SEPT 94

a) La regla de Barrow permite hallar el valor de una integral definida:

$$\boxed{\int_a^b f(x)\,dx = F(b)-F(a)}$$ siendo $F'(x)=f(x)$, Es decir, que $F(x)$ es una función primitiva de $f(x)$.

b) $u(x)=3x^3+3x^2 \longrightarrow u'(x)=9x^2+6x$

$v(x)=x^3-3x^2 \longrightarrow v'(x)=3x^2-6x=f(x) \Rightarrow$ $\boxed{v(x) \text{ es primitiva de } f(x)}$

c) $\int_0^4 (3x^2-6x)\,dx = \left[x^3-3x^2\right]_0^4 = 4^3-3\cdot 4^2 = \boxed{16}$

JUN 95

a) (Como en Sept 94)

b) $F(x)=x^4+ax^3+bx$

$(1,2)\in F(x) \Rightarrow 2=1^4+a\cdot 1^3+b\cdot 1 \;;\; 2=1+a+b \;;\; a+b=1$

$\int_1^2 f(x)\,dx=10 \Rightarrow F(2)-F(1)=10 \Rightarrow (2^4+a\cdot 2^3+b\cdot 2)-(1^4+a\cdot 1^3+b\cdot 1)=10 \Rightarrow$

$\Rightarrow 16+8a+2b-1-a-b=10 \;;\; 7a+b=-5$

$$\begin{cases} a+b=1 \\ 7a+b=-5 \end{cases}$$
$6a \quad =-6 \longrightarrow \boxed{a=-1} \longrightarrow \boxed{b=2}$

SEPT 95

a) $F(x)$ es una primitiva de una función $f(x)$ si su derivada resulta dicha función:

F primitiva de $f \iff F'(x)=f(x)$

b) $f(x)=e^{2x}-2x^2+8 \longrightarrow f'(x)=2e^{2x}-4x=h(x) \Rightarrow$ $\boxed{f \text{ es primitiva de } h}$

c) (El enunciado de la regla de Barrow como en Sept 94)

$\int_0^1 (2e^{2x}-4x)\,dx = \left[e^{2x}-2x^2\right]_0^1 = (e^{2\cdot 1}-2\cdot 1^2)-(e^0-0) = e^2-2-1 = \boxed{e^2-3}$

JUN 96

a) $\int (x+1)(3x-2)\,dx = \int (3x^2+x-2)\,dx = \boxed{x^3+\frac{x^2}{2}-2x+K}$

b) $F(x)=x^3+2x^2+2 \longrightarrow F'(x)=3x^2+4x \neq f(x) \Rightarrow$ $\boxed{F \text{ no es primitiva de } f}$

c) (El enunciado de la regla de Barrow como en Sept 94)

$\int_0^1 (x+1)(3x-2)\,dx = \int_0^1 (3x^2+x-2)\,dx = \left[x^3+\frac{x^2}{2}-2x\right]_0^1 = 1+\frac{1}{2}-2 = \boxed{-\frac{1}{2}}$

SEPT 96

a) (Como en Sept 95)

b) $F(x)=ax^3+bx^2-2x \longrightarrow F'(x)=f(x)=3ax^2-2bx-2$

$(1,9)\in f(x) \Rightarrow 9=3a\cdot 1^2-2b\cdot 1-2 \;;\; 3a-2b=11$

$\int_0^1 f(x)\,dx=1 \Rightarrow \left[ax^3+bx^2-2x\right]_0^1=1 \Rightarrow a+b-2=1 \;;\; a+b=3$

$$\begin{cases} 3a-2b=11 \\ a+b=3 \end{cases} \longrightarrow \boxed{a=17/5}\;\boxed{b=-2/5}$$

JUN 97

a) (Como en SEPT 94)

b) $F(x) = x^4 - 2x^2 + cx \longrightarrow F'(x) = 4x^3 - 4x + c = f(x) = ax^3 + bx + c \Rightarrow \boxed{\begin{matrix} a=4 \\ b=-4 \end{matrix}}$

$\int_0^1 f(x)dx = 1 \Rightarrow \int_0^1 (4x^3 - 4x + c)dx = 1 \; ; \; \left[x^4 - 2x^2 + cx\right]_0^1 = 1 \; ; \; 1 - 2 + c = 1 \Rightarrow \boxed{c = 2}$

SEPT 97

a) $\int (x^2 + \frac{2}{x^2})dx = \int (x^2 + 2x^{-2})dx = \frac{x^4}{4} + 2\frac{x^{-1}}{-1} = \boxed{\frac{x^4}{4} - \frac{2}{x} + K}$

b) (Continuando como en SEPT 94)

$\int_1^2 f(x)dx = \left[\frac{x^4}{4} - \frac{2}{x}\right]_1^2 = \left(\frac{16}{4} - \frac{2}{2}\right) - \left(\frac{1}{4} - 2\right) = \boxed{\frac{19}{4}}$

JUN 98

a) $\int (x + \frac{a}{x^3})dx = \int (x + ax^{-3})dx = \frac{x^2}{2} + a\frac{x^{-1}}{-1} = \boxed{\frac{x^2}{2} - \frac{a}{x} + K}$

b) $F(x)$ primitiva de $f(x) \Leftrightarrow F'(x) = f(x)$

$G'(x) = (F(x) + 2x)' = F'(x) + 2 = f(x) + 2 \neq f(x) \Rightarrow$ $\boxed{G \text{ no es primitiva de } f}$

c) $\int_1^2 f(x)dx = 1'5 \rightarrow \left[\frac{x^2}{2} - \frac{a}{x}\right]_1^2 = 1'5 \; ; \; \left(\frac{4}{2} - \frac{a}{2}\right) - \left(\frac{1}{2} - a\right) = 1'5 \; ; \; 2 - \frac{a}{2} - \frac{1}{2} + a = 1'5$

$4 - a - 1 + 2a = 3 \; ; \; \boxed{a = 0}$

SEPT 98

a) $F_1(x) = 4e^{4x} + ax \longrightarrow F_1'(x) = 16e^{4x} + a \neq f(x) \Rightarrow$ F_1 no es primitiva de f

$F_2(x) = e^{4x} + ax \longrightarrow F_2'(x) = 4e^{4x} + a = f(x) \Rightarrow$ F_2 sí " " " "

b) $\int_0^1 f(x)dx = e^4 \rightarrow \left[4e^{4x} + ax\right]_0^1 = e^4 \; ; \; 4e^4 + a - 4 = e^4 \; ; \; \boxed{a = 4 - 3e^4}$

JUN 99

a) $\int_1^2 f(x)dx = \int_1^2 (ae^{x/3} + \frac{1}{x^2})dx = \int_1^2 (ae^{x/3} + x^{-2})dx = \left[3ae^{x/3} + \frac{x^{-1}}{-1}\right]_1^2 = \left[3ae^{x/3} - \frac{1}{x}\right]_1^2 =$

$= \left(3ae^{2/3} - \frac{1}{2}\right) - \left(3ae^{1/3} - 1\right) = \boxed{3a\left(e^{2/3} - e^{1/3}\right) + \frac{1}{2}}$

b) $F(x) = 3ae^{x/3} - \frac{1}{x} + K$

$F(1) = 0 \longrightarrow 0 = 3ae^{1/3} - 1 + K$

$F(2) = \frac{1}{2} \longrightarrow \frac{1}{2} = 3ae^{2/3} - \frac{1}{2} + K$

$\Rightarrow \begin{cases} 3ae^{1/3} + K = 1 \\ 3ae^{2/3} + K = 1 \end{cases}$

$3a(e^{1/3} - e^{2/3}) = 0 \Rightarrow \boxed{a = 0}$

SEPT 99

a) $\int xe^{x/2}dx = 2xe^{x/2} - \int 2e^{x/2}dx = \boxed{2xe^{x/2} - 4e^{x/2} + K}$

$\boxed{\begin{matrix} u = x \rightarrow du = dx \\ dv = e^{x/2}dx \rightarrow v = 2e^{x/2} \end{matrix}}$

b) $\int_0^2 f(x)dx = \left[2xe^{x/2} - 4e^{x/2}\right]_0^2 = (4e - 4e) - (0 - 4) = \boxed{4}$

c) F, G primitivas de $f \Rightarrow F'(x) = G'(x) = f(x)$

$H'(x) = (F(x) - G(x))' = F'(x) - G'(x) = f(x) - f(x) = 0 \neq x^2$. $\boxed{\text{No es posible}}$

JUN 00

(Enunciado como en Sept 94)

$$\int_0^1 e^x(x+1)\,dx = \Big[(x+1)e^x\Big]_0^1 - \int_0^1 e^x dx = \Big[(x+1)e^x - e^x\Big]_0^1 = \Big[xe^x\Big]_0^1 = \boxed{e}$$

$$\boxed{\begin{array}{l} u = x+1 \rightarrow du = dx \\ dv = e^x dx \rightarrow v = e^x \end{array}}$$

SEPT 00

$f(x) = \ln x$ $\quad \boxed{\text{dom} f = (0,+\infty)}$

$\ln x = 0 \rightarrow x = 1$

	0		1		$+\infty$
$f(x)$		$-$		$+$	

$$\text{Area} = \int_1^e \ln x\,dx = \Big[x\ln x\Big]_1^e - \int_1^e x\,\frac{1}{x}\,dx = \Big[x\ln x - x\Big]_1^e = (e\ln e - e) - (1\cdot\ln 1 - 1) =$$

$$= \cancel{e} - \cancel{e} - 0 + 1 = \boxed{1}$$

$$\boxed{\begin{array}{l} u = \ln x \rightarrow du = \frac{1}{x}dx \\ dv = dx \rightarrow v = x \end{array}}$$

JUN 01

a) $f(x) = x^2 + bx$

$$F(x) = \int f(x)dx = \int (x^2+bx)dx = \frac{x^3}{3} + \frac{bx^2}{2} + K$$

$F(0) = 2 \longrightarrow \boxed{2 = K}$

$F(3) = 20 \longrightarrow 20 = \frac{27}{3} + \frac{9b}{2} + K \longrightarrow 20 = 9 + \frac{9b}{2} + 2 \; ; \; 40 = 18 + 9b + 4$

$\boxed{b = 2}$

$\boxed{F(x) = \frac{x^3}{3} + x^2 + 2}$

b) $f(x) = x^2 - x$ $\quad$ dom $f = \mathbb{R}$

$x^2 - x = 0 \rightarrow x = 0, \; x = 1$

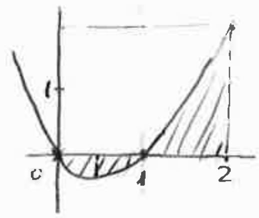
	$-\infty$		0		1		$+\infty$
f		$+$		$-$		$+$	

$f'(x) = 2x - 1$

$f'(x) = 0 \rightarrow x = 1/2$

		1/2	
f'	$-$		$+$
f	$\searrow$	MIN	$\nearrow$

$$\text{Area} = -\int_0^1 (x^2-x)dx + \int_1^2 (x^2-x)\,dx =$$



x	y
0	0
MIN 1/2	-1/4
1	0
2	2

$$= -\Big[\frac{x^3}{3} - \frac{x^2}{2}\Big]_0^1 + \Big[\frac{x^3}{3} - \frac{x^2}{2}\Big]_1^2 =$$

$$= -\left(\frac{1}{3} - \frac{1}{2}\right) + 0 + \left(\frac{8}{3} - \frac{4}{2}\right) - \left(\frac{1}{3} - \frac{1}{2}\right) = \frac{6}{3} - \frac{2}{2} = \boxed{1}$$

SEPT 01

a) $$\int (x+a)e^{\frac{x}{2}+1}dx = 2(x+a)e^{\frac{x}{2}+1} - 2\int e^{\frac{x}{2}+1}dx = 2(x+a)e^{\frac{x}{2}+1} - 4e^{\frac{x}{2}+1} =$$

$$= \boxed{(2x+2a-4)e^{\frac{x}{2}+1} + K}$$

$$\boxed{\begin{array}{l} u = x+a \rightarrow du = dx \\ dv = e^{\frac{x}{2}+1}dx \rightarrow v = 2e^{\frac{x}{2}+1} \end{array}}$$

b) $$\int_{-2}^2 f(x)dx = 8 \Rightarrow \Big[(2x+2a-4)e^{\frac{x}{2}+1}\Big]_{-2}^2 = 8 \; ; \; (\cancel{4}+2a-\cancel{4})e^2 - (-4+2a-4)e^0 = 8 \; ;$$

$$2ae^2 + \cancel{8} - 2a = \cancel{8} \; ; \; 2a(e^2-1) = 0 \Rightarrow \boxed{a = 0}$$

$a = 0$: $f(x) = x\,e^{x/2+1}$

$$(2xe^{x/2+1})' = 2e^{x/2+1} + \cancel{2}x\cdot\frac{1}{\cancel{2}}e^{x/2+1} = (x+2)e^{x/2+1} \neq f(x) \Rightarrow \boxed{\text{No es primitiva}}$$

JUN02

a) $f(x) = 3ax^2 + \dfrac{2a}{x^3} + 5 \quad (x>0)$

$F(x) = \int f(x)dx = \int (3ax^2 + 2ax^{-3} + 5)dx = ax^3 + 2a\dfrac{x^{-2}}{-2} + 5x + K = ax^3 - \dfrac{a}{x^2} + 5x + K$

$F(1) = 6 \Rightarrow 6 = \cancel{a} - \cancel{a} + 5 + K \Rightarrow (K=1)$

$F(2) = 42 \Rightarrow 42 = 8a - \dfrac{a}{4} + 10 + K \;;\; 42 = 8a - \dfrac{a}{4} + 10 + 1 \;;\; 168 = 32a - a + 40 + 4 \;;$

$124 = 31a \Rightarrow \boxed{a=4}$

b) $f(x) = 12x^2 + \dfrac{8}{x^3} + 5 \quad (x>0)$

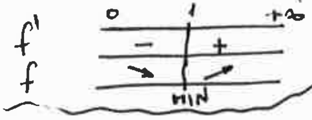
$\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^+} \left(12x^2 + \dfrac{8}{x^3} + 5\right) = 0 + \dfrac{8}{0^+} + 5 = 0 + \infty + 5 = +\infty$

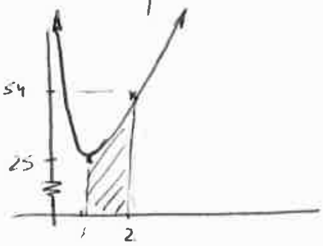
$\lim\limits_{x\to +\infty} f(x) = \lim\limits_{x\to +\infty} \left(12x^2 + \dfrac{8}{x^3} + 5\right) = +\infty + \dfrac{8}{+\infty} + 5 = +\infty + 0 + 5 = +\infty$

Como $x>0$, $f(x)$ no se anula en este dominio, ya que los tres sumandos que lo forman $12x^2$, $\dfrac{8}{x^3}$, 5 son positivos.

$f'(x) = 24x + \dfrac{-8\cdot 3x^2}{x^6} = 24x - \dfrac{24}{x^4}$

$f'(x) = 0 \rightarrow 24x - \dfrac{24}{x^4} = 0 \;;\; x^5 = 1$; $x = 1$





x	y
0^+	$+\infty$
1	25
2	54
$+\infty$	$+\infty$

MIN (x = 1)

$\text{Area} = \int_1^2 \left(12x^2 + \dfrac{8}{x^3} + 5\right)dx = \int_1^2 (12x^2 + 8x^{-3} + 5)\,dx = \left[4x^3 + 8\dfrac{x^{-2}}{-2} + 5x\right]_1^2 =$

$= \left[4x^3 - \dfrac{4}{x^2} + 5x\right] = (32 - 1 + 10) - (\cancel{4} - \cancel{4} + 5) = 41 - 5 = \boxed{36}$

También se podría haber hecho con los datos del apartado (a):

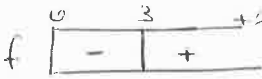
$\text{Area} = \int_1^2 f(x)dx = \left[F(x)\right]_1^2 = F(2) - F(1) = 42 - 6 = \boxed{36}$

SEPT02

a) $\int f(x)dx = \int (x^3 - 27 + axe^{x^2})\,dx = \boxed{\dfrac{x^4}{4} - 27x + \dfrac{a}{2}e^{x^2} + K}$

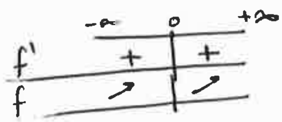
b) $a=0$: $f(x) = x^3 - 27 \quad (x \geq 0)$

$f(x) = 0 \rightarrow x^3 - 27 = 0 \;;\; x = 3$



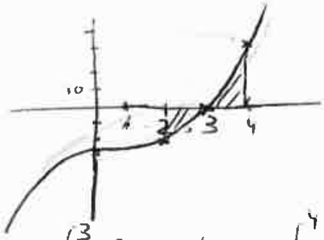
$f'(x) = 3x^2$

$f'(x) = 0 \rightarrow x = 0$



Es siempre creciente

x	y
0	−27
2	−19
3	0
4	37



$\text{Area} = -\int_2^3 f(x)dx + \int_3^4 f(x)dx = -\int_2^3 (x^3 - 27)\,dx + \int_3^4 (x^3 - 27)\,dx =$

$= -\left[\dfrac{x^4}{4} - 27x\right]_2^3 + \left[\dfrac{x^4}{4} - 27x\right]_3^4 = -\left(\dfrac{81}{4} - 81\right) + \left(\dfrac{16}{4} - 54\right) + \left(\dfrac{256}{4} - 108\right) - \left(\dfrac{81}{4} - 81\right)$

$= \dfrac{55}{2} = \boxed{27'5}$

JUN 03

a) $\int f(x)dx = \int\left(25 - x^2 + \frac{a}{x^2}\right)dx = \int(25 - x^2 + a x^{-2})dx = 25x - x^3/3 + a\frac{x^{-1}}{-1} =$

$= \boxed{25x - \frac{x^3}{3} - \frac{a}{x} + K}$

$f'(x) = -2x + \frac{-a\cdot 2x}{x^4} = -2x - \frac{2a}{x^3}$

$f'(1) = -2 \Rightarrow -2 = -2 - 2a \;;\; 2a = 0 \;;\; \boxed{a = 0}$

b) $f(x) = 25 - x^2$

$f(x) = 0 \rightarrow 25 - x^2 = 0 \;;\; x = \pm 5$

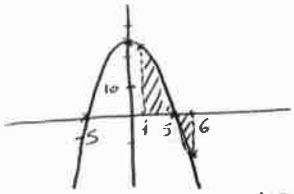
	$-\infty$	-5	5	$+\infty$
$f(x)$	$-$	$+$	$-$	

$f'(x) = -2x$

$f'(x) = 0 \rightarrow x = 0$

		0	
f'	$+$		$-$
f	$\nearrow$	MAX	$\searrow$

	x	y
	-5	0
MAX	0	25
	1	24
	5	0
	6	-11



$\text{Área} = \int_1^5 (25 - x^2)dx - \int_5^6 (25 - x^2)dx = \left[25x - \frac{x^3}{3}\right]_1^5 - \left[25x - \frac{x^3}{3}\right]_5^6 =$

$= \left(125 - \frac{125}{3}\right) - \left(25 - \frac{1}{3}\right) - \left(150 - \frac{216}{3}\right) + \left(125 - \frac{125}{3}\right) = \boxed{64}$

SEPT 03

a) $F(x) = \int f(x)dx = \int\left(x - \frac{27}{x^2} + e^{\frac{x}{2}+1}\right)dx = \int\left(x - 27\cdot x^{-1} + e^{\frac{x}{2}+1}\right)dx =$

$= \frac{x^2}{2} - 27\frac{x^{-2}}{-2} + 2e^{\frac{x}{2}+1} = \frac{x^2}{2} - \frac{27}{2x^2} + 2e^{\frac{x}{2}+1} + K$

$F(2) = 15'5 \Rightarrow 15'5 = \frac{4}{2} - \frac{27}{8} + 2e^2 + K \;;\; K = 15'5 - 2 + \frac{27}{8} - 2e^2 = \boxed{2'0969} = \boxed{16'875 - 2e^2}$

$\boxed{F(x) = \frac{x^2}{2} - \frac{27}{2x^2} + 2e^{\frac{x}{2}+1} + 16'875 - 2e^2}$

b) $f(x) = x - \frac{27}{x^2} \quad (x > 0)$

$f(x) = 0 \Rightarrow x - \frac{27}{x^2} = 0 \;;\; x = \frac{27}{x^2} \;;\; x^3 = 27 \;;\; x = 3$

	0	3	$+\infty$
f	$-$	$+$	

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}\left(x - \frac{27}{x^2}\right) = 0 - \frac{27}{0^+} = -\infty$

$\lim_{x\to +\infty} f(x) = \lim_{x\to +\infty}\left(x - \frac{27}{x^2}\right) = +\infty - \frac{27}{+\infty} = +\infty - 0 = +\infty$

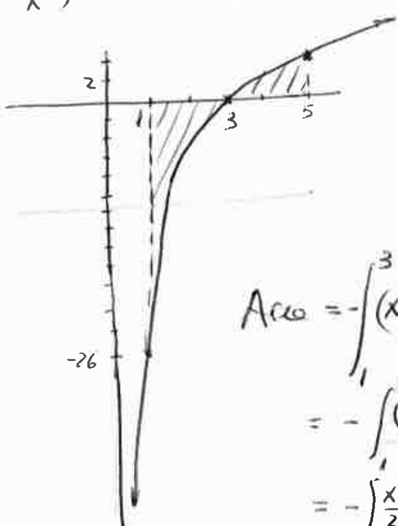
$f'(x) = 1 - \frac{-27\cdot 2x}{x^4} = 1 + \frac{54}{x^3}$

$f'(x) = 0 \rightarrow 1 + \frac{54}{x^3} = 0 \;;\; x = \sqrt[3]{-54}$

$\notin (0, +\infty)$

	0		$+\infty$
f'		$+$	
f		$\nearrow$	

x	y
0^+	$-\infty$
1	-26
3	0
5	$3'92$
$+\infty$	$+\infty$



$\text{Área} = -\int_1^3\left(x - \frac{27}{x^2}\right)dx + \int_3^5\left(x - \frac{27}{x^2}\right)dx =$

$= -\int_1^3 (x - 27x^{-2})dx + \int_3^5 (x - 27x^{-2})dx =$

$= -\left[\frac{x^2}{2} - 27\frac{x^{-1}}{-1}\right]_1^3 + \left[\frac{x^2}{2} - 27\frac{x^{-1}}{-1}\right]_3^5 =$

$= -\left[\frac{x^2}{2} + \frac{27}{x}\right]_1^3 + \left[\frac{x^2}{2} + \frac{27}{x}\right]_3^5 = -\left(\frac{9}{2} + 9\right) + \left(\frac{1}{2} + 27\right) + \left(\frac{25}{2} + \frac{27}{5}\right) - \left(\frac{9}{2} + 9\right) = \boxed{18'4}$

JUN 04

a) $F(x) = \int f(x)\,dx = \int (27 - x^3 + 3e^{2x-1})\,dx = 27x - \frac{x^4}{4} + \frac{3}{2}e^{2x-1} + K$

$F(1) = 26'75 \Rightarrow 26'75 = 27 - \frac{1}{4} + \frac{3}{2}e + K \Rightarrow K = -\frac{3}{2}e \rightarrow \boxed{F(x) = 27x - \frac{x^4}{4} + \frac{3}{2}e^{2x-1} - \frac{3}{2}e}$

b) $f(x) = 27 - x^3$

$f(x) = 0 \rightarrow 27 - x^3 = 0 \ ; \ x = 3$

	$-\infty$	3	$+\infty$
f	+		−

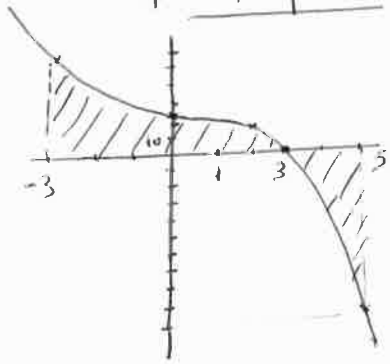
$f'(x) = -3x^2$

$f'(x) = 0 \rightarrow x = 0$

	$-\infty$	0	$+\infty$
f'	−		−
f	↘		↘

Es siempre decreciente.

x	y
−3	54
0	27
2	19
3	0
5	−98



$\text{Area} = \int_{-3}^{3}(27 - x^3)\,dx - \int_{3}^{5}(27 - x^3)\,dx = \left[27x - \frac{x^4}{4}\right]_{-3}^{3} - \left[27x - \frac{x^4}{4}\right]_{3}^{5} =$

$= \left(81 - \frac{81}{4}\right) - \left(-81 - \frac{81}{4}\right) - \left(135 - \frac{625}{4}\right) + \left(81 - \frac{81}{4}\right) = \boxed{244}$

SEPT 04

a) $f(x) = \frac{a}{x} + 32x^2 - x^3 \longrightarrow f'(x) = \frac{-a}{x^2} + 64x - 3x^2$

$f'(-1) = -10 \Rightarrow -10 = -a - 64 - 3 \ ; \ \boxed{a = -57}$

b) $f(x) = 3x^2 - x^3$

$f(x) = 0 \rightarrow 3x^2 - x^3 = 0 \ ; \ x^2(3 - x) = 0 \ ; \ x = \begin{cases} 0 \\ 3 \end{cases}$

	$-\infty$	0		3	$+\infty$
f(x)	+		+		−

$f'(x) = 6x - 3x^2$

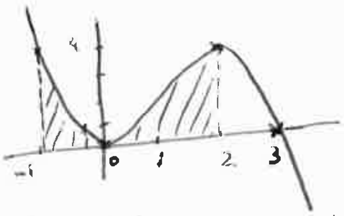
$f'(x) = 0 \rightarrow 6x - 3x^2 = 0$

$3x(2 - x) = 0$

$x = \begin{cases} 0 \\ 2 \end{cases}$

		0		2	
f'	−		+		−
f	↘		↗		↘

	x	y
	−1	4
MIN	0	0
MAX	2	4
	3	0



$\text{Area} = \int_{-1}^{2}(3x^2 - x^3)\,dx = \left[x^3 - \frac{x^4}{4}\right]_{-1}^{2} = \left(8 - \frac{16}{4}\right) - \left(-1 - \frac{1}{4}\right) = \boxed{5'25}$

JUN 05

a) $f(x) = \frac{4}{x^2} + 8x - x^2 - 12 \ ; \quad f'(x) = \frac{-8x}{x^4} + 8 - 2x = \frac{-8}{x^3} + 8 - 2x$

$f'(2) = \frac{-8}{8} + 8 - 4 = \boxed{3}$

b) $f(x) = 8x - x^2 - 12$

$f(x) = 0 \rightarrow -x^2 + 8x - 12 = 0 \ ; \ x = \frac{-8 \pm \sqrt{64 - 48}}{-2} = \frac{-8 \pm 4}{-2} \begin{cases} 2 \\ 6 \end{cases}$

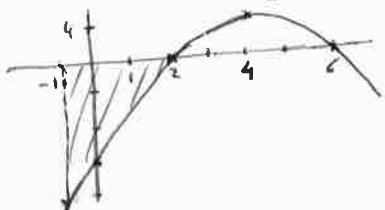
	$-\infty$	2		6	$+\infty$
f(x)	−		+		−

$f'(x) = 8 - 2x$

$f'(x) = 0 \rightarrow x = 4$

		4	
f'	+		−
f	↗	MAX	↘

	x	y
	−1	−21
	0	−12
	2	0
MAX	4	4
	6	0



$\text{Area} = -\int_{-1}^{2}(8x - x^2 - 12)\,dx = -\left[4x^2 - \frac{x^3}{3} - 12x\right]_{-1}^{2} = -\left(16 - \frac{8}{3} - 24\right) + \left(4 + \frac{1}{3} + 12\right) = \boxed{27}$

SEPT 05

$f(x) = x + \frac{4}{x^2} \quad (x>0)$

a) $F(x) = \int f(x)\,dx = \int \left(x + \frac{4}{x^2}\right) dx = \int (x + 4\cdot x^{-2})\,dx = \frac{x^2}{2} + 4\frac{x^{-1}}{-1} = \boxed{\frac{x^2}{2} - \frac{4}{x} + K}$

$\left.\begin{matrix} x=2 \\ y=5 \end{matrix}\right| \to \quad 5 = \frac{2^2}{2} - \frac{4}{2} + K \;;\; 5 = \cancel{2} - \cancel{2} + K \;;\; K=5 \qquad \boxed{F(x) = \frac{x^2}{2} - \frac{4}{x} + 5}$

b) $f(x) = x + \frac{4}{x^2} \qquad \text{dom} f = (0, +\infty)$

$\lim_{x \to 0^+} f(x) = 0 + \frac{4}{+0} = +\infty$

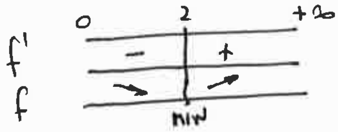
$\lim_{x \to +\infty} f(x) = +\infty + \frac{4}{+\infty} = +\infty + 0 = +\infty$

$y=0 \to \quad 0 = x + \frac{4}{x^2} \;;\; -\frac{4}{x^2} = x \;;\; -4 = x^3 \;;\; x = \sqrt[3]{-4} \notin (0,+\infty)$

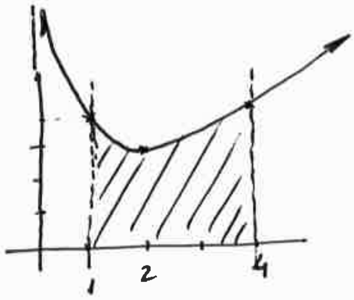
Por lo tanto no corta a los ejes.

$f'(x) = 1 + \frac{-4\cdot 2x}{x^4} = 1 - \frac{8}{x^3}$

$f'(x)=0 \to \quad 0 = 1 - \frac{8}{x^3} \;;\; \frac{8}{x^3} = 1 \;;\; 8 = x^3 \;;\; x = \sqrt[3]{8} = 2$



Tiene un mínimo en $x=2 \to y = 2 + \frac{4}{4} = 3 \quad \boxed{(2,3)}$



	x	y
	1	5
MIN	2	3
	4	4'25

$\text{Area} = \int_1^4 \left(x + \frac{4}{x^2}\right) dx =$

$= \left[\frac{x^2}{2} - \frac{4}{x}\right]_1^4 =$

$= (8-1) - \left(\frac{1}{2} - 4\right) = \boxed{10'5}$

JUN 06

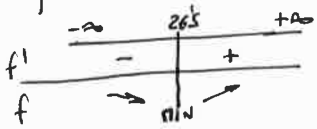
a) $f(x) = x^2 + \frac{1}{x^2} - 53x + 150 \longrightarrow f'(x) = 2x + \frac{-2x}{x^4} - 53 = 2x - \frac{2}{x^3} - 53$

$f'(-0'5) = 2\cdot(-0'5) - \frac{2}{(-0'5)^3} - 53 = \boxed{-38}$

b) $y = x^2 - 53x + 150$

$y' = 2x - 53$

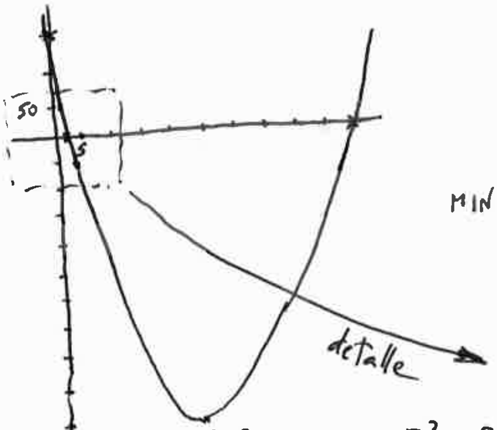
$y' = 0 \to x = 53/2 = 26'5$



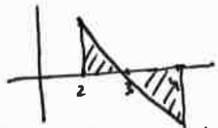
$x=0 \to y = 150$

$y=0 \to x^2 - 53x + 150 = 0$

$x = \begin{matrix} \nearrow 50 \\ \searrow 3 \end{matrix}$



	x	y
	0	150
	50	0
	3	0
MIN	26'5	-552'25
	2	48
	4	-46



$\text{Area} = \int_2^3 (x^2 - 53x + 150)\,dx - \int_3^4 (x^2 - 53x + 150)\,dx = \left[\frac{x^3}{3} - \frac{53x^2}{2} + 150x\right]_2^3 - \left[\frac{x^3}{3} - \frac{53x^2}{2} + 150x\right]_3^4 =$

$= \left(\frac{441}{2} - \frac{590}{3}\right) - \left(\frac{592}{3} - \frac{441}{2}\right) = \boxed{47}$

SEPT 06

$f(x) = x^3 - 81x^2 \Rightarrow f'(x) = 3x^2 - 162x$

a) $F(x) = \frac{x^4}{4} - 27x^3 + K$

$F(4) = f'(54) \Rightarrow \frac{4^4}{4} - 27\cdot 4^3 + K = 3\cdot 54^2 - 162\cdot 54 \rightarrow K = 1664 \rightarrow \boxed{F(x) = \frac{x^4}{4} - 27x^3 + 1664}$

b) $y = x^3 - 81x^2$

$x=0 \rightarrow y=0$

$y=0 \rightarrow x^3 - 81x^2 = 0$

$x^2(x-81) = 0 \rightarrow x = 0,\ 81$

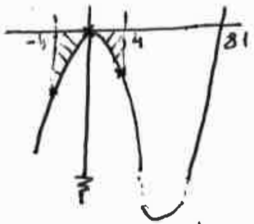
$y' = 3x^2 - 162x$

$y' = 0 \Rightarrow 3x^2 - 162x = 0$

$3x(x-54) = 0 \rightarrow x = 0,\ 54$

	$-\infty$	0	54	$+\infty$
f'	+	−	+	
f	↗	↘	↗	
		MAX	MIN	

	x	y
MAX	0	0
	81	0
MIN	54	−78732
	−4	−1360
	4	−1232



$$\text{Area} = -\int_{-4}^{4} (x^3 - 81x^2)\,dx = -\left[\frac{x^4}{4} - 27x^3\right]_{-4}^{4} = -\left(\frac{4^4}{4} - 27\cdot 4^3\right) + \left(\frac{(-4)^4}{4} - 27\cdot(-4)^3\right) = \boxed{3456}$$

JUN 07

a) $f(x) = 4x - x^2 + \frac{2}{x^3}$

$f'(x) = 4 - 2x + \frac{-2\cdot 3x^2}{x^6} = 4 - 2x - \frac{6}{x^4}$

$f'(-2) = 4 - 2\cdot(-2) - \frac{6}{(-2)^4} = \boxed{7'625} = \frac{61}{8}$

b) $y = 4x - x^2$

$x=0 \rightarrow y=0$

$y=0 \rightarrow 4x - x^2 = 0$

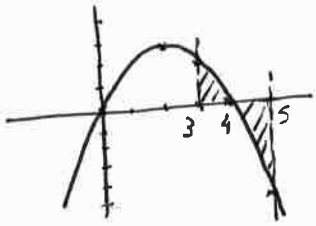
$x(4-x) = 0 \rightarrow x = 4,\ 0$

$y' = 4 - 2x$

$y' = 0 \rightarrow x = 2$

	$-\infty$	2	$+\infty$
f'	+	−	
f	↗	↘	

	x	y
	0	0
	4	0
MAX	2	4
	3	3
	5	−5



$$\text{Area} = \int_3^4 (4x - x^2)\,dx - \int_4^5 (4x - x^2)\,dx = \left[2x^2 - \frac{x^3}{3}\right]_3^4 - \left[2x^2 - \frac{x^3}{3}\right]_4^5 =$$

$$= \left(2\cdot 4^2 - \frac{4^3}{3}\right) - \left(2\cdot 3^2 - \frac{3^3}{3}\right) - \left(2\cdot 5^2 - \frac{5^3}{3}\right) + \left(2\cdot 4^2 - \frac{4^3}{3}\right) = \boxed{4}$$

SEPT 07 $f(x) = -x^2+7x-12 \longrightarrow f'(x) = -2x+7$

a) $F(x) = -\frac{x^3}{3} + \frac{7x^2}{2} - 12x + K$

$F(6) = f'(6) \Rightarrow -\frac{6^3}{3} + \frac{7\cdot 6^2}{2} - 12\cdot 6 + K = -2\cdot 6+7 \Rightarrow K=13 \Rightarrow \boxed{F(x) = -\frac{x^3}{3} + \frac{7x^2}{2} - 12x + 13}$

b) $y = -x^2+7x-12$

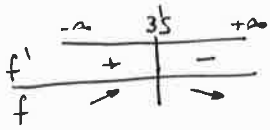
$x=0 \rightarrow y=-12$

$y=0 \rightarrow -x^2+7x-12=0$

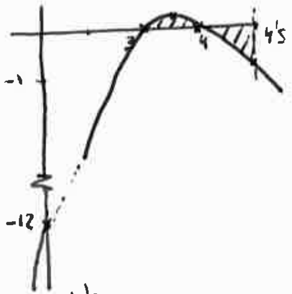
$x = \frac{-7\pm\sqrt{49-48}}{-2} = \frac{-7\pm 1}{-2} < \begin{matrix} 3 \\ 4 \end{matrix}$

$y' = -2x+7$

$y'=0 \Rightarrow x = 3'5$



x	y
0	-12
3	0
4	0
MAX 3'5	0'25
4'5	-0'75



$$Area = \int_3^4 (-x^2+7x-12)dx - \int_4^{4'5} (-x^2+7x-12)dx = \left[-\frac{x^3}{3} + \frac{7x^2}{2} - 12x\right]_3^4 - \left[-\frac{x^3}{3} + 7\frac{x^2}{2} - 12x\right]_4^{4'5} =$$

$$= \left[-\frac{4^3}{3} + \frac{7\cdot 4^2}{2} - 12\cdot 4\right] - \left[-\frac{3^3}{3} + \frac{7\cdot 3^2}{2} - 12\cdot 3\right] - \left[-\frac{4'5^3}{3} + \frac{7\cdot 4'5^2}{2} - 12\cdot 4'5\right] + \left[-\frac{4^3}{3} + 7\frac{4^2}{2} - 12\cdot 4\right.$$

$= \boxed{\frac{1}{3}}$

JUN 08 a) $f(x) = 2x^3 - 6x^2 + \frac{3}{x^4} \longrightarrow f'(x) = 6x^2 - 12x + \frac{-3\cdot 4x^3}{x^8} = 6x^2 - 12x - \frac{12}{x^5}$

$f'(-2) = 6\cdot(-2)^2 - 12\cdot(-2) - \frac{12}{(-2)^5} = \frac{387}{8} = \boxed{48'375}$

b) $y = 2x^3 - 6x^2$

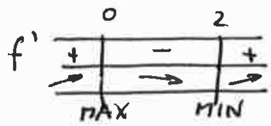
$x=0 \rightarrow y=0$

$y=0 \rightarrow 2x^3-6x^2=0$

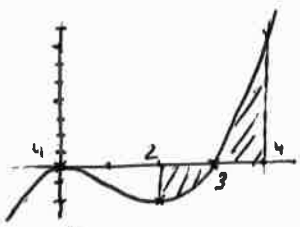
$2x^2(x-3)=0 \rightarrow x = \begin{matrix} 0 \\ 3 \end{matrix}$

$y' = 6x^2-12x$

$y'=0 \rightarrow 6x^2-12x=0 ;\quad 6x(x-2)=0 ;\quad x = \begin{matrix} 0 \\ 2 \end{matrix}$



	x	y
MAX	0	0
	3	0
MIN	2	-8
	4	32



$$Area = -\int_2^3 (2x^3-6x^2)dx + \int_3^4 (2x^3-6x^2)dx = -\left[\frac{2x^4}{4} - 2x^3\right]_2^3 + \left[\frac{2x^4}{4} - 2x^3\right]_3^4 =$$

$$= -\left(\frac{2\cdot 3^4}{4} - 2\cdot 3^3\right) + \left(\frac{2\cdot 2^4}{4} - 2\cdot 2^3\right) + \left(\frac{2\cdot 4^4}{4} - 2\cdot 4^3\right) - \left(\frac{2\cdot 3^4}{4} - 2\cdot 3^3\right) = \boxed{19}$$

SEPT 08 $f(x)=3x^2-6x \longrightarrow f'(x)=6x-6$

a) $F(x)=x^3-3x^2+K$

$F(2)=f'(3) \Rightarrow 2^3-3\cdot 2^2+K=6\cdot 3-6 \Rightarrow K=16 \Rightarrow \boxed{F(x)=x^3-3x^2+16}$

b) $y=3x^2-6x$

$x=0 \rightarrow y=0$

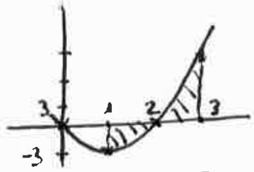
$y=0 \rightarrow 3x^2-6x=0 \; ; \; 3x(x-2)=0 \; ; \; x= 0, \; 2$

$y'=6x-6$

$y'=0 \Rightarrow x=1$

	$-\infty$	1	$+\infty$
f'	$-$		$+$
f	$\searrow$	MIN	$\nearrow$

	x	y
	0	0
	2	0
MIN	1	-3
	3	9



$$\text{Area} = -\int_1^2 (3x^2-6x)\,dx + \int_2^3 (3x^2-6x)\,dx = -\left[x^3-3x^2\right]_1^2 + \left[x^3-3x^2\right]_2^3 =$$

$$= -(8-12)+(1-3)+(27-27)-(8-12) = \boxed{16}$$

JUN 09 $f(x)=\dfrac{a}{x^2}+x^2 \longrightarrow f'(x)=\dfrac{-a\cdot 2x}{x^4}+2x = \dfrac{-2a}{x^3}+2x$

a) $f'(2)=1 \Rightarrow 1=\dfrac{-2a}{8}+4 \; ; \; 8=-2a+32 \; ; \; 2a=24 \; ; \; \boxed{a=12}$

b) $f(x)=\dfrac{16}{x^2}+x^2$

$\text{dom} f: \mathbb{R}-\{0\}$

No corta al eje Y

$y=0 \rightarrow \dfrac{16}{x^2}+x^2=0 \; ; \; 16+x^4=0 \; ; \; x^4=-16$ No corta al eje X.

$\lim_{x\to 0^-} f(x) = \dfrac{16}{0^+}+0^2=+\infty$ | Asíntota vertical $x=0$

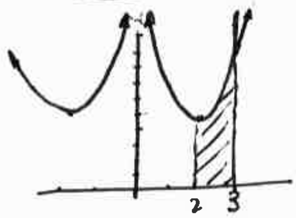
$\lim_{x\to 0^+} f(x) = \dfrac{16}{0^+}+0^2=+\infty$

$f'(x)=\dfrac{-16\cdot 2x}{x^4}+2x = \dfrac{-32}{x^3}+2x$

$f'(x)=0 \Rightarrow 0=-\dfrac{32}{x^3}+2x \; ; \; \dfrac{32}{x^3}=2x \; ; \; 16=x^4 \; ; \; x=\pm 2$

	$-\infty$	-2		0		2	$+\infty$
f'	$-$		$+$		$-$		$+$
f	$\searrow$	MIN	$\nearrow$		$\searrow$	MIN	$\nearrow$

	x	y
MIN	-2	5
MIN	2	5
	3	10'7



$$\text{Area}=\int_2^3 \left(\frac{16}{x^2}+x^2\right)dx = \int_2^3 (16\cdot x^{-2}+x^2)\,dx =$$

$$= \left[16\,\frac{x^{-1}}{-1}+\frac{x^3}{3}\right]_2^3 = \left[-\frac{16}{x}+\frac{x^3}{3}\right]_2^3 = \left(-\frac{16}{3}+9\right)-\left(-8+\frac{8}{3}\right)=\boxed{19}$$

SEPT 09

$f(x) = 5+\frac{1}{x^2} \rightarrow f'(x) = 0 + \frac{-2x}{x^4} = -\frac{2}{x^3}$

a) $f'(2) = \frac{-2}{2^3} = \boxed{-\frac{1}{4}}$

b) $y = 5+\frac{1}{x^2}$

$\text{dom} f = \mathbb{R}-\{0\} \longrightarrow$ No corta el eje y

$y=0 \Rightarrow 5+\frac{1}{x^2}=0 \;;\; 5=-\frac{1}{x^2}$

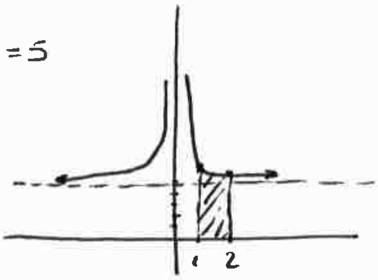
$5x^2=-1$

No corta el eje X.

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = 5+\frac{1}{+0} = +\infty$

$\lim_{x\to -\infty} f(x) = \lim_{x\to -\infty} f(x) = 5+\frac{1}{+\infty} = 5+0=5$

x	y
1	6
2	5'25
-1	6
-2	5'25



$$\text{Area} = \int_1^2 \left(5+\frac{1}{x^2}\right)dx = \int_1^2 (5+x^{-2})\,dx = \left[5x+\frac{x^{-1}}{-1}\right]_1^2 = \left[5x-\frac{1}{x}\right]_1^2 = \left(10-\frac{1}{2}\right)-(5-1) = \boxed{5'5}$$

JUN 10
F. General

$f(x) = x^2-4x$

a) $F(x) = \frac{x^3}{3}-2x^2+K$

$F(3)=0 \Rightarrow 0 = \frac{3^3}{3}-2\cdot 3^2+K \;;\; K=9 \longrightarrow \boxed{F(x) = \frac{x^3}{3}-2x^2+9}$

b) $y = x^2-4x$ $\qquad \text{dom} f = \mathbb{R}$

$x=0 \rightarrow y=0$

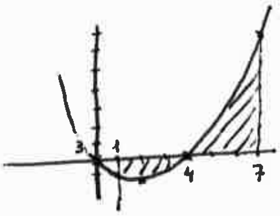
$y=0 \rightarrow x^2-4x=0 \;;\; x(x-4)=0 \;;\; x = 0,\ 4$

$y' = 2x-4$

$y'=0 \Rightarrow 2x-4=0 \;;\; x=2$ Posible MAX/MIN

	$-\infty$	2	$+\infty$
f'	$-$		$+$
f	↘	MIN	↗

	x	y
	0	0
	4	0
MIN	2	-4
	1	-3
	7	21



$$\text{Area} = -\int_1^4 (x^2-4x)\,dx + \int_4^7 (x^2-4x)\,dx = -\left(\frac{x^3}{3}-2x^2\right)_1^4 + \left(\frac{x^3}{3}-2x^2\right)_4^7 =$$

$$= -\left(\frac{4^3}{3}-2\cdot 4^2\right) + \left(\frac{1}{3}-2\right) + \left(\frac{7^3}{3}-2\cdot 7^2\right) - \left(\frac{4^3}{3}-2\cdot 4^2\right) = \boxed{36}$$

JUN 10
F. Específica

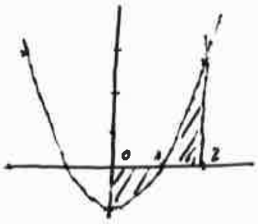
$f(x) = x^2-1$

a) $F(x) = \frac{x^3}{3} - x + K$

$F(3)=10 \Rightarrow 10 = \frac{3^3}{3} - 3 + K \Rightarrow K=4 \rightarrow \boxed{F(x) = \frac{x^3}{3} - x + 4}$

b) $y = x^2-1$

	x	y
MIN	0	-1
	1	0
	-1	0
	2	3
	-2	3



$$\text{Area} = -\int_0^1 (x^2-1)dx + \int_1^2 (x^2-1)dx =$$
$$= -\left(\frac{x^3}{3}-x\right)_0^1 + \left(\frac{x^3}{3}-x\right)_1^2 =$$
$$= -\left(\frac{1}{3}-1\right) + \left(\frac{8}{3}-2\right) - \left(\frac{1}{3}-1\right) = \boxed{2}$$

JUL 10
F. General

$f(x) = x^2+1$

a) $F(x) = \frac{x^3}{3} + x + K$

$F(3)=10 \Rightarrow 10 = \frac{3^3}{3} + 3 + K \Rightarrow K=-2 \rightarrow \boxed{F(x) = \frac{x^3}{3} + x - 2}$

b) $y = x^2+1$

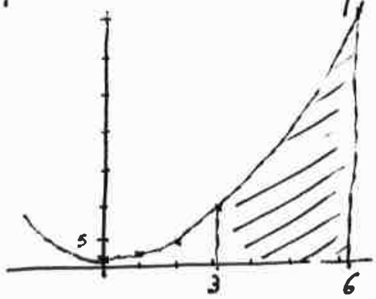
$y=0 \Rightarrow x^2+1=0$; $x^2=-1$ No corta al eje y

$y' = 2x$

$y'=0 \rightarrow 2x=0$; $x=0$

	0	
f'	−	+
f	↘ MIN	↗

	x	y
MIN	0	1
	1	2
	2	5
	-1	2
	-2	5
	3	10
	6	37



$$\text{Area} = \int_3^6 (x^2+1)dx = \left(\frac{x^3}{3}+x\right)_3^6 = \left(\frac{6^3}{3}+6\right) - \left(\frac{3^3}{3}+3\right) = \boxed{66}$$

JUL 10
F. Específica

$f(x) = 2x - x^2$

a) $F(x) = x^2 - \frac{x^3}{3} + K$

$F(3) = 100 \Rightarrow 100 = 9 - \frac{27}{3} + K$; $K=100$ $\boxed{F(x) = x^2 - \frac{x^3}{3} + 100}$

b) $y = 2x - x^2$

$x=0 \rightarrow y=0$

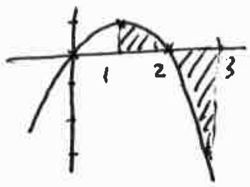
$y=0 \rightarrow 0 = 2x - x^2$; $0 = x(2-x)$; $x = 0, 2$

$y' = 2-2x$

$y'=0 \Rightarrow 2-2x=0$; $x=1$ Posible MAX/MIN

	$-\infty$	1	$+\infty$
f'	+		−
f	↗	MAX	↘

	x	y
	0	0
	2	0
MAX	1	1
	3	-3



$$\text{Area} = \int_1^2 (2x-x^2)dx - \int_2^3 (2x-x^2)dx = \left(x^2 - \frac{x^3}{3}\right)_1^2 - \left(x^2 - \frac{x^3}{3}\right)_2^3 =$$
$$= \left(4-\frac{8}{3}\right) - \left(1-\frac{1}{3}\right) - \left(9-\frac{27}{3}\right) + \left(4-\frac{8}{3}\right) = \boxed{5}$$

JUN 11
fase
General

$f(x) = 3x^2 - 6x + 10$ $\boxed{\text{dom} f = \mathbb{R}}$

a) $F(x) = \int (3x^2 - 6x + 10)\,dx = x^3 - 3x^2 + 10x + K$

$F(1) = 10 \Rightarrow 1^3 - 3\cdot 1^2 + 10\cdot 1 + K = 10$; $1 - 3 + 10 + K = 10 \Rightarrow K = 2$

$\boxed{F(x) = x^3 - 3x^2 + 10x + 2}$

b) $f'(x) = 6x - 6$

$f'(x) = 0 \rightarrow 6x - 6 = 0$; $x = 1$

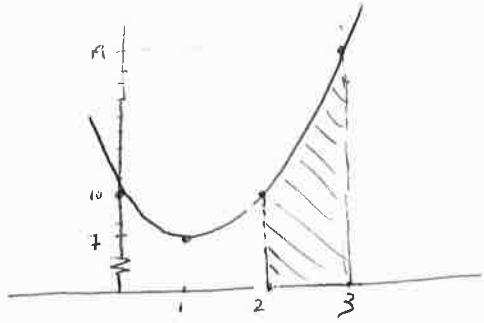
	$-\infty$	1	$+\infty$
f'	$-$		$+$
f	$\searrow$	MIN REL	$\nearrow$

$y = 0 \rightarrow 3x^2 - 6x + 10 = 0$

$x = \dfrac{6 \pm \sqrt{36 - 120}}{2}$ $\nexists$ No corta al eje X

$x = 0 \rightarrow y = 10$

	x	y
	$-\infty$	$+\infty$
	0	10
MIN	1	7
	2	10
	3	19
	$+\infty$	$+\infty$



$\text{Area} = \int_2^3 (3x^2 - 6x + 10)\,dx = \left[x^3 - 3x^2 + 10x\right]_2^3 = (27 - 27 + 30) - (8 - 12 + 20) = \boxed{14}$

JUN 11
fase
Específica

$f(x) = x^2 - 6x + 8$ $\boxed{\text{dom} f = \mathbb{R}}$

a) $F(x) = \int (x^2 - 6x + 8)\,dx = \dfrac{x^3}{3} - 3x^2 + 8x + K$

$F(3) = 10 \Rightarrow \dfrac{3^3}{3} - 3\cdot 3^2 + 8\cdot 3 + K = 10$; $K = -6$

$\boxed{F(x) = \dfrac{x^3}{3} - 3x^2 + 8x - 6}$

b) $f'(x) = 2x - 6$

$f'(x) = 0 \rightarrow x = 3$

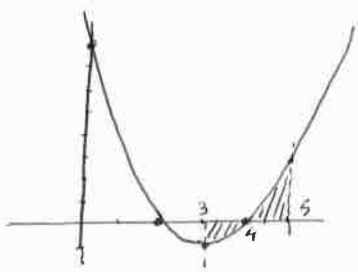
	$-\infty$	3	$+\infty$
f'	$-$		$+$
f	$\searrow$	MIN REL	$\nearrow$

$x = 0 \rightarrow y = 8$

$y = 0 \rightarrow x^2 - 6x + 8 = 0$

$x = \dfrac{6 \pm \sqrt{36 - 32}}{2} = \dfrac{6 \pm 2}{2} \begin{cases} 4 \\ 2 \end{cases}$

	x	y
	$-\infty$	$+\infty$
	0	8
	2	0
MIN	3	-1
	4	0
	5	$+3$
	$+\infty$	$+\infty$



$\text{Area} = -\int_3^4 (x^2 - 6x + 8)\,dx + \int_4^5 (x^2 - 6x + 8)\,dx = -\left(\dfrac{x^3}{3} - 3x^2 + 8x\right)_3^4 + \left(\dfrac{x^3}{3} - 3x^2 + 8x\right)_4^5 =$

$= -\left(\dfrac{4^3}{3} - 3\cdot 4^2 + 8\cdot 4\right) + \left(\dfrac{1}{3} - 3 + 8\right) + \left(\dfrac{5^3}{3} - 3\cdot 5^2 + 8\cdot 5\right) - \left(\dfrac{4^3}{3} - 3\cdot 4^2 + 8\cdot 4\right) = \boxed{2}$

JUL 11
Fase General

$f(x) = x^2 - 2x$ $\boxed{\text{dom} f = \mathbb{R}}$

a) $F(x) = \int (x^2-2x)\,dx = \frac{x^3}{3} - x^2 + K$

$F(6) = 40 \Rightarrow \frac{6^3}{3} - 6^2 + K = 40$; $K = 4$

$\boxed{F(x) = \frac{x^3}{3} - x^2 + 4}$

b) $f'(x) = 2x - 2$

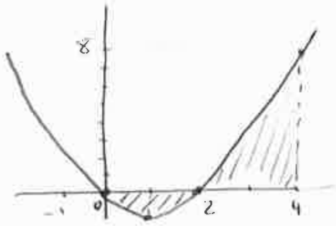
$f'(x) = 0 \rightarrow 2x - 2 = 0$; $x = 1$

	$-\infty$	1	$+\infty$
f'	$-$		$+$
f	$\searrow$	MIN REL.	$\nearrow$

$x = 0 \rightarrow y = 0$

$y = 0 \rightarrow x^2 - 2x = 0$; $x = 0$, $x = 2$

	x	y
	$-\infty$	$+\infty$
	0	0
MIN	1	-1
	2	0
	4	8
	$+\infty$	$+\infty$



$$\text{Area} = -\int_0^2 (x^2-2x)\,dx + \int_2^4 (x^2-2x)\,dx = -\left(\frac{x^3}{3} - x^2\right)_0^2 + \left(\frac{x^3}{3} - x^2\right)_2^4 =$$

$$= -\left(\frac{8}{3} - 4\right) + \left(\frac{64}{3} - 16\right) - \left(\frac{8}{3} - 4\right) = \boxed{8}$$

JUL 11
Fase Específica

$f(x) = 1 - \frac{1}{x^2}$ $\boxed{\text{dom} f = \mathbb{R} - \{0\}}$

a) $F(x) = \int \left(1 - \frac{1}{x^2}\right)dx = \int (1 - x^{-2})\,dx = x - \frac{x^{-1}}{-1} = x + \frac{1}{x} + K$

$F(1) = 3 \Rightarrow 1 + \frac{1}{1} + K = 3$; $K = 1$

$\boxed{F(x) = x + \frac{1}{x} + 1}$

b) $f'(x) = -\frac{-2x}{x^4} = \frac{2}{x^3}$

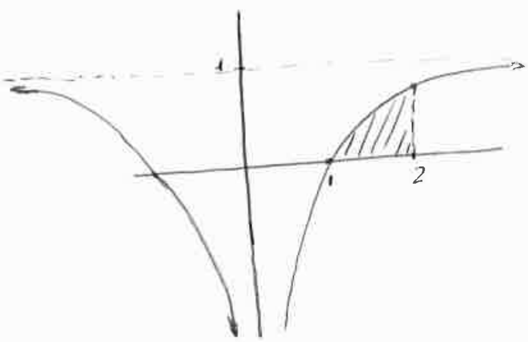
$f'(x) = 0 \rightarrow \frac{2}{x^3} = 0$ $\nexists$

	$-\infty$	0	$+\infty$
f'	$-$		$+$
f	$\searrow$		$\nearrow$

$x = 0 \rightarrow \nexists$ No corta al eje Y

$y = 0 \rightarrow 0 = 1 - \frac{1}{x^2}$; $\frac{1}{x^2} = 1$; $1 = x^2$; $x = \pm 1$

x	y
$-\infty$	1
-1	0
0^-	$-\infty$
0^+	$-\infty$
1	0
2	0'75
$+\infty$	1



$$\text{Area} = \int_1^2 \left(1 - \frac{1}{x^2}\right)dx = \int_1^2 (1 - x^{-2})\,dx = \left[x - \frac{x^{-1}}{-1}\right]_1^2 = \left[x + \frac{1}{x}\right]_1^2 = \left(2 + \frac{1}{2}\right) - \left(1 + \frac{1}{1}\right) = \boxed{0'5}$$

JUN 12 fase General

$f(x) = x^3 - 12x$

a) $F(x) = \int (x^3 - 12x)\,dx = \frac{x^4}{4} - 6x^2 + C$

$F(2) = 1 \Rightarrow 1 = \frac{16}{4} - 24 + C \;;\; C = 25 - \frac{16}{4} = 21$

$\boxed{F(x) = \frac{x^4}{4} - 6x^2 + 21}$

b) $y = x^3 - 12x$

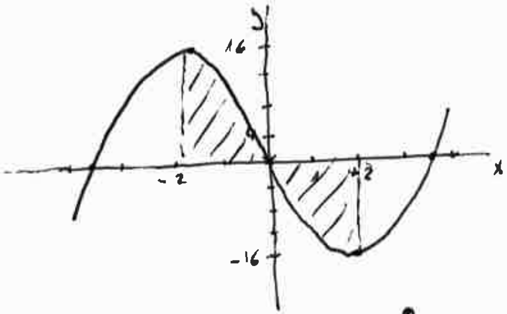
$x = 0 \rightarrow y = 0$

$y = 0 \rightarrow x^3 - 12x = 0 \;;\; x(x^2 - 12) = 0 \rightarrow x = 0$; $\rightarrow x^2 = 12 \;;\; x = \pm\sqrt{12}$

$y' = 3x^2 - 12$

$y' = 0 \Rightarrow 3x^2 - 12 = 0 \;;\; x^2 = 4 \;;\; x = \pm 2$

		-2		$+2$	
y'	$+$		$-$		$+$
y	$\nearrow$	MAX	$\searrow$	MIN	$\nearrow$



	x	y
	0	0
	$\sqrt{12}$	0
	$-\sqrt{12}$	0
MAX	-2	16
MIN	2	-16

$$\text{Area} = \int_{-2}^{0} f(x)\,dx - \int_{0}^{2} f(x)\,dx = 2\cdot\int_{-2}^{0} f(x)\,dx = 2\cdot[F(0) - F(-2)] =$$

$$= 2\left[21 - \left(\frac{16}{4} - 24 + 21\right)\right] = \boxed{40 \text{ u.s.}}$$

JUN 12 fase Específica

$f(x) = 5x - x^2 - 4$

a) $F(x) = \int (5x - x^2 - 4)\,dx = \frac{5x^2}{2} - \frac{x^3}{3} - 4x + C$

$F(3) = 0 \Rightarrow 0 = \frac{45}{2} - 9 - 12 + C \;;\; C = 21 - \frac{45}{2} = -\frac{3}{2}$

$\boxed{F(x) = \frac{5x^2}{2} - \frac{x^3}{3} - 4x - \frac{3}{2}}$

b) $y = 5x - x^2 - 4$

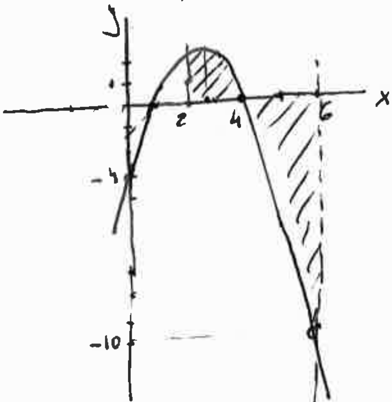
$x = 0 \rightarrow y = -4$

$y = 0 \rightarrow 0 = 5x - x^2 - 4 \;;\; x^2 - 5x + 4 = 0 \;;\; x = \frac{5 \pm \sqrt{25 - 16}}{2} = \begin{cases} 4 \\ 1 \end{cases}$

$y' = 5 - 2x$

$y' = 0 \rightarrow 5 - 2x = 0 \;;\; x = 5/2$

		$5/2$	
y'	$+$		$-$
y	$\nearrow$	MAX	$\searrow$



	x	y
	0	-4
	4	0
	1	0
MAX	$5/2$	2'25
	6	-10

$$\text{Area} = \int_{2}^{4} f(x)\,dx - \int_{4}^{6} f(x)\,dx =$$

$$= \left(\frac{5\cdot16}{2} - \frac{64}{3} - 16 - \frac{3}{2}\right) - \left(\frac{5\cdot4}{2} - \frac{8}{3} - 8 - \frac{3}{2}\right) -$$

$$- \left(\frac{5\cdot36}{2} - \frac{216}{3} - 24 - \frac{3}{2}\right) + \left(\frac{5\cdot16}{2} - \frac{64}{3} - 16 - \frac{3}{2}\right) = \boxed{12 \text{ u.s.}}$$

JUL 12 fase General

a) $f(x) = 3-x$

$F(x) = \int (3-x)\,dx = 3x - \frac{x^2}{2} + C$

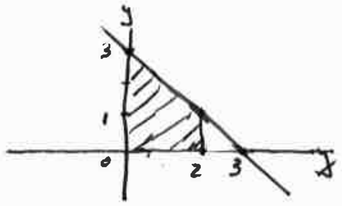
$F(0) = 2 \Rightarrow 2 = 0 - 0 + C \;;\; C = 2$

$\Rightarrow F(x) = \boxed{3x - \frac{x^2}{2} + 2}$

b) $y = 3-x$

$x=0 \rightarrow y=3$

$y=0 \rightarrow x=3$



x	y
0	3
3	0
2	1

$\text{Área} = \int_0^2 f(x)\,dx = \left(3\cdot 2 - \frac{4}{2}\right) - 0 = \boxed{4 \text{ u.s.}}$

También: Área Trapecio $= \frac{B+b}{2}\cdot h = \frac{3+1}{2}\cdot 2 = 4$ u.s. ✓

JUL 12 fase Específica

a) $f(x) = 4-x^2$

$F(x) = \int (4-x^2)\,dx = 4x - \frac{x^3}{3} + C$

$F(3) = 5 \Rightarrow 5 = 12 - \frac{27}{3} + C \;;\; C = \frac{27}{3} - 7 = 2$

$\Rightarrow F(x) = \boxed{4x - \frac{x^3}{3} + 2}$

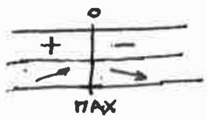
b) $y = 4-x^2$

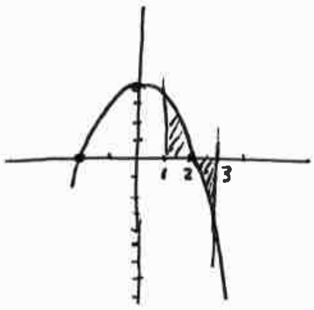
$x=0 \rightarrow y=4$

$y=0 \rightarrow 4-x^2=0 \;;\; x=\pm 2$

$y' = -2x$

$y'=0 \rightarrow x=0$





	x	y
MAX	0	4
	2	0
	-2	0
	3	-5
	1	3

$\text{Área} = \int_1^2 f(x)\,dx - \int_2^3 f(x)\,dx =$

$= \left(8 - \frac{8}{3}\right) - \left(4 - \frac{1}{3}\right) - \left(12 - \frac{27}{3}\right) + \left(8 - \frac{8}{3}\right) = \boxed{4 \text{ u.s.}}$

JUN 13 fase General

a) $f(x) = 4x - 2x^2$

$F(x) = \int (4x - 2x^2)\,dx = 2x^2 - \frac{2x^3}{3} + C$

$F(6) = 0 \Rightarrow 0 = 2\cdot 36 - 2\cdot\frac{216}{3} + C \;;\; C = 72$

$F(x) = \boxed{2x^2 - \frac{2x^3}{3} + 72}$

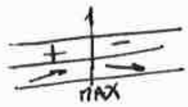
b) $y = 4x - 2x^2$

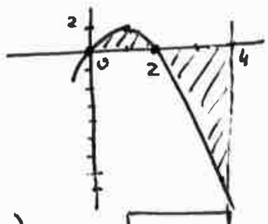
$x=0 \rightarrow y=0$

$y=0 \rightarrow 4x-2x^2=0 \;;\; 2x(2-x)=0 \;;\; x = 0,\ 2$

$y' = 4-4x$

$y'=0 \Rightarrow 4-4x=0 \;;\; x=1$





	x	y
	0	0
	2	0
MAX	1	2
	4	-16

$\text{Área} = \int_0^2 f(x)\,dx - \int_2^4 f(x)\,dx =$

$= \left(2\cdot 4 - \frac{2\cdot 8}{3}\right) - (0-0) =$

$- \left(2\cdot 16 - \frac{2\cdot 64}{3}\right) - \left(2\cdot 4 - \frac{2\cdot 8}{3}\right) = \boxed{16 \text{ u.s.}}$

JUN13 fase específica

a) $f(x) = e^{x/3}$

$F(x) = \int e^{x/3}\,dx = 3e^{x/3} + C$

$F(0) = 4 \Rightarrow 4 = 3e^0 + C \;;\; C = 1$

$F(x) = \boxed{3e^{x/3} + 1}$

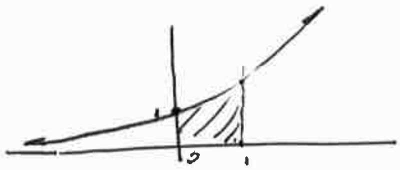
b) $y = e^{x/3}$

$x=0 \to y = 1$

$y=0 \to e^{x/3} = 0$ Absurdo.

	$-\infty$	$+\infty$
y'	$+$	
y	$\nearrow$	

$\lim\limits_{x\to-\infty} f(x) = e^{-\infty} = 0 \;;\; \lim\limits_{x\to+\infty} f(x) = e^{+\infty} = +\infty$



x	y
$-\infty$	0
0	1
1	$e^{1/3}$
$+\infty$	$+\infty$

$\text{Area} = \int_0^1 e^{x/3}\,dx = \left[3e^{x/3}+1\right]_0^1 = (3e^{1/3}+1) - (3e^0+1) = \boxed{3e^{1/3} - 3}$ u.s.

JUL 13 fase General

a) $f(x) = \frac{2}{x}$ $\quad \text{dom} f = \mathbb{R} - \{0\}$

$F(x) = \int \frac{2}{x}\,dx = 2\ln x + C$

$F(1) = 2 \Rightarrow 2 = 2\ln 1 + C \;;\; C = 2$

$F(x) = \boxed{2\ln x + 2}$

b) $y = \frac{2}{x}$

$x=0 \to y = \frac{2}{0}$ Absurdo

$\lim\limits_{x\to 0^-} \frac{2}{x} = \frac{2}{-0} = -\infty$

$\lim\limits_{x\to 0^+} \frac{2}{x} = \frac{2}{+0} = +\infty$

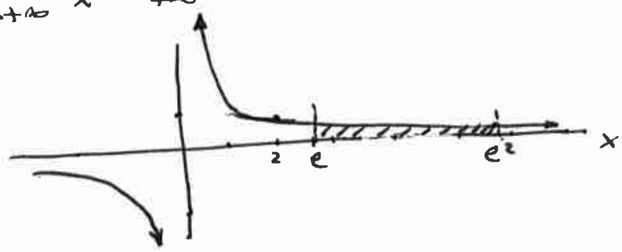
$y=0 \to 0 = \frac{2}{x} \;;\; 0 = 2$ Absurdo

$y' = \frac{-2}{x^2}$

$y'=0 \to \frac{-2}{x^2} = 0 \;;\; -2 = 0$ Absurdo

	$-\infty$	0	$+\infty$
y'	$-$		$-$
y	$\searrow$		$\searrow$

$\lim\limits_{x\to-\infty} \frac{2}{x} = \frac{2}{-\infty} = 0 \;;\; \lim\limits_{x\to+\infty} \frac{2}{x} = \frac{2}{+\infty} = 0$



$\text{Area} = \int_e^{e^2} \frac{2}{x}\,dx =$

$= \left[2\ln x\right]_e^{e^2} = 2\ln e^2 - 2\ln e = 2\cdot 2 - 2\cdot 1 = \boxed{2 \text{ u.s.}}$

JUL 13
fase
Específica

a) $f(x) = 8x - 2x^3$

$F(x) = \int (8x-2x^3)\,dx = 4x^2 - \frac{2x^4}{4} + C = 4x^2 - \frac{x^4}{2} + C$

$F(2) = 9 \Rightarrow 9 = 4\cdot 4 - \frac{16}{2} + C \; ; \; C = 1$

$F(x) = \boxed{4x^2 - \frac{x^4}{2} + 1}$

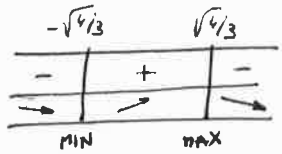
b) $y = 8x - 2x^3$

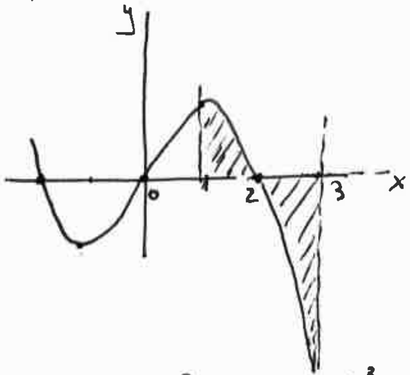
$x=0 \rightarrow y=0$

$y=0 \rightarrow 8x-2x^3=0 \; ; \; 2x(4-x^2)=0 \; ; \; x = \begin{cases} 0 \\ \pm 2 \end{cases}$

$y' = 8 - 6x^2$

$y'=0 \Rightarrow 0 = 8-6x^2 \; ; \; 6x^2 = 8 \; ; \; x^2 = \frac{4}{3} \; ; \; x = \pm\sqrt{4/3}$





x	y
0	0
2	0
−2	0
MIN $-\sqrt{4/3}$	$-32/3\sqrt{3}$
MAX $\sqrt{4/3}$	$+32/3\sqrt{3}$
1	6
3	−30

$\text{Area} = \int_1^2 f(x)\,dx - \int_2^3 f(x)\,dx = \left[4x^2 - \frac{x^4}{2}\right]_1^2 - \left[4x^2 - \frac{x^4}{2}\right]_2^3 =$

$= \left(4\cdot 4 - \frac{16}{2}\right) - \left(4\cdot 1 - \frac{1}{2}\right) - \left(4\cdot 9 - \frac{81}{2}\right) + \left(4\cdot 4 - \frac{16}{2}\right) = \boxed{17 \text{ u.s.}}$

JUN 14
fase
General

a) $f(x) = \frac{9}{(2+x)^2} - 1 \qquad \text{dom} f: \mathbb{R} - \{-2\}$

$F(x) = \int \left(\frac{9}{(2+x)^2} - 1\right) dx = \int \left(9(2+x)^{-2} - 1\right) dx = 9\frac{(2+x)^{-1}}{-1} - x + C =$

$= -\frac{9}{2+x} - x + C$

$F(1) = 1 \Rightarrow 1 = -\frac{9}{3} - 1 + C \; ; \; C = 5$

$F(x) = \boxed{-\frac{9}{2+x} - x + 5}$

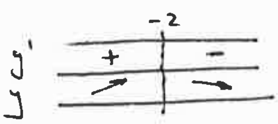
b) $y = \frac{9}{(2+x)^2} - 1$

$x=0 \rightarrow y = \frac{9}{4} - 1 = \frac{5}{4}$

$y=0 \rightarrow 0 = \frac{9}{(2+x)^2} - 1 \; ; \; 1 = \frac{9}{(2+x)^2} \; ; \; (2+x)^2 = 9 \; ; \; 2+x = \pm 3 \; ; \; x = \begin{cases} 1 \\ -5 \end{cases}$

$y' = \frac{-9\cdot 2(2+x)}{(2+x)^4} = \frac{-18}{(2+x)^3}$

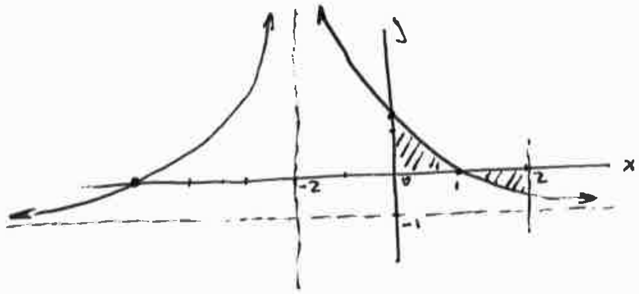
$y'=0 \Rightarrow 0 = \frac{-18}{(2+x)^3} \; ; \; 0 = -18 \quad \text{Absurdo}$



$\lim_{x\to -2^-} f(x) = \frac{9}{+0} - 1 = +\infty$

$\lim_{x\to -2^-} f(x) = \frac{9}{+0} - 1 = +\infty$

$\lim_{x\to -\infty} f(x) = \frac{9}{+\infty} - 1 = -1 \quad ; \quad \lim_{x\to +\infty} f(x) = \frac{9}{+\infty} - 1 = -1$



x	y
$-\infty$	-1
-5	0
-2^-	$+\infty$
-2^+	$+\infty$
0	$5/4$
1	0
$+\infty$	-1
2	$-7/16$

$$\text{Area} = \int_0^1 f(x)dx - \int_1^2 f(x)dx = \left[-\frac{9}{2+x} - x\right]_0^1 - \left[-\frac{9}{2+x} - x\right]_1^2 =$$

$$= \left(-\frac{9}{3} - 1\right) - \left(-\frac{9}{2} - 0\right) - \left(-\frac{9}{4} - 2\right) + \left(-\frac{9}{3} - 1\right) = \boxed{49/3 \text{ u.s.}}$$

JUN 14
fase
Específica

a) $f(x) = \dfrac{-1}{\sqrt{x-1}}$ $\quad \text{dom} f = (1, +\infty)$

$$F(x) = \int \frac{-1}{\sqrt{x-1}}\,dx = \int -(x-1)^{-1/2}dx = -\frac{(x-1)^{1/2}}{1/2} + C = C - 2\sqrt{x-1}$$

$$F(5) = 1 \Rightarrow 1 = C - 2\sqrt{4} \;;\; 1 = C - 4 \;;\; C = 5 \Rightarrow$$

$$\Rightarrow F(x) = \boxed{5 - 2\sqrt{x-1}}$$

b) $y = \dfrac{-1}{\sqrt{x-1}}$

$$\lim_{x\to 1^+} \frac{-1}{\sqrt{x-1}} = \frac{-1}{0} = -\infty$$

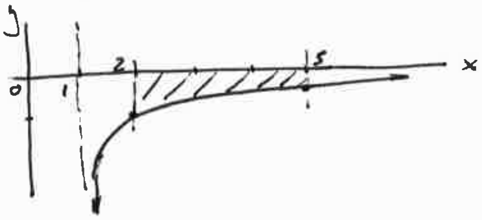
$y = 0 \rightarrow 0 = \dfrac{-1}{\sqrt{x-1}}$; $0 = -1$ Absurdo

$$y' = \frac{+1 \cdot \frac{1}{2\sqrt{x-1}}}{x-1} = \frac{1}{2(x-1)\sqrt{x-1}}$$

$y' = 0 \Rightarrow 0 = \dfrac{1}{2(x-1)\sqrt{x-1}}$; $0 = 1$ Absurdo

	1		$+\infty$
y'		$+$	
y		$\nearrow$	

$$\lim_{x\to+\infty} \frac{-1}{\sqrt{x-1}} = \frac{-1}{+\infty} = 0$$



x	y
1^+	$-\infty$
2	-1
5	$-1/2$
$+\infty$	0

$$\text{Area} = -\int_2^5 \frac{-1}{\sqrt{x-1}}\,dx = -\left[-2\sqrt{x-1}\right]_2^5 = 2\sqrt{4} - 2\sqrt{1} = \boxed{4 \text{ u.s.}}$$

JUL 14 fase General

a) $f(x) = 2x + ax^3$

$F(x) = \int (2x + ax^3)\,dx = x^2 + \frac{ax^4}{4} + C$

$F(1) = 4 \Rightarrow 4 = 1 + \frac{a}{4} + C$
$F(2) = 22 \Rightarrow 22 = 4 + \frac{16a}{4} + C$
$\rightarrow$ $a + 4C = 12$, $16a + 4C = 72$

$15a = 60 \Rightarrow a = 4$

$4 + 4C = 12$

$C = 2$

$F(x) = x^2 + \frac{4x^4}{4} + 2 = \boxed{x^2 + x^4 + 2}$

b) $y = 2x + 4x^3$

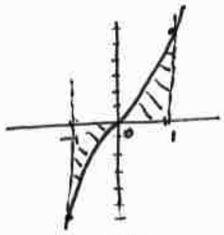
$x = 0 \rightarrow y = 0$

$y = 0 \rightarrow 2x + 4x^3 = 0 \;;\; 2x(1 + 2x^2) = 0$ → $2x = 0 \;;\; x = 0$ → $1 + 2x^2 = 0 \;;\; 2x^2 = -1$ Absurdo

$y' = 2 + 12x^2$

$y' = 0 \rightarrow 2 + 12x^2 = 0 \;;\; 12x^2 = -2$ Absurdo

	$-\infty$ $\quad$ $+\infty$
y'	+
y	↗



x	y
-1	-6
0	0
1	6

$\text{Area} = -\int_{-1}^{0} (2x + 4x^3)\,dx + \int_0^1 (2x + 4x^3)\,dx = 2\int_0^1 (2x + 4x^3)\,dx = \left[2(x^2 + x^4)\right]_0^1 = \boxed{4 \text{ u.s}}$

JUL 14 fase Específica

a) $f(x) = (x-1)^2(2x-5) = (x^2 - 2x + 1)(2x - 5) = 2x^3 - 9x^2 + 12x - 5$

$F(x) = \int (2x^3 - 9x^2 + 12x - 5)\,dx = \frac{2x^4}{4} - 3x^3 + 6x^2 - 5x + C$

$F(2) = 1 \Rightarrow 1 = \frac{2 \cdot 16}{4} - 3 \cdot 8 + 6 \cdot 4 - 5 \cdot 2 + C \;;\; C = 3$

$\Rightarrow F(x) = \boxed{\frac{x^4}{2} - 3x^3 + 6x^2 - 5x + 3}$

b) $y = (x-1)^2(2x-5)$

$x = 0 \rightarrow y = -5$

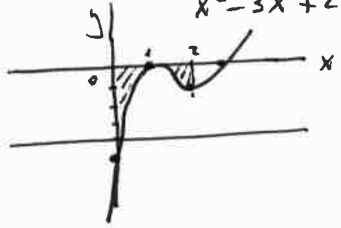
$y = 0 \rightarrow (x-1)^2(2x-5) = 0$ → $(x-1)^2 = 0 \;;\; x = 1$ → $2x - 5 = 0 \;;\; x = 5/2$

$y' = 6x^2 - 18x + 12$

$y' = 0 \Rightarrow 6x^2 - 18x + 12 = 0$

$x^2 - 3x + 2 = 0 \;;\; x = 1, 2$

	1	2	
y'	+	−	+
y	↗	↘	↗
	MAX	MIN	



	x	y
	0	-5
MAX	1	0
MIN	2	-1
	5/2	0

$\text{Area} = -\int_0^2 f(x)\,dx = -\left(\frac{x^4}{2} - 3x^3 + 6x^2 - 5x\right)_0^2 =$

$= -\left(\frac{16}{2} - 3 \cdot 8 + 6 \cdot 4 - 10\right) = \boxed{2 \text{ u.s.}}$

JUN 15 fase general

$f(x) = 3 + 8x + 15x^2$

a) $F(x) = \int (3 + 8x + 15x^2)\,dx = 3x + 4x^2 + 5x^3 + C$

$F(0) = 100 \Longrightarrow C = 100$; $\boxed{F(x) = 5x^3 + 4x^2 + 3x + 100}$

b) $y = 15x^2 + 8x + 3$

$y' = 30x + 8$

$y' = 0 \rightarrow x = -\frac{8}{30} = -\frac{4}{15} \longrightarrow y = 1'9\widehat{3}$

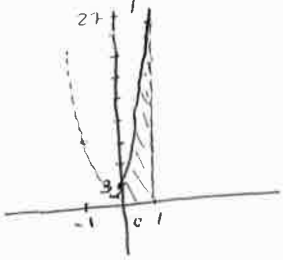
	x	y
	0	3
	1	26
V	$-4/15$	$1'9\widehat{3}$
	-1	10

$y = 0 \rightarrow 15x^2 + 8x + 3 = 0$

$x = \frac{-8 \pm \sqrt{64 - 120}}{30}$ ✗ No corta al eje X

$\text{Area} = \int_0^1 (15x^2 + 8x + 3)\,dx =$

$= \left[5x^3 + 4x^2 + 3x\right]_0^1 = \boxed{12}$ u.s.



JUN 15 fase específica

$f(x) = \frac{10}{(x+1)^2}$

a) $F(x) = \int \frac{10}{(x+1)^2}\,dx = 10 \int (x+1)^{-2}\,dx = 10\,\frac{(x+1)^{-1}}{-1} = \frac{-10}{x+1} + C$

$F(4) = 0 \Longrightarrow 0 = \frac{-10}{4+1} + C$; $C = 2$; $\boxed{F(x) = 2 - \frac{10}{x+1}}$

b) $y = \frac{10}{(x+1)^2}$ Dominio $= \mathbb{R} - \{-1\}$

$y' = \frac{-10 \cdot 2(x+1)}{(x+1)^4} = \frac{-20}{(x+1)^3}$

$y' = 0 \rightarrow \frac{-20}{(x+1)^3} = 0$; $-20 = 0$ ✗ No tiene extremos relativos.

	$-\infty$	-1	$+\infty$
y'	+		−
y	↗		↘

$\lim_{x \to -\infty} \frac{10}{(x+1)^2} = \frac{10}{+\infty} = 0$

$\lim_{x \to +\infty} \frac{10}{(x+1)^2} = \frac{10}{+\infty} = 0$ $\longrightarrow$ Asíntota Horizontal $y = 0$

$\lim_{x \to -1^-} \frac{10}{(x+1)^2} = \frac{10}{0^+} = +\infty$

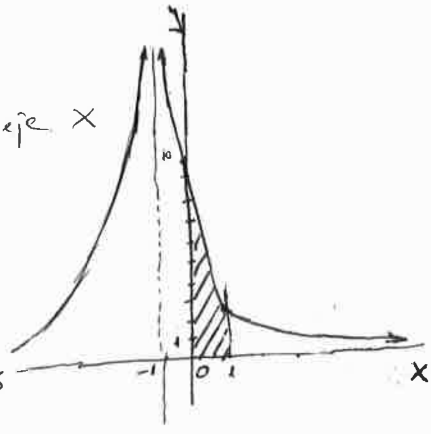
$\lim_{x \to -1^+} \frac{10}{(x+1)^2} = \frac{10}{0^+} = +\infty$ $\longrightarrow$ Asíntota Vertical $x = -1$

$y = 0 \rightarrow \frac{10}{(x+1)^2} = 0$; $10 = 0$ ✗ No corta al eje X

x	y
$-\infty$	0
-1^-	$+\infty$
-1^+	$+\infty$
0	10
1	$2'5$
3	$1'25$
$+\infty$	0

$\text{Area} = \int_0^1 \frac{10}{(x+1)^2}\,dx = \left[\frac{-10}{x+1}\right]_0^1 =$

$= \frac{-10}{2} + \frac{10}{1} = \boxed{5}$ u.s.



JUL 15
fase
general

$f(x) = x^2 - 3x + 2$

a) $F(x) = \int (x^2-3x+2)\,dx = \frac{x^3}{3} - \frac{3x^2}{2} + 2x + C$

$F(3) = 2 \Rightarrow \frac{27}{3} - \frac{3\cdot 9}{2} + 2\cdot 3 = C \;;\; C = \frac{3}{2} \;;\; \boxed{F(x) = \frac{x^3}{3} - \frac{3x^2}{2} + 2x + \frac{3}{2}}$

b) $y = x^2 - 3x + 2$

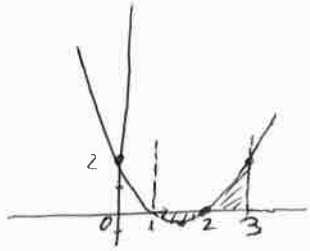
$y' = 2x - 3$

$y' = 0 \rightarrow x = 3/2 \Rightarrow y = \frac{9}{4} - \frac{9}{2} + 2 = -\frac{1}{4}$

$y = 0 \rightarrow x^2 - 3x + 2 = 0$

$x = \frac{3 \pm \sqrt{9-8}}{2} = \frac{3 \pm 1}{2} \begin{matrix} \nearrow 2 \\ \searrow 1 \end{matrix}$

x	y
0	2
1	0
√ 3/2	-1/4
2	0
3	2



	1		2	
y	+	−	+	

$\text{Area} = -\int_1^2 (x^2-3x+2)\,dx + \int_2^3 (x^2-3x+2)\,dx =$

$= -\left[\frac{x^3}{3} - \frac{3x^2}{2} + 2x\right]_1^2 + \left[\frac{x^3}{3} - \frac{3x^2}{2} + 2x\right]_2^3 =$

$= -\frac{8}{3} + \frac{12}{2} - 4 + \frac{1}{3} - \frac{3}{2} + 2 + \frac{27}{3} - \frac{27}{2} + 6 - \frac{8}{3} + \frac{12}{2} - 4 = \boxed{1}$ u.s.

JUL 15
fase
específica

$f(x) = x^2 - x$

a) $F(x) = \int (x^2 - x)\,dx = \frac{x^3}{3} - \frac{x^2}{2} + C$

$F(6) = 50 \Rightarrow \frac{6^3}{3} - \frac{6^2}{2} + C = 50 \;;\; C = -4 \;;\; \boxed{F(x) = \frac{x^3}{3} - \frac{x^2}{2} - 4}$

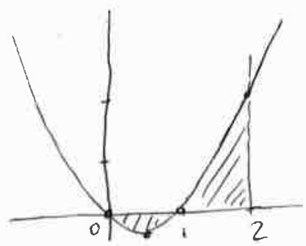
b) $y = x^2 - x$

$y' = 2x - 1$

$y' = 0 \rightarrow x = 1/2 \rightarrow y = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4}$

$y = 0 \rightarrow x^2 - x = 0 \;;\; x(x-1) = 0 \;;\; x = \begin{matrix} \nearrow 0 \\ \searrow 1 \end{matrix}$

x	y
0	0
√ 1/2	-1/4
1	0
2	2



	0		1	
y	+	−	+	

$\text{Area} = -\int_0^1 (x^2 - x)\,dx + \int_1^2 (x^2 - x)\,dx =$

$= -\left(\frac{x^3}{3} - \frac{x^2}{2}\right)_0^1 + \left(\frac{x^3}{3} - \frac{x^2}{2}\right)_1^2 =$

$= -\frac{1}{3} + \frac{1}{2} + \frac{8}{3} - \frac{4}{2} - \frac{1}{3} + \frac{1}{2} = \boxed{1}$ u.s.

JUN 16 fase General

$$f(x) = \frac{20}{(x+2)^2}$$

a) $F(x) = \int \frac{20}{(x+2)^2}dx = 20\int (x+2)^{-2}dx = 20\,\frac{(x+2)^{-1}}{-1} + C = -\frac{20}{x+2} + C$

$F(3) = 0 \Rightarrow 0 = -\frac{20}{5} + C \;;\; C = 4 \Rightarrow \boxed{F(x) = 4 - \frac{20}{x+2}}$

b) $f(x) = \frac{20}{(x+2)^2}$ $\quad dom f = \mathbb{R} - \{-2\}$

$\lim_{x\to -\infty} \frac{20}{(x+2)^2} = \frac{20}{+\infty} = 0$
$\lim_{x\to +\infty} \frac{20}{(x+2)^2} = \frac{20}{+\infty} = 0$ | Asíntota Horizontal $y=0$

$\lim_{x\to -2^-} f(x) = \frac{20}{(-0)^2} = \frac{20}{+0} = +\infty$
$\lim_{x\to -2^+} f(x) = \frac{20}{(+0)^2} = \frac{20}{+0} = +\infty$ | Asíntota Vertical

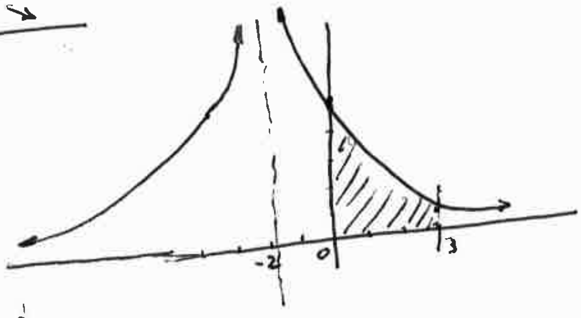
$y=0 \rightarrow 0 = \frac{20}{(x+2)^2} \;;\; 0 = 20$ Absurdo No corta al eje X

$f'(x) = \frac{-20\cdot 2\cdot (x+2)}{(x+2)^4} = \frac{-40}{(x+2)^3}$

$f'(x) = 0 \rightarrow 0 = \frac{-40}{(x+2)^3} \;;\; 0 = -40$ Absurdo No tiene extremos.

	$-\infty$ -2	$+\infty$
f'	+	−
f	↗	↘

x	y
$-\infty$	0
−4	5
-2^-	$+\infty$
-2^+	$+\infty$
0	5
3	0'8
$+\infty$	0



$$Area = \int_0^3 \frac{20}{(x+2)^2}dx = \left[\frac{-20}{x+2}\right]_0^3 = \frac{-20}{5} + \frac{20}{2} = \boxed{6}\; u^2$$

JUN 16 fase Específica

$f(x) = 0'6 - 0'01x$

a) $F(x) = \int (0'6 - 0'01x)\,dx = 0'6x - 0'01\frac{x^2}{2} + C = 0'6x - 0'005x^2 + C$

$F(0) = 0'2 \Rightarrow 0'2 = 0 - 0 + C \;;\; C = 0'2 \Rightarrow \boxed{F(x) = 0'6x - 0'005x^2 + 0'2}$

b) $f(x) = 0'6 - 0'01x$ $\quad dom f = \mathbb{R}$

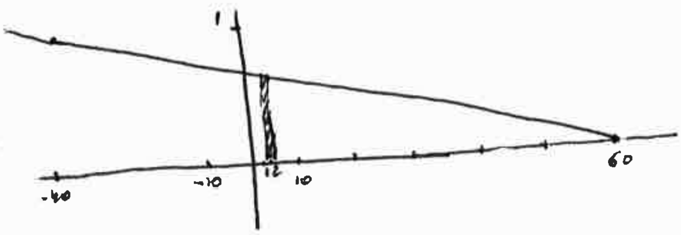
$f'(x) = -0'01$

$f'(x) = 0 \rightarrow -0'01 = 0$ Absurdo No tiene extremos

	$-\infty$	$+\infty$
f'	−	
f	→	

$y=0 \rightarrow 0 = 0'6 - 0'01x \;;\; 0'01x = 0'6 \;;\; x = \frac{0'6}{0'01} = 60$

x	y
−40	1
0	0'6
1	0'59
2	0'58
60	0



$$\text{Área} = \int_1^2 (0'6 - 0'01x)\,dx = \left[0'6x - 0'005x^2\right]_1^2 = (1'2 - 0'02) - (0'6 - 0'005) = 0'605 - 0'02 = \boxed{0'603}\ u^2$$

JUL 16 fase General

$f(x) = x^3 - 5x^2 + 4x$

a) $F(x) = \int (x^3 - 5x^2 + 4x)\,dx = \frac{x^4}{4} - \frac{5x^3}{3} + 2x^2 + C$

$F(2) = 1 \Rightarrow 1 = 4 - \frac{40}{3} + 8 + C \ ; \ C = \frac{7}{3} \Rightarrow \boxed{F(x) = \frac{x^4}{4} - \frac{5x^3}{3} + 2x^2 + \frac{7}{3}}$

b) $f'(x) = 3x^2 - 10x + 4$

$f'(x) = 0 \Rightarrow x = \frac{10 \pm \sqrt{100 - 48}}{6} = \frac{10 \pm \sqrt{52}}{6}$ → $2'87$, $0'465$

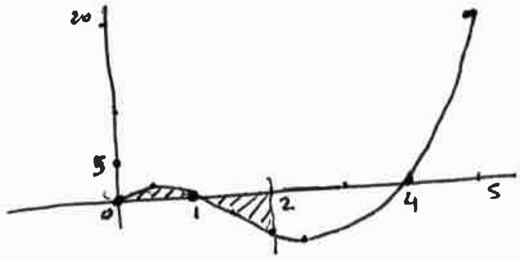
	$-\infty$	$0'465$		$2'87$	$+\infty$
f'	+		−		+
f	↗	MAX	↘	MIN	↗

$y = 0 \rightarrow x^3 - 5x^2 + 4x = 0$

$x(x^2 - 5x + 4) = 0$ → $x = 0$; $x^2 - 5x + 4 = 0$

$x = \frac{5 \pm \sqrt{25 - 16}}{2} = \frac{5 \pm 3}{2}$ → 4, 1

	x	y
	0	0
MAX	0'465	0'879
	1	0
	2	−4
MIN	2'87	−6'06
	4	0
	5	20



$$\text{Área} = \int_0^1 (x^3 - 5x^2 + 4x)\,dx - \int_1^2 (x^3 - 5x^2 + 4x)\,dx = \left[\frac{x^4}{4} - \frac{5x^3}{3} + 2x^2\right]_0^1 - \left[\frac{x^4}{4} - \frac{5x^3}{3} + 2x^2\right]_1^2 =$$

$$= \left(\frac{1}{4} - \frac{1}{3} + 2\right) - 0 - \left(4 - \frac{40}{3} + 8\right) + \left(\frac{1}{4} - \frac{1}{3} + 2\right) = \frac{31}{6} = \boxed{5'16}\ u^2$$

JUL 16 fase Específica

$f(x) = 4x - x^3$

a) $F(x) = \int (4x - x^3)\,dx = 2x^2 - \frac{x^4}{4} + C$

$F(2) = 7 \Rightarrow 7 = 8 - 4 + C \Rightarrow C = 3 \Rightarrow \boxed{F(x) = 2x^2 - \frac{x^4}{4} + 3}$

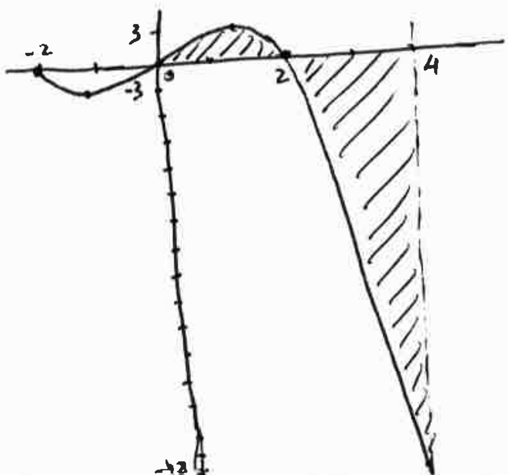
b) $f'(x) = 4 - 3x^2$

$f'(x) = 0 \Rightarrow 0 = 4 - 3x^2 \ ; \ 3x^2 = 4 \ ; \ x^2 = \frac{4}{3} \ ; \ x = \pm \frac{2}{\sqrt{3}} = \pm 1'15$

	$-\infty$	$-2/\sqrt{3}$		$2/\sqrt{3}$	$+\infty$
f'	−		+		−
f	↘	MIN	↗	MAX	↘

$y = 0 \Rightarrow 0 = 4x - x^3 \ ; \ 0 = x(4 - x^2)$ → $x = 0$; $x = \pm 2$

	x	y
	−2	0
MIN	$-2/\sqrt{3}$	−3'08
	0	0
MAX	$2/\sqrt{3}$	3'08
	2	0
	4	−48



$$\text{Area} = \int_0^2 (4x-x^3)\,dx - \int_2^4 (4x-x^3)\,dx = \left[2x^2-\frac{x^4}{4}\right]_0^2 - \left[2x^2-\frac{x^4}{4}\right]_2^4 =$$

$$= (8-4) - 0 - (32-64) + (8-4) = \boxed{40}\ u^2$$

Modelo 17 $f(x) = 3x^2-6x$

a) $F(x) = \int (3x^2-6x)\,dx = x^3-3x^2+C$

$F(1)=10 \Rightarrow 10 = 1-3+C \ ; \ C=12 \Rightarrow \boxed{F(x) = x^3-3x^2+12}$

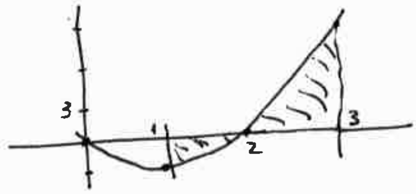
b) $f'(x) = 6x-6$

$f'(x)=0 \Rightarrow 6x-6=0 \ ; \ x=1$

	$-\infty$	1	$+\infty$
f'	$-$		$+$
f	$\searrow$	MIN	$\nearrow$

$y=0 \Rightarrow 0=3x^2-6x \ ; \ 0=3x(x-2)$ $\to x=0$, $\to x=2$

	x	y
	0	0
MIN	1	-3
	2	0
	3	9



$$\text{Area} = -\int_1^2 (3x^2-6x)\,dx + \int_2^3 (3x^2-6x)\,dx = -\left[x^3-3x^2\right]_1^2 + \left[x^3-3x^2\right]_2^3 =$$

$$= -(8-12) + (1-3) + (27-27) - (8-12) = \boxed{6}\ u^2$$