Cálculo Diferencial en exámenes BI – NM (Extracto)

Definición e Interpretación geométrica de la derivada

Mayo 00 P1

Let $f(x) = x^3$.

(a) Evaluate $\dfrac{f(5+h) - f(5)}{h}$ for $h = 0.1$.

(b) What number does $\dfrac{f(5+h) - f(5)}{h}$ approach as h approaches zero?

Mayo 02 P1

En la **Figura 1** aparecen las gráficas de las funciones f_1 , f_2 , f_3 , f_4 .

En la **Figura 2** se muestran las gráficas de las derivadas de las funciones de la **Figura 1**, por ejemplo, la derivada de f_1 aparece en el diagrama (d).

Figura 1

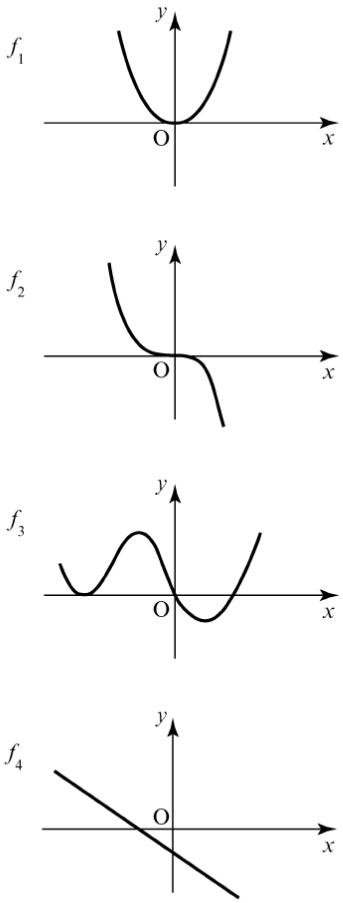
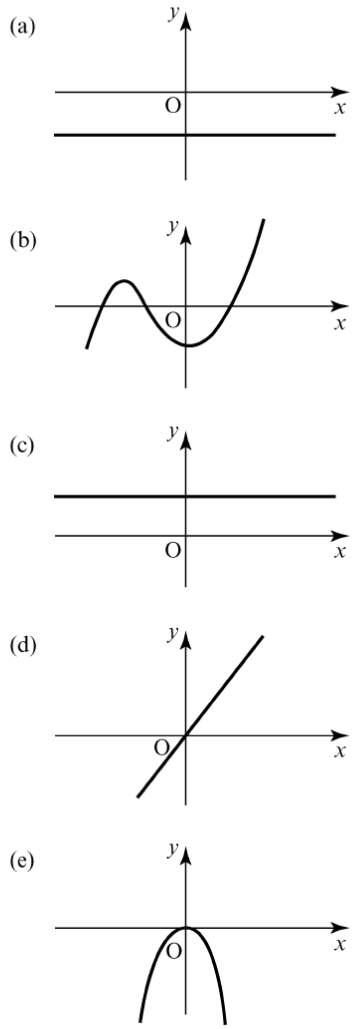


Figura 2

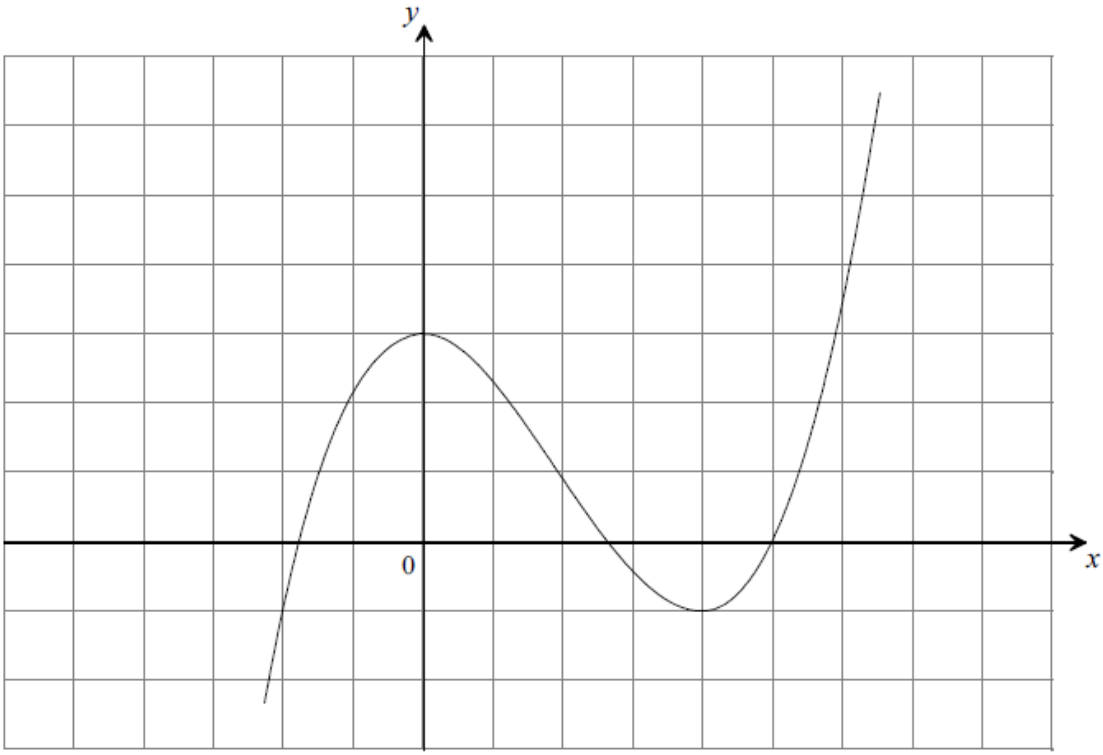


Rellene la tabla siguiente efectuando la correspondencia entre cada una de las funciones y su derivada.

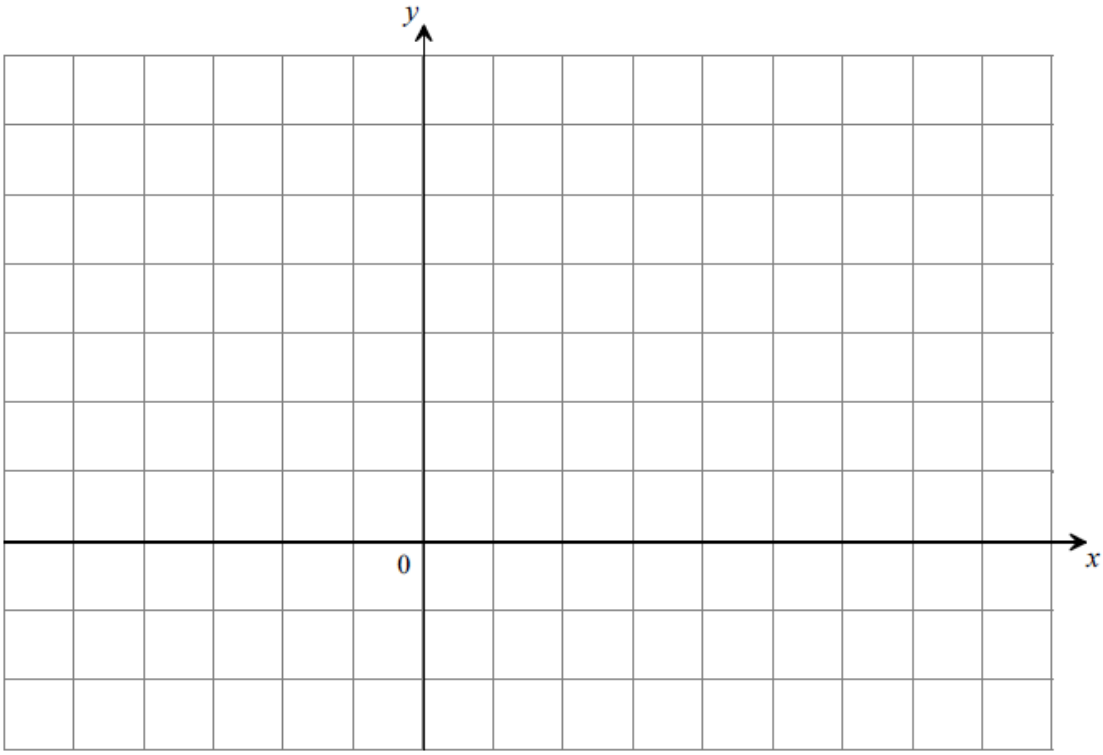
Función	Diagrama de la derivada
f_1	(d)
f_2	
f_3	
f_4	

Nov 03
P1

La siguiente figura muestra la gráfica de la función $y = f(x)$.

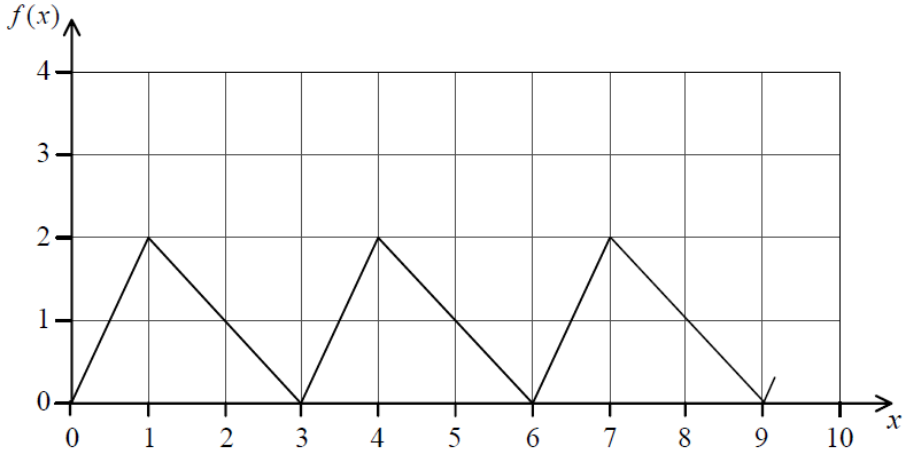


Dibuje de forma aproximada, sobre la cuadrícula inferior, la gráfica de $y = f'(x)$.



Nov 04 P1

A continuación se muestra parte de la gráfica de la función periódica f. El dominio de f es $0 \le x \le 15$ y el período es 3.



(a) Halle

(i) $f(2)$;

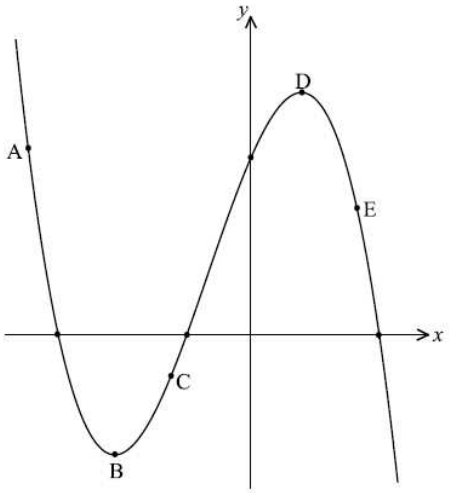
(ii) $f'(6{,}5)$;

(iii) $f'(14)$.

(b) ¿Cuántas soluciones tiene la ecuación $f(x) = 1$ en el dominio dado?

Mayo 06 P1

The following diagram shows part of the curve of a function f. The points A, B, C, D and E lie on the curve, where B is a minimum point and D is a maximum point.



(a) Complete the following table, noting whether $f'(x)$ is positive, negative or zero at the given points.

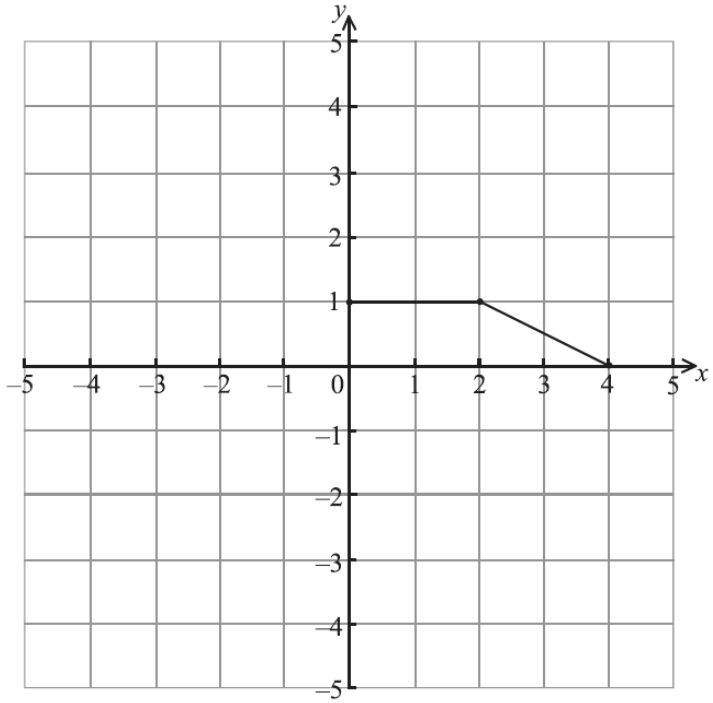
	A	B	E
$f'(x)$			

(b) Complete the following table, noting whether $f''(x)$ is positive, negative or zero at the given points.

	A	C	E
$f''(x)$			

May 07 P1

The graph of the function $y = f(x)$, $0 \le x \le 4$, is shown below.



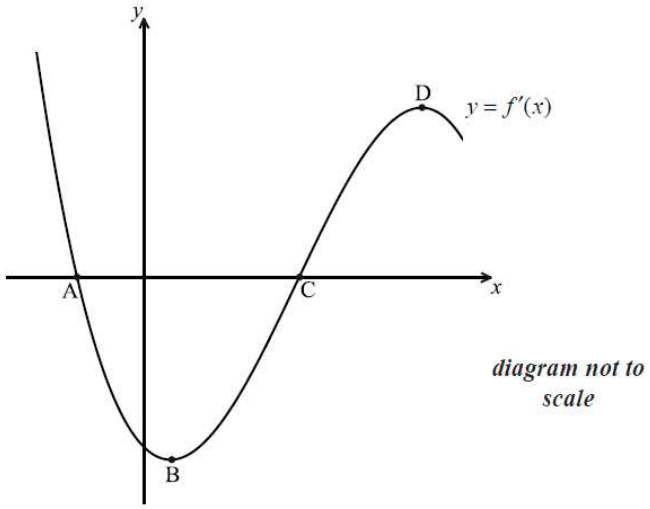
(a) Write down the value of

(i) $f'(1)$;

(ii) $f'(3)$.

Nov 08 P1

The diagram shows part of the graph of $y = f'(x)$. The x-intercepts are at points A and C. There is a minimum at B, and a maximum at D.



(a) (i) Write down the value of $f'(x)$ at C.

(ii) **Hence**, show that C corresponds to a minimum on the graph of f, *i.e.* it has the same x-coordinate.

(b) Which of the points A, B, D corresponds to a maximum on the graph of f?

(c) Show that B corresponds to a point of inflexion on the graph of f.

Mayo 09 P2

Let $f(x) = x^3 - 4x + 1$.

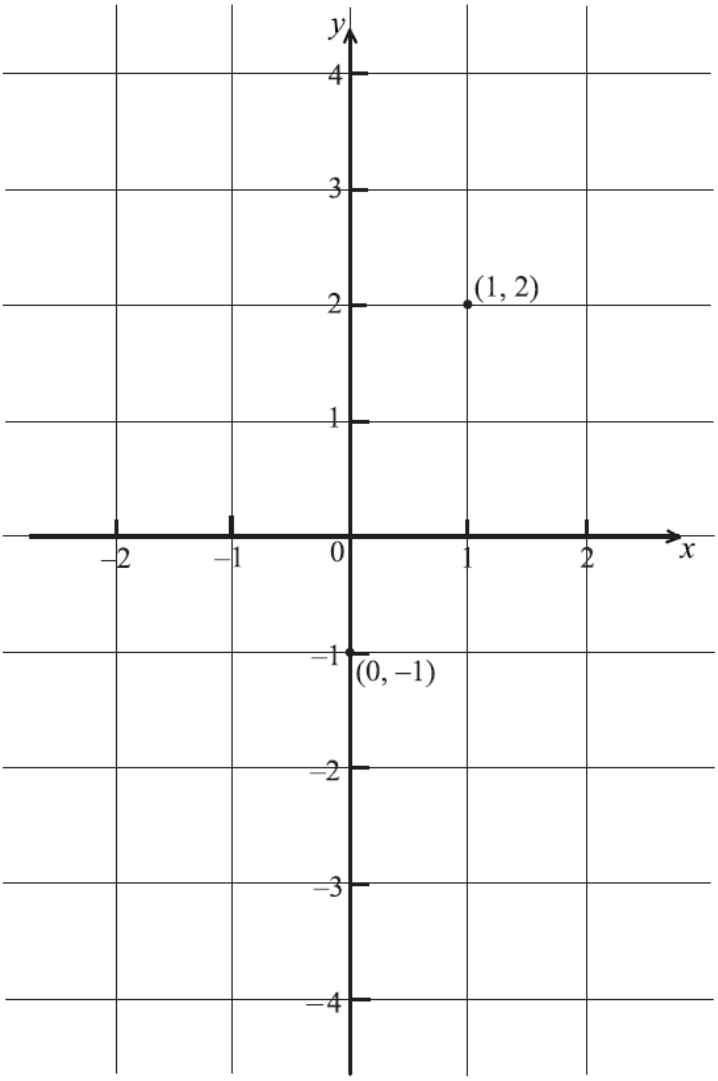
(a) Expand $(x+h)^3$.

(b) Use the formula $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ to show that the derivative of $f(x)$ is $3x^2 - 4$.

Nov 07 P1

On the axes below, sketch a curve $y = f(x)$ which satisfies the following conditions.

x	$f(x)$	$f'(x)$	$f''(x)$
$-2 \le x < 0$		negative	positive
0	-1	0	positive
$0 < x < 1$		positive	positive
1	2	positive	0
$1 < x \le 2$		positive	negative



Recta tangente / Recta Normal

Mayo 00 P1

Find the coordinates of the point on the graph of $y = x^2 - x$ at which the tangent is parallel to the line $y = 5x$.

Mayo 06 P1

Considere la función $f : x \mapsto 3x^2 - 5x + k$.

(a) Escriba $f'(x)$.

La ecuación de la tangente a la gráfica de f en $x = p$ es $y = 7x - 9$. Halle el valor de

(b) p;

(c) k.

May 07 P1

Sea $f(x) = 3\cos 2x + \operatorname{sen}^2 x$.

(a) Compruebe que $f'(x) = -5\operatorname{sen} 2x$.

(b) En el intervalo $\frac{\pi}{4} \le x \le \frac{3\pi}{4}$, una normal a la gráfica de f tiene por ecuación $x = k$. Halle el valor de k.

May 07 P1

Let $f(x) = x^3 - 3x^2 - 24x + 1$.

The tangents to the curve of f at the points P and Q are parallel to the x-axis, where P is to the left of Q.

(a) Calculate the coordinates of P and of Q.

Let N_1 and N_2 be the normals to the curve at P and Q respectively.

(b) Write down the coordinates of the points where

(i) the tangent at P intersects N_2;

(ii) the tangent at Q intersects N_1.

Nov 07 P1

Consider the function $f(x) = 4x^3 + 2x$. Find the equation of the normal to the curve of f at the point where $x = 1$.

Nov 09 P2

Consider the curve with equation $f(x) = px^2 + qx$, where p and q are constants. The point A(1, 3) lies on the curve. The tangent to the curve at A has gradient 8. Find the value of p and of q.

Mayo 10 TZ2 P1#5

Sea $f(x) = kx^4$. El punto $P(1, k)$ pertenece a la curva de f. En P, la recta normal a la curva es paralela a $y = -\frac{1}{8}x$. Halle el valor de k.

Mayo 14 TZ1 P2#7

Let $f(x) = \frac{g(x)}{h(x)}$, where $g(2) = 18$, $h(2) = 6$, $g'(2) = 5$, and $h'(2) = 2$. Find the equation of the normal to the graph of f at $x = 2$.

Cálculo de funciones derivadas

Mayo 03 P1

Sea $f(x)=e^{\frac{x}{3}}+5\cos^2 x$. Halle $f'(x)$

Nov 04 P1

Sea $f(x)=6\sqrt[3]{x^2}$. Halle $f'(x)$.

Nov 04 P2

Sea $f(x)=1+3\cos(2x)$ para $0\le x\le \pi$, donde x viene expresado en radianes.

(a) (i) Halle $f'(x)$.

(ii) Halle los valores de x para los cuales $f'(x)=0$, expresando las respuestas en función de π.

Nov 05 P1

Let $f(x)=(2x+7)^3$ and $g(x)=\cos^2(4x)$. Find

(a) $f'(x)$;

(b) $g'(x)$.

Mayo 06 P1

(a) Let $f(x)=e^{5x}$. Write down $f'(x)$.

(b) Let $g(x)=\sin 2x$. Write down $g'(x)$.

(c) Let $h(x)=e^{5x}\sin 2x$. Find $h'(x)$.

Mayo 09 P1

Sean $f(x)=e^{-3x}$ y $g(x)=\text{sen}\left(x-\frac{\pi}{3}\right)$.

(a) Escriba

(i) $f'(x)$;

(ii) $g'(x)$.

(b) Sea $h(x)=e^{-3x}\,\text{sen}\left(x-\frac{\pi}{3}\right)$. Halle el valor exacto de $h'\left(\frac{\pi}{3}\right)$.

Nov 09 P2

Let $f(x)=\cos 2x$ and $g(x)=\ln(3x-5)$.

(a) Find $f'(x)$.

(b) Find $g'(x)$.

(c) Let $h(x)=f(x)\times g(x)$. Find $h'(x)$.

Muestra 14 P1#7

Given that $f(x)=\frac{1}{x}$, answer the following.

(a) Find the first four derivatives of $f(x)$.

(b) Write an expression for $f^{(n)}(x)$ in terms of x and n.

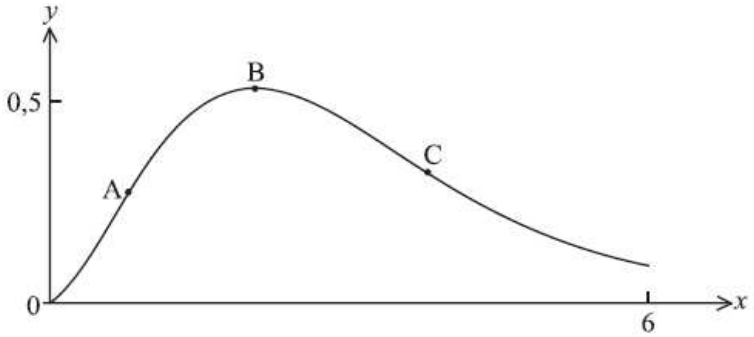
Mayo 11 TZ2 P1#4

Sea $h(x)=\frac{6x}{\cos x}$. Halle $h'(0)$.

Máximos / Mínimos y Puntos de Inflexión

Mayo 06 P1

El siguiente diagrama presenta la gráfica de $f(x) = x^2 e^{-x}$ para $0 \le x \le 6$. Existen puntos de inflexión en A y C y existe un máximo en B.



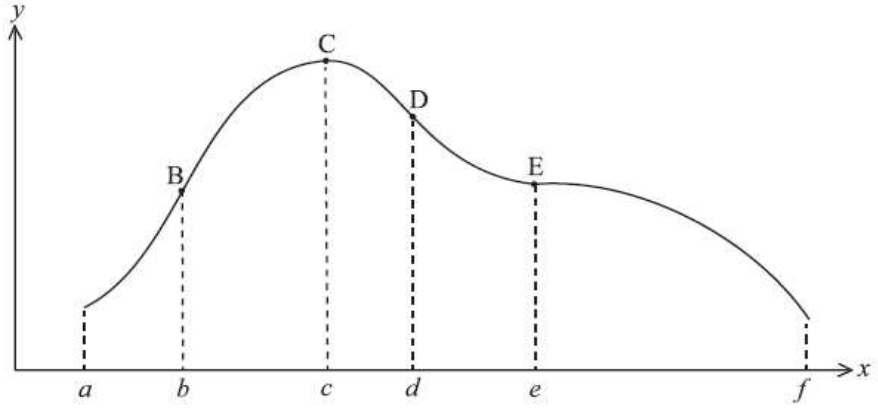
(a) Utilizando la regla del producto para la derivación, halle $f'(x)$.

(b) Halle el valor **exacto** de la **coordenada** y de B.

(c) La derivada segunda de f es $f''(x) = (x^2 - 4x + 2)e^{-x}$. Utilice este resultado para hallar el valor **exacto** de la coordenada x de C.

Mayo 06 P1

El diagrama siguiente presenta la gráfica de una función g.



La pendiente de la curva tiene un valor máximo en el punto B y un valor mínimo en el punto D. La tangente es horizontal en los puntos C y E.

(a) Complete la siguiente tabla, indicando si la derivada primera g' es positiva o negativa y si la derivada segunda g'' es positiva o negativa.

Intervalo	g'	g''
$a < x < b$		
$e < x < f$		

(b) Complete la tabla que aparece a continuación indicando los puntos de la gráfica descritos por las siguientes condiciones.

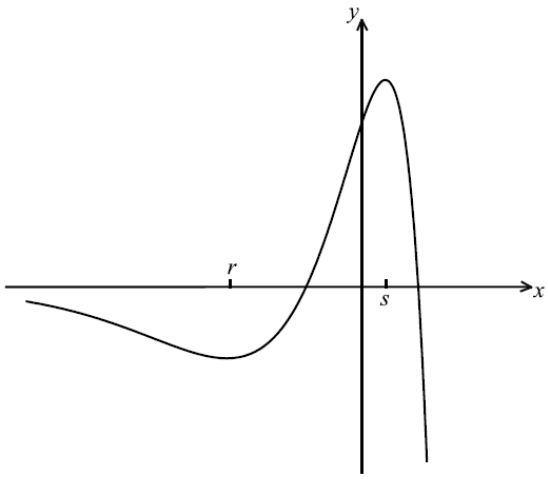
Condiciones	Punto
$g'(x) = 0$, $g''(x) < 0$	
$g'(x) < 0$, $g''(x) = 0$	

Mayo 08 P2

Let $f(x) = e^x(1-x^2)$.

(a) Show that $f'(x) = e^x(1-2x-x^2)$.

Part of the graph of $y = f(x)$, for $-6 \le x \le 2$, is shown below. The x-coordinates of the local minimum and maximum points are r and s respectively.



(b) Write down the **equation** of the horizontal asymptote.

(c) Write down the value of r and of s.

(d) Let L be the normal to the curve of f at P(0, 1). Show that L has equation $x + y = 1$.

Nov 10 P2#7

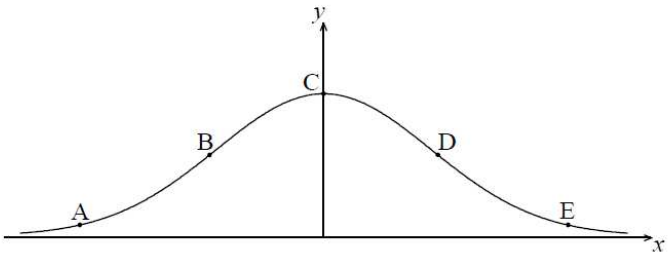
Sea $f'(x) = -24x^3 + 9x^2 + 3x + 1$.

(a) En la gráfica de f hay dos puntos de inflexión. Escriba las coordenadas x de estos puntos.

(b) Sea $g(x) = f''(x)$. Explique por qué en la gráfica de g no hay ningún punto de inflexión.

Mayo 11 TZ1 P2#9

The following diagram shows the graph of $f(x) = e^{-x^2}$.



The points A, B, C, D and E lie on the graph of f. Two of these are points of inflexion.

(a) Identify the **two** points of inflexion.

(b) (i) Find $f'(x)$.

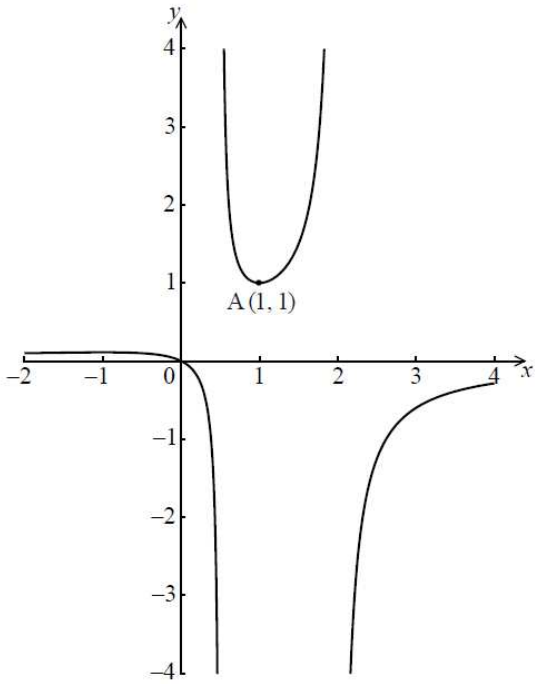
(ii) Show that $f''(x) = (4x^2 - 2)e^{-x^2}$.

(c) Find the x-coordinate of each point of inflexion.

(d) Use the second derivative to show that one of these points is a point of inflexion.

Mayo 12
TZ2
P1#10

Sea $f(x)=\dfrac{x}{-2x^2+5x-2}$, para $-2\le x\le 4$, $x\ne\dfrac{1}{2}$, $x\ne 2$. A continuación se muestra la gráfica de f.



La gráfica de f presenta un mínimo local en $\mathrm{A}(1,1)$ y un máximo local en B.

(a) Utilice la regla del cociente para comprobar que $f'(x)=\dfrac{2x^2-2}{(-2x^2+5x-2)^2}$.

(b) A partir de lo anterior, halle las coordenadas de B.

(c) Sabiendo que la recta $y=k$ no corta a la gráfica de f, halle los posibles valores de k.

Mayo 13
TZ1
P1#10

Consider $f(x)=\ln(x^4+1)$.

(a) Find the value of $f(0)$.

(b) Find the set of values of x for which f is increasing.

The second derivative is given by $f''(x)=\dfrac{4x^2(3-x^4)}{(x^4+1)^2}$.

The equation $f''(x)=0$ has only three solutions, when $x=0$, $\pm\sqrt[4]{3}$ $(\pm 1.316\ldots)$.

(c) (i) Find $f''(1)$.

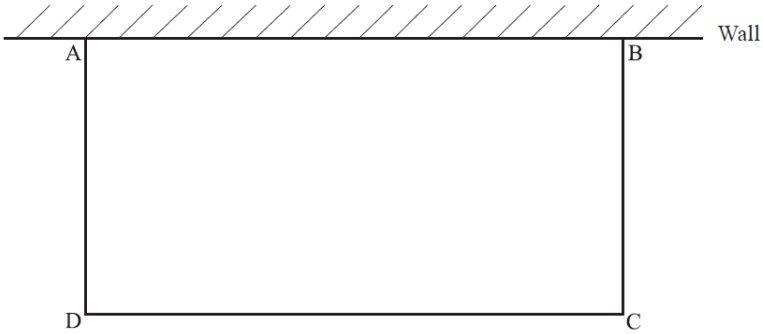
(ii) **Hence**, show that there is no point of inflexion on the graph of f at $x=0$.

(d) There is a point of inflexion on the graph of f at $x=\sqrt[4]{3}$ $(x=1.316\ldots)$. Sketch the graph of f, for $x\ge 0$.

Aplicación de la derivada en la resolución de Problemas

Nov 05 P1

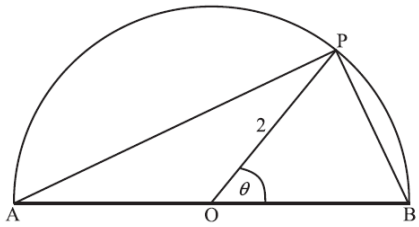
The following diagram shows a rectangular area ABCD enclosed on three sides by 60 m of fencing, and on the fourth by a wall AB.



Find the width of the rectangle that gives its maximum area.

Mayo 08 P1

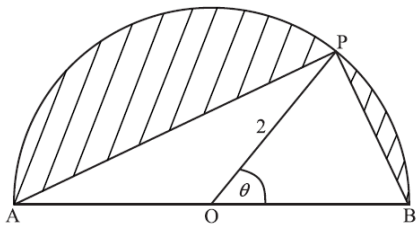
The following diagram shows a semicircle centre O, diameter [AB], with radius 2. Let P be a point on the circumference, with $P\hat{O}B = \theta$ radians.



(a) Find the area of the triangle OPB, in terms of θ.

(b) Explain why the area of triangle OPA is the same as the area triangle OPB.

Let S be the total area of the two segments shaded in the diagram below.



(c) Show that $S = 2(\pi - 2\sin\theta)$.

(d) Find the value of θ when S is a local minimum, justifying that it is a minimum.

(e) Find a value of θ for which S has its greatest value.

Mayo 10 TZ2 P2#7

El número n de bacterias que hay en un plato en el instante t minutos viene dado por $n = 800e^{0.13t}$.

(a) Halle el valor de n para $t = 0$.

(b) Halle la tasa de variación a la que está aumentando n en el instante $t = 15$.

(c) Transcurridos k minutos, la tasa de incremento de n es mayor que 10 000 bacterias por minuto. Halle el valor mínimo de k, donde $k \in \mathbb{Z}$.

Mayo 14 TZ1 P2#5

The population of deer in an enclosed game reserve is modelled by the function $P(t) = 210\sin(0.5t - 2.6) + 990$, where t is in months, and $t = 1$ corresponds to 1 January 2014.

(a) Find the number of deer in the reserve on 1 May 2014.

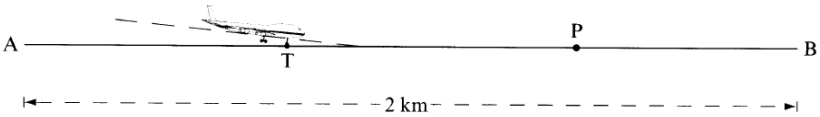
(b) (i) Find the rate of change of the deer population on 1 May 2014.

(ii) Interpret the answer to part (i) with reference to the deer population size on 1 May 2014.

Cinemática

Mayo 00 P2#2

The main runway at *Concordville* airport is 2 km long. An aeroplane, landing at *Concordville*, touches down at point T, and immediately starts to slow down. The point A is at the southern end of the runway. A marker is located at point P on the runway.



Not to scale

As the aeroplane slows down, its distance, s, from A, is given by

$$s = c + 100t - 4t^2,$$

where t is the time in seconds after touchdown, and c metres is the distance of T from A.

(a) The aeroplane touches down 800 m from A, (*i.e.* $c = 800$).

(i) Find the distance travelled by the aeroplane in the first 5 seconds after touchdown.

(ii) Write down an expression for the velocity of the aeroplane at time t seconds after touchdown, and hence find the velocity after 5 seconds.

The aeroplane passes the marker at P with a velocity of 36 m s^{-1}. Find

(iii) how many seconds after touchdown it passes the marker;

(iv) the distance from P to A.

(b) Show that if the aeroplane touches down before reaching the point P, it can stop before reaching the northern end, B, of the runway.

Nov 00 P2#2

A rock-climber slips off a rock-face and falls vertically. At first he falls freely, but after 2 seconds a safety rope slows him down. The height h metres of the rock-climber after t seconds of the fall is given by:

$$h = 50 - 5t^2, \qquad 0 \le t \le 2$$
$$h = 90 - 40t + 5t^2, \qquad 2 \le t \le 5$$

(a) Find the height of the rock-climber when $t = 2$.

(b) Sketch a graph of h against t for $0 \le t \le 5$.

(c) Find $\frac{dh}{dt}$ for:

(i) $0 \le t \le 2$

(ii) $2 \le t \le 5$

(d) Find the velocity of the rock-climber when $t = 2$.

(e) Find the times when the velocity of the rock-climber is zero.

(f) Find the minimum height of the rock-climber for $0 \le t \le 5$.

Mayo 04 TZ2 P1#7

El desplazamiento, s metros, de un coche a los t segundos, respecto a un punto fijo A, viene dado por $s = 4t + 5 - 5e^{-t}$.

(a) Halle una expresión para la velocidad (en ms^{-1}) a los t segundos.

(b) Halle la aceleración (en ms^{-2}) en A.

Nov 04 P1#9

El desplazamiento s metros de un coche, t segundos después de pasar por un punto fijo A, viene dado por

$$s = 10t - 0{,}5t^2.$$

(a) Calcule la velocidad cuando $t = 0$.

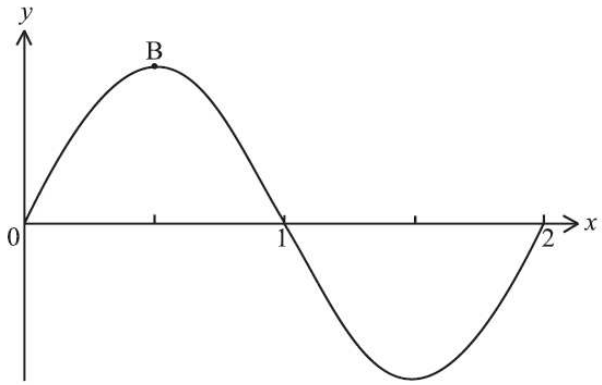
(b) Calcule el valor de t cuando la velocidad es cero.

(c) Calcule el desplazamiento del coche respecto a A cuando la velocidad es cero.

Actividades con uso de Calculadora Gráfica

Nov 05 P1

Let $f(x) = 6\sin \pi x$, and $g(x) = 6e^{-x} - 3$, for $0 \le x \le 2$. The graph of f is shown on the diagram below. There is a maximum value at B$(0.5, b)$.



(a) Write down the value of b.

(b) On the same diagram, sketch the graph of g.

(c) Solve $f(x) = g(x)$, for $0.5 \le x \le 1.5$.

Mayo 06 TZ2 P1#14

El desplazamiento s metros en un tiempo t segundos está dado por

$$s = 5\cos 3t + t^2 + 10, \text{ para } t \ge 0.$$

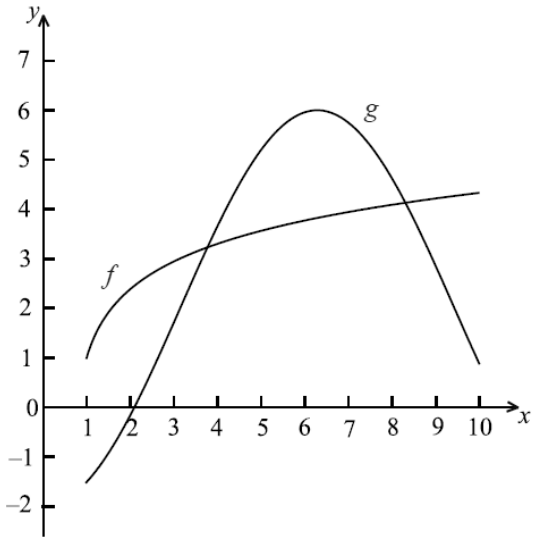
(a) Escriba el valor mínimo de s.

(b) Halle la aceleración, a, en el tiempo t.

(c) Halle el valor de t cuando el valor **máximo** de a ocurre por primera vez.

Mayo 08 P2

The following diagram shows the graphs of $f(x) = \ln(3x - 2) + 1$ and $g(x) = -4\cos(0.5x) + 2$, for $1 \le x \le 10$.



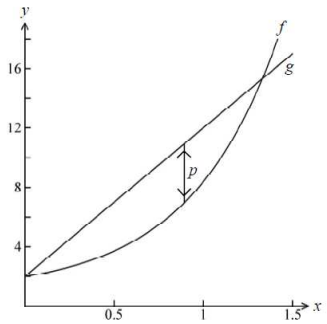
(b) (i) Find $f'(x)$.

(ii) Find $g'(x)$.

(c) There are two values of x for which the gradient of f is equal to the gradient of g. Find both these values of x.

Muestra 06 P2

The diagram below shows the graphs of $f(x)=1+e^{2x}$, $g(x)=10x+2$, $0\le x\le 1.5$

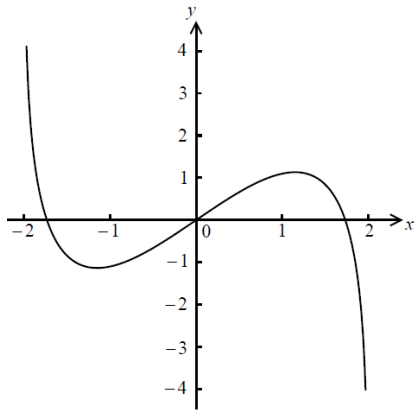


(a) (i) Write down an expression for the vertical distance p between the graphs of f and g.

(ii) Given that p has a maximum value for $0\le x\le 1.5$, find the value of x at which this occurs.

Mayo 10 TZ2 P2#10

Considere $f(x)=x\ln(4-x^2)$, para $-2<x<2$. A continuación se muestra la gráfica de f.



(a) Sean P y Q puntos pertenecientes a la curva de f en los cuales la tangente a la gráfica de f es paralela al eje x.

(i) Halle la coordenada x de P y de Q.

(ii) Considere $f(x)=k$. Escriba todos los valores de k para los cuales existen exactamente dos soluciones.

Sea $g(x)=x^3\ln(4-x^2)$, para $-2<x<2$.

(b) Compruebe que $g'(x)=\dfrac{-2x^4}{4-x^2}+3x^2\ln(4-x^2)$.

(c) Dibuje aproximadamente la gráfica de g'.

(d) Considere $g'(x)=w$. Escriba todos los valores de w para los cuales existen exactamente dos soluciones.

Nov 11 P2#10

Sea $f(x)=\dfrac{20x}{e^{0,3x}}$, para $0\le x\le 20$.

(a) Dibuje aproximadamente la gráfica de f.

(b) (i) Escriba la coordenada x del punto máximo en la gráfica de f.

(ii) Escriba el intervalo en el cual f es creciente.

(c) Compruebe que $f'(x)=\dfrac{20-6x}{e^{0,3x}}$.

(d) Halle el intervalo en el cual la razón de cambio de f es creciente.