Vectores de R^3 en exámenes BI-NS

Nov 01

(b) Find the vector $\boldsymbol{v} = (\boldsymbol{i} + 3\boldsymbol{j} - 2\boldsymbol{k}) \times (2\boldsymbol{i} + \boldsymbol{j} + 3\boldsymbol{k})$.

(c) If $\boldsymbol{a} = \boldsymbol{i} + 3\boldsymbol{j} - 2\boldsymbol{k}$, $\boldsymbol{b} = 2\boldsymbol{i} + \boldsymbol{j} + 3\boldsymbol{k}$ and $\boldsymbol{u} = m\boldsymbol{a} + n\boldsymbol{b}$ where m, n are scalars, and $\boldsymbol{u} \neq 0$, show that $\boldsymbol{v}$ is perpendicular to $\boldsymbol{u}$ for all m and n.

b) $\vec v = (\vec i + 3\vec j - 2\vec k) \times (2\vec i + \vec j + 3\vec k) = \begin{vmatrix} \vec i & \vec j & \vec k \\ 1 & 3 & -2 \\ 2 & 1 & 3 \end{vmatrix} = \boxed{11\vec i - 7\vec j - 5\vec k} = (11, -7, -5)$

c) $\vec u = m\vec a + n\vec b$, $\vec v = \vec a \times \vec b$ | $\vec u \cdot \vec v = (m\vec a + n\vec b)\cdot(\vec a \times \vec b) = m\,\vec a\cdot(\vec a\times\vec b) + n\,\vec b\cdot(\vec a\times\vec b) =$

$= m\cdot 0 + m\cdot 0 = \boxed{0}$. Esto es, que $\vec u \perp \vec v$.

Mayo 02

Halle el ángulo entre los vectores $\boldsymbol{v} = \boldsymbol{i} + \boldsymbol{j} + 2\boldsymbol{k}$ y $\boldsymbol{w} = 2\boldsymbol{i} + 3\boldsymbol{j} + \boldsymbol{k}$. Indique su respuesta en radianes.

$$\cos\alpha = \frac{\vec v\cdot\vec w}{|\vec v|\,|\vec w|} = \frac{(1,1,2)\cdot(2,3,1)}{\sqrt{1+1+4}\,\sqrt{4+9+1}} = \frac{2+3+2}{\sqrt6\,\sqrt{14}} = \frac{7}{\sqrt{84}} \Rightarrow \alpha = \boxed{0'7017 \text{ rad}}$$

Mayo 02

Los puntos A, B, C y D tienen las siguientes coordenadas:

A : (1, 3, 1) B : (1, 2, 4) C : (2, 3, 6) D : (5, – 2, 1).

(a) (i) Calcule el producto vectorial $\overrightarrow{AB} \times \overrightarrow{AC}$, exprese su respuesta en función de los vectores unitarios $\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$.

(ii) Halle el área del triángulo ABC.

$\vec{AB} = (0,-1,3)$, $\vec{AC} = (1,0,5)$ | $\vec{AB}\times\vec{AC} = \begin{vmatrix} \vec i & \vec j & \vec k \\ 0 & -1 & 3 \\ 1 & 0 & 5 \end{vmatrix} = \boxed{-5\vec i + 3\vec j + \vec k}$

$$\text{Área } \widehat{ABC} = \frac{|\vec{AB}\times\vec{AC}|}{2} = \frac{\sqrt{25+9+1}}{2} = \boxed{\frac{\sqrt{35}}{2}}$$

Mayo 03

Dados $\boldsymbol{a} = \boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k}$, $\boldsymbol{b} = -3\boldsymbol{i} + 2\boldsymbol{j} + 2\boldsymbol{k}$ y $\boldsymbol{c} = 2\boldsymbol{i} - 3\boldsymbol{j} + 4\boldsymbol{k}$, halle $(\boldsymbol{a}\times\boldsymbol{b})\cdot\boldsymbol{c}$

$$(\vec a\times\vec b)\cdot\vec c = \begin{vmatrix} \vec i & \vec j & \vec k \\ 1 & 2 & -1 \\ -3 & 2 & 2 \end{vmatrix} \cdot (2,-3,4) = \begin{vmatrix} 2 & -3 & 4 \\ 1 & 2 & -1 \\ -3 & 2 & 2 \end{vmatrix} = 8+8-9+24+6+4 = \boxed{41}$$

Nov 03

Considere los puntos $A(1, 2, -4)$, $B(1, 5, 0)$ y $C(6, 5, -12)$. Halle el área de ΔABC

$\vec{AB} = (0,3,4)$, $\vec{AC} = (5,3,-8)$ | $\vec{AB}\times\vec{AC} = \begin{vmatrix} \vec i & \vec j & \vec k \\ 0 & 3 & 4 \\ 5 & 3 & -8 \end{vmatrix} = (-36, 20, -15)$

$$\text{Área } \widehat{ABC} = \frac{|\vec{AB}\times\vec{AC}|}{2} = \frac{\sqrt{(-36)^2+20^2+(-15)^2}}{2} = \boxed{\frac{\sqrt{1921}}{2}}$$

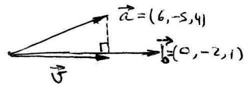
Mayo 04

Suponiendo que $\boldsymbol{a} = (\boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k}) \times (-2\boldsymbol{i} + 3\boldsymbol{k})$,

(a) halle $\boldsymbol{a}$;

(b) halle la proyección vectorial de $\boldsymbol{a}$ sobre el vector $-2\boldsymbol{j} + \boldsymbol{k}$

$\vec a = \begin{vmatrix} \vec i & \vec j & \vec k \\ 1 & 2 & 1 \\ -2 & 0 & 3 \end{vmatrix} = \boxed{6\vec i - 5\vec j + 4\vec k} = (6,-5,4)$



$\longrightarrow$ $\vec v \parallel \vec b \Rightarrow \vec v = r\cdot(0,-2,1) = (0,-2r,r)$

$|\vec v| = \dfrac{\vec a\cdot\vec b}{|\vec b|} \Rightarrow \sqrt{0^2+(-2r)^2+r^2} = \dfrac{(6,-5,4)\cdot(0,-2,1)}{\sqrt{0^2+(-2)^2+1^2}} \Rightarrow$

$\Rightarrow \sqrt{4r^2+r^2} = \dfrac{0+10+4}{\sqrt5}$; $\sqrt{5r^2} = \dfrac{14}{\sqrt5}$; $5r = 14 \Rightarrow r = \dfrac{14}{5} \Rightarrow \boxed{\vec v = \left(0, -\frac{28}{5}, \frac{14}{5}\right)}$

Mayo 04 Given that $\boldsymbol{a} = (\boldsymbol{i}+2\boldsymbol{j}+\boldsymbol{k})$, $\boldsymbol{b} = (\boldsymbol{i}-3\boldsymbol{j}+2\boldsymbol{k})$ and $\boldsymbol{c} = (2\boldsymbol{i}+\boldsymbol{j}-2\boldsymbol{k})$, calculate $(\boldsymbol{a}-\boldsymbol{b})\cdot(\boldsymbol{b}\times\boldsymbol{c})$

$$(\vec a-\vec b)\cdot(\vec b\times\vec c) = (0,5,-1)\cdot\begin{vmatrix} \vec i & \vec j & \vec k\\ 1 & -3 & 2\\ 2 & 1 & -2\end{vmatrix} = \begin{vmatrix} 0 & 5 & -1\\ 1 & -3 & 2\\ 2 & 1 & -2\end{vmatrix} = -1+20-6+10 = \boxed{23}$$

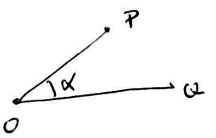
También: $(\vec a-\vec b)\cdot(\vec b\times\vec c) = \vec a\cdot(\vec b\times\vec c) - \vec b\cdot(\vec b\times\vec c) = \vec a\cdot(\vec b\times\vec c) - 0 = \begin{vmatrix} 1 & 2 & 1\\ 1 & -3 & 2\\ 2 & 1 & -2\end{vmatrix} =$

$= 6+1+8+6+4-2 = 23$ ✓

Mayo 05 Los vectores de posición de los puntos P y Q son $\begin{pmatrix}2\\-3\\1\end{pmatrix}$ y $\begin{pmatrix}2\\2\\-4\end{pmatrix}$ respectivamente. El origen está en O. Halle

(a) el ángulo POQ;

(b) el área del triángulo OPQ.



$\vec{OP} = (2,-3,1)$
$\vec{OQ} = (2,2,-4)$

$$\cos\alpha = \frac{(2,-3,1)\cdot(2,2,-4)}{\sqrt{4+9+1}\cdot\sqrt{4+4+16}} = \frac{4-6-4}{\sqrt{14}\sqrt{24}} = \frac{-6}{\sqrt{336}} \Rightarrow \boxed{\alpha = 121{,}23^\circ}$$

$$\vec{OP}\times\vec{OQ} = \begin{vmatrix} \vec i & \vec j & \vec k\\ 2 & -3 & 1\\ 2 & 2 & -4\end{vmatrix} = (10,10,10)$$

$$\text{Area} = \frac{|\vec{OP}\times\vec{OQ}|}{2} = \frac{\sqrt{100+100+100}}{2} = \frac{10\sqrt3}{2} = \boxed{5\sqrt3}$$

Mayo 05 The vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ are defined by $\boldsymbol{a} = \begin{pmatrix}3\\2\\-1\end{pmatrix}$, $\boldsymbol{b} = \begin{pmatrix}1\\5\\2\end{pmatrix}$, and $\boldsymbol{c} = \begin{pmatrix}2\\y\\3\end{pmatrix}$.

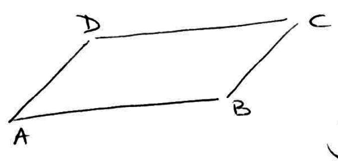
Given that $\boldsymbol{c}$ is perpendicular to $2\boldsymbol{a}-\boldsymbol{b}$, find the value of y.

$2\vec a-\vec b = (5,-1,-4)$

$\vec c \perp (2\vec a-\vec b) \Rightarrow \vec c\cdot(2\vec a-\vec b)=0 \Rightarrow 10-y-12=0 \Rightarrow \boxed{y=-2}$

Nov 05 The parallelogram ABCD has vertices $A(3, 2, 0)$, $B(7, -1, -1)$, $C(10, -3, 0)$ and $D(6, 0, 1)$. Calculate the area of the parallelogram.

$\vec{AB} = (4,-3,-1)$ $\quad\left(\vec{DC} = (4,-3,1)\right.$
$\vec{AD} = (3,-2,1)$ $\quad\left.\vec{BC} = (3,-2,1)\right)$



Esta es la buena posición del paralelogramo, ya que $\vec{AB} = \vec{DC}$; $\vec{AD} = \vec{BC}$

$$\vec{AB}\times\vec{AD} = \begin{vmatrix} \vec i & \vec j & \vec k\\ 4 & -3 & -1\\ 3 & -2 & 1\end{vmatrix} = (-5,-7,1)$$

$$\text{Area } \overline{ABCD} = |\vec{AB}\times\vec{AD}| = \sqrt{25+49+1} = \boxed{\sqrt{125}}$$

Muestra 06 = 08

A triangle has its vertices at $A(-1, 3, 2)$, $B(3, 6, 1)$ and $C(-4, 4, 3)$.

(a) Show that $\overrightarrow{AB} \bullet \overrightarrow{AC} = -10$.

(b) Show that, to three significant figures, $\cos B\hat{A}C = -0.591$.

$$\overrightarrow{AB} = (4,3,-1) \quad \overrightarrow{AC} = (-3,1,1) \quad \Big| \quad \overrightarrow{AB}\cdot\overrightarrow{AC} = -12+3-1 = -10 \checkmark$$

$$\cos B\hat{A}C = \frac{\overrightarrow{AB}\cdot\overrightarrow{AC}}{|\overrightarrow{AB}|\cdot|\overrightarrow{AC}|} = \frac{-10}{\sqrt{16+9+1}\sqrt{9+1+1}} = \frac{-10}{\sqrt{286}} = -0'591 \checkmark$$

Mayo 06

Let A be the point $(2, -1, 0)$, B the point $(3, 0, 1)$ and C the point $(1, m, 2)$, where $m \in \mathbb{Z}$, $m < 0$.

(a) (i) Find the scalar product $\overrightarrow{BA} \bullet \overrightarrow{BC}$.

(ii) Hence, given that $A\hat{B}C = \arccos\frac{\sqrt{2}}{3}$, show that $m = -1$.

$$\overrightarrow{BA} = (-1,-1,-1) \quad \overrightarrow{BC} = (-2,m,1) \quad \Big| \quad \overrightarrow{BA}\cdot\overrightarrow{BC} = 2-m-1 = 1-m$$

$$\cos A\hat{B}C = \frac{\overrightarrow{BA}\cdot\overrightarrow{BC}}{|\overrightarrow{BA}||\overrightarrow{BC}|} = \frac{1-m}{\sqrt{1+1+1}\sqrt{4+m^2+1}} = \frac{1-m}{\sqrt{3}\sqrt{m^2+5}} = \frac{1-m}{\sqrt{3m^2+15}}$$

$$\cos A\hat{B}C = \frac{\sqrt{2}}{3} \Rightarrow \frac{\sqrt{2}}{3} = \frac{1-m}{\sqrt{3m^2+15}} \ ; \ \frac{2}{9} = \frac{1-2m+m^2}{3m^2+15} \ ; \ 6m^2+30 = 9-18m+9m^2 \ ;$$

$$0 = 3m^2-18m-21 \ ; \ 0 = m^2-6m-7 \ ; \ m = \begin{cases} \cancel{7} & \text{(dice que } m<0) \\ \boxed{-1} \checkmark \end{cases}$$

Mayo 06

Let $\boldsymbol{a} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$, $\boldsymbol{b} = \begin{pmatrix} -1 \\ p \\ 6 \end{pmatrix}$ and $\boldsymbol{c} = \begin{pmatrix} 2 \\ -4 \\ 3 \end{pmatrix}$.

(a) Find $\boldsymbol{a} \times \boldsymbol{b}$.

(b) Find the value of p, given that $\boldsymbol{a} \times \boldsymbol{b}$ is parallel to $\boldsymbol{c}$.

$$\vec{a}\times\vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & 0 \\ -1 & p & 6 \end{vmatrix} = (6,-12,2p+1)$$

$$c \parallel \vec{a}\times\vec{b} \Rightarrow \frac{6}{2} = \frac{-12}{-4} = \frac{2p+1}{3} \Rightarrow \frac{2p+1}{3} = 3 \Rightarrow \boxed{p=4}$$

Mayo 07

Consider the vectors $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}, \boldsymbol{d}$

$$\boldsymbol{a} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}, \boldsymbol{b} = \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix}, \boldsymbol{c} = \begin{pmatrix} 3 \\ 1 \\ \lambda \end{pmatrix}, \boldsymbol{d} = \begin{pmatrix} \mu \\ -2 \\ 1 \end{pmatrix}.$$

Let $\boldsymbol{s} = (\boldsymbol{a} \bullet \boldsymbol{b})\boldsymbol{c} + \boldsymbol{d}$, where $\boldsymbol{s}$ is perpendicular to $\boldsymbol{a}$.

Find an expression for λ in terms of μ.

$\vec{a}\cdot\vec{b} = 2+6-5 = 3$

$\vec{s} = (\vec{a}\cdot\vec{b})\vec{c} + \vec{d} = 3(3,1,\lambda) + (\mu,-2,1) = (9+\mu, 1, 3\lambda+1)$

$\vec{s} \perp \vec{a} \Rightarrow \vec{s}\cdot\vec{a} = 0 \rightarrow (9+\mu)\cdot 2 + 1\cdot 3 + (3\lambda+1)\cdot(-1) = 0$; $18+2\mu+3-3\lambda-1 = 0$;

$20+2\mu-3\lambda = 0 \Rightarrow \lambda = \boxed{\dfrac{20+2\mu}{3}}$

Mayo 07

Considere los vectores $\boldsymbol{a} = \boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k}$, $\boldsymbol{b} = \boldsymbol{i} + 2\boldsymbol{j} + 4\boldsymbol{k}$ y $\boldsymbol{c} = 2\boldsymbol{i} - 5\boldsymbol{j} - \boldsymbol{k}$.

(a) Sabiendo que $\boldsymbol{c} = m\boldsymbol{a} + n\boldsymbol{b}$, donde $m, n \in \mathbb{Z}$, halle el valor de m y de n.

(b) Halle un vector unitario $\boldsymbol{u}$ que sea normal a $\boldsymbol{a}$ y también a $\boldsymbol{b}$.

a)

$\vec{c} = m\vec{a} + n\vec{b} \Rightarrow (2,-5,-1) = (m,-m,m) + (n,2n,4n) \Rightarrow$

$\Rightarrow \left.\begin{array}{l} 2 = m+n \\ -5 = -m+2n \\ -1 = m+4n \end{array}\right\}$

$A = \begin{pmatrix} 1 & 1 \\ -1 & 2 \\ 1 & 4 \end{pmatrix}$ $\quad \begin{vmatrix} 1 & 1 \\ -1 & 2 \end{vmatrix} = 3 \neq 0 \Rightarrow r(A) = 2$

$A^* = \left(\begin{array}{cc|c} 1 & 1 & 2 \\ -1 & 2 & -5 \\ 1 & 4 & -1 \end{array}\right)$ $\quad \begin{vmatrix} 1 & 1 & 2 \\ -1 & 2 & -5 \\ 1 & 4 & -1 \end{vmatrix} = -2-8-5-4-1+20 = 0 \Rightarrow r(A^*) = 2$

$n = 2$

S. Compat. Determ.

$\left.\begin{array}{l} m+n = 2 \\ -m+2n = -5 \\ \cancel{m+4n = -1} \end{array}\right\} \rightarrow m = \dfrac{\begin{vmatrix} 2 & 1 \\ -5 & 2 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ -1 & 2 \end{vmatrix}} = \dfrac{9}{3} = \boxed{3}$ $\quad n = \dfrac{\begin{vmatrix} 1 & 2 \\ -1 & -5 \end{vmatrix}}{3} = \dfrac{-3}{3} = \boxed{-1}$

b) $\vec{a}\times\vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & 1 \\ 1 & 2 & 4 \end{vmatrix} = (-6,-3,3)$

$|\vec{a}\times\vec{b}| = \sqrt{36+9+9} = \sqrt{54} = 3\sqrt{6}$

$\Rightarrow \vec{u} = \dfrac{1}{3\sqrt{6}}(-6,-3,3) = \boxed{\left(\dfrac{-2}{\sqrt{6}}, \dfrac{-1}{\sqrt{6}}, \dfrac{1}{\sqrt{6}}\right)}$

Mayo 07

Sabiendo que $\boldsymbol{a} = 2\boldsymbol{i} - \boldsymbol{j} - \boldsymbol{k}$, $\boldsymbol{b} = 2\boldsymbol{i} + \boldsymbol{j} - 2\boldsymbol{k}$ y $\boldsymbol{c} = -\boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k}$ son los vectores de posición de los puntos A, B y C respectivamente, calcule el área del triángulo ABC.

$\vec{AB} = (0,2,-1)$

$\vec{AC} = (-3,2,0)$

$\vec{AB}\times\vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 2 & -1 \\ -3 & 2 & 0 \end{vmatrix} = (2,3,6)$

Área $\widehat{ABC} = \dfrac{\sqrt{4+9+36}}{2} = \boxed{\dfrac{7}{2}}$

Muestra 08

Consider the points $A(1, 2, 1)$, $B(0, -1, 2)$, $C(1, 0, 2)$ and $D(2, -1, -6)$.

(a) Find the vectors $\vec{AB}$ and $\vec{BC}$.

(b) Calculate $\vec{AB}\times\vec{BC}$.

(c) Hence, or otherwise find the area of triangle ABC.

$$\vec{AB} = (-1, -3, 1) \quad \vec{BC} = (1, 1, 0) \quad \Big| \quad \vec{AB}\times\vec{BC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & -3 & 1 \\ 1 & 1 & 0 \end{vmatrix} = (-1, 1, 2)$$

$$\text{Area } \hat{ABC} = \frac{\sqrt{1+1+4}}{2} = \boxed{\frac{\sqrt{6}}{2}}$$

Mayo 08

Given any two non-zero vectors $\boldsymbol{a}$ and $\boldsymbol{b}$, show that $|\boldsymbol{a}\times\boldsymbol{b}|^2 = |\boldsymbol{a}|^2|\boldsymbol{b}|^2 - (\boldsymbol{a}\bullet\boldsymbol{b})^2$

$$|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|\operatorname{sen}\alpha \longrightarrow |\vec{a}\times\vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2\operatorname{sen}^2\alpha$$

$$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\alpha \longrightarrow + (\vec{a}\cdot\vec{b})^2 = |\vec{a}|^2|\vec{b}|^2\cos^2\alpha$$

$$|\vec{a}\times\vec{b}|^2 + (\vec{a}\cdot\vec{b})^2 = |\vec{a}|^2|\vec{b}|^2\cdot(\operatorname{sen}^2\alpha+\cos^2\alpha)$$

$$|\vec{a}\times\vec{b}|^2 + (\vec{a}\cdot\vec{b})^2 = |\vec{a}|^2|\vec{b}|^2 \Rightarrow$$

$$\Rightarrow |\vec{a}\times\vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2 - (\vec{a}\cdot\vec{b})^2 \quad \checkmark$$

Nov 08

The angle between the vector $\boldsymbol{a} = \boldsymbol{i} - 2\boldsymbol{j} + 3\boldsymbol{k}$ and the vector $\boldsymbol{b} = 3\boldsymbol{i} - 2\boldsymbol{j} + m\boldsymbol{k}$ is 30°.

Find the values of m.

$$\cos 30^\circ = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} \Rightarrow \frac{\sqrt{3}}{2} = \frac{(1,-2,3)\cdot(3,-2,m)}{\sqrt{1+4+9}\,\sqrt{9+4+m^2}} \; ; \; \frac{\sqrt{3}}{2} = \frac{3+4+3m}{\sqrt{14}\,\sqrt{m^2+13}} \; ;$$

$$\frac{\sqrt{3}}{2} = \frac{3m+7}{\sqrt{14m^2+182}} \; ; \; \frac{3}{4} = \frac{9m^2+42m+49}{14m^2+182} \; ; \; 42m^2+546 = 36m^2+168m+196 \; ;$$

$$6m^2 - 168m + 350 = 0 \; ; \; 3m^2 - 84m + 175 = 0 \; ; \; \boxed{m = \begin{cases} 25{,}73 \\ 2{,}27 \end{cases}}$$

Mayo 16
TZ2
P2#9

OACB es un paralelogramo en el que $\vec{OA} = \boldsymbol{a}$ y $\vec{OB} = \boldsymbol{b}$, donde $\boldsymbol{a}$ y $\boldsymbol{b}$ son vectores no nulos.

(a) Muestre que

(i) $|\vec{OC}|^2 = |\boldsymbol{a}|^2 + 2\boldsymbol{a}\cdot\boldsymbol{b} + |\boldsymbol{b}|^2$;

(ii) $|\vec{AB}|^2 = |\boldsymbol{a}|^2 - 2\boldsymbol{a}\cdot\boldsymbol{b} + |\boldsymbol{b}|^2$.

(b) Sabiendo que $|\vec{OC}| = |\vec{AB}|$, demuestre que OACB es un rectángulo.

Mayo 09
TZ1
P1#8

A triangle has vertices $A(1, -1, 1)$, $B(1, 1, 0)$ and $C(-1, 1, -1)$.

Show that the area of the triangle is $\sqrt{6}$.

$\vec{AB} = (0, 2, -1)$

$\vec{AC} = (-2, 2, -2)$

$\vec{AB} \times \vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 2 & -1 \\ -2 & 2 & -2 \end{vmatrix} = (-2, 2, 4)$

Area $\widehat{ABC} = \dfrac{\sqrt{4+4+16}}{2} = \dfrac{\sqrt{24}}{2} = \dfrac{2\sqrt{6}}{2} = \boxed{\sqrt{6}}$ ✓

Mayo 10
TZ2
P1#3

Los tres vectores $\boldsymbol{a}$, $\boldsymbol{b}$ y $\boldsymbol{c}$ vienen dados por

$$\boldsymbol{a} = \begin{pmatrix} 2y \\ -3x \\ 2x \end{pmatrix},\ \boldsymbol{b} = \begin{pmatrix} 4x \\ y \\ 3-x \end{pmatrix},\ \boldsymbol{c} = \begin{pmatrix} 4 \\ -7 \\ 6 \end{pmatrix} \text{ donde } x, y \in \mathbb{R}.$$

(a) Si $\boldsymbol{a} + 2\boldsymbol{b} - \boldsymbol{c} = 0$, halle el valor de x y de y.

(b) Halle el valor exacto de $|\boldsymbol{a} + 2\boldsymbol{b}|$.

$\vec{a} + 2\vec{b} - \vec{c} = 0 \Rightarrow \begin{cases} 2y + 8x - 4 = 0 \\ -3x + 2y + 7 = 0 \\ 2x + 6 - 2x - 6 = 0 \end{cases} \rightarrow \begin{cases} 8x + 2y = 4 \\ -3x + 2y = -7 \\ 0 = 0 \end{cases}$

$x = \dfrac{\begin{vmatrix} 4 & 2 \\ -7 & 2 \end{vmatrix}}{\begin{vmatrix} 8 & 2 \\ -3 & 2 \end{vmatrix}} = \dfrac{8+14}{16+6} = \dfrac{22}{22} = \boxed{1}$ $\quad y = \dfrac{\begin{vmatrix} 8 & 4 \\ -3 & -7 \end{vmatrix}}{22} = \dfrac{-56+12}{22} = \boxed{-2}$

$|\vec{a} + 2\vec{b}| = |\vec{c}| = \sqrt{16+49+36} = \boxed{\sqrt{101}}$

Mayo 10 TZ2 P1#12

(a) Considere los vectores $\boldsymbol{a} = 6\boldsymbol{i} + 3\boldsymbol{j} + 2\boldsymbol{k}$, $\boldsymbol{b} = -3\boldsymbol{j} + 4\boldsymbol{k}$.

(i) Halle el coseno del ángulo que forman los vectores $\boldsymbol{a}$ y $\boldsymbol{b}$.

(ii) Halle $\boldsymbol{a} \times \boldsymbol{b}$.

$$\cos\alpha = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\,|\vec{b}|} = \frac{(6,3,2)\cdot(0,-3,4)}{\sqrt{36+9+4}\,\sqrt{0+9+16}} = \frac{-9+8}{35} = \boxed{-\frac{1}{35}}$$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 6 & 3 & 2 \\ 0 & -3 & 4 \end{vmatrix} = (18,-24,-18) = \boxed{18\vec{i} - 24\vec{j} - 18\vec{k}}$$

Nov 10 P2 #9

Considere los vectores $\boldsymbol{a} = \operatorname{sen}(2\alpha)\,\boldsymbol{i} - \cos(2\alpha)\,\boldsymbol{j} + \boldsymbol{k}$ y $\boldsymbol{b} = \cos\alpha\,\boldsymbol{i} - \operatorname{sen}\alpha\,\boldsymbol{j} - \boldsymbol{k}$, donde $0 < \alpha < 2\pi$.

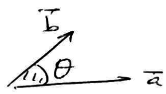
Sea θ el ángulo que forman los vectores $\boldsymbol{a}$ y $\boldsymbol{b}$.

(a) Exprese $\cos\theta$ en función de α.

(b) Halle el ángulo agudo α para el cual los dos vectores son perpendiculares.

(c) Para $\alpha = \frac{7\pi}{6}$, determine el producto vectorial de $\boldsymbol{a}$ y $\boldsymbol{b}$ y comente el significado geométrico de este resultado.

$$\vec{a} = (\operatorname{sen}(2\alpha), -\cos(2\alpha), 1) \qquad \vec{b} = (\cos\alpha, -\operatorname{sen}\alpha, -1)$$



$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\,|\vec{b}|} = \frac{\operatorname{sen}(2\alpha)\cos\alpha + \cos(2\alpha)\operatorname{sen}\alpha - 1}{\sqrt{\operatorname{sen}^2(2\alpha) + \cos^2(2\alpha) + 1}\,\sqrt{\cos^2\alpha + \operatorname{sen}^2\alpha + 1}} =$$

$$= \frac{\operatorname{sen}(2\alpha+\alpha) - 1}{\sqrt{2}\,\sqrt{2}} = \boxed{\frac{\operatorname{sen}(3\alpha) - 1}{2}}$$

$$\vec{a} \perp \vec{b} \Rightarrow \cos\theta = 0 \Rightarrow \frac{\operatorname{sen}(3\alpha) - 1}{2} = 0 \longrightarrow \operatorname{sen}(3\alpha) = 1$$

$$\operatorname{sen}(3\alpha) = 1 \Rightarrow 3\alpha = \frac{\pi}{2} + 2\pi N \Rightarrow \alpha = \frac{\pi}{6} + \frac{2\pi}{3}N \longrightarrow \boxed{\alpha = \pi/6}$$

$$\alpha = \frac{7\pi}{6} \longrightarrow \cos\theta = \frac{\operatorname{sen}\left(3\cdot\frac{7\pi}{6}\right) - 1}{2} = \frac{\operatorname{sen}\left(\frac{7\pi}{2}\right) - 1}{2} = \frac{-1-1}{2} = -1$$

$\cos\theta = -1 \Rightarrow$ $\vec{a}$ y $\vec{b}$ son paralelos de sentido opuesto.

Por lo tanto $\boxed{\vec{a}\times\vec{b} = \vec{0}}$

Mayo 11 TZ2 P2#11

Los puntos $\mathrm{P}(-1, 2, -3)$, $\mathrm{Q}(-2, 1, 0)$, $\mathrm{R}(0, 5, 1)$ y S forman un paralelogramo, siendo S diagonalmente opuesto a Q.

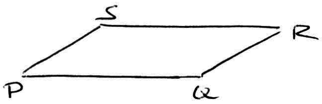
(a) Halle las coordenadas de S.

(b) El producto vectorial $\overrightarrow{\mathrm{PQ}} \times \overrightarrow{\mathrm{PS}} = \begin{pmatrix} -13 \\ 7 \\ m \end{pmatrix}$. Halle el valor de m.

(c) A partir de lo anterior, calcule el área del paralelogramo PQRS.

$\vec{QR} = (2,4,1)$

$S = P + (2,4,1) = \boxed{(1,6,-2)}$



$\vec{PQ} = (-1,-1,3)$

$\vec{PS} = (2,4,1)$

$\vec{PQ} \times \vec{PS} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & -1 & 3 \\ 2 & 4 & 1 \end{vmatrix} = (-13, 7, -2) \Rightarrow \boxed{m=-2}$

$\text{Area } \widehat{PQRS} = |\vec{PQ} \times \vec{PS}| = \sqrt{169+49+4} = \boxed{\sqrt{222}}$

Nov 11
P1#12

(a) Compruebe que, para vectores no nulos $\boldsymbol{a}$ y $\boldsymbol{b}$,

(i) si $|\boldsymbol{a}-\boldsymbol{b}| = |\boldsymbol{a}+\boldsymbol{b}|$, entonces $\boldsymbol{a}$ y $\boldsymbol{b}$ son perpendiculares;

(ii) $|\boldsymbol{a}\times\boldsymbol{b}|^2 = |\boldsymbol{a}|^2|\boldsymbol{b}|^2 - (\boldsymbol{a}\bullet\boldsymbol{b})^2$.

(b) Los puntos A, B y C tienen vectores de posición $\boldsymbol{a}$, $\boldsymbol{b}$ y $\boldsymbol{c}$.

(i) Compruebe que el área del triángulo ABC es $\frac{1}{2}|\boldsymbol{a}\times\boldsymbol{b}+\boldsymbol{b}\times\boldsymbol{c}+\boldsymbol{c}\times\boldsymbol{a}|$.

(ii) A partir de lo anterior, compruebe que la distancia más corta entre B y AC es

$$\frac{|\boldsymbol{a}\times\boldsymbol{b}+\boldsymbol{b}\times\boldsymbol{c}+\boldsymbol{c}\times\boldsymbol{a}|}{|\boldsymbol{c}-\boldsymbol{a}|}.$$

a) (i) idéntico a NOV 02
(ii) " " MAYO 08

b) $\vec{AB} = \vec{b} - \vec{a} = (b_1-a_1, b_2-a_2, b_3-a_3)$

$\vec{AC} = \vec{c} - \vec{a} = (c_1-a_1, c_2-a_2, c_3-a_3)$

$\vec{AB} \times \vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ b_1-a_1 & b_2-a_2 & b_3-a_3 \\ c_1-a_1 & c_2-a_2 & c_3-a_3 \end{vmatrix} =$ (Aplicando propiedades de los determinantes)

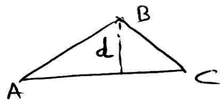
$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} - \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \end{vmatrix} - \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ b_1 & b_2 & b_3 \\ a_1 & a_2 & a_3 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ a_1 & a_2 & a_3 \end{vmatrix} =$

$= \vec{b}\times\vec{c} - \vec{a}\times\vec{c} - \vec{b}\times\vec{a} + \vec{0} =$

$= \vec{b}\times\vec{c} + \vec{c}\times\vec{a} + \vec{a}\times\vec{b}$

$\text{Area } \widehat{ABC} = \frac{|\vec{AB}\times\vec{AC}|}{2} = \frac{|\vec{b}\times\vec{c}+\vec{c}\times\vec{a}+\vec{a}\times\vec{b}|}{2}$ ✓

$\text{Area } \widehat{ABC} = \frac{\text{base} \cdot \text{altura}}{2} = \frac{|\vec{AC}|\cdot d}{2} = \frac{|\vec{c}-\vec{a}|\cdot d}{2}$



$\frac{d\cdot|\vec{c}-\vec{a}|}{\cancel{2}} = \frac{|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}|}{\cancel{2}}$

$d = \frac{|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}|}{|\vec{c}-\vec{a}|}$ ✓

Mayo 12
TZ2
P1#2

Halle los valores de x para los cuales los vectores $\begin{pmatrix} 1 \\ 2\cos x \\ 0 \end{pmatrix}$ y $\begin{pmatrix} -1 \\ 2\operatorname{sen} x \\ 1 \end{pmatrix}$ son perpendiculares, siendo $0 \le x \le \frac{\pi}{2}$.

$(1, 2\cos x, 0) \perp (-1, 2\sin x, 1) \Rightarrow -1 + 4\sin x\cos x = 0 \Rightarrow$

$\Rightarrow 4\sin x \cos x = 1 \quad ; \quad 2\sin(2x) = 1 \quad ; \quad \sin(2x) = \frac{1}{2}$

$\sin(2x) = \frac{1}{2} \Rightarrow 2x = \begin{cases} 30^\circ + 360N \\ 150^\circ + 360N \end{cases} \Rightarrow x = \begin{cases} 15^\circ + 180^\circ N \\ 75^\circ + 180^\circ N \end{cases} = \begin{cases} \frac{\pi}{12} + \pi N \\ \frac{5\pi}{12} + \pi N \end{cases}$

$0 \le x \le \frac{\pi}{2} \Rightarrow \boxed{x = \begin{cases} \pi/12 \\ 5\pi/12 \end{cases}}$

Muestra 14 P1#8

The vectors $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$ satisfy the equation $\boldsymbol{a}+\boldsymbol{b}+\boldsymbol{c}=\boldsymbol{0}$. Show that $\boldsymbol{a}\times\boldsymbol{b}=\boldsymbol{b}\times\boldsymbol{c}=\boldsymbol{c}\times\boldsymbol{a}$.

$\vec{a}+\vec{b}+\vec{c}=\vec{0}$

- $\vec{a}\times(\vec{a}+\vec{b}+\vec{c}) = \vec{a}\times\vec{0} \Rightarrow \vec{a}\times\vec{a} + \vec{a}\times\vec{b} + \vec{a}\times\vec{c} = \vec{0} \; ; \; \vec{0} + \vec{a}\times\vec{b} + \vec{a}\times\vec{c} = \vec{0} \; ;$
  $\vec{a}\times\vec{b} = -\vec{a}\times\vec{c} \; ; \; \vec{a}\times\vec{b} = \vec{c}\times\vec{a}$ ✓
- $\vec{b}\times(\vec{a}+\vec{b}+\vec{c}) = \vec{b}\times\vec{0} \Rightarrow \vec{b}\times\vec{a} + \vec{b}\times\vec{b} + \vec{b}\times\vec{c} = \vec{0} \; ; \; \vec{b}\times\vec{a} + \vec{0} + \vec{b}\times\vec{c} = \vec{0} \; ;$
  $\vec{b}\times\vec{c} = -\vec{b}\times\vec{a} \; ; \; \vec{b}\times\vec{c} = \vec{a}\times\vec{b}$ ✓

Mayo 15 TZ1 P2#8

Let $\boldsymbol{v} = \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}$ and $\boldsymbol{w} = \begin{pmatrix} 4 \\ \lambda \\ 10 \end{pmatrix}$.

(a) Find the value of λ for $\boldsymbol{v}$ and $\boldsymbol{w}$ to be parallel.

(b) Find the value of λ for $\boldsymbol{v}$ and $\boldsymbol{w}$ to be perpendicular.

(c) Find the two values of λ if the angle between $\boldsymbol{v}$ and $\boldsymbol{w}$ is 10°.

Mayo 16 TZ1 P1#8

O, A, B and C are distinct points such that $\overrightarrow{OA}=\boldsymbol{a}$, $\overrightarrow{OB}=\boldsymbol{b}$ and $\overrightarrow{OC}=\boldsymbol{c}$.

It is given that $\boldsymbol{c}$ is perpendicular to $\overrightarrow{AB}$ and $\boldsymbol{b}$ is perpendicular to $\overrightarrow{AC}$.

Prove that $\boldsymbol{a}$ is perpendicular to $\overrightarrow{BC}$.

Mayo 16 TZ2 P2#9

OACB es un paralelogramo en el que $\overrightarrow{OA}=\boldsymbol{a}$ y $\overrightarrow{OB}=\boldsymbol{b}$, donde $\boldsymbol{a}$ y $\boldsymbol{b}$ son vectores no nulos.

(a) Muestre que

(i) $|\overrightarrow{OC}|^2 = |\boldsymbol{a}|^2 + 2\boldsymbol{a}\cdot\boldsymbol{b} + |\boldsymbol{b}|^2$;

(ii) $|\overrightarrow{AB}|^2 = |\boldsymbol{a}|^2 - 2\boldsymbol{a}\cdot\boldsymbol{b} + |\boldsymbol{b}|^2$.

(b) Sabiendo que $|\overrightarrow{OC}| = |\overrightarrow{AB}|$, demuestre que OACB es un rectángulo.