# Cálculo Diferencial e Integral con uso de Calculadora Gráfica en exámenes BI - NM

**Mayo 00 P2**

**In this question you should note that radians are used throughout.**

(a) (i) Sketch the graph of $y = x^2 \cos x$, for $0 \le x \le 2$ making clear the approximate positions of the positive $x$-intercept, the maximum point and the end-points.

(ii) Write down the **approximate** coordinates of the positive $x$-intercept, the maximum point and the end-points.

(b) Find the **exact value** of the positive $x$-intercept for $0 \le x \le 2$.

Let $R$ be the region in the first quadrant enclosed by the graph and the $x$-axis.

(c) (i) Shade $R$ on your diagram.

(ii) Write down an integral which represents the area of $R$.

(d) Evaluate the integral in part (c)(ii), either by using a graphic display calculator, or by using the following information.

$$\frac{\mathrm{d}}{\mathrm{d}x}(x^2 \sin x + 2x \cos x - 2 \sin x) = x^2 \cos x.$$

**Mayo 00 P2**

(a) Sketch the graph of $y = \pi \sin x - x$, $-3 \le x \le 3$, on millimetre square paper, using a scale of 2 cm per unit on each axis.

Label and number both axes and indicate clearly the approximate positions of the $x$-intercepts and the local maximum and minimum points.

(b) Find the solution of the equation

$$\pi \sin x - x = 0, \qquad x > 0.$$

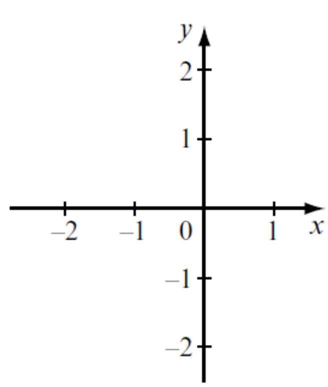
(c) Find the indefinite integral

$$\int (\pi \sin x - x)\mathrm{d}x$$

and hence, or otherwise, calculate the area of the region enclosed by the graph, the $x$-axis and the line $x = 1$.

**Mayo 01 P1**

(a) On the following diagram, sketch the graphs of $y = \mathrm{e}^x$ and $y = \cos x$ for $-2 \le x \le 1$.




(b) The equation $\mathrm{e}^x = \cos x$ has a solution between $-2$ and $-1$.

Find this solution.

**Mayo 01 P2**

Let $f(x) = \sin(1 + \sin x)$.

(a) (i) Sketch the graph of $y = f(x)$, for $0 \le x \le 6$.

(ii) Write down the $x$-coordinates of all minimum and maximum points of $f$, for $0 \le x \le 6$. Give your answers correct to **four** significant figures.

(b) Let $S$ be the region in the first quadrant completely enclosed by the graph of $f$ and **both** coordinate axes.

(i) Shade $S$ on your diagram.

(ii) Write down the integral which represents the area of $S$.

(iii) Evaluate the area of $S$ to **four** significant figures.

(c) Give reasons why $f(x) \ge 0$ for all values of $x$.

**Nov 01 P2**

The function $f$ is defined by $f : x \mapsto x^3 e^{-x}$.

As $x$ increases from $0$, the graph of $f$ rises to a maximum value and then decreases steadily, approaching a limiting value asymptotically.

(a) Sketch the graph of $f$ for $0 \le x \le 10$, choosing a suitable scale for the $y$-axis. (There is no need to draw an accurate, scaled graph.) The $y$-intercept, the position of the maximum and the asymptotic behaviour should be clear from your graph.

(b) (i) Shade on your graph the area represented by

$$\int_0^5 x^3 e^{-x} \, dx .$$

(ii) Evaluate this integral.

(c) Give the equation of the horizontal asymptote to the graph of $f$.

(d) Consider the equation $f(x) = 1$.

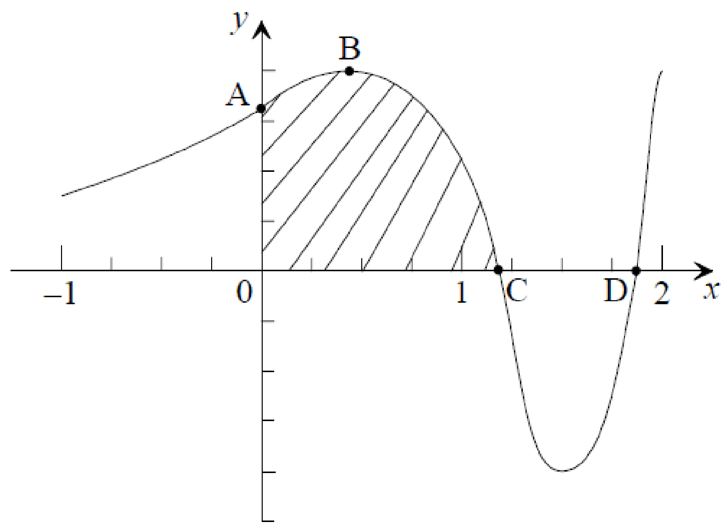
(i) Draw a line on your graph illustrating why this equation has more than one solution.

(ii) Write down the solutions of $f(x) = 1$.

(e) Give the coordinates of the maximum point on the graph of $f$.

**Nov 02**
**P2**

El diagrama a continuación muestra un esbozo de la gráfica de la función $y = \operatorname{sen}\left(e^x\right)$ donde $-1 \le x \le 2$, y $x$ está en radianes. La gráfica corta al eje de las $y$ en A, y al eje de las $x$ en C y en D. Tiene un punto de máximo en B.




(a) Halle las coordenades de A.

(b) Las coordenades de C se pueden expresar como $(\ln k, 0)$. Halle el valor **exacto** de $k$.

(c) (i) Escriba la ordenada $y$ de B.

(ii) Halle $\frac{dy}{dx}$.

(iii) A partir de allí, muestre que en el punto B, $x = \ln\frac{\pi}{2}$.

(d) (i) Escriba la integral que representa el área de la región sombreada.

(ii) Evalúe esta integral.

(e) (i) Copie el diagrama anterior a su cuadernillo de respuestas. (No es necesario copiar el sombreado.) Trace la gráfica de $y = x^3$ en el diagrama.

(ii) Las dos gráficas se cortan en el punto P. Halle la abscisa $x$ de P.

**Mayo 04**
**P2**

La derivada de la función $f$ viene dada por $f'(x) = e^x + x - 5$. El punto $(1, e-2)$ pertenece a la gráfica de $f(x)$.

(a) Compruebe que $f(x) = e^x + \frac{1}{2}x^2 - 5x + 2{,}5$.

(b) Dibuje aproximadamente la gráfica de $y = f(x)$ para $-3 < x < 3$.

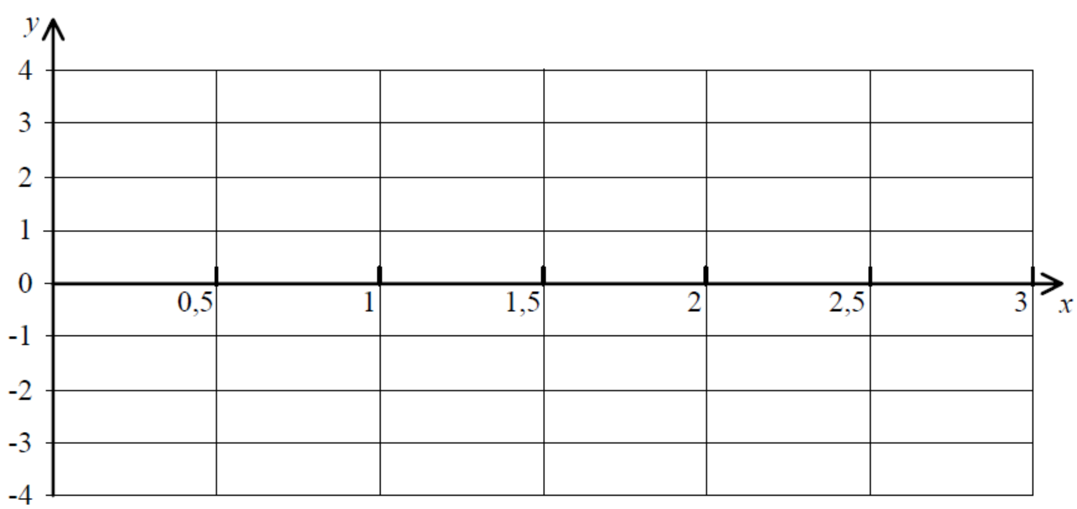
(c) Halle el valor mínimo de $f(x)$.

(d) Halle el área encerrada por la gráfica de $y = f(x)$, los ejes de coordenadas y la recta $x = 2$.

**Nov 04**
**P1**

Sea $f(x) = 2 + \cos(2x) - 2\mathrm{sen}(0{,}5x)$ para $0 \le x \le 3$, donde $x$ viene expresado en radianes.

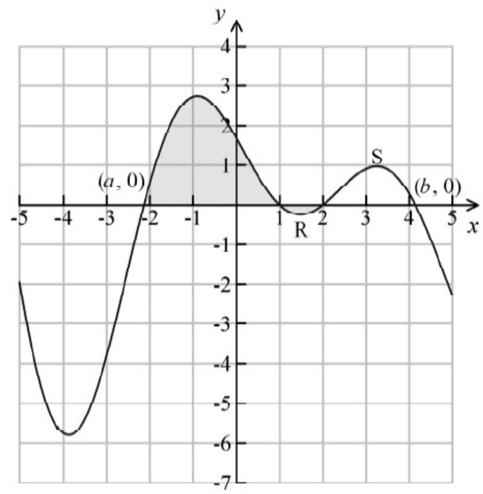
(a) Sobre el siguiente sistema de coordenadas, dibuje aproximadamente la curva $y = f(x)$, indicando claramente el punto P de la curva donde la derivada es cero.




(b) Escriba las soluciones de $f(x) = 0$.

**Nov 04**
**P2**

Sea $h(x) = (x-2)\operatorname{sen}(x-1)$ para $-5 \le x \le 5$. A continuación se muestra la curva de $h(x)$. Existe un mínimo local en R y un máximo local en S. La curva corta al eje $x$ en los puntos $(a, 0)$ $(1, 0)$ $(2, 0)$ and $(b, 0)$.




(a) Halle los valores exactos de

(i) $a$;

(ii) $b$.

Las regiones entre la curva y el eje $x$ para $a \le x \le 2$ están sombreadas según se muestra en la figura.

(b) (i) Escriba una expresión que represente el área **total** de las regiones sombreadas.

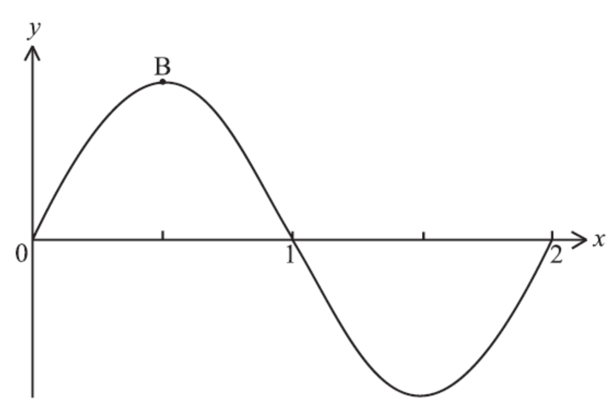
(ii) Calcule esta área total.

(c) (i) La ordenada de R es $-0,240$. Halle la ordenada de S.

(ii) A partir de lo anterior, o de cualquier otro modo, halle el conjunto de valores de $k$ para los cuales la ecuación $(x-2)\operatorname{sen}(x-1) = k$ tiene **cuatro** soluciones distintas.

**Nov 05**
**P1**

Let $f(x) = 6\sin \pi x$, and $g(x) = 6e^{-x} - 3$, for $0 \le x \le 2$. The graph of $f$ is shown on the diagram below. There is a maximum value at B(0.5, $b$).




(a) Write down the value of $b$.

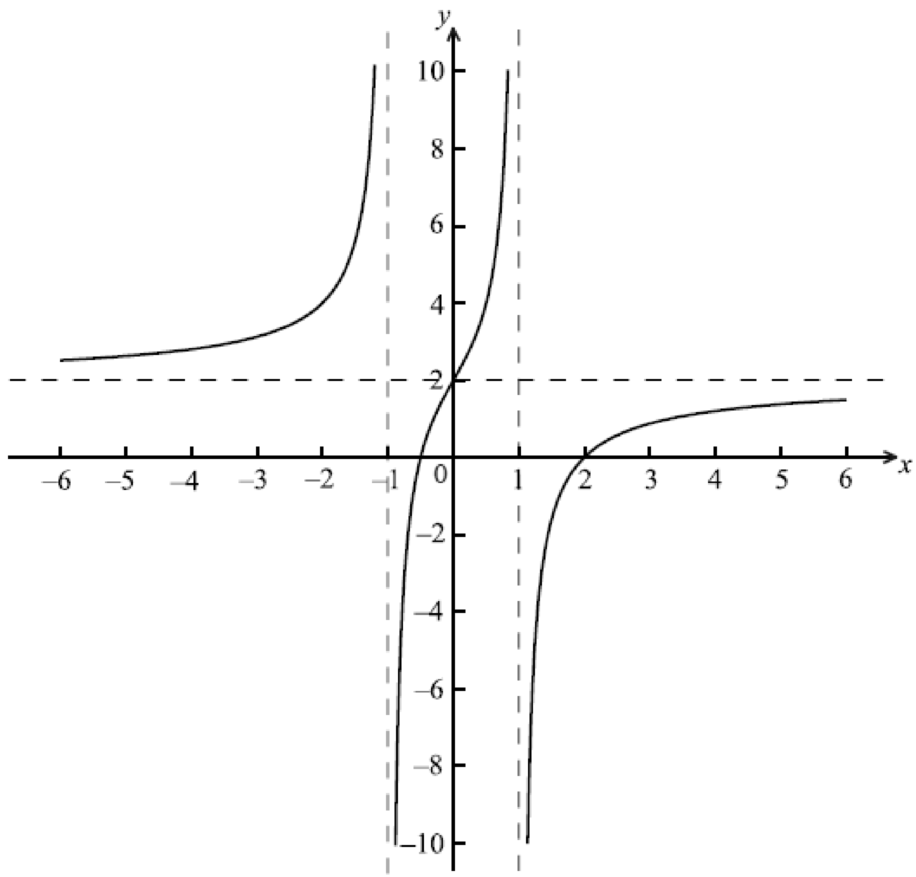
(b) On the same diagram, sketch the graph of $g$.

(c) Solve $f(x) = g(x)$, for $0.5 \le x \le 1.5$.

**May 07**
**P2**

Sea $f(x) = p - \dfrac{3x}{x^2 - q^2}$, donde $p,\ q \in \mathbb{R}^+$.

A continuación se muestra una parte de la gráfica de $f$, incluyendo las asíntotas.




(a) Las ecuaciones de las asíntotas son, respectivamente, $x = 1$, $x = -1$, $y = 2$. Escriba el valor de

(i) $p$; (ii) $q$.

(b) Sea $R$ la región delimitada por la gráfica de $f$, el eje $x$ y el eje $y$.

(i) Halle la intersección de $f$ con el semieje $x$ negativo.

(ii) A partir de lo anterior, halle el volumen resultante cuando se hace girar la región $R$ $360^\circ$ en torno al eje $x$.

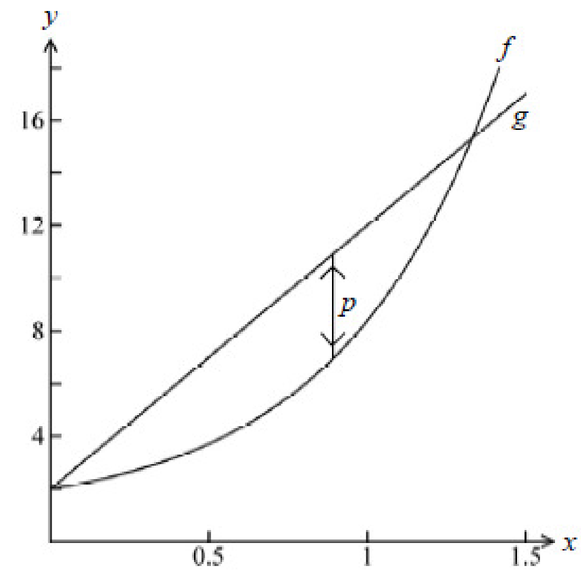
(c) (i) Compruebe que $f'(x) = \dfrac{3(x^2+1)}{(x^2-1)^2}$.

(ii) A partir de lo anterior, compruebe que la gráfica de $f$ no tiene ni máximos ni mínimos.

(d) Sea $g(x) = f'(x)$. Sea $A$ el área de la región delimitada por la gráfica de $g$ y por el eje $x$, entre $x = 0$ y $x = a$, donde $a > 0$. Sabiendo que $A = 2$, halle el valor de $a$.

**Muestra 06 P2**

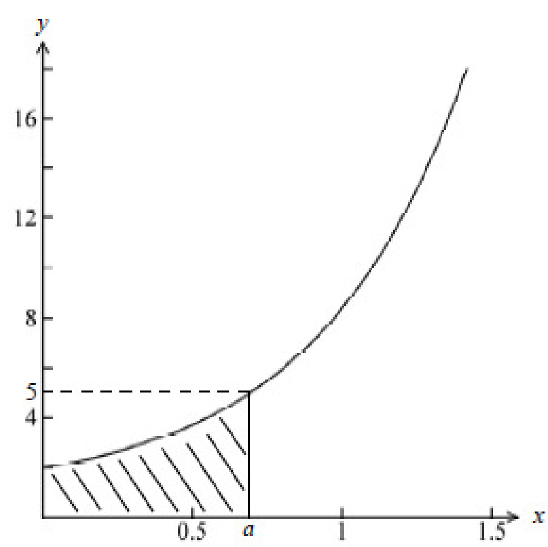
The diagram below shows the graphs of $f(x) = 1 + e^{2x}$, $g(x) = 10x + 2$, $0 \le x \le 1.5$




(a) (i) Write down an expression for the vertical distance $p$ between the graphs of $f$ and $g$.

(ii) Given that $p$ has a maximum value for $0 \le x \le 1.5$, find the value of $x$ at which this occurs.

The graph of $y = f(x)$ only is shown in the diagram below. When $x = a$, $y = 5$.




(b) (i) Find $f^{-1}(x)$.

(ii) **Hence** show that $a = \ln 2$.

(c) The region shaded in the diagram is rotated through $360°$ about the $x$-axis. Write down an expression for the volume obtained.

**Nov 06 P2**

La función $f$ se define como $f(x) = (2x+1)\,e^{-x}$, $0 \le x \le 3$. El punto $P(0, 1)$ está sobre la gráfica de $f(x)$ la cual presenta un punto máximo en Q.

(a) Dibuje aproximadamente la gráfica de $y = f(x)$, rotulando los puntos P y Q.

(b) (i) Compruebe que $f'(x) = (1-2x)\,e^{-x}$.

(ii) Halle las coordenadas **exactas** del punto Q.

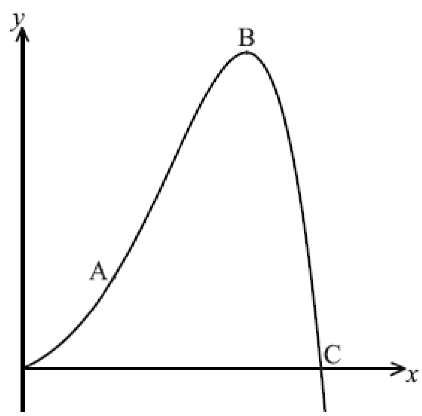
(c) La ecuación $f(x) = k$, donde $k \in \mathbb{R}$, tiene dos soluciones. Escriba el rango de valores de $k$.

(d) Sabiendo que $f''(x) = e^{-x}(-3+2x)$, compruebe que la curva de $f$ tiene solamente un punto de inflexión.

(e) Sea R el punto sobre la curva de $f$ que tiene la coordenada $x$ igual a 3. Halle el área de la región limitada por la curva y la recta (PR).

**May 07 P2**

The function $f$ is defined as $f(x) = e^x \sin x$, where $x$ is in radians. Part of the curve of $f$ is shown below.




There is a point of inflexion at A, and a local maximum point at B. The curve of $f$ intersects the $x$-axis at the point C.

(a) Write down the $x$-coordinate of the point C.

(b) (i) Find $f'(x)$.

(ii) Write down the value of $f'(x)$ at the point B.

(c) Show that $f''(x) = 2e^x \cos x$.

(d) (i) Write down the value of $f''(x)$ at A, the point of inflexion.

(ii) Hence, calculate the coordinates of A.

(e) Let $R$ be the region enclosed by the curve and the $x$-axis, between the origin and C.

(i) Write down an expression for the area of $R$.

(ii) Find the area of $R$.

**Nov 07 P2**

Consider the function $f(x) = e^{(2x-1)} + \frac{5}{(2x-1)}$, $x \neq \frac{1}{2}$.

(a) Sketch the curve of $f$ for $-2 \le x \le 2$, including any asymptotes.

(b) (i) Write down the equation of the vertical asymptote of $f$.

(ii) Write down which one of the following expressions does **not** represent an area between the curve of $f$ and the $x$-axis.

$$\int_1^2 f(x)\,dx$$

$$\int_0^2 f(x)\,dx$$

(iii) Justify your answer.

(c) The region between the curve and the $x$-axis between $x = 1$ and $x = 1.5$ is rotated through $360^\circ$ about the $x$-axis. Let $V$ be the volume formed.

(i) Write down an expression to represent $V$.

(ii) Hence write down the value of $V$.

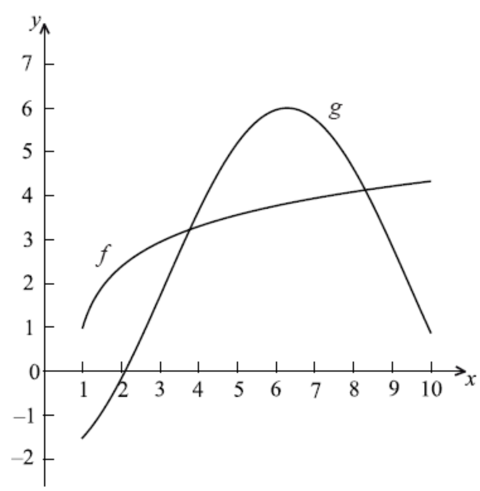
(d) Find $f'(x)$.

(e) (i) Write down the value of $x$ at the minimum point on the curve of $f$.

(ii) The equation $f(x) = k$ has no solutions for $p \le k < q$. Write down the value of $p$ and of $q$.

**Mayo 08 P2**

The following diagram shows the graphs of $f(x) = \ln(3x-2) + 1$ and $g(x) = -4\cos(0.5x) + 2$, for $1 \le x \le 10$.




(a) Let $A$ be the area of the region **enclosed** by the curves of $f$ and $g$.

(i) Find an expression for $A$.

(ii) Calculate the value of $A$. *[6 ma*

(b) (i) Find $f'(x)$.

(ii) Find $g'(x)$. *[4 ma*

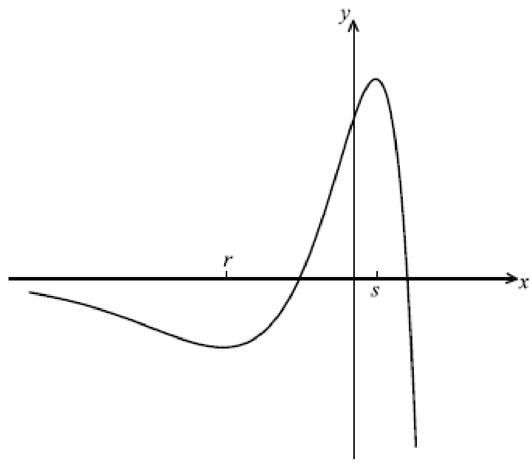
(c) There are two values of $x$ for which the gradient of $f$ is equal to the gradient of $g$. Find both these values of $x$. *[4 ma*

**Mayo 08 P2**

Let $f(x) = e^x(1-x^2)$.

(a) Show that $f'(x) = e^x(1-2x-x^2)$.

Part of the graph of $y = f(x)$, for $-6 \le x \le 2$, is shown below. The $x$-coordinates of the local minimum and maximum points are $r$ and $s$ respectively.




(b) Write down the **equation** of the horizontal asymptote.

(c) Write down the value of $r$ and of $s$.

(d) Let $L$ be the normal to the curve of $f$ at $P(0, 1)$. Show that $L$ has equation $x + y = 1$.

(e) Let $R$ be the region enclosed by the curve $y = f(x)$ and the line $L$.

(i) Find an expression for the area of $R$.

(ii) Calculate the area of $R$.

**Mayo 09 P2**

Sea $f(x) = 3\,\text{sen}\,x + 4\cos x$, para $-2\pi \le x \le 2\pi$.

(a) Dibuje aproximadamente la gráfica de $f$.

(b) Escriba

(i) la amplitud;

(ii) el período;

(iii) la intersección con el eje $x$ que se encuentra entre $-\frac{\pi}{2}$ y 0.

(c) A partir de lo anterior, escriba $f(x)$ en la forma $p\,\text{sen}\,(qx + r)$.

(d) Escriba un valor de $x$ para el cual $f'(x) = 0$.

(e) Escriba los dos valores de $k$ para los cuales la ecuación $f(x) = k$ tiene exactamente dos soluciones.

(f) Sea $g(x) = \ln(x+1)$, para $0 \le x \le \pi$. Hay un valor de $x$, entre 0 y 1, para el cual la pendiente de $f$ es igual a la pendiente de $g$. Halle dicho valor de $x$.

**Nov 09**
**P2**

Let $f(x) = 5\cos\frac{\pi}{4}x$ and $g(x) = -0.5x^2 + 5x - 8$, for $0 \le x \le 9$.

(a) On the same diagram, sketch the graphs of $f$ and $g$.

(b) Consider the graph of $f$. Write down

(i) the $x$-intercept that lies between $x = 0$ and $x = 3$;

(ii) the period;

(iii) the amplitude.

(c) Consider the graph of $g$. Write down

(i) the two $x$-intercepts;

(ii) the equation of the axis of symmetry.

(d) Let $R$ be the region enclosed by the graphs of $f$ and $g$. Find the area of $R$.