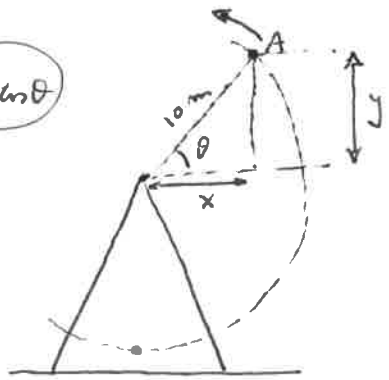
$$\frac{d\overline{PH}}{dt} = \frac{1}{2\sqrt{h^2+1600}}\cdot 2h\cdot\frac{dh}{dt} = \frac{10h}{\sqrt{h^2+1600}}$$

$$h=30 \rightarrow \frac{d\overline{PH}}{dt} = \frac{10\cdot 30}{\sqrt{30^2+1600}} = \boxed{6 \text{ m/sg}}$$

N09

$\sin\theta = \frac{y}{10}$; $y = 10\sin\theta$

$$\frac{dy}{dt} = 10\cos\theta\,\frac{d\theta}{dt} = 10\cos\theta\cdot 3 = 30\cos\theta$$



a) $y = 6\text{ m} \Rightarrow \sin\theta = \frac{6}{10} = 0'6$; $\cos\theta = \sqrt{1-0'6^2} = 0'8$

$$y=6 \rightarrow \frac{dy}{dt} = 30\cdot 0'8 = \boxed{24 \text{ m/min}}$$

Otra forma

$\theta = \text{arc}\sin\left(\frac{y}{10}\right)$

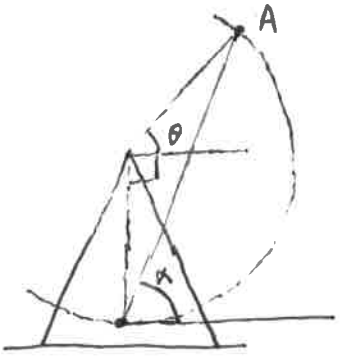
$$\frac{d\theta}{dt} = \frac{1}{\sqrt{1-\left(\frac{y}{10}\right)^2}}\cdot\frac{1}{10}\frac{dy}{dt} = \frac{1}{\sqrt{100-y^2}}\frac{dy}{dt} \; ; \; \frac{dy}{dt} = \sqrt{100-y^2}\cdot\frac{d\theta}{dt} = 3\sqrt{100-y^2}$$

$$y=6 \rightarrow \frac{dy}{dt} = 3\sqrt{100-6^2} = 24 \text{ m/min} \checkmark$$

b) α es un ángulo semi-inscrito, será entonces mitad del ángulo central:

$$\alpha = \frac{\theta+\pi/2}{2} = \frac{\theta}{2}+\frac{\pi}{4}$$

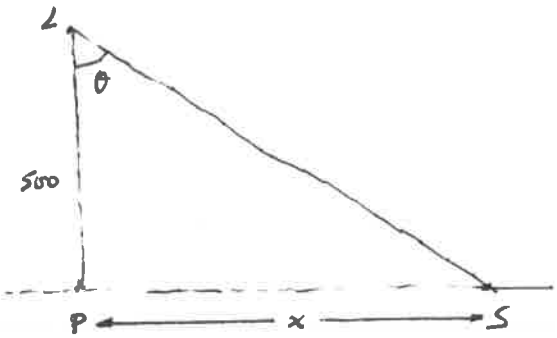
$$\frac{d\alpha}{dt} = \frac{1}{2}\frac{d\theta}{dt} = \frac{1}{2}\cdot 3 = \boxed{1'5 \text{ rad/sg}}$$



M10

$\operatorname{tg}\theta = \frac{x}{500}$

$\theta = \operatorname{arctg}\left(\frac{x}{500}\right)$



a) $\frac{d\theta}{dt} = \frac{1}{1+\left(\frac{x}{500}\right)^2} \cdot \frac{1}{500} \cdot \frac{dx}{dt}$;

$\frac{dx}{dt} = \frac{500^2+x^2}{500} \quad \frac{d\theta}{dt} = \frac{500^2+x^2}{500} \cdot 8\pi$

$x = 2000 \rightarrow \quad \frac{dx}{dt} = \frac{500^2+2000^2}{500} \cdot 8\pi = 213628{,}3$

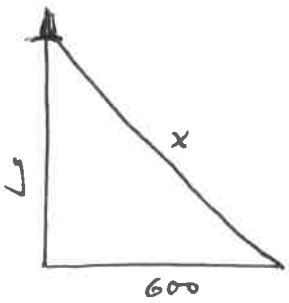
$\frac{dx}{dt} \simeq 214000 \text{ m/min}$ ✓

b) $\frac{d^2x}{dt^2} = \frac{8\pi}{500} \cdot 2x \cdot \frac{dx}{dt} = \frac{4x}{125}\frac{dx}{dt} = \frac{4x}{125} \cdot \frac{500^2+x^2}{500} \cdot 8\pi$

$x = 2000 \rightarrow \quad \frac{d^2x}{dt^2} = \boxed{13672211 \text{ m/min}^2}$

M11

$x = \sqrt{y^2+600^2}$



$\frac{dx}{dt} = \frac{1}{2\sqrt{y^2+600^2}} \cdot 2y \frac{dy}{dt} =$

$= \frac{y}{\sqrt{y^2+600^2}} \cdot 300 = \frac{300\,y}{\sqrt{y^2+600^2}}$

$y = 800 \text{ m} \rightarrow \quad \frac{dx}{dt} = \frac{300 \cdot 800}{\sqrt{800^2+600^2}} = \boxed{240 \text{ m/sg}}$

N11

$V = \pi R^2 H$

$\frac{dV}{dt} = 2\pi R H \frac{dR}{dt} + \pi R^2 \frac{dH}{dt} =$

$= 2\pi R H \cdot (-0'5) + \pi R^2 \cdot 3 =$

$= -\pi R H + 3\pi R^2 = \pi R (3R - H)$

Si $R > \frac{H}{3}$, estará aumentando de volumen.

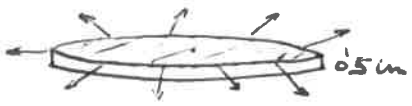
Si $R < \frac{H}{3}$, " disminuyendo de volumen.

$R = 40 \text{ mm}$
$H = 200 \text{ mm}$ $\rightarrow \quad \frac{dV}{dt} = \pi \cdot 40 \cdot (3 \cdot 40 - 200) = -10053{,}1 \text{ mm}^3/\text{sglo}$

El volumen estará disminuyendo a un ritmo de $\boxed{10053 \text{ mm}^3/\text{sglo}}$

M13

$V = \pi R^2 H = 0'5\pi R^2$



$\frac{dV}{dt} = 0'5\pi \cdot 2R \frac{dR}{dt} = \pi R \frac{dR}{dt}$

$\frac{dR}{dt} = \frac{1}{\pi R}\frac{dV}{dt} = \frac{4}{\pi R}$

$R = 20\,m \rightarrow \frac{dR}{dt} = \frac{4}{20\pi} = \frac{1}{5\pi} = \boxed{0'0637\ m/sg}$

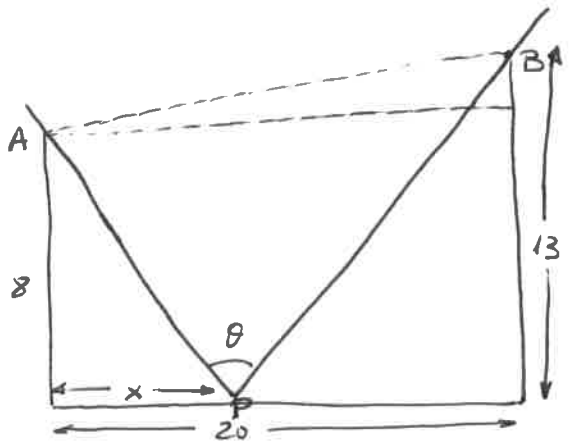
M13

a)

$\overline{AB} = \sqrt{20^2 + (13-8)^2} = \sqrt{425}$

$\overline{AP} = \sqrt{x^2 + 8^2} = \sqrt{x^2+64}$

$\overline{BP} = \sqrt{(20-x)^2 + 13^2} = \sqrt{x^2 - 40x + 569}$



$\cos\theta = \frac{-\overline{AB}^2 + \overline{AP}^2 + \overline{BP}^2}{2\,\overline{AP}\cdot\overline{BP}}$

$\cos\theta = \frac{-425 + (x^2+64) + (x^2-40x+569)}{2\sqrt{x^2+64}\cdot\sqrt{x^2-40x+569}} =$

$= \frac{2x^2 - 40x + 208}{2\sqrt{x^2+64}\sqrt{x^2-40x+569}} = \frac{x^2 - 20x + 104}{\sqrt{x^2+64}\sqrt{x^2-40x+569}} \rightarrow \boxed{\theta = \arccos \frac{x^2-20x+104}{\sqrt{x^2+64}\sqrt{x^2-40x+569}}}$

b) $x=0 \rightarrow \theta = \arccos \frac{104}{\sqrt{64}\sqrt{569}} = \boxed{0'994\ rad}$ También: $\theta = \frac{\pi}{2} - \text{arctg}\frac{13}{20} = 0'994$ ✓

$x=20 \rightarrow \theta = \arccos \frac{104}{\sqrt{464}\sqrt{169}} = \boxed{1'190\ rad}$ También: $\theta = \frac{\pi}{2} - \text{arctg}\frac{8}{20} = 1'190$ ✓

También:

$\theta = \boxed{\pi - \text{arctg}\frac{8}{x} - \text{arctg}\frac{13}{20-x}}$

d) $\frac{d\theta}{dx} = -\frac{1}{1+\left(\frac{8}{x}\right)^2}\cdot\frac{-8}{x^2} - \frac{1}{1+\left(\frac{13}{20-x}\right)^2}\cdot\frac{-13\cdot(-1)}{(20-x)^2} = \frac{8}{x^2+64} - \frac{13}{(20-x)^2+13^2} =$

$= \frac{8}{x^2+64} - \frac{13}{x^2-40x+569} = \frac{8x^2 - 320x + 4552 - 13x^2 - 832}{(x^2+64)(x^2-40x+569)} = \frac{-5x^2-320x+3720}{(x^2+64)(x^2-40x+569)} =$

$= \frac{5(744 - 64x - x^2)}{(x^2+64)(x^2-40x+569)}$ ✓

e) $\frac{d\theta}{dx} = 0 \Rightarrow x = \frac{64 \pm \sqrt{64^2 + 4\cdot 744}}{-2} = \frac{64 \pm \sqrt{7072}}{-2} =$ ~~-74'05~~ / $\boxed{10'0476}$

f) $\frac{d\theta}{dt} = \frac{d\theta}{dx}\cdot\frac{dx}{dt} = \frac{5(744-64x-x^2)}{(x^2+64)(x^2-40x+569)}\cdot 0'5$

$x = 10\,m \rightarrow \frac{d\theta}{dt} = \frac{2'5\cdot(744-640-100)}{(100+64)(100-400+569)} = \boxed{2'27\times10^{-4}\ rad/seg}$