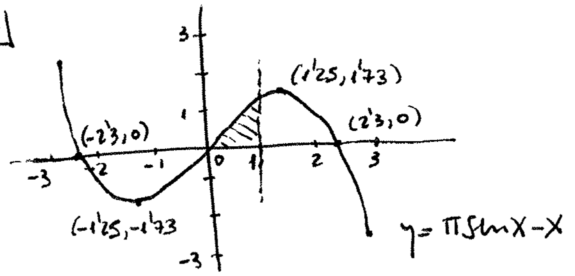
M00 P2#4

a)



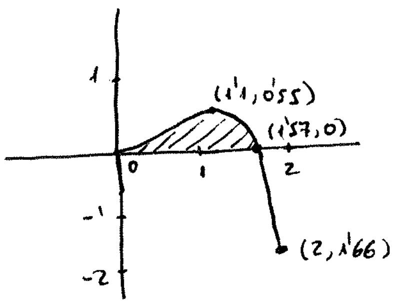
b) $\pi \operatorname{sen} x - x = 0 \Rightarrow \boxed{x = 2'31}$ Hecho con calculadora Gráfica

c) $\int (\pi \operatorname{sen} x - x)\,dx = -\pi \cos x - \frac{x^2}{2} + C$

$\int_0^1 (\pi \operatorname{sen} x - x)\,dx = \boxed{0'944}$

N00 P2#3

a)



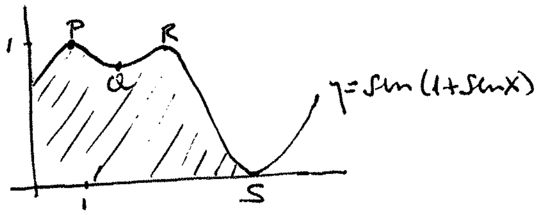
$y = x^2 \cos x$

b) $y = 0 \rightarrow x^2 \cos x = 0$ → $x = 0$; $\cos x = 0$; $\boxed{x = \pi/2}$

c) $\int_0^{\pi/2} x^2 \cos x\,dx = \boxed{0'467}$ Hecho con calculadora Gráfica

M01 P2#4

a)



Máximo P en $x = 0'6075$

Mínimo Q en $x = 1'571$

Máximo R en $x = 2'534$

Mínimo S en $x = 4'712$

b) $\int_0^{4'712} \operatorname{sen}(1+\operatorname{sen} x)\,dx = \boxed{3'517}$

c) $-1 \le \operatorname{sen} x \le 1 \Rightarrow 0 \le 1 + \operatorname{sen} x \le 2$

El intervalo $[0,2]$ está incluido en $[0,\pi] \Rightarrow \operatorname{sen}(1+\operatorname{sen} x) \ge 0$ ✓

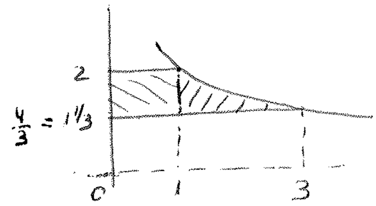
M00 P1

$f(x) = \int \cos x\,dx = \sin x + K$

$f(\pi/2) = -2 \Rightarrow -2 = \sin\frac{\pi}{2} + K$; $-2 = 1 + K$; $K = 3 \longrightarrow \boxed{f(x) = \sin x + 3}$

N00 P1

Área Rectángulo $= 1 \cdot (2 - 4/3) = \boxed{\frac{2}{3}}$



Área Total $= \frac{2}{3} + \int_1^3 \left[\left(1 + \frac{1}{x}\right) - \frac{4}{3}\right] dx = \frac{2}{3} + \left[-\frac{1}{3}x + \ln x\right]_1^3 = \frac{2}{3} + (-1 + \ln 3) - \left(-\frac{1}{3} + \ln 1\right) = \boxed{\ln 3}$

N00 P1

$f(x) = (2x+5)^3$

a) $f'(x) = 3(2x+5)^2 \cdot 2 = \boxed{6(2x+5)^2}$

b) $\int (2x+5)^3\,dx = \frac{1}{2}\int 2(2x+5)^3\,dx = \frac{1}{2}\,\frac{(2x+5)^4}{4} = \boxed{\frac{(2x+5)^4}{8} + K}$

N00 P1

$f(x) = \int (-2x+3)\,dx = -x^2 + 3x + K$

$(1,1) \in f \Rightarrow 1 = -1^2 + 3 \cdot 1 + K$; $K = -1 \Rightarrow \boxed{f(x) = -x^2 + 3x - 1}$

M01 P1

a) $\int \sin(3x+7)\,dx = \frac{1}{3}\int 3\sin(3x+7)\,dx = \boxed{-\frac{1}{3}\cos(3x+7) + K}$

b) $\int e^{-4x}\,dx = \frac{1}{-4}\int -4e^{-4x}\,dx = \boxed{-\frac{1}{4}e^{-4x} + K}$

N01 P2

a) $f(x) = x(x-a)(x-b)$

Corta al eje X en: $-3 \longrightarrow \boxed{a=-3}$, 0, $5 \longrightarrow \boxed{b=5}$ $\Rightarrow \boxed{f(x) = x(x+3)(x-5)}$

b) $f(x) = -x(x+3)(x-5) = -x^3 + 2x^2 + 15x$

$f'(x) = -3x^2 + 4x + 15$

$f'(x) = 0 \Rightarrow 3x^2 - 4x - 15 = 0$, $x = \frac{4 \pm \sqrt{16+180}}{6} = \frac{4 \pm 14}{6}$ → 3, $-5/3$

$Q(3, f(3))$ $f(3) = -3(3+3)(3-5) = \boxed{36}$

c) $x = 0 \to f(0) = 0$, $f'(0) = 15$ $y - 0 = 15(x - 0)$; $\boxed{y = 15x}$

$\left.\begin{aligned} y &= -x^3 + 2x^2 + 15x \\ y &= 15x \end{aligned}\right\}$ $\cancel{15x} = -x^3 + 2x^2 + \cancel{15x}$; $x^3 - 2x^2 = 0$; $x^2(x-2) = 0$; $x =$ 0, $\boxed{2}$

d) Área $= \int_0^5 (-x^3 + 2x^2 + 15x)\,dx = \left[-\frac{x^4}{4} + \frac{2x^3}{3} + \frac{15x^2}{2}\right]_0^5 = -\frac{625}{4} + \frac{250}{3} + \frac{375}{2} \simeq \boxed{115}$

N01
P2

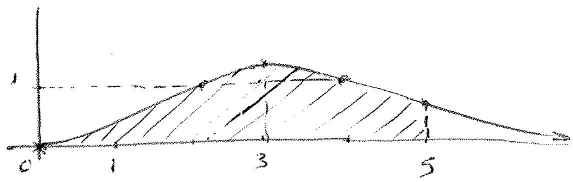
a) $f(x) = x^3 e^{-x}$

$f'(x) = 3x^2 e^{-x} + x^3 \cdot (-e^{-x}) = x^2(3-x)e^{-x}$

$f'(x) = 0 \Rightarrow x^2(3-x)e^{-x} = 0 \quad ; \quad x = \begin{cases} 0 \\ 3 \end{cases}$

	$-\infty$	0	3	$+\infty$
f'	+	+	−	
f	↗	↗	↘	

$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} x^3 e^{-x} = 0$



x	y	
0	0	
3	1'34	← Máximo relativo
5	0'84	
$+\infty$	0	← $\boxed{\text{Asíntota Horizontal } y=0}$ c)

b) $\int_0^5 x^3 e^{-x} dx = \left[-x^3 e^{-x}\right]_0^5 + \int_0^5 3x^2 e^{-x} dx = \left[-x^3e^{-x} - 3x^2 e^{-x}\right]_0^5 + \int_0^5 6xe^{-x} dx =$

$\boxed{\begin{matrix} u = x^3 \to du = 3x^2 dx \\ dv = e^{-x}dx \to v = -e^{-x} \end{matrix}}$ $\boxed{\begin{matrix} u = 3x^2 \to du = 6x dx \\ dv = e^{-x}dx \to v = -e^{-x} \end{matrix}}$ $\boxed{\begin{matrix} u = 6x \to du = 6 dx \\ dv = e^{-x}dx \to v = -e^{-x} \end{matrix}}$

$= \left[-x^3e^{-x} - 3x^2e^{-x} - 6xe^{-x}\right] + \int_0^5 6e^{-x}dx = \left[-x^3e^{-x} - 3x^2e^{-x} - 6xe^{-x} - 6e^{-x}\right]_0^5 =$

$= -\left[(x^3 + 3x^2 + 6x + 6)e^{-x}\right]_0^5 = -236e^{-5} + 6 = 6 - \frac{236}{e^5} = \boxed{4'41}$

También se podía haber hecho con calculadora gráfica

d) $\left.\begin{matrix} y = 1 \\ y = x^3 e^{-x} \end{matrix}\right\} \quad x^3 e^{-x} = 1 \longrightarrow \boxed{x = \begin{cases} 1'86 \\ 4'54 \end{cases}}$ Hecho con calculadora gráfica

e) $\boxed{\text{Máximo } (3, 1'34)}$

N02
P2#3

$y = \operatorname{sen}(e^x)$

a) $x = 0 \to y = \operatorname{sen}(e^0) = \operatorname{sen} 1 \simeq 0'841$ $\quad \boxed{A(0, 0'841)}$

b) $y = 0 \to \operatorname{sen}(e^x) = 0 \Rightarrow e^x = \pi \Rightarrow x = \ln \pi$ $\quad \boxed{K = \pi}$

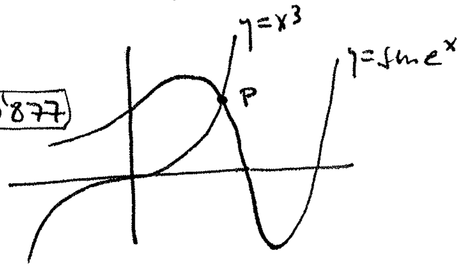
c) El máximo valor de la función seno es $\boxed{1}$:

$y' = e^x \cdot \cos(e^x)$

$y' = 0 \to e^x \cdot \cos(e^x) = 0 \begin{cases} \to e^x = 0 \ \text{✗} \\ \to \cos(e^x) = 0 \to e^x = \pi/2 \Rightarrow \boxed{x = \ln \pi/2} \end{cases}$

d) $\text{Área} = \int_0^{\ln \pi} \operatorname{sen}(e^x)\, dx = \boxed{0'906}$ Hecho con calculadora Gráfica

e) $\left.\begin{matrix} y = x^3 \\ y = \operatorname{sen} e^x \end{matrix}\right\} \quad x^3 = \operatorname{sen} e^x \Rightarrow \boxed{x = 0'877}$



Hecho con Calculadora Gráfica

Nov 01 P2

a) $f(x) = -0,5x^2 + 2x + 2,5$

$f'(x) = -x + 2$

$f'(0) = 2$

b) $x = 0 \to f(0) = 2'5$

$\to f'(0) = 2 \to$ pendiente recta normal $= -\frac{1}{2}$

Recta Normal: $y - 2'5 = -\frac{1}{2}x$; $y = 2'5 - \frac{x}{2}$: $\boxed{y = \frac{5-x}{2}}$

c) $\left.\begin{array}{l} y = -0'5x^2 + 2x + 2'5 \\ y = \frac{5-x}{2} \end{array}\right\}$ $-0'5x^2 + 2x + 2'5 = \frac{5-x}{2}$; $-x^2 + 4x + 5 = 5 - x$; $-x^2 + 5x = 0$

$\boxed{x = \begin{cases} 0 \\ 5 \end{cases}}$

d) $x = 5 \to y = -0'5 \cdot 5^2 + 2 \cdot 5 + 2'5 = 0$; $\boxed{P(5,0)}$

e) $$\text{Area} = \int_0^5 \left| \frac{5-x}{2} - (-0'5x^2 + 2x + 2'5) \right| dx$$

f) $$\text{Area} = \int_0^5 \left(\cancel{\frac{5}{2}} - \frac{x}{2} + 0'5x^2 - 2x - \cancel{2'5}\right) dx =$$

$$= \int_0^5 -\frac{1}{2}(x - x^2 + 4x)\,dx = -\frac{1}{2}\int_0^5 (-x^2 + 5x)\,dx = -\frac{1}{2}\left[-\frac{x^3}{3} + \frac{5x^2}{2}\right]_0^5 = -\frac{1}{2}\left(\frac{125}{3} + \frac{125}{2}\right) = \boxed{\frac{125}{12}} = \boxed{10'4}$$

Nov 01 P1

$f(x) = 2\,\text{sen}(5x-3)$

a) $f'(x) = 2\cos(5x-3) \cdot 5 = \boxed{10\cos(5x-3)}$

b) $\int f(x)\,dx = \int 2\,\text{sen}(5x-3)\,dx = \frac{2}{5}\int 5\,\text{sen}(5x-3)\,dx = \boxed{-\frac{2}{5}\cos(5x-3) + K}$

M02 P2

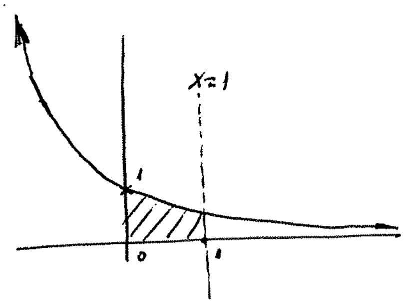
a) $$\int_0^1 e^{-kx}\,dx = \frac{1}{-k}\left[e^{-kx}\right]_0^1 = -\frac{1}{k}\left[e^{-k} - e^0\right] = -\frac{1}{k}\left(e^{-k} - 1\right) = \boxed{\frac{1 - e^{-k}}{k}}$$ ✓

b) $y = e^{-0'5x}$

$x = 0 \to y = e^0 = 1$

$\lim_{x \to -\infty} e^{-0'5x} = e^{+\infty} = +\infty$

$\lim_{x \to +\infty} e^{-0'5x} = e^{-\infty} = \frac{1}{e^{+\infty}} = \frac{1}{+\infty} = 0$



$$\text{Area} = \int_0^1 e^{-0'5x}\,dx = \frac{1 - e^{-0'5}}{0'5} = \boxed{2(1 - e^{-0'5})} \simeq 0'787$$

c) $y = e^{-kx} \to \boxed{y' = -k e^{-kx}}$

$P(1, 0'8) \to 0'8 = e^{-k} \Rightarrow -k = \ln 0'8$; $\boxed{K = -\ln 0'8} = \boxed{0'223}$

$x = 1 \to y' = -k \cdot e^{-k} = -\ln 0'8 \cdot 0'8 = \boxed{-0'8 \ln 0'8} = \boxed{-0'179}$

M02 P1

$f'(x) = \frac{1}{x+1} - 0'5\,\mathrm{sen}\,x$

$f(x) = \int\left(\frac{1}{x+1} - 0'5\,\mathrm{sen}\,x\right)dx = \ln|x+1| + 0'5\cos x + K$

$P(0,2) \in f \Rightarrow \quad 2 = \ln(0+1) + 0'5\cos 0 + K \;;\quad 2 = 0 + 0'5 + K \;;\quad K = 1'5$

$\boxed{f(x) = \ln|x+1| + 0'5\cos x + 1'5}$

NOV 02 P1

$\text{Area} = \int_0^{3\pi/4} \mathrm{sen}\,x\,dx = \left[-\cos x\right]_0^{3\pi/4} = -\cos\frac{3\pi}{4} + \cos 0 = +\frac{\sqrt{2}}{2} + 1 = \boxed{1+\frac{\sqrt{2}}{2}}$

NOV 02 P1

$f(x) = \sqrt{x^3}$

a) $f'(x) = \frac{1}{2\sqrt{x^3}} \cdot 3x^2 = \frac{3x^2}{2\sqrt{x^3}} = \frac{3x}{2\sqrt{x}} = \boxed{\frac{3\sqrt{x}}{2}}$

También: $f(x) = x^{3/2} \longrightarrow f'(x) = \frac{3}{2}x^{\frac{3}{2}-1} = \frac{3}{2}x^{1/2} = \frac{3\sqrt{x}}{2}$ ✓

b) $\int f(x)dx = \int\sqrt{x^3}\,dx = \int x^{3/2}\,dx = \frac{x^{3/2+1}}{3/2+1} = \frac{x^{5/2}}{5/2} = \boxed{\frac{2\sqrt{x^5}}{5} + K}$

M03 P1

$\int_1^3 g(x)dx = 10 \longrightarrow$ a) $\int_1^3 \frac{1}{2}g(x)dx = \frac{1}{2}\int_1^3 g(x)dx = \frac{1}{2}\cdot 10 = \boxed{5}$

b) $\int_1^3 (g(x)+4)\,dx = \int_1^3 g(x)dx + \left[4x\right]_1^3 = 10 + 12 - 4 = \boxed{18}$

NOV 03 P2

$f(x) = 1 + e^{-2x}$

a) $f'(x) = \boxed{-2e^{-2x}}$

$f'(x) = 0 \Rightarrow -2e^{-2x} = 0 \;;\quad \frac{-2}{e^{2x}} = 0 \;;\quad -2 = 0$ $f'(x)$ no se anula.

Al ser $f'(x)$ una función continua, no anularse significa que su signo es siempre el mismo, como $e^{-2x} > 0 \Rightarrow f'(x) = -2e^{-2x} \leq 0$

Por lo que $\boxed{f(x) \text{ es siempre decreciente}}$

b) $x = -\frac{1}{2} \rightarrow y = 1 + e^{-2\cdot\frac{-1}{2}} = \boxed{1+e}$

$\rightarrow y' = \boxed{-2e}$

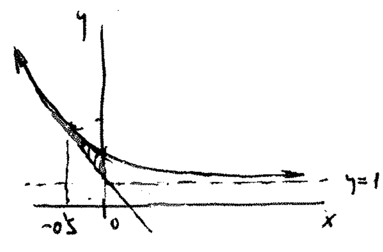
c) $y - (1+e) = -2e\left(x + \frac{1}{2}\right) \;;\quad y = 1 + \cancel{e} - 2ex - \cancel{e}$ $\quad\boxed{y = -2ex + 1}$

d) $y = 1 + e^{-2x}$

$x = 0 \rightarrow y = 1 + e^0 = 2$

$\lim_{x\to-\infty}(1+e^{-2x}) = 1 + e^{+\infty} = +\infty$

$\lim_{x\to+\infty}(1+e^{-2x}) = 1 + e^{-\infty} = 1 + 0 = 1$



$\text{Area} = \int_{-0'5}^{0}\left[(1+e^{-2x}) - (-2ex+1)\right]dx = \int_{-0'5}^{0}(\cancel{1} + e^{-2x} + 2ex - \cancel{1})\,dx =$

$= \left[\frac{1}{-2}e^{-2x} + ex^2\right]_{-0'5}^{0} = \left(-\frac{1}{2}\right) - \left(-\frac{e}{2} + \frac{e}{4}\right) = -\frac{1}{2} + \frac{e}{2} - \frac{e}{4} = \frac{-2+2e-e}{4} = \boxed{\frac{e-2}{4}} \approx 0'180$

N03 P1

a) $\int (1+3\operatorname{sen}(x+2))\,dx = \boxed{x-3\cos(x+2)+k}$

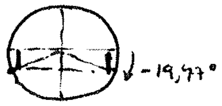
b) $f(x)=0 \longrightarrow 1+3\operatorname{sen}(x+2)=0$; $3\operatorname{sen}(x+2)=-1$; $\operatorname{sen}(x+2)=-\frac{1}{3}$

$$x+2=\begin{cases}-0'3398+N\cdot 2\pi\\ 3'4814+N\cdot 2\pi\end{cases} \Rightarrow x=\begin{cases}-2-0'3398+N\cdot 2\pi\\ -2+3'4814+N\cdot 2\pi\end{cases} \Rightarrow x=\begin{cases}-2'3398+N\cdot 2\pi\\ 1'4814+N\cdot 2\pi\end{cases}$$

$\boxed{a=1'48}$

También se podía haber hecho en grados para después pasarlo a radianes:

$$\operatorname{sen}(x+2)=-\frac{1}{3} \longrightarrow x+2=\begin{cases}-19'47^\circ\\ 199,47^\circ \equiv 199,47\cdot\frac{\pi}{180}=3'4814\end{cases} \longrightarrow x=3'4814-2=\boxed{1'48}$$



N03 P1

$\frac{dy}{dx}=x^3+2x-1$; $y=\int (x^3+2x-1)\,dx = \frac{x^4}{4}+x^2-x+k$

$\left.\begin{matrix}x=2\\ y=13\end{matrix}\right| \longrightarrow 13=\frac{16}{4}+4-2+k$; $k=7$ $\quad \boxed{y=\frac{x^4}{4}+x^2-x+7}$

M04 P1

a) $\frac{d}{dx}(3x^4-5x+1)=\boxed{12x^3-5}$

b) $\int (3x^4-5x+1)\,dx = \boxed{\frac{3x^5}{5}-\frac{5x^2}{2}+x+k}$

M04 P2

a) $y=2+\frac{1}{x-1}$ $\qquad$ $\operatorname{dom} f=\mathbb{R}-\{1\}$

b) $\lim_{x\to 1^-} f(x)=\lim_{x\to 1^-}\left(2+\frac{1}{x-1}\right)=2+\frac{1}{-0}=2-\infty=-\infty$

$\lim_{x\to 1^+} f(x)=\lim_{x\to 1^+}\left(2+\frac{1}{x-1}\right)=2+\frac{1}{+0}=2+\infty=+\infty$

$\longrightarrow$ Asíntota vertical $x=1$

$\lim_{x\to -\infty} f(x)=\lim_{x\to -\infty}\left(2+\frac{1}{x-1}\right)=2+\frac{1}{-\infty}=2+0=2$

$\lim_{x\to +\infty} f(x)=\lim_{x\to +\infty}\left(2+\frac{1}{x-1}\right)=2+\frac{1}{+\infty}=2+0=2$

$\longrightarrow$ Asíntota Horizontal $y=2$

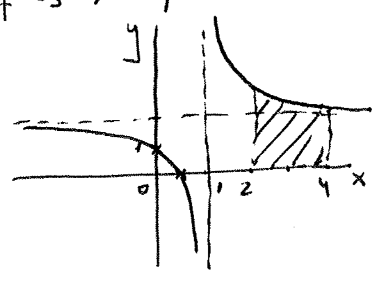
$x=0 \longrightarrow y=2+\frac{1}{-1}=1$ $\quad\boxed{(0,1)}$

$y=0 \longrightarrow 0=2+\frac{1}{x-1}$; $-2=\frac{1}{x-1}$; $-2x+2=1$; $-2x=-1$; $x=\frac{1}{2}$ $\quad\boxed{\left(\frac{1}{2},0\right)}$

c) $\boxed{f'(x)=\frac{-1}{(x-1)^2}}$

	$-\infty$	1	$+\infty$
f'	$-$		$-$
f	$\searrow$		$\searrow$

$f'(x)$ es siempre negativo, por lo que f es siempre decreciente, por lo que no puede tener extremos.



d) $\text{Area}=\int_2^4\left(2+\frac{1}{x-1}\right)dx=\left[2x+\ln(x-1)\right]_2^4=$

$=(8+\ln 3)-(4+\ln 1)=\boxed{4+\ln 3}=$

$=\boxed{5'10}$

N04
P2#5ii

$h(x)=(x-2)\,\text{sen}(x-1) \qquad (-5\le x\le 5)$

a) $y=0 \to (x-2)\,\text{sen}(x-1)=0$ → $x-2=0$; $x=2$ ✓
→ $\text{sen}(x-1)=0$; $x-1=\begin{cases}0 \to x=1 ✓\\ \pi \to b=\boxed{\pi+1}\\ -\pi \to a=\boxed{1-\pi}\end{cases}$

b) $\text{Area}=\int_{1-\pi}^{1}(x-2)\,\text{sen}(x-1)\,dx-\int_{1}^{2}(x-2)\,\text{sen}(x-1)\,dx = 5'141-(-0'1585)=\boxed{5'30}$

c) Ordenada de $S=\boxed{0'973}$

$(x-2)\,\text{sen}(x-1)=k$
con cuatro soluciones : $\boxed{k\in(-0'240,\ 0'973)}$ Hecho todo con calculadora gráfica

M05
P1#15

a) $\frac{d}{dx}(f(x)+g(x))=f'(x)+g'(x)=7+4=\boxed{11}$ en $x=4$

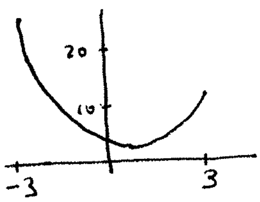
b) $\int_1^3 (g'(x)+6)\,dx = \Big[g(x)+6x\Big]_1^3=(g(3)+18)-(g(1)+6)=$
$=(2+18)-(1+6)=\boxed{13}$

M04
TZ2
P2#2

$f'(x)=e^x+x-5$

a) $f(x)=\int(e^x+x-5)\,dx=e^x+\frac{x^2}{2}-5x+C$
$f(1)=e-2 \to \cancel{e}+\frac12-5+C=\cancel{e}-2$; $C=\frac52$ ⟹ $\boxed{f(x)=e^x+\frac{x^2}{2}-5x+\frac52}$

b)



c) Mínimo en $(1'31,\ 0'514)$

d) $\int_0^2\left(e^x+\frac{x^2}{2}-5x+\frac52\right)dx=\boxed{2'72}$

M05
P1#6

a) $f(x)=(3x+4)^5 \longrightarrow f'(x)=5(3x+4)^4\cdot 3=\boxed{15(3x+4)^4}$

b) $\int(3x+4)^5\,dx=\frac13\,\frac{(3x+4)^6}{6}=\boxed{\frac18(3x+4)^6+C}$

M05
P1#8

$\frac{dy}{dx}=3x^2-5 \longrightarrow y=\int(3x^2-5)\,dx=x^3-5x+C$
$\left.\begin{matrix}x=2\\y=6\end{matrix}\right| \longrightarrow 6=8-10+C$; $C=8$ ⟹ $\boxed{y=x^3-5x+8}$

N05
P2#5ii

$\int_0^k \text{sen}\,2x\,dx=\left[-\frac12\cos 2x\right]_0^k=-\frac12\cos 2k+\frac12=\frac{1-\cos 2k}{2}$

$\frac{1-\cos 2k}{2}=0'85$; $1-\cos 2k=1'7$; $\cos 2k=-0'7 \Rightarrow 2k=\begin{cases}2'346 \to k=\boxed{1'17}\\ 3'937 \to k=\cancel{1'97}\end{cases}$
porque es $>\pi/2$

N04 P1

$$\int_0^3 f(x)\,dx = 8$$

a) $\int_0^3 2f(x)\,dx = 2\int_0^3 f(x)\,dx = 2\cdot 8 = \boxed{16}$

$$\int_0^3 (f(x)+2)\,dx = \int_0^3 f(x)dx + \int_0^3 2\,dx = 8 + [2x]_0^3 = 8+6 = \boxed{14}$$

b) $t = x-2 \rightarrow dt = dx$ $\qquad x = c \rightarrow t = c-2$

$\qquad x = d \rightarrow t = d-2$

$$\int_c^d f(x-2)\,dx = 8 \rightarrow \int_{c-2}^{d-2} f(t)\,dt = 8 \rightarrow \begin{cases} c-2=0 \\ d-2=3 \end{cases} \Rightarrow \boxed{\begin{matrix} c=2 \\ d=5 \end{matrix}}$$

N04 P1

$$f'(x) = e^{-2x} + \frac{1}{1-x}$$

$$f(x) = \int\left(e^{-2x} + \frac{1}{1-x}\right)dx = \frac{1}{-2}e^{-2x} - \ln(1-x) + K$$

$P(0,4) \in f(x) \Rightarrow 4 = -\frac{1}{2}e^0 - \ln 1 + K$; $4 = -\frac{1}{2} + K$; $K = \frac{9}{2}$ $\Rightarrow \boxed{f(x) = -\frac{1}{2}e^{-2x} - \ln(1-x) + \frac{9}{2}}$

M05 P2

$$f(x) = \frac{1}{2}\sin 2x + \cos x \qquad (0 \le x \le 2\pi)$$

a) $f'(x) = \frac{1}{2}\cdot\cos 2x\cdot 2 - \sin x = \boxed{\cos 2x - \sin x} = \cos^2 x - \sin^2 x - \sin x = 1 - \sin^2 x - \sin^2 x - \sin x =$

$= -2\sin^2 x - \sin x + 1$

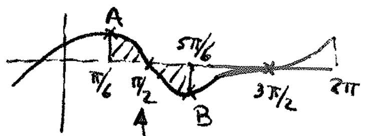
$2\sin^2 x + \sin x - 1 = 0$

$\sin x = \frac{-1 \pm \sqrt{1+8}}{4} = \frac{-1\pm 3}{4}$ $\begin{matrix} \nearrow 1/2 \\ \searrow -1 \end{matrix}$ $\rightarrow 2\sin^2 x + \sin x - 1 = 2\left(\sin x - \frac{1}{2}\right)(\sin x + 1)$

$f'(x) = 0 \Rightarrow \sin x = 1/2 \Rightarrow x = 30° \equiv \pi/6$ rad ; $150° = 5\pi/6$ rad

$\qquad \sin x = -1 \Rightarrow x = 270° = 3\pi/2$ rad

	0 — π/6	π/6 — 5π/6	5π/6 — 3π/2	3π/2 — 2π
f'	+	−	+	+
f	↗	↘	↗	↗



b) Máximo Relativo en A: $\boxed{x = \pi/6}$

c) $f(x) = 0 \rightarrow \frac{1}{2}\sin 2x + \cos x = 0$

$\frac{1}{2}\cdot 2\sin x\cos x + \cos x = 0 \rightarrow \cos x(\sin x + 1) = 0 \rightarrow \cos x = 0 \Rightarrow x = 90° = \frac{\pi}{2}$; $\sin x = -1 \rightarrow 270° = \frac{3\pi}{2}$

$$\text{Area} = \int_{\pi/6}^{\pi/2}\left(\frac{1}{2}\sin 2x + \cos x\right)dx - \int_{\pi/6}^{\pi/2}\left(\frac{1}{2}\sin 2x + \cos x\right)dx =$$

$$= \left[-\frac{1}{4}\cos(2x) + \sin x\right]_{\pi/6}^{\pi/2} - \left[-\frac{1}{4}\cos(2x) + \sin x\right]_{\pi/2}^{5\pi/6} =$$

$$= \left(-\frac{1}{4}\cos\pi + \sin\frac{\pi}{2}\right) - \left(-\frac{1}{4}\cos\frac{\pi}{3} + \sin\frac{\pi}{6}\right) - \left(-\frac{1}{4}\cos\frac{5\pi}{3} + \sin 5\pi/6\right) + \left(-\frac{1}{4}\cos\pi + \sin\frac{\pi}{2}\right) =$$

$$= -\frac{1}{4}\cdot(-1) + 1 + \frac{1}{4}\cdot\frac{1}{2} - \frac{1}{2} + \frac{1}{4}\cdot\frac{1}{2} - \frac{1}{2} - \frac{1}{4}\cdot(-1) + 1 = \boxed{\frac{7}{4}}$$

N05 P1#8

$$\int_3^K \frac{1}{x-2}dx = [\ln(x-2)]_3^K = \ln(K-2) - \ln 1 = \ln(K-2) = \ln 7 \Rightarrow \boxed{K=9}$$

M05 P1

a) $\frac{d}{dx}(f(x)+g(x))$ en $x=4$ $= f'(4)+g'(4) = 7+4 = \boxed{11}$

b) $\int_1^3 (g'(x)+6)\,dx = \left[g(x)+6x\right]_1^3 = (g(3)+18) - (g(1)+6) = (2+18)-(1+6) = \boxed{13}$

M05 P1

$\frac{dy}{dx} = 3x^2-5 \Rightarrow y = \int (3x^2-5)\,dx = x^3-5x+K$

$P(2,6) \in f(x) \Rightarrow 6 = 2^3-5\cdot 2+K$; $K=8$ $\boxed{y = x^3-5x+8}$

M05 P1

$f(x) = (3x+4)^5$

a) $f'(x) = 5\cdot(3x+4)^4\cdot 3 = \boxed{15(3x+4)^4}$

b) $\int (3x+4)^5\,dx = \frac{1}{3}\frac{(3x+4)^6}{6} = \boxed{\frac{(3x+4)^6}{18}+K}$

N05 P1

$\int_0^K \text{sen}\, 2x\,dx = 0'85 \rightarrow \left[-\frac{1}{2}\cos(2x)\right]_0^K = 0'85$; $-\frac{1}{2}\cos(2K) + \frac{1}{2}\cos 0 = 0'85$

$-\frac{1}{2}\cos(2K) + \frac{1}{2} = 0'85$

$-\cos(2K)+1 = 1'7$

$\cos(2K) = -0'7$

$\cos^{-1}(-0'7) = 2'3426$ rad

$2K = 2'3462 \Rightarrow \boxed{K=1'17}$

N05 P1

$\int_3^K \frac{1}{x-2}\,dx = \ln 7 \rightarrow \left[\ln(x-2)\right]_3^K = \ln 7$

$\ln(K-2) - \ln 1 = \ln 7$; $\ln(K-2) = \ln 7 \Rightarrow \boxed{K=9}$

M06 P2

$f(x) = -\frac{3}{4}x^2+x+4$

a) $f'(x) = -\frac{3}{4}\cdot 2x+1 = \boxed{-\frac{3}{2}x+1}$

$(2,3)$: $x=2 \rightarrow y=3$

$\rightarrow y' = -\frac{3}{2}\cdot 2+1 = -2 \rightarrow$ pendiente normal $= \frac{-1}{-2} = \frac{1}{2}$

$\boxed{y-3 = \frac{1}{2}(x-2)}$

$y-3 = \frac{x}{2}-1 \rightarrow \boxed{y = \frac{x+4}{2}}$

$\left.\begin{array}{l} y = -\frac{3}{4}x^2+x+4 \\ y = \frac{x+4}{2} \end{array}\right\} \rightarrow \frac{x+4}{2} = \frac{-3x^2}{4}+x+4$

$2x+8 = -3x^2+4x+16$

$3x^2-2x-8 = 0$

$x = \frac{2\pm\sqrt{4+96}}{6} = \frac{2\pm 10}{6}$ $\begin{array}{l} 2 \\ -4/3 \rightarrow y = \frac{-4/3+4}{2} = \frac{4}{3} \end{array}$ $\boxed{P(-4/3, 4/3)}$

b) Area $= \int_{-1}^2 \left(-\frac{3}{4}x^2+x+4\right)dx = \left[-\frac{x^3}{4}+\frac{x^2}{2}+4x\right]_{-1}^2 = \left(-\frac{8}{4}+\frac{4}{2}+8\right) - \left(\frac{1}{4}+\frac{1}{2}-4\right) = \boxed{11'25}$

$\boxed{\text{Volumen} = \pi\int_{-1}^2 \left(-\frac{3}{4}x^2+x+4\right)^2 dx}$

c) $\int_1^K f(x)\,dx = \left[-\frac{x^3}{4}+\frac{x^2}{2}+4x\right]_1^K = \left(-\frac{K^3}{4}+\frac{K^2}{2}+4K\right) - \left(-\frac{1}{4}+\frac{1}{2}+4\right) = \boxed{-\frac{K^3}{4}+\frac{K^2}{2}+4K-\frac{17}{4}}$

M06 P2

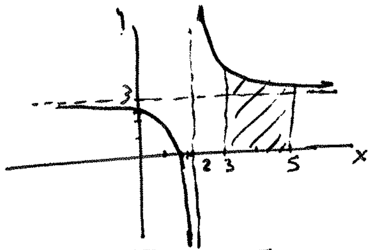
a) $f(x) = 3x-5$

$y = 3x-5 \;;\; x = 3y-5 \;;\; x+5 = 3y \;;\; y = \frac{x+5}{3}$ $\quad \boxed{f^{-1}(x) = \frac{x+5}{3}}$

b) $\left.\begin{array}{l} g^{-1}(x) = x+2 \\ f(x) = 3x-5 \end{array}\right\}$ $\quad (g^{-1}\circ f)(x) = g^{-1}(f(x)) = g^{-1}(3x-5) = 3x-5+2 = \boxed{3x-3}$

c) $(f^{-1}\circ g)(x) = (g^{-1}\circ f)(x) \Rightarrow \frac{x+3}{3} = 3x-3 \;;\; x+3 = 9x-9 \;;\; \boxed{x = \frac{3}{2}}$

d) $h(x) = \frac{3x-5}{x-2}$



Asíntota Vertical $x=2$
Asíntota Horizontal $y=3$

e) $\int \frac{3x-5}{x-2}\,dx = \int \left(3 + \frac{1}{x-2}\right)dx = \boxed{3x + \ln(x-2) + K}$

$\int_3^5 h(x)\,dx = \left[3x + \ln(x-2)\right]_3^5 = (15 + \ln 3) - (9 + \ln 1) = \boxed{6 + \ln 3}$

M06 P1

a) $\text{Volumen} = \pi \int_0^2 (2x-x^2)^2\,dx$

b) $V = \pi \int_0^2 (4x^2 - 4x^3 + x^4)\,dx = \pi \left(\frac{4x^3}{3} - \frac{\cancel{4}x^4}{\cancel{4}} + \frac{x^5}{5}\right)_0^2 = \pi\left(\frac{32}{3} - 16 + \frac{32}{5}\right) = \boxed{\frac{16\pi}{15}}$

Muestra 06/08 P2

$f(x) = -0'5x^2 + 2x + 2'5$

a) $f'(x) = \boxed{-x+2}$

$f'(0) = \boxed{2}$

b) $x=0 \rightarrow y = 2'5$
$\quad \hookrightarrow y' = 2 \rightarrow$ pendiente normal $= -\frac{1}{2}$ $\quad y - 2'5 = -\frac{1}{2}x \;;\; \boxed{y = -0'5x + 2'5}$ ✓

c) $f(x) = g(x) \Rightarrow -0'5x^2 + 2x + \cancel{2'5} = -0'5x + \cancel{2'5}$

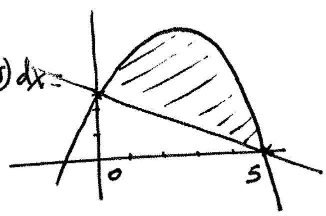
$0 = 0'5x^2 - 2'5x$

$0 = 0'5x(x-5) \Rightarrow x = \begin{cases} 0 \\ 5 \end{cases} \Rightarrow y = -0'5\cdot 5 + 2'5 = 0 \quad \boxed{P(5,0)}$

d)

$\text{Area} = \int_0^5 \left[(-0'5x^2 + 2x + 2'5) - (-0'5x + 2'5)\right]dx =$

$= \int_0^5 (-0'5x^2 + 2'5x)\,dx$



$\text{Area} = \left[-0'5\frac{x^3}{3} + 2'5\frac{x^2}{2}\right]_0^5 = -0'5\,\frac{125}{3} + 2'5\,\frac{25}{2} = \boxed{\frac{125}{12}}$

Muestra 06/08 P1

$f(x) = 2\sin(5x-3)$

a) $f'(x) = 10\cos(5x-3)$

$\boxed{f''(x) = -50\sin(5x-3)}$

b) $\int 2\sin(5x-3)\,dx = \boxed{-\frac{2}{5}\cos(5x-3) + K}$

M07 TZ2 P2#5

$$f(x) = p - \frac{3x}{x^2-q^2}$$

a) $\lim_{x\to q} f(x) = p - \frac{3q}{0} = \infty \Rightarrow$ Asíntota vertical $x=q \Rightarrow \boxed{q=1}$

$\lim_{x\to\infty} f(x) = p - 0 = p \Rightarrow$ Asíntota Horizontal $y=p \Rightarrow \boxed{p=2}$

b) $f(x) = 2 - \frac{3x}{x^2-1}$

$y=0 \Rightarrow 2 - \frac{3x}{x^2-1} = 0$; $2 = \frac{3x}{x^2-1}$; $2x^2-2=3x$; $2x^2-3x-2=0$;

$x = \frac{3\pm\sqrt{9+16}}{4} = \frac{3\pm5}{4}$ ↗ $2 \rightarrow (2,0)$ ✓ ↘ $-1/2 \rightarrow \boxed{(-1/2,0)}$

$V = \pi\int_{-1/2}^{0}\left(2-\frac{3x}{x^2-1}\right)^2 dx = \boxed{2'52}$ Hecho con calculadora gráfica

c) $f'(x) = 0 - \frac{3(x^2-1)-3x\cdot 2x}{(x^2-1)^2} = -\frac{3x^2-3-6x^2}{(x^2-1)^2} = \frac{3x^2+3}{(x^2-1)^2} = \frac{3(x^2+1)}{(x^2-1)^2}$ ✓

$f'(x)=0 \rightarrow \frac{3(x^2+1)}{(x^2-1)^2}=0$; $3(x^2+1)=0$; $x^2+1=0$; $x^2=-1$ ∄ No tiene extremos locales ✓

d) $g(x) = \frac{3(x^2+1)}{(x^2-1)^2}$

Como $g(x)>0$ para todo x de su dominio :

Area $A = \int_0^a \frac{3(x^2+1)}{(x^2-1)^2}dx = \left[2-\frac{3x}{x^2-1}\right]_0^a = \left(2-\frac{3a}{a^2-1}\right)-(2-0) = -\frac{3a}{a^2-1}$

Area $A=2 \Rightarrow 2 = \frac{-3a}{a^2-1}$; $2a^2-2=-3a$; $2a^2+3a-2=0$;

$a = \frac{-3\pm\sqrt{9+16}}{4} = \frac{-3\pm5}{4}$ ↗ $\boxed{1/2}$ ↘ $\cancel{-2}$ porque $a>0$

M07 TZ1 P2#1B

$f(x) = e^x \sin x$

a) $y=0 \rightarrow e^x\sin x = 0$ ↗ $e^x=0$ ∄ ↘ $\sin x=0$; $x = \begin{cases}0\\ \boxed{\pi}\end{cases}$ $C(\pi,0)$

b) $f'(x) = e^x\sin x + e^x\cos x = e^x(\sin x+\cos x)$

En el punto B , $\boxed{f'(x)=0}$

c) $f''(x) = e^x(\sin x+\cos x) + e^x(\cos x-\sin x) = 2e^x\cos x$ ✓

d) En el punto A , $\boxed{f''(x)=0}$

$f''(x)=0 \Rightarrow 2e^x\cos x=0$ ↗ $e^x=0$ ∄ ↘ $\cos x=0$; $x = \begin{cases}\boxed{\pi/2}\\ \cancel{3\pi/2}\end{cases}$ porque estaría a la dcha de C $A(\pi/2, e^{\pi/2})$

e) Area $= \int_0^\pi e^x\sin x\,dx = \boxed{12'1}$ Hecho con calculadora gráfica

Muestra 06 P2#3A

a) $p = (10x+2)-(1+e^{2x})$

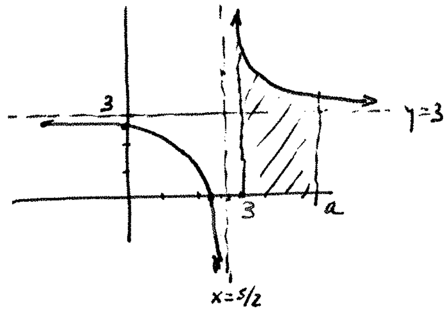
$p' = 10-2e^{2x}$; $p'=0 \Rightarrow e^{2x}=5$; $2x=\ln 5$; $x = \boxed{\frac{1}{2}\ln 5}$

b) $x = 1+e^{2y}$; $x-1=e^{2y}$; $2y=\ln(x-1)$ $\boxed{y=\frac{1}{2}\ln(x-1)}$; $\boxed{f^{-1}(x)=\frac{1}{2}\ln(x-1)}$

$f^{-1}(5) = \frac{1}{2}\ln(5-1) = \frac{1}{2}\ln 4 = \frac{1}{2}\ln 2^2 = \frac{1}{\cancel{2}}\cdot\cancel{2}\ln 2 = \ln 2$ ✓ $\rightarrow \boxed{V=\pi\int^{\ln 2}(1+e^{2x})^2dx}$

M07 P2

$f(x) = 3 + \dfrac{1}{2x-5}$



$\boxed{\begin{array}{l}\text{Asíntota Vertical } x = \frac{5}{2} \\ \quad " \quad \text{Horizontal } y = 3\end{array}}$

$x = 0 \longrightarrow y = 3 + \frac{1}{-5} = 2'8$

$\boxed{(0, 2'8)}$

$y = 0 \rightarrow 0 = 3 + \dfrac{1}{2x-5}$

$-3 = \dfrac{1}{2x-5}$; $-6x + 15 = 1$; $-6x = -14$; $x = \frac{7}{3} = 2'\widehat{3} \longrightarrow \boxed{(7/3, 0)}$

$$\int \left(9 + \frac{6}{2x-5} + \frac{1}{(2x-5)^2}\right) dx = \int \left(9 + \frac{6}{2x-5} + (2x-5)^{-2}\right) dx = 9x + 3\ln(2x-5) + \frac{1}{2}\,\frac{(2x-5)^{-1}}{-1} =$$

$$= \boxed{9x + 3\ln(2x-5) - \frac{1}{2}\,\frac{1}{2x-5} + K}$$

$$V = \pi \int_3^a \left(3 + \frac{1}{2x+5}\right)^2 dx = \pi \left[9x + 3\ln(2x-5) - \frac{1}{4x-10}\right]_3^a =$$

$$= \pi\left[9a + 3\ln(2a-5) - \frac{1}{4a-10}\right] - \pi\left[27 + 3\ln 1 - \frac{1}{2}\right] =$$

$$= \pi\left[9a + 3\ln(2a-5) - \frac{1}{4a-10} - 26'5\right]$$

$$V = \pi\left(\frac{28}{3} + 3\ln 3\right) \longrightarrow \begin{cases} 2a - 5 = 3 \longrightarrow a = 4 \\ 9a - \frac{1}{4a-10} - 26'5 = \frac{28}{3} \end{cases}$$

$$36 - \frac{1}{6} - 26'5 = \frac{28}{3}$$

$$\frac{28}{3} = \frac{28}{3} \quad ✓ \quad \Longrightarrow \boxed{a = 4}$$

M07 P1

$\int_1^3 f(x)dx = 5 \rightarrow$ a) $\int_1^3 2f(x)dx = 2\int_1^3 f(x)dx = 2 \cdot 5 = \boxed{10}$

b) $\int_1^3 (3x^2 + f(x))dx = \left[x^3\right]_1^3 + \int_1^3 f(x)dx = 27 - 1 + 5 = \boxed{31}$

M07 P1

$f'(x) = 12x^2 - 2 \longrightarrow f(x) = \int (12x^2 - 2)dx = \frac{12x^3}{3} - 2x + K = 4x^3 - 2x + K$

$f(-1) = 1 \Longrightarrow 1 = -4 + 2 + K \Longrightarrow K = 3$ $\quad \boxed{f(x) = 4x^3 - 2x + 3}$

N07 P1

a) b) Area A $= \int_{3\pi/2}^{2\pi} \cos x\, dx = \left[\operatorname{sen} x\right]_{3\pi/2}^{2\pi} = 0 - (-1) = \boxed{1}$

c) Area B $= -\int_{4\pi/3}^{3\pi/2} \cos x\, dx = \left[-\operatorname{sen} x\right]_{4\pi/3}^{3\pi/2} = -(-1) + \left(-\frac{\sqrt{3}}{2}\right) = 1 - \frac{\sqrt{3}}{2} = \boxed{\frac{2-\sqrt{3}}{2}}$

Area Total $= 1 + \frac{2-\sqrt{3}}{2} = \boxed{\frac{4-\sqrt{3}}{2}}$

M08 P1

$$V = \pi \int_0^{\pi/2} \left[\sqrt{3}\operatorname{sen} x \sqrt{\cos x}\right]^2 dx = \pi \int_0^{\pi/2} 3\operatorname{sen}^2 x \cdot \cos x\, dx = \pi\left[\operatorname{sen}^3 x\right]_0^{\pi/2} = \pi\left[1 - 0\right] = \boxed{\pi}$$

N06 P1 #15

$$V = \pi \int_0^a (\sqrt{x})^2 dx = \pi \int_0^a x\, dx = \left[\pi \frac{x^2}{2}\right]_0^a = \frac{\pi a^2}{2}$$

$V = 0'845\pi \Longrightarrow \frac{a^2}{2} = 0'845$; $a^2 = 1'69$; $\boxed{a = 1'3}$

M08 P1

$\int_1^5 3f(x)\,dx = 12$

a) $\int_5^1 f(x)\,dx = \frac{1}{3}\int_5^1 3f(x)\,dx = -\frac{1}{3}\int_1^5 3f(x)\,dx = -\frac{1}{3}\cdot 12 = \boxed{-4}$

b) $\int_1^2 (x+f(x))\,dx + \int_2^5 (x+f(x))\,dx = \int_1^5 (x+f(x))\,dx = \int_1^5 x\,dx + \int_1^5 f(x)\,dx =$

$= \left[\frac{x^2}{2}\right]_1^5 + \frac{1}{3}\int_1^5 3f(x)\,dx = \frac{25}{2} - \frac{1}{2} + \frac{1}{3}\cdot 12 = 12+4 = \boxed{16}$

M08 P1

a) $\int \frac{1}{2x+3}\,dx = \boxed{\frac{1}{2}\ln(2x+3) + K}$

b) $\int_0^3 \frac{1}{2x+3}\,dx = \ln\sqrt{P} \Rightarrow \left[\frac{1}{2}\ln(2x+3)\right]_0^3 = \ln\sqrt{P}$; $\frac{1}{2}\ln 9 - \frac{1}{2}\ln 3 = \ln\sqrt{P}$;

$\Rightarrow \frac{1}{2}[\ln 9 - \ln 3] = \ln\sqrt{P} \Rightarrow \frac{1}{2}\ln\left(\frac{9}{3}\right) = \ln\sqrt{P} \Rightarrow \ln(3^{1/2}) = \ln\sqrt{P}$

$\boxed{P=3}$

Muestra 08 P1

$y = \sin(2x) \longrightarrow 2T = 2\pi \Rightarrow \boxed{T=\pi}$ El periodo es π $\Rightarrow$ $\boxed{m = \pi/2}$

Area $= \int_0^{\pi/2} \sin(2x)\,dx = \left[-\frac{\cos(2x)}{2}\right]_0^{\pi/2} = -\frac{\cos(\pi)}{2} + \frac{\cos(0)}{2} = \frac{+1}{2} + \frac{1}{2} = \boxed{1}$

Muestra 08 P1

a) $\int_1^2 (3x^2-2)\,dx = \left[x^3-2x\right]_1^2 = (8-4)-(1-2) = 4+1 = \boxed{5}$

b) $\int_0^1 2e^{2x}\,dx = \left[e^{2x}\right]_0^1 = e^2 - e^0 = \boxed{e^2-1}$

M09 P2

$f(x) = x(x-5)^2$

a) Area R $= \int_0^5 x(x-5)^2\,dx = \int_0^5 (x^3 - 10x^2 + 25x)\,dx = \left[\frac{x^4}{4} - \frac{10x^3}{3} + \frac{25x^2}{2}\right]_0^5 = \boxed{\frac{625}{12}}$

b) Volumen $= \pi\int_0^5 [x(x-5)^2]^2\,dx = \pi\int_0^5 x^2(x-5)^4\,dx = \pi\int_0^5 x^2(x^4 - 20x^3 + 150x^2 - 500x + 625)\,dx =$

$= \pi\int_0^5 (x^6 - 20x^5 + 150x^4 - 500x^3 + 625x^2)\,dx = \pi\left[\frac{x^7}{7} - \frac{20x^6}{6} + \frac{150x^5}{5} - \frac{500x^4}{4} + \frac{625x^3}{3}\right]_0^5 =$

$= \pi\left(5^7/7 - 10\cdot 5^6/3 + 30\cdot 5^5 - 125\cdot 5^4 + 625\cdot 5^3/3\right) = \boxed{2340}$

c) $\int_0^a x(a-x)\,dx = \int_0^a (ax - x^2)\,dx = \left[\frac{ax^2}{2} - \frac{x^3}{3}\right]_0^a = \frac{a^3}{2} - \frac{a^3}{3} = \frac{a^3}{6}$

$\frac{a^3}{6} = \frac{625}{12} \Rightarrow a^3 = 312'5 \Rightarrow \boxed{a = 6'79}$

M09 P1

$V = 32\pi \longrightarrow 32\pi = \pi\int_0^a (\sqrt{x})^2\,dx = \pi\int_0^a x\,dx = \pi\left[\frac{x^2}{2}\right]_0^a = \frac{\pi a^2}{2}$

$32\cancel{\pi} = \frac{\cancel{\pi} a^2}{2} \Rightarrow a^2 = 64 \Rightarrow \boxed{a=8}$

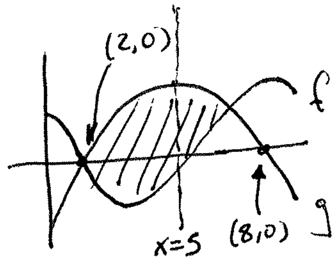
N09 P2

b) $f(x)=0 \rightarrow \boxed{x=2}$
Periodo $= \boxed{5}$
Amplitud $= \boxed{8}$

c) $g(x)=0 \rightarrow \boxed{(2,0)\ (8,0)}$
Eje simetría : $\boxed{x=5}$

Hecho todo con calculadora Gráfica

d) $f(x)=g(x) \Rightarrow x=2,\ x=6'79 \rightarrow$ Area $= \int_2^{6'79} g(x)-f(x)\,dx = \boxed{2'76}$



M09 P1

$$f(x) = \frac{ax}{x^2+1}$$

a) $f(-x) = \frac{a(-x)}{(-x)^2+1} = \frac{-ax}{x^2+1} = -f(x)$ ✓

b) $f''(x)=0 \Rightarrow \frac{2ax(x^2-3)}{(x^2+1)^3} = 0 \Rightarrow 2ax(x^2-3)=0 \Rightarrow x = 0,\ \sqrt{3},\ -\sqrt{3}$ $\boxed{(0,0);\ (\sqrt{3}, a\sqrt{3}/4);\ (-\sqrt{3}, -a\sqrt{3}/4)}$ Puntos Inflexión

c) $\text{Area} = \int_3^7 f(x)\,dx = \left[\frac{a}{2}\ln(x^2+1)\right]_3^7 = \frac{a}{2}\ln 50 - \frac{a}{2}\ln 10 = \frac{a}{2}\ln\left(\frac{50}{10}\right) = \boxed{\frac{a}{2}\ln 5}$

$$\int_4^8 2f(x-1)\,dx = \int_3^7 2f(x)\,dx = \cancel{2}\frac{a}{\cancel{2}}\ln 5 = \boxed{a\ln 5}$$

N09 P1

$f(x) = \sqrt{x} \longrightarrow f'(x) = \frac{1}{2\sqrt{x}}$

a) $P(4,2)$: $x=4 \to y=2$; $y' = \frac{1}{2\sqrt{4}} = \frac{1}{4} \to$ Pendiente Normal $= -4$ $y-2=-4(x-4)$: $\boxed{y = 18-4x}$ ✓

b) $y=0 \Rightarrow 18-4x=0$; $\boxed{x = 9/2}$

c) $\text{Area} = \int_0^4 \sqrt{x}\,dx + \int_4^{9/2}(18-4x)\,dx = \left[\frac{x^{3/2}}{3/2}\right]_0^4 + \left[18x-2x^2\right]_4^{9/2} = \left[\frac{2\sqrt{x^3}}{3}\right]_0^4 + \left[18x-2x^2\right]_4^{9/2} =$

$$= \frac{16}{3} + \left(81 - \frac{81}{2}\right) - (72-32) = \boxed{\frac{35}{6}}$$

d) $V = \pi\int_0^4 (\sqrt{x})^2 dx + \pi\int_4^{9/2}(18-4x)^2 dx = \pi\left[\frac{x^2}{2}\right]_0^4 + \pi\left[\frac{1}{-4}\frac{(18-4x)^3}{3}\right]_4^{9/2} =$

$$= 8\pi + \frac{2}{3}\pi = \boxed{\frac{26}{3}\pi}$$

M10 P1

a) $6+6\sin x = 6 \Rightarrow 6\sin x = 0 \Rightarrow \sin x = 0 \Rightarrow \boxed{x = 0,\ \pi}$

$6+6\sin x = 0 \Rightarrow 6\sin x = -6 \Rightarrow \sin x = -1 \Rightarrow \boxed{x = 3\pi/2}$

b) $y=0 \Rightarrow 6+6\sin x = 0 \Rightarrow \boxed{x = 3\pi/2}$

c) $\text{Area} = \int_0^{3\pi/2}(6+6\sin x)\,dx = \left[6x - 6\cos x\right]_0^{3\pi/2} = \left(9\pi - 6\cos\frac{3\pi}{2}\right) - (0 - 6\cos 0) = \boxed{9\pi+6} = K$

d) $f(x) = 6+6\sin x \longrightarrow g(x) = 6+6\sin\left(x-\frac{\pi}{2}\right)$

Se trata de una traslación horizontal derecha según el vector guía : $\begin{pmatrix}\pi/2\\0\end{pmatrix}$

e) $\int_p^{p+\frac{3\pi}{2}} g(x)\,dx = K = \int_0^{3\pi/2} f(x)\,dx$

El recinto que determina $f(x)$ con el eje X entre 0 y $3\pi/2$ será el mismo que el trasladado mediante el vector $(\pi/2, 0)$ para la función $g(x)$.

Por lo tanto : $\boxed{p = \frac{\pi}{2}}$ ó $\boxed{p = 0}$

M10 P1

$f''(x) = 3x-1$

a) $B\left(-\frac{4}{3}, \frac{358}{27}\right)$ es un máximo.

$f''\left(-\frac{4}{3}\right) = 3\cdot\left(-\frac{4}{3}\right)-1 = -5 < 0 \Rightarrow$ f es convexa en B $\Rightarrow$ máximo ✓

b) $f'(x) = \int (3x-1)\,dx = \frac{3x^2}{2} - x + p$

Como f tiene en A un mínimo y un máximo en B, en ambos puntos la derivada debe ser nula:

Mínimo en $A(2,4) \Rightarrow f'(2)=0$; $\frac{3\cdot 2^2}{2} - 2 + p = 0$; $p=-4$ ✓

ó también:

Máximo en $B\left(-\frac{4}{3}, \frac{358}{27}\right) \Rightarrow f'\left(-\frac{4}{3}\right)=0$; $\frac{3\cdot\left(-4/3\right)^2}{2} - \left(-\frac{4}{3}\right) + p = 0$; $p=-4$ ✓

c) $f(x) = \int \left(\frac{3x^2}{2} - x - 4\right) dx = \frac{x^3}{2} - \frac{x^2}{2} - 4x + C$

$A(2,4) \Rightarrow f(2)=4$; $\frac{2^3}{2} - \frac{2^2}{2} - 4\cdot 2 + C = 4$; $C=10$ $\Rightarrow \boxed{f(x) = \frac{x^3}{2} - \frac{x^2}{2} - 4x + 10}$

ó también:

$B\left(-\frac{4}{3}, \frac{358}{27}\right) \Rightarrow f\left(-\frac{4}{3}\right) = \frac{358}{27}$ - - - - -

M10 TZ1 P1#6

$$V = \pi \int_{-2}^{2} \left(\sqrt{16-4x^2}\right)^2 dx = \pi \int_{-2}^{2} (16-4x^2)\,dx = \pi\left(16x - \frac{4x^3}{3}\right)_{-2}^{2} =$$

$$= \pi\left(16\cdot 2 - \frac{4\cdot 2^3}{3}\right) - \pi\left(-16\cdot 2 - \frac{4\cdot(-2)^3}{3}\right) = \boxed{\frac{128\pi}{3}}$$

También:

$$V = 2\pi \int_{0}^{2} \left(\sqrt{16-4x^2}\right)^2 dx = 2\pi \int_{0}^{2} (16-4x^2)\,dx = 2\pi\left(16x - \frac{4x^3}{3}\right)_{0}^{2} = \frac{128\pi}{3}$$ ✓

M10 TZ1 P2#9

$f(x) = Ae^{kx} + 3$

a) $f(0)=13 \Rightarrow 13 = A\cdot e^0 + 3$; $13 = A+3$; $A=10$ ✓

b) $f(15) = 3'49 \Rightarrow 3'49 = 10e^{15k} + 3$; $0'49 = 10e^{15k}$; $0'049 = e^{15k}$; $15k = \ln 0'049 \Rightarrow \boxed{k = -0'201}$

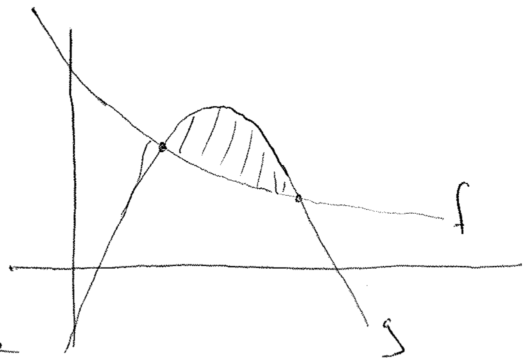
c) $f(x) = 10e^{-0'201x} + 3$

$f'(x) = 10e^{-0'201x}\cdot(-0'201) = \boxed{-2'01\cdot e^{-0'201x}}$

Como $e^{-0'201x} > 0$ para todo $x \in \mathbb{R}$, $f'(x) < 0 \Rightarrow$ f es una función decreciente ✓

$\lim_{x\to+\infty} \left(10e^{-0'201x} + 3\right) = 10e^{-\infty} + 3 = 10\cdot 0 + 3 = 3 \longrightarrow \boxed{\text{Asíntota Horizontal } y=3}$

d) $\left.\begin{array}{l} g(x) = -x^2 + 12x - 24 \\ f(x) = 10e^{-0'201x} + 3 \end{array}\right\} \longrightarrow x = \{$



$\text{Area} = \int (-x^2+12x-24) - \left(10e^{-0'201x} + 3\right) dx =$

$= \boxed{19'5}$

Hecho todo con calculadora gráfica

M10
TZ2
P2#6

$f(x) = e^x \sin 2x + 10$

a) $x = \boxed{2'31}$

b) $p = \boxed{1'02}$

$q = \boxed{2'59}$

Todo hecho con calculadora gráfica

c) $\displaystyle\int_{1'02}^{2'59} f(x)\,dx = \boxed{9'96}$

No es el área de la región sombreada porque entre p y q la función pasa de ser positiva a negativa. El área se calcularía con: $\displaystyle\int_{1'02}^{2'31} f(x)\,dx - \int_{2'31}^{2'59} f(x)\,dx$

N10
P1#6

$f(x) = \displaystyle\int \sin(2x-3)\,dx = -\frac{1}{2}\cos(2x-3) + C$

$f\left(\frac{3}{2}\right) = 4 \Rightarrow -\frac{1}{2}\cos\left(2\cdot\frac{3}{2}-3\right) + C = 4$; $-\frac{1}{2}\cos 0 + C = 4$; $-\frac{1}{2} + C = 4$;

$C = 4'5 \Rightarrow \boxed{f(x) = -\frac{1}{2}\cos(2x-3) + 4'5}$

N10
P1#10

$f(x) = x^3$

a) $x=a \longrightarrow f(a) = a^3$; $\left.\begin{matrix} P(a, a^3) \\ Q\left(\frac{2}{3}, 0\right)\end{matrix}\right| \rightarrow \vec{QP} = \left(a-\frac{2}{3}, a^3\right) \rightarrow$ pendiente $= \dfrac{a^3}{a-\frac{2}{3}}$ ✓

$f'(x) = 3x^2$; $f'(a) = 3a^2$

$f'(a) =$ pendiente $\Rightarrow 3a^2 = \dfrac{a^3}{a-\frac{2}{3}}$; $3a^3 - 2a^2 = a^3$; $\cancel{2}a^3 - \cancel{2}a^2 = 0$;

$a^2(a-1) = 0 \Rightarrow a = \begin{cases} \cancel{0} \\ \boxed{1} \end{cases}$ porque $a > 0$

b) Area $= \displaystyle\int_{-2}^{K} [x^3 - (3x-2)]\,dx = \int_{-2}^{K} (x^3 - 3x + 2)\,dx = \left[\frac{x^4}{4} - \frac{3x^2}{2} + 2x\right]_{-2}^{K} =$

$= \left(\frac{K^4}{4} - \frac{3K^2}{2} + 2K\right) - \left(\frac{16}{4} - \frac{12}{2} - 4\right) = \frac{K^4}{4} - \frac{3K^2}{2} + 2K + 6$

Area $= 2K + 4 \Rightarrow \frac{K^4}{4} - \frac{3K^2}{2} + \cancel{2K} + 6 = \cancel{2K} + 4$; $\frac{K^4}{4} - \frac{3K^2}{2} + 2 = 0$;

$K^4 - 6K^2 + 8 = 0$ ✓

N10
P2#8

$f(x) = x \ln(4-x^2)$

a) $y=0 \Rightarrow x \cdot \ln(4-x^2)=0 \Rightarrow \boxed{x=0}$

$\ln(4-x^2)=0 \; ; \; 4-x^2=1 \; ; \; \boxed{x=\pm\sqrt{3}}$ $\boxed{\begin{array}{l} a=-\sqrt{3} \\ b=\sqrt{3} \end{array}}$

b) $c = \boxed{1'15}$ Hecho con calculadora gráfica.

c) $V = \pi \int_0^{1'15} [x \ln(4-x^2)]^2 dx = \boxed{2'16}$ Hecho con calculadora gráfica

d) $\text{Area} = -\int_{-\sqrt{3}}^{0} x \ln(4-x^2) dx + \int_0^{1'15} x \ln(4-x^2) dx = \boxed{2'07}$ Hecho con calculadora gráfica

M11
TZ1
P2#6

$\cos x^2 = e^x \longrightarrow x = \begin{cases} -1'10 \\ 0 \end{cases}$ Hecho todo con calculadora Gráfica.

$\text{Area} = \int_{-1'10}^{0} (\cos x^2 - e^x) = \boxed{0'282}$

M11
TZ2
P2#7

$y = \int (10e^{2x} - 5) dx = 5e^{2x} - 5x + C$

$\left.\begin{array}{l} x=0 \\ y=8 \end{array}\right| \Rightarrow 8 = 5 \cdot e^0 - 5 \cdot 0 + C \; ; \; C=3 \Rightarrow y = 5e^{2x} - 5x + 3$

$x=1 \Rightarrow y = 5e^2 - 5 + 3 = \boxed{5e^2 - 2}$

M11
TZ2
P1#8

$f(x) = 2x^2 \longrightarrow f'(x) = 4x$

a) $x=1 \longrightarrow f(1)=2$, $f'(1)=4$ $\quad y-2 = 4(x-1) \; ; \; y-2 = 4x-4 \; ; \; y = 4x-2$ ✓

b) $\left.\begin{array}{l} y=4x-2 \\ y=0 \end{array}\right| \longrightarrow \boxed{x=\frac{1}{2}}$

c) $\text{Area} = \int_0^{1/2} 2x^2 dx + \int_{1/2}^{1} 2x^2 - (4x-2) dx = \left[\frac{2x^3}{3}\right]_0^{1/2} + \left[\frac{2x^3}{3} - 2x^2 + 2x\right]_{1/2}^{1} =$

$= \frac{2/8}{3} + \left(\frac{2}{3} - 2 + 2\right) - \left(\frac{2/8}{3} - \frac{2}{4} + 1\right) = \frac{2}{3} + \frac{2}{4} - 1 = \boxed{\frac{1}{6}}$ u.s.

N11
P1#4

$f(x) = \int (3x^2+2) dx = x^3 + 2x + C$

$f(2)=5 \Rightarrow 5 = 8 + 4 + C \; ; \; C = -7 \Rightarrow f(x) = \boxed{x^3 + 2x - 7}$

N11 P1#10

$f(x) = \frac{1}{4}x^2+2 \rightarrow f'(x) = \frac{1}{4}\cdot 2x = \frac{x}{2}$

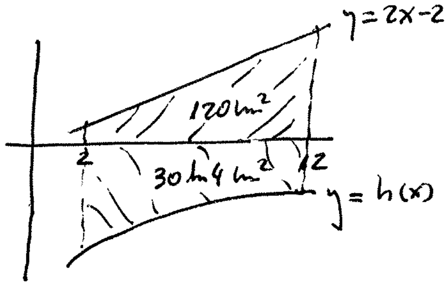
a) $P(4,6) \rightarrow f'(4) = \frac{4}{2} = 2$ $\quad y-6 = 2(x-4)$; $y-6 = 2x-8$; $\boxed{y = 2x-2}$

b) $\text{Area} = \int_2^{12} \frac{90}{3x+4}\,dx = \left[30\ln(3x+4)\right]_2^{12} = 30\ln 40 - 30\ln 10 = 30(\ln 40 - \ln 10) =$

$= 30\ln\frac{40}{10} = \boxed{30\ln 4}$

c) $y = 2x-2$

x	y
2	2
12	22



$\text{Area} = \boxed{120+30\ln 4}\ u^2$

M12 TZ1 P1#6

$\int_0^5 \frac{2}{2x+5}\,dx = \left[\ln(2x+5)\right]_0^5 = \ln 15 - \ln 5 = \ln\frac{15}{5} = \ln 3 \rightarrow \boxed{k=3}$

M12 TZ2 P1#8

a) $f(x) = a(x-2)^2+1$ $\quad \boxed{\begin{matrix} h=2 \\ k=1 \end{matrix}}$

$f(0) = 13 \Rightarrow 13 = a\cdot(-2)^2+1$; $12 = 4a$; $\boxed{a=3}$

b) $f(x) = 3(x-2)^2+1 = 3(x^2-4x+4)+1 = \boxed{3x^2-12x+13}$

c) $\text{Area} = \int_2^4 (3x^2-12x+13)\,dx = \left[x^3-6x^2+13x\right]_2^4 = (64-96+52)-(8-24+26) = \boxed{10}$

N12 P1#3

a) $\int_4^{10}(x-4)\,dx = \left[\frac{(x-4)^2}{2}\right]_4^{10} = \frac{36}{2} - 0 = \boxed{18}$

b) $V = \pi\int_4^{10}\left(\sqrt{x-4}\right)^2 dx = \pi\int_4^{10}(x-4)\,dx = \boxed{18\pi}$

M12 TZ1 P2#4

a) $y = (x-1)\sin x$ $\quad \left(0 \le x \le \frac{5\pi}{2}\right)$

$y = 0 \Rightarrow (x-1)\sin x = 0$ $\rightarrow x-1 = 0$; $\boxed{x=1}$

$\rightarrow \sin x = 0$; $\boxed{x = \begin{cases} 0 \\ \pi \\ 2\pi \end{cases}} \leftarrow \boxed{k = 2\pi}$

b) $V = \pi\int_{\pi}^{2\pi}(x-1)^2\sin^2 x\,dx = \boxed{69'6}$ Hecho con calculadora gráfica

N12
P1#10

a) $f(x) = \dfrac{6x}{x+1} \longrightarrow f'(x) = \dfrac{6(x+1) - 6x}{(x+1)^2} = \boxed{\dfrac{6}{(x+1)^2}}$

b) $g(x) = \ln \dfrac{6x}{x+1} \longrightarrow g'(x) = \dfrac{1}{\frac{6x}{x+1}} \cdot \left(\dfrac{6x}{x+1}\right)' = \dfrac{\cancel{x+1}}{\cancel{6x}} \cdot \dfrac{\cancel{6}}{(x+1)^{\cancel{2}}} = \dfrac{1}{x(x+1)}$ ✓

c) $\text{Area} = \displaystyle\int_{1/5}^{k} \dfrac{1}{x(x+1)}\,dx = \left[\ln \dfrac{6x}{x+1}\right]_{1/5}^{k} = \ln \dfrac{6k}{k+1} - \ln \dfrac{6/5}{1/5+1} =$

$= \ln \dfrac{6k}{k+1} - \ln 1 = \ln \dfrac{6k}{k+1}$

$\text{Area} = \ln 4 \Rightarrow \dfrac{6k}{k+1} = 4 \;;\; 6k = 4k+4 \;;\; 2k = 4 \;;\; \boxed{k = 2}$

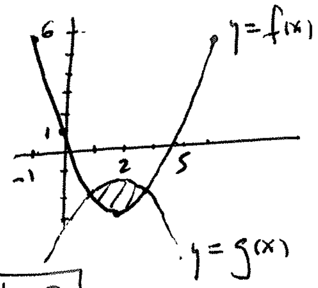
NOTA: El dato de que $k > \frac{1}{5}$ es para poner correctamente los extremos de integración.

N12
P2#9

$f(x) = x^2 - 4x + 1$

a)

	x	y
	-1	6
	0	1
V	2	-3
	5	6



b) $y = (x-2)^2 - 3 \longrightarrow \boxed{p = 2}$

c) $y = x^2 - 4x + 1 \xrightarrow{\text{Simetría Eje X}} y = -x^2 + 4x - 1 \xrightarrow{\text{Traslación } \binom{0}{6}} y = -x^2 + 4x - 1 + 6 =$
$= -x^2 + 4x + 5$ ✓

d) $\begin{cases} y = x^2 - 4x + 1 \\ y = -x^2 + 4x - 5 \end{cases}$ $\quad x^2 - 4x + 1 = -x^2 + 4x + 5 \;;\quad 2x^2 - 8x - 4 = 0$

$x^2 - 4x - 2 = 0$

$\boxed{x = 2 \pm \sqrt{6}}$

e) $\text{Area} = \displaystyle\int_{2-\sqrt{6}}^{2+\sqrt{6}} (-x^2 + 4x - 5) - (x^2 - 4x + 1)\,dx = \boxed{39'2}$ Hecho con Calculadora Gráfica

N13
TZ2
P1#7

a) $\text{Area} = \dfrac{\pi \cdot 2^2}{4} = \pi \Rightarrow \boxed{\displaystyle\int_0^2 f(x)\,dx = -\pi}$

b) $\text{Area} = 3\pi$

$\displaystyle\int_2^6 f(x)\,dx = 3\pi - \pi = \boxed{2\pi}$

M13 TZ1 P1#6

$f(x)=\int \frac{12}{2x-5}\,dx = 6\ln(2x-5)+C$

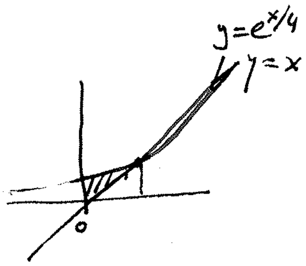
$f(4)=0 \longrightarrow 0=6\ln 3 + C \; ; \; C=-6\ln 3 \Longrightarrow f(x)=6\ln(2x-5)-6\ln 3 = \boxed{6\ln\frac{2x-5}{3}}$

M13 TZ2 P2#10

$f(x)=e^{x/4}$ $\quad -5\le x\le 5$

$g(x)=mx$

a) $m=1$

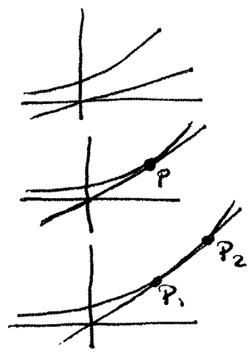


$\left.\begin{array}{l} y=e^{x/4} \\ y=x \end{array}\right\} \longrightarrow x=1'43$

Hecho con calculadora gráfica

$\text{Area} = \int_0^{1'43} (e^{x/4}-x)\,dx = \boxed{0'697}$ Hecho con calculadora gráfica

b)
- Para valores positivos de 'm', pequeños, no se cortarán:
- Para un cierto valor de 'm', la recta será tangente a la curva, teniendo un único punto de corte.
- Para valores mayores de 'm', se cortarán en dos puntos:



Obviamente, el área tomará su máximo valor para cuando sean tangentes:

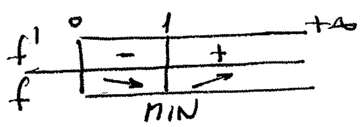
Si llamamos 'a' al valor de x donde se produce la tangencia:

$\left.\begin{array}{l} f(a)=g(a) \Rightarrow e^{a/4}=ma \\ f'(a)=g'(a) \Rightarrow \frac{1}{4}e^{a/4}=m \end{array}\right\} \Rightarrow \frac{1}{4}\cancel{m}a=\cancel{m} \Rightarrow a=4 \Rightarrow \boxed{m=\frac{1}{4}e}$

N13 P1#10

a) $f(x)=\frac{\ln^2 x}{2} \longrightarrow f'(x)=\frac{1}{2}\cdot 2\ln x\cdot\frac{1}{x}=\frac{\ln x}{x}$ ✓

b) $f'(x)=0 \Rightarrow \frac{\ln x}{x}=0 \; ; \; \ln x = 0 \; ; \; x=1$



Tiene un mínimo en $\boxed{(1,0)}$

c) $f'(x)=0 \Rightarrow x=1$ $\quad \boxed{p=1}$

d) $f'(x)=g(x) \Rightarrow \frac{\ln x}{x}=\frac{1}{x} \; ; \; \ln x=1 \; ; \; x=e$ $\quad \boxed{q=e}$

e) $\text{Area}=\int_1^e \left(\frac{1}{x}-\frac{\ln x}{x}\right)dx = \left[\ln x - \frac{\ln^2 x}{2}\right]_1^e =$

$=\left(\ln e - \frac{\ln^2 e}{2}\right)-\left(\ln 1-\frac{\ln^2 1}{2}\right) = 1-\frac{1}{2}=\frac{1}{2}$ ✓

N13 P1#4

$\int_1^6 f(x)\,dx = 8$

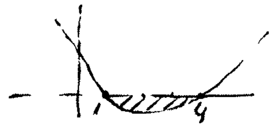
a) $\int_1^6 2f(x)\,dx = 2\int_1^6 f(x)\,dx = \boxed{16}$

b) $\int_1^6 (f(x)+2)\,dx = \int_1^6 f(x)\,dx + \int_1^6 2\,dx = 8 + \left[2x\right]_1^6 = 8+12-2 = \boxed{18}$

N13 P2#2

$f(x) = (x-1)(x-4)$

a) $y=0 \rightarrow \boxed{x = \begin{cases} 1 \\ 4 \end{cases}}$



b) $V = \pi\int_1^4 (x-1)^2(x-4)^2\,dx = \boxed{25'4} = 8'1\pi$ Hecho con calculadora gráfica

N13 P2#3

a) $f(x) = \sqrt[3]{x^4} - \frac{1}{2} = x^{4/3} - \frac{1}{2} \longrightarrow f'(x) = \frac{4}{3}x^{1/3} = \boxed{\frac{4}{3}\sqrt[3]{x}}$

b) $\int\left(x^{4/3} - \frac{1}{2}\right)dx = \frac{x^{7/3}}{7/3} - \frac{x}{2} + C = \boxed{\frac{3}{7}\sqrt[3]{x^7} - \frac{x}{2} + C}$

Muestra 14 P1#5

a) $\int \frac{e^x}{1+e^x}\,dx = \boxed{\ln(1+e^x) + C}$

b) $\int \operatorname{sen} 3x \cos 3x\,dx = \boxed{\frac{1}{2}\operatorname{sen}^2 3x + C}$

También:

$\int \operatorname{sen} 3x \cos 3x\,dx = -\frac{1}{2}\cos^2 3x + C$ ✓

También:

$\int \operatorname{sen} 3x \cos 3x\,dx = \frac{1}{2}\int \operatorname{sen} 6x\,dx = -\frac{1}{12}\cos 6x + C$ ✓

Muestra 14 P2#9

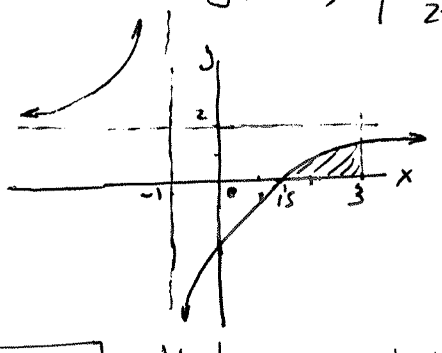
a) $h(x) = \frac{2x-1}{x+1}$

$x = \frac{2y-1}{y+1}$; $xy + x = 2y - 1$; $x+1 = y(2-x)$; $y = \frac{x+1}{2-x}$; $\boxed{h^{-1}(x) = \frac{x+1}{2-x}}$

b) Asíntota Vertical $\boxed{x=-1}$

Asíntota Horizontal $\boxed{y=2}$

$y=0 \rightarrow \boxed{x = 1'5}$



c) $\text{Area} = \int_{1'5}^3 \frac{2x-1}{x+1}\,dx = \boxed{2'06}$ Hecho con calculadora gráfica

$$\boxed{V = \pi\int_{1'5}^3 \left(\frac{2x-1}{x+1}\right)^2 dx}$$

M14
TZ1
P1#3

a) $\int_1^2 (x^2)^2\,dx = \int_1^2 x^4\,dx = \left[\frac{x^5}{5}\right]_1^2 = \frac{32}{5} - \frac{1}{5} = \boxed{\frac{31}{5}}$

b) $V = \pi \int_1^2 (x^2)^2\,dx = \pi \cdot \frac{31}{5} = \boxed{\frac{31\pi}{5}}$

M14
TZ1
P1#6

$\int_\pi^a \cos 2x\,dx = \left[\frac{1}{2}\sin 2x\right]_\pi^a = \frac{1}{2}\sin 2a - \frac{1}{2}\sin 2\pi = \frac{1}{2}\sin 2a$

$\frac{1}{2}\sin 2a = \frac{1}{2} \Rightarrow \sin 2a = 1 \rightarrow 2a = \begin{cases} \pi/2 \rightarrow a = \cancel{\pi/4} \quad \text{porque } a \in (\pi, 2\pi) \\ 5\pi/2 \rightarrow \boxed{a = \frac{5\pi}{4}} \end{cases}$

M14
TZ2
P1#10

a) $f(x) = \frac{2x}{x^2+5} \longrightarrow f'(x) = \frac{2(x^2+5) - 2x \cdot 2x}{(x^2+5)^2} = \frac{2x^2+10-4x^2}{(x^2+5)^2} = \frac{10-2x^2}{(x^2+5)^2}$ ✓

b) $\int \frac{2x}{x^2+5}\,dx = \boxed{\ln(x^2+5) + C}$

c) $\text{Area} = \int_{\sqrt{5}}^{q} \frac{2x}{x^2+5}\,dx = \ln(q^2+5) - \ln(5+5) = \ln \frac{q^2+5}{10}$

$\text{Area} = \ln 7 \Rightarrow \frac{q^2+5}{10} = 7 \;;\; q^2 = 65 \;;\; \boxed{q = \sqrt{65}}$

M14
TZ2
P1#5

$h'(x) = 4\cos 2x \rightarrow h(x) = \int 4\cos 2x\,dx = 2\sin 2x + C$

$h(\pi/12) = 5 \Rightarrow 5 = 2\sin\frac{2\pi}{12} + C \;;\; 5 = 2\sin\frac{\pi}{6} + C \;;\; 5 = 2\cdot\frac{1}{2} + C \;;\; C = 4$

$\Rightarrow h(x) = \boxed{2\sin 2x + 4}$

M14
TZ2
P2#2

$y = 5 - x^2$

a) $y = 0 \Rightarrow x = \pm\sqrt{5}$ $\boxed{\begin{matrix} A = -\sqrt{5} \\ B = \sqrt{5} \end{matrix}}$

b) $V = \pi \int_{-\sqrt{5}}^{\sqrt{5}} (5-x^2)^2\,dx = \boxed{187}$ Hecho con calculadora gráfica

N14
P1#6

$\text{Area} = \int_0^4 \frac{x}{x^2+1}\,dx = \left[\frac{1}{2}\ln(x^2+1)\right]_0^4 = \frac{1}{2}\ln 17 - \frac{1}{2}\ln 1 = \boxed{\frac{1}{2}\ln 17} = \boxed{\ln\sqrt{17}}$

M15
TZ1
P1#7

$\text{Area} = \int_{3\pi/2}^{b} \cos x\,dx = \left[\sin x\right]_{3\pi/2}^{b} = \sin b - \sin\frac{3\pi}{2} = \sin b - (-1) = \sin b + 1$

$\text{Area} = 1 - \frac{\sqrt{3}}{2} \Rightarrow \sin b + \cancel{1} = \cancel{1} - \frac{\sqrt{3}}{2} \;;\; \sin b = -\frac{\sqrt{3}}{2} \Rightarrow b = \begin{cases} \cancel{4\pi/3} \quad \text{porque } b > \frac{3\pi}{2} \\ \boxed{5\pi/3} \end{cases}$

M15
TZ1
P2#10

a) Si f tiene un máximo local en $x=p$, su derivada en $x=p$ debe ser nula, esto sucede en $x=0$, $x=4$, $x=6$.
Al tratarse de un máximo, no de un mínimo, la derivada debe ser positiva a la izquierda de p y negativa a su derecha.
Esto es lo que sucede en $x=6 \Rightarrow \boxed{p=6}$

b) $\boxed{f'(2)=-2}$

c) $g(x)=\ln f(x) \longrightarrow g'(x)=\frac{1}{f(x)}\cdot f'(x)=\frac{f'(x)}{f(x)}$

$g'(2)=\frac{f'(2)}{f(2)}=\boxed{\frac{-2}{3}}$

d) $\ln 3+\int_2^a g'(x)\,dx=\ln 3+\left[g(x)\right]_2^a=\ln 3+g(a)-g(2)=\ln 3+g(a)-\ln f(2)=$
$=\cancel{\ln 3}+g(a)-\cancel{\ln 3}=g(a)$ ✓

e) $\text{Área } A=-\int_2^4 g'(x)\,dx=-g(4)+g(2)=0'66$
$\text{Área } B=\int_4^5 g'(x)\,dx=g(5)-g(4)=0'21$;

$g(4)-g(2)=-0'66$
$+\ g(5)-g(4)=0'21$
$g(5)-g(2)=-0'45 \Rightarrow$

$\Rightarrow g(5)=g(2)-0'45=\ln f(2)-0'45=\boxed{\ln 3-0'45}$

M15
TZ2
P1#4

a) $g(x)=\frac{\ln x}{x} \longrightarrow g'(x)=\frac{\frac{1}{x}\cdot x-\ln x\cdot 1}{x^2}=\boxed{\frac{1-\ln x}{x^2}}$

b) $\int \frac{\ln x}{x}\,dx=\boxed{\frac{\ln^2 x}{2}+C}$

M15
TZ2
P2#8

a) $\left.\begin{array}{l} y=\frac{9}{x+2} \\ y=3x^2 \end{array}\right\}$ $3x^2=\frac{9}{x+2}$; $3x^3+6x^2=9$; $x^3+2x^2-3=0$

$$\begin{array}{c|cccc} & 1 & 2 & 0 & -3 \\ 1 & & 1 & 3 & 3 \\ \hline & 1 & 3 & 3 & 0 \end{array}$$

$\boxed{x=1} \longrightarrow \boxed{y=3}$

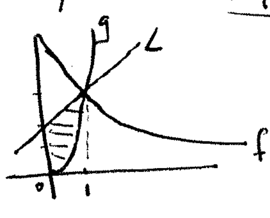
p=1
q=3

b) $f(x)=\frac{9}{x+2} \longrightarrow f'(x)=\boxed{\frac{-9}{(x+2)^2}}$

$f'(p)=f'(1)=\frac{-9}{3^2}=\boxed{-1}$

c) Recta Normal en $(1,3)$: $y-3=-\frac{1}{-1}(x-1)$; $y-3=x-1$; $\boxed{y=x+2}$

$x=0 \longrightarrow y=2$ $\boxed{Q(0,2)}$



d) $\text{Área}=\int_0^1 (x+2)-3x^2\,dx=\left[\frac{x^2}{2}+2x-x^3\right]_0^1=$
$=\frac{1}{2}+2-1=\boxed{1'5}$

M15
TZ2
P1 #10

a) $f'(x)$ es negativa en $x \in (0, d) \Rightarrow$ $\boxed{f \text{ es decreciente en } 0 \leq x \leq d}$

b) Si f tiene un mínimo local en un valor de x, debe de anularse f' en dicho valor, ser negativa f' a su izquierda y positiva a su derecha.
Esto sucede en $\boxed{x = d}$

c) $$\text{Area} = \int_a^0 f'(x)dx - \int_0^d f'(x)dx = \left[f(x)\right]_a^0 - \left[f(x)\right]_0^d =$$

$$= f(0) - f(a) - f(d) + f(0) = 2f(0) - 3 - (-1) = 2f(0) - 2$$

$\text{Area} = 15 \Rightarrow \quad 2f(0) - 2 = 17 \quad ; \quad \boxed{f(0) = \frac{19}{2}}$