M00 P1 #3

$z_1 = a_{\pi/4}$, $z_2 = b_{\pi/3}$ $\;\Big|\;$ $\left(\frac{z_1}{z_2}\right)^3 = \left(\frac{a_{\pi/4}}{b_{\pi/3}}\right)^3 = \left[\left(\frac{a}{b}\right)_{\pi/4-\pi/3}\right]^3 = \left[\left(\frac{a}{b}\right)_{-\pi/12}\right]^3 = \left(\frac{a}{b}\right)^3_{-3\pi/12} =$

$$= \left(\frac{a^3}{b^3}\right)_{-\pi/4} = \frac{a^3}{b^3}\left(\cos\frac{-\pi}{4} + i\sin\frac{-\pi}{4}\right) = \frac{a^3}{b^3}\left(\frac{\sqrt{2}}{2} - i\cdot\frac{\sqrt{2}}{2}\right) = \boxed{\frac{\sqrt{2}a^3}{2b^3} - i\frac{\sqrt{2}a^3}{2b^3}}$$

N00 P1 #10

$$(1+ki)^2 + k(1+ki) + 5 = 0$$

$$1 + 2ki - k^2 + k + k^2 i + 5 = 0$$

$$(-k^2+k+6) + i(k^2+2k) = 0 \;\rightarrow\; -k^2+k+6 = 0 \;\Rightarrow\; k = \cancel{3},\ -2$$

$$\rightarrow\; k^2+2k = 0 \;\Rightarrow\; k = -2,\ \cancel{0} \;\Rightarrow\; \boxed{k=-2}$$

M01 P1 #10

Si tiene el factor $z+2i$, tendrá también el factor $z-2i$. La tercera raíz deberá ser un nº real 'r', es decir, el factor $z-r$. Además dispondrá del factor 2, ya que el monomio de mayor exponente lo tiene:

$$2\cdot(z-r)(z+2i)(z-2i) = (2z-2r)(z^2-4i^2) = (2z-2r)(z^2+4) =$$

$$= 2z^3 + 8z - 2rz^2 - 8r = 2z^3 \underbrace{-2r}_{-3}z^2 + 8z \underbrace{-8r}_{-12}$$

$$\left.\begin{array}{l} -2r = -3 \\ -8r = -12 \end{array}\right\} \rightarrow r = 3/2 \;\Rightarrow\; 2z^3 - 3z^2 + 8z - 12 = \boxed{2\left(z-\tfrac{3}{2}\right)(z+2i)(z-2i)}$$

<u>otra forma</u>:

	2	-3	8	-12
$2i$		$4i$	$-8-6i$	12
	2	$-3+4i$	$-6i$	0
$-2i$		$-4i$	$6i$	
	2	-3	0	
$3/2$		3		
	2	0		

N00 P1 #18

$$|z+16| = 4|z+1|$$

$$z = x+iy \;\rightarrow\; z+16 = x+16+iy \;\Rightarrow\; |z+16| = \sqrt{(x+16)^2+y^2}$$

$$\rightarrow\; z+1 = x+1+iy \;\Rightarrow\; |z+1| = \sqrt{(x+1)^2+y^2}$$

$$\sqrt{(x+16)^2+y^2} = 4\sqrt{(x+1)^2+y^2} \;;\quad (x+16)^2+y^2 = 16\cdot\left((x+1)^2+y^2\right);$$

$$x^2 + \cancel{32x} + 256 + y^2 = 16x^2 + \cancel{32x} + 16 + 16y^2$$

$$240 = 15x^2 + 15y^2$$

$$240 = 15(x^2+y^2)$$

$$16 = x^2+y^2 \;\Rightarrow\; |z| = \sqrt{x^2+y^2} = \boxed{4}$$

M01 P1 #14

$$z=(b+i)^2 = b^2+2bi+i^2 = (b^2-1)+2bi \Rightarrow tg\,\alpha = \frac{2b}{b^2-1}$$

$$\arg z = 60^\circ \Rightarrow tg\,60^\circ = \frac{2b}{b^2-1} \; ; \; \sqrt3 = \frac{2b}{b^2-1} \; ; \; \sqrt3\, b^2 - \sqrt3 = 2b \; ;$$

$$\sqrt3\, b^2 - 2b - \sqrt3 = 0 \; ; \quad b = \frac{2\pm\sqrt{4+12}}{2\sqrt3} = \frac{2\pm 4}{2\sqrt3} = \begin{cases} \frac{6}{2\sqrt3} = \frac{3}{\sqrt3} = \boxed{\sqrt3} \\ \cancel{\frac{-2}{2\sqrt3}} \end{cases}$$

No vale porque $b>0$

N01 P1#2

$$i(z+2) = 1-2z$$

$$iz+2i = 1-2z \; ; \quad z(2+i) = 1-2i \; ; \quad z = \frac{1-2i}{2+i} = \frac{(1-2i)(2-i)}{(2+i)(2-i)} = \frac{2-i-4i+2i^2}{4-\cancel{2i}+\cancel{2i}-i^2} =$$

$$= \frac{\cancel{2}-5i-\cancel{2}}{4+1} = \boxed{-i}$$

N01 P2#4

a)

$$\begin{array}{c|cccccc} & 1 & 0 & 0 & 0 & 0 & -1 \\ 1 & & 1 & 1 & 1 & 1 & 1 \\ \hline & 1 & 1 & 1 & 1 & 1 & \boxed{0} \end{array}$$

$$z^5-1 = \boxed{(z-1)(z^4+z^3+z^2+z+1)}$$

b)

$$z^5-1=0 \Rightarrow z = \sqrt[5]{1_0} = \left(\sqrt[5]{1}\right)_{\frac{0}{5}+\frac{2\pi k}{5}} = 1_{\frac{2\pi k}{5}} = \boxed{\begin{cases} 1_0 = 1 \\ 1_{2\pi/5} = \cos\frac{2\pi}{5} + i\sin\frac{2\pi}{5} \\ 1_{4\pi/5} = \cos\frac{4\pi}{5} + i\sin\frac{4\pi}{5} \\ 1_{6\pi/5} = 1_{-4\pi/5} = \cos\frac{-4\pi}{5} + i\sin\frac{-4\pi}{5} \\ 1_{8\pi/5} = 1_{-2\pi/5} = \cos\frac{-2\pi}{5} + i\sin\frac{-2\pi}{5} \end{cases}}$$

c)

$$z^5-1 = (z-1)\left(z-\cos\frac{2\pi}{5}-i\sin\frac{2\pi}{5}\right)\left(z-\cos\frac{4\pi}{5}-i\sin\frac{4\pi}{5}\right)\left(z-\cos\frac{-4\pi}{5}-i\sin\frac{-4\pi}{5}\right)\left(z-\cos\frac{-2\pi}{5}-i\sin\frac{-2\pi}{5}\right)$$

El desarrollo de $z^4+z^3+z^2+z+1$ es el producto de los últimos 4 factores:

$$\left(z-\cos\frac{2\pi}{5}-i\sin\frac{2\pi}{5}\right)\left(z-\cos\frac{-2\pi}{5}-i\sin\frac{-2\pi}{5}\right) = \left(z-\cos\frac{2\pi}{5}-i\sin\frac{2\pi}{5}\right)\left(z-\cos\frac{2\pi}{5}+i\sin\frac{2\pi}{5}\right) =$$

$$= \left(z-\cos\frac{2\pi}{5}\right)^2 - \left(i\sin\frac{2\pi}{5}\right)^2 = z^2 - 2z\cos\frac{2\pi}{5} + \cos^2\frac{2\pi}{5} + \sin^2\frac{2\pi}{5} = z^2-2z\cos\frac{2\pi}{5}+1$$

Igualmente, los otros dos factores resultarían: $z^2-2z\cos\frac{4\pi}{5}+1$

Por lo tanto: $z^4+z^3+z^2+z+1 = \boxed{\left(z^2-2z\cos\frac{2\pi}{5}+1\right)\left(z^2-2z\cos\frac{4\pi}{5}+1\right)} \simeq$

$$\simeq (z^2-0'6180z+1)(z^2+1'6180z+1)$$

M02 P1#3

a) $8i = \boxed{8_{\pi/2}}$

b)

$$z = \sqrt[3]{8i} = \sqrt[3]{8_{\pi/2}} = \left(\sqrt[3]{8}\right)_{\frac{\pi}{6}+\frac{2\pi k}{3}} = \begin{cases} \boxed{2_{\pi/6}} \longrightarrow 2\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right) = 2\left(\frac{\sqrt3}{2}+i\cdot\frac12\right) = \boxed{\sqrt3+i} \\ 2_{5\pi/6} \\ 2_{7\pi/6} \end{cases}$$

N02 P2#2

a) $\cos\frac{\pi}{4} - i\sin\frac{\pi}{4} = \cos\frac{-\pi}{4} + i\sin\frac{-\pi}{4} = 1_{-\pi/4}$ de la misma forma se obtiene $1_{-\pi/24}$

$$z = \frac{\left(1_{-\pi/4}\right)^2\left(1_{\pi/3}\right)^3}{\left(1_{-\pi/24}\right)^4} = \frac{1_{-\pi/2}\cdot 1_{\pi}}{1_{-\pi/6}} = 1_{-\pi/2+\pi+\pi/6} = \boxed{1_{2\pi/3}}$$

b) $\left(1_{2\pi/3}\right)^3 = \left(1^3\right)_{2\pi} = 1_{2\pi} = 1_0 = 1$ ✓

c)

$$(1+2z)(2+z^2) = 2+z^2+4z+2z^3 = 2+\left(1_{2\pi/3}\right)^2+4\cdot 1_{2\pi/3}+2\cdot\left(1_{2\pi/3}\right)^3 = 2+1_{4\pi/3}+4\cdot 1_{2\pi/3}+2\cdot 1 =$$

$$= 2+\cos\frac{4\pi}{3}+i\sin\frac{4\pi}{3}+4\cos\frac{2\pi}{3}+4i\sin\frac{2\pi}{3}+2 = \boxed{\frac32 + i\,\frac{3\sqrt3}{2}}$$

M03 / N03
P2 #3 / P2#2

$$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta \quad , \; n \in \mathbb{Z}^+$$

$\underline{n=1}$: $(\cos\theta + i\sin\theta)^1 = \cos(1\theta) + i\sin(1\theta)$

$\cos\theta + i\sin\theta = \cos\theta + i\sin\theta$

$\underline{n=K}$: Suponiendo cierto que: $(\cos\theta + i\sin\theta)^K = \cos K\theta + i\sin K\theta$, Tengo que demostrar que: $(\cos\theta + i\sin\theta)^{K+1} = \cos(K+1)\theta + i\sin(K+1)\theta$

$$(\cos\theta + i\sin\theta)^{K+1} = (\cos\theta + i\sin\theta)^K \cdot (\cos\theta + i\sin\theta) = (\cos K\theta + i\sin K\theta)(\cos\theta + i\sin\theta) =$$

$$= \cos K\theta \cos\theta + i\cos K\theta \sin\theta + i\sin K\theta\cos\theta + i^2 \sin K\theta \sin\theta =$$

$$= (\cos K\theta\cos\theta - \sin K\theta \sin\theta) + i(\cos K\theta\sin\theta + \sin K\theta\cos\theta) =$$

$$= \cos(K\theta + \theta) + i\sin(K\theta + \theta) = \cos(K+1)\theta + i\sin(K+1)\theta$$

M03
P1 #11

$$\sqrt{z} = \frac{2}{1-i} + 1 - 4i = \frac{2(1+i)}{(1-i)(1+i)} + 1 - 4i = \frac{\cancel{2}(1+i)}{\cancel{2}} + 1 - 4i = 2 - 3i$$

$$z = (2-3i)^2 = 4 - 12i + 9i^2 = \boxed{-5-12i}$$

M03
P2#3

i) $$\frac{1}{z} = \frac{1}{\cos\theta + i\sin\theta} = \frac{1\cdot(\cos\theta - i\sin\theta)}{(\cos\theta + i\sin\theta)(\cos\theta - i\sin\theta)} = \frac{\cos\theta - i\sin\theta}{\cos^2\theta - \cancel{i\cos\theta\sin\theta} + \cancel{i\cos\theta\sin\theta} - i^2\sin^2\theta} =$$

$$= \frac{\cos\theta - i\sin\theta}{\cos^2\theta + \sin^2\theta} = \frac{\cos(-\theta) + i\sin(-\theta)}{1} = \cos(-\theta) + i\sin(-\theta)$$

ii) $$z^n + z^{-n} = z^n + \left(\frac{1}{z}\right)^n = (1_\theta)^n + (1_{-\theta})^n = 1_{n\theta} + 1_{-n\theta} =$$

$$= \cos n\theta + i\sin n\theta + \cos(-n\theta) + i\sin(-n\theta) =$$

$$= \cos n\theta + \cancel{i\sin n\theta} + \cos n\theta - \cancel{i\sin n\theta} = 2\cos n\theta$$

iii) $$(z + z^{-1})^5 = z^5 + 5z^4z^{-1} + 10z^3z^{-2} + 10z^2z^{-3} + 5zz^{-4} + z^{-5} =$$

$$= \boxed{z^5 + 5z^3 + 10z + 10z^{-1} + 5z^{-3} + z^{-5}}$$

iv) $z + z^{-1} = 2\cos\theta$; $z^3 + z^{-3} = 2\cos 3\theta$; $z^5 + z^{-5} = 2\cos 5\theta$

$$(2\cos\theta)^5 = 2\cos 5\theta + 5\cdot 2\cos 3\theta + 10 \cdot 2\cos\theta$$

$$32\cos^5\theta = 2\cos 5\theta + 10\cos 3\theta + 20\cos\theta$$

$$\cos^5\theta = \frac{1}{32}(2\cos 5\theta + 10\cos 3\theta + 20\cos\theta) = \boxed{\frac{1}{16}(\cos 5\theta + 5\cos 3\theta + 10\cos\theta)}$$

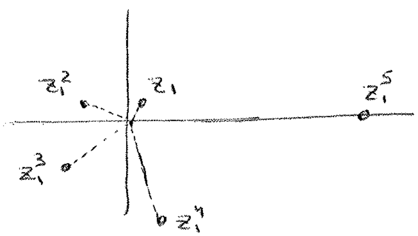
$\boxed{a=1}$, $\boxed{b=5}$, $\boxed{c=10}$

N03
P2#2

i) $z_1 = 2_{2\pi/5}$ $\quad (2_{2\pi/5})^5 - 32 = (2^5)_{2\pi} - 32 = 32_{2\pi} - 32 = 32 - 32 = 0$

ii) $z_1^2 = \boxed{4_{4\pi/5}}$; $z_1^3 = \boxed{8_{6\pi/5}}$; $z_1^4 = \boxed{16_{8\pi/5}}$; $z_1^5 = \boxed{32}$

iii)



M04 P1 TZ2 #6

$$z = 1 + \frac{i}{i-\sqrt3} = 1+\frac{i(i+\sqrt3)}{(i-\sqrt3)(i+\sqrt3)} = 1+\frac{i^2+i\sqrt3}{i^2-(\sqrt3)^2} = 1+\frac{-1+i\sqrt3}{-4} =$$

$$= \frac{-4-1+i\sqrt3}{-4} = \frac{-5+i\sqrt3}{-4} = \boxed{\frac54 - i\frac{\sqrt3}{4}}$$

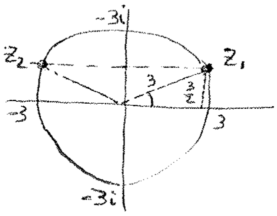
M04 P2 TZ1 #3

a) $|z|=|z-3i| \Rightarrow \sqrt{x^2+y^2}=\sqrt{x^2+(y-3)^2}$; $\cancel{x^2}+\cancel{y^2} = \cancel{x^2}+\cancel{y^2}-6y+9 \Rightarrow y=\frac96=\frac32$

b) $z = x+\frac32 i$

$|z|=3 \Rightarrow \sqrt{x^2+\frac94}=3$; $x^2+\frac94=9$; $x^2=\frac{27}{4} \Rightarrow x=\pm\frac{3\sqrt3}{2}$ $\quad \boxed{\begin{array}{l} z_1=\frac{3\sqrt3}{2}+\frac32 i \\ z_2=-\frac{3\sqrt3}{2}+\frac32 i \end{array}}$

Esta es otra forma de hallarlo.



$\operatorname{sen}\alpha = \frac{3/2}{3}=\frac12 \Rightarrow \arg z_1=\frac\pi6$

$\arg z_2 = \pi-\frac\pi6 = \boxed{\frac{5\pi}{6}}$

c) $\arg\left(\frac{z_1^k z_2}{2i}\right) = \arg\frac{(3_{\pi/6})^k\cdot 3_{5\pi/6}}{2_{\pi/2}} = \arg\frac{(3^k)_{k\pi/6}\cdot 3_{5\pi/6}}{2_{\pi/2}} = \frac{k\pi}{6}+\frac{5\pi}{6}-\frac\pi2 = \frac{k\pi+2\pi}{6}=\frac{k+2}{6}\pi$

$\frac{k+2}{6}\pi=\pi \Rightarrow \frac{k+2}{6}=1\Rightarrow \boxed{k=4}$

N04 P1 #4

$(a+i)(2-bi)=7-i$

$2a-abi+2i-bi^2=7-i \rightarrow \begin{cases} 2a+b=7 \\ 2-ab=-1 \end{cases}$; $\begin{cases} 2a+b=7 \rightarrow b=7-2a \\ ab=3 \end{cases}$

$a(7-2a)=3$; $7a-2a^2=3$; $0=2a^2-7a+3$; $a=\frac{7\pm\sqrt{49-24}}{4}=\frac{7\pm5}{4}$ $\begin{array}{l} \boxed{3}\Rightarrow\boxed{b=1} \\ \cancel{1/2} \text{ porque } a\in\mathbb{Z} \end{array}$

N04 P1 #13

$z^3-8i=0 \Rightarrow z=\sqrt[3]{8i}=\sqrt[3]{8_{90°}}=\left(\sqrt[3]{8}\right)_{\frac{90}{3}+\frac{360k}{3}} = 2_{30+120k}=$

$$= \begin{cases} z_{30} = \boxed{2(\cos30+i\sin30)} = \boxed{\sqrt3+i} \\ z_{150} = \boxed{2(\cos150+i\sin150)} = \boxed{-\sqrt3+i} \\ z_{270} = \boxed{2(\cos270°+i\sin270°)} = \boxed{-2i} \end{cases}$$

N04 P2 #1

a) $z^n+\frac1{z^n} = z^n+z^{-n} = \cos n\theta + i\sin n\theta + \cos(-n\theta)+i\sin(-n\theta)=$

$=\cos n\theta + \cancel{i\sin n\theta}+\cos n\theta - \cancel{i\sin n\theta} = 2\cos n\theta$

b) $\left(z+\frac1z\right)^4 = z^4+4z^3\frac1z+6z^2\frac1{z^2}+4z\frac1{z^3}+\frac1{z^4} = z^4+4z^2+6+4\cdot\frac1{z^2}+\frac1{z^4}=$

$=\left(z^4+\frac1{z^4}\right)+4\left(z^2+\frac1{z^2}\right)+6$

$(2\cos\theta)^4 = 2\cos4\theta+4\cdot2\cos2\theta+6 \Rightarrow \cos^4\theta = \frac{2\cos4\theta+8\cos2\theta+6}{16}=\frac18(\cos4\theta+4\cos2\theta+3)$

M05 P1 TZ2 #7

$(z+2)(z-(-3+2i))(z-(-3-2i)) = (z+2)(z+3-2i)(z+3+2i) = (z+2)((z+3)^2-4i^2)=$

$=(z+2)(z^2+6z+9+4) = (z+2)(z^2+6z+13) = z^3+6z^2+13z+2z^2+12z+26=$

$=\boxed{z^3+8z^2+25z+26} \rightarrow \boxed{a=8,\ b=25,\ c=26}$

M05 P1 TZ2 #11

$|z| = 2\sqrt{5} \Rightarrow \quad z \cdot z^* = (2\sqrt{5})^2 = 20$

$\frac{25}{z} - \frac{15}{z^*} = 1-8i \;;\quad \frac{25z^* - 15z}{z \cdot z^*} = 1-8i \;;\quad \frac{25z^* - 15z}{20} = 1-8i \;;\quad \frac{5z^* - 3z}{4} = 1-8i \;;$

$5z^* - 3z = 4-32i \longrightarrow 5a - 5bi - 3a - 3bi = 4-32i \;;\quad 2a - 8bi = 4-32i \;;$

$a - 4bi = 2-16i \Rightarrow \boxed{\begin{matrix} a=2 \\ b=4 \end{matrix}} \;;\; z = 2+4i$

M05 P1 TZ1 #11

$$\frac{z}{3+4i} + \frac{z-1}{5i} = \frac{5}{3-4i}$$

$$\frac{1}{3+4i} = \frac{3-4i}{(3+4i)(3-4i)} = \frac{3-4i}{9+16} = \frac{3-4i}{25}$$

$$\frac{1}{5i} = \frac{1 \cdot i}{5i \cdot i} = -\frac{i}{5}$$

$$\frac{5}{3-4i} = \frac{5(3+4i)}{(3-4i)(3+4i)} = \frac{15+20i}{9+16} = \frac{15+20i}{25}$$

$\frac{z}{3+4i} + \frac{z-1}{5i} = \frac{5}{3-4i} \Rightarrow \frac{z(3-4i)}{25} - \frac{(z-1)i}{5} = \frac{15+20i}{25} \;;\; z(3-4i) - 5(z-1)i = 15+20i \;;$

$z(3-4i) - 5zi + 5i = 15+20i \;;\quad z(3-4i-5i) = 15+15i \;;\quad z = \frac{15+15i}{3-9i} \;;$

$z = \frac{5+5i}{1-3i} = \frac{(5+5i)(1+3i)}{(1-3i)(1+3i)} = \frac{5+15i+5i-15}{1+9} = \frac{-10+20i}{10} = \boxed{-1+2i}$

N05 P1 #6

$z_1 + z_2 = 3 \longrightarrow \frac{a}{1+i} + \frac{b}{1-2i} = 3 \;;\quad a(1-2i) + b(1+i) = 3(1+i)(1-2i) \;;$

$a - 2ai + b + bi = 3(1-2i+i+2) \;;\quad (a+b) + (b-2a)i = 9-3i \Rightarrow$

$\Rightarrow \left.\begin{matrix} a+b=9 \\ b-2a=-3 \end{matrix}\right\} \longrightarrow \left.\begin{matrix} 2a+2b=18 \\ b-2a=-3 \end{matrix}\right\}$

$3b = 15 \Rightarrow \boxed{b=5} \rightarrow \boxed{a=4}$

N05 P2 #5

$z_1 \cdot z_2 = (2+i)(3+i) = 6+2i+3i+i^2 = \boxed{5+5i}$

$z_1 = 2+i = (\sqrt{5})_{\operatorname{arctg}\frac{1}{2}}$

$z_2 = 3+i = (\sqrt{10})_{\operatorname{arctg}\frac{1}{3}}$

$z_1 \cdot z_2 = 5+5i = (\sqrt{50})_{\operatorname{arctg}1} = (\sqrt{50})_{\pi/4}$

$\Rightarrow \frac{\pi}{4} = \operatorname{arctg}\frac{1}{2} + \operatorname{arctg}\frac{1}{3}$ ✓

M06 P1 #2

a) $z_2 = 1+\sqrt{3}i = \boxed{2_{\pi/3}}$

b) $|z_1 \cdot z_2^3| = |r_{\pi/4} \cdot (2_{\pi/3})^3| = |r_{\pi/4} \cdot 8_\pi| = |8r_{\pi/4+\pi}| = |8r_{5\pi/4}| = 8r$

$8r = 2 \Rightarrow \boxed{r = \frac{1}{4}}$

M06
P2#2

a) $z^3 = (\cos\theta + i\operatorname{sen}\theta)^3 = \cos^3\theta + 3\cos^2\theta\, i\operatorname{sen}\theta + 3\cos\theta\, i^2\operatorname{sen}^2\theta + i^3\operatorname{sen}^3\theta =$

$= \boxed{(\cos^3\theta - 3\operatorname{sen}^2\theta\cos\theta) + i\,(3\cos^2\theta\operatorname{sen}\theta - \operatorname{sen}^3\theta)}$

$z^3 = (1_\theta)^3 = (1^3)_{3\theta} = \cos 3\theta + i\operatorname{sen}3\theta$

$\cos 3\theta = \cos^3\theta - 3\operatorname{sen}^2\theta\cos\theta = \cos^3\theta - 3(1-\cos^2\theta)\cos\theta = \cos^3\theta - 3\cos\theta + 3\cos^3\theta =$

$= 4\cos^3\theta - 3\cos\theta$

$\operatorname{sen}3\theta = 3\cos^2\theta\operatorname{sen}\theta - \operatorname{sen}^3\theta = 3(1-\operatorname{sen}^2\theta)\operatorname{sen}\theta - \operatorname{sen}^3\theta = 3\operatorname{sen}\theta - 3\operatorname{sen}^3\theta - \operatorname{sen}^3\theta =$

$= 3\operatorname{sen}\theta - 4\operatorname{sen}^3\theta$

b)

$$\frac{\operatorname{sen}3\theta - \operatorname{sen}\theta}{\cos 3\theta + \cos\theta} = \frac{3\operatorname{sen}\theta - 4\operatorname{sen}^3\theta - \operatorname{sen}\theta}{4\cos^3\theta - 3\cos\theta + \cos\theta} = \frac{2\operatorname{sen}\theta - 4\operatorname{sen}^3\theta}{4\cos^3\theta - 2\cos\theta} = \frac{\operatorname{sen}\theta - 2\operatorname{sen}^3\theta}{2\cos^3\theta - \cos\theta} =$$

$$= \frac{\operatorname{sen}\theta\,(1 - 2\operatorname{sen}^2\theta)}{\cos\theta\,(2\cos^2\theta - 1)} = \operatorname{tg}\theta \cdot \frac{1 - 2(1-\cos^2\theta)}{2\cos^2\theta - 1} = \operatorname{tg}\theta \cdot \frac{1 - 2 + 2\cos^2\theta}{2\cos^2\theta - 1} =$$

$$= \operatorname{tg}\theta \cdot \frac{\cancel{2\cos^2\theta - 1}}{\cancel{2\cos^2\theta - 1}} = \operatorname{tg}\theta$$

c) $\operatorname{sen}\theta = \frac{1}{3} \Rightarrow \cos\theta = \pm\sqrt{1 - \frac{1}{9}} = \pm\frac{2\sqrt{2}}{3} =$ $\frac{2\sqrt{2}}{3}$; $\cancel{-\frac{2\sqrt{2}}{3}}$ Porque $-\frac{\pi}{4} < \theta < \pi/4$

$$\operatorname{tg}3\theta = \frac{\operatorname{sen}3\theta}{\cos 3\theta} = \frac{3\cdot\frac{1}{3} - 4\cdot\left(\frac{1}{3}\right)^3}{4\left(\frac{2\sqrt{2}}{3}\right)^3 - 3\cdot\frac{2\sqrt{2}}{3}} = \frac{1 - \frac{4}{27}}{\frac{64\sqrt{2}}{27} - 2\sqrt{2}} = \frac{27-4}{64\sqrt{2} - 54\sqrt{2}} = \boxed{\frac{23}{10\sqrt{2}}} = \frac{23\sqrt{2}}{20}$$

N06
P1#10

$$\begin{cases} 2z_1 + 3z_2 = 7 \\ z_1 + iz_2 = 4 + 4i \end{cases} \qquad \begin{cases} 2z_1 + 3z_2 = 7 \\ -2z_1 - 2iz_2 = -8 - 8i \end{cases}$$

$(3-2i)z_2 = -1 - 8i$

$$z_2 = \frac{-1-8i}{3-2i} = \frac{(-1-8i)(3+2i)}{(3-2i)(3+2i)} = \frac{-3 - 2i - 24i + 16}{9+4} = \boxed{1 - 2i}$$

$2z_1 + 3(1-2i) = 7$; $2z_1 = 7 - 3 + 6i$; $z_1 = \boxed{2 + 3i}$

M07
P2 TZ2
#5

a) $$\frac{u}{v} = \frac{1+\sqrt{3}i}{1+i} = \frac{(1+\sqrt{3}i)(1-i)}{(1+i)(1-i)} = \frac{1 - i + \sqrt{3}i + \sqrt{3}}{1+1} = \frac{\sqrt{3}+1}{2} + i\,\frac{\sqrt{3}-1}{2}$$

$u = 1 + \sqrt{3}i$ $\begin{cases} m = \sqrt{1+3} = 2 \\ \operatorname{tg}\alpha = \sqrt{3} \to \alpha = \pi/3 \end{cases}$ $u = 2_{\pi/3}$

$v = 1 + i$ $\begin{cases} m = \sqrt{1+1} = \sqrt{2} \\ \operatorname{tg}\alpha = \frac{1}{1} \to \alpha = \pi/4 \end{cases}$ $v = (\sqrt{2})_{\pi/4}$

$$\Rightarrow \frac{u}{v} = \frac{2_{\pi/3}}{(\sqrt{2})_{\pi/4}} = \left(\frac{2}{\sqrt{2}}\right)_{\pi/3 - \pi/4} = (\sqrt{2})_{\pi/12}$$

$$\frac{u}{v} = \frac{\sqrt{3}+1}{2} + i\,\frac{\sqrt{3}-1}{2} = \sqrt{2}\left(\cos\frac{\pi}{12} + i\operatorname{sen}\frac{\pi}{12}\right) \Rightarrow \begin{cases} \operatorname{sen}\pi/12 = \frac{\sqrt{3}-1}{2\sqrt{2}} \\ \cos\pi/12 = \frac{\sqrt{3}+1}{2\sqrt{2}} \end{cases} \Rightarrow \operatorname{tg}\pi/12 = \frac{\sqrt{3}-1}{\sqrt{3}+1} = \boxed{2 - \sqrt{3}}$$

b) $\underline{n=1}$ $(1+\sqrt{3}i)^1 = 2^1\left(\cos\frac{\pi}{3} + i\operatorname{sen}\pi/3\right)$

$\underline{n=k}$ Suponiendo cierto que $(1+\sqrt{3}i)^k = 2^k\left(\cos\frac{k\pi}{3} + i\operatorname{sen}\frac{k\pi}{3}\right)$, vamos a demostrar que $(1+\sqrt{3}i)^{k+1} = 2^{k+1}\cdot\left(\cos\frac{(k+1)\pi}{3} + i\operatorname{sen}\frac{(k+1)\pi}{3}\right)$

$(1+\sqrt{3}i)^{k+1} = (1+\sqrt{3}i)^k(1+\sqrt{3}i) = 2^k\left(\cos\frac{k\pi}{3} + i\operatorname{sen}\frac{k\pi}{3}\right)\cdot 2\left(\cos\frac{\pi}{3} + i\operatorname{sen}\frac{\pi}{3}\right) =$

$= 2^{k+1}\left(\cos\frac{k\pi}{3}\cos\frac{\pi}{3} + \cos\frac{k\pi}{3}\, i\operatorname{sen}\frac{\pi}{3} + i\operatorname{sen}\frac{k\pi}{3}\cos\frac{\pi}{3} - \operatorname{sen}\frac{k\pi}{3}\operatorname{sen}\frac{\pi}{3}\right) =$

$= 2^{k+1}\cdot\left(\cos\left(\frac{k\pi}{3} + \frac{\pi}{3}\right) + i\operatorname{sen}\left(\frac{k\pi}{3} + \frac{\pi}{3}\right)\right) = 2^{k+1}\cdot\left(\cos\frac{(k+1)\pi}{3} + i\operatorname{sen}\frac{(k+1)\pi}{3}\right)$

c) $z=\frac{\sqrt{2}\,v+u}{\sqrt{2}\,v-u}=\frac{\sqrt{2}(1+i)+1+\sqrt{3}i}{\sqrt{2}(1+i)-1-\sqrt{3}i}=\frac{(\sqrt{2}+1)+i(\sqrt{2}+\sqrt{3})}{(\sqrt{2}-1)+i(\sqrt{2}-\sqrt{3})}=\frac{[(\sqrt{2}+1)+i(\sqrt{2}+\sqrt{3})][(\sqrt{2}-1)-i(\sqrt{2}-\sqrt{3})]}{[(\sqrt{2}-1)+i(\sqrt{2}-\sqrt{3})][(\sqrt{2}-1)-i(\sqrt{2}-\sqrt{3})]}=$

$$=\frac{(\sqrt{2}+1)(\sqrt{2}-1)-i(\sqrt{2}+1)(\sqrt{2}-\sqrt{3})+i(\sqrt{2}+\sqrt{3})(\sqrt{2}-1)+(\sqrt{2}+\sqrt{3})(\sqrt{2}-\sqrt{3})}{(\sqrt{2}-1)^2+(\sqrt{2}-\sqrt{3})^2}\Rightarrow$$

$$\Rightarrow \operatorname{Re}[z]=\frac{(\sqrt{2}+1)(\sqrt{2}-1)+(\sqrt{2}+\sqrt{3})(\sqrt{2}-\sqrt{3})}{(\sqrt{2}-1)^2+(\sqrt{2}-\sqrt{3})^2}=\frac{2-1+2-3}{2-2\sqrt{2}+1+2-2\sqrt{6}+3}=\frac{0}{8-2\sqrt{2}-2\sqrt{6}}=0\ ✓$$

M07 P2 TZ1 #5

a)

$$\begin{array}{r|rrrr} & 1 & -3 & -3 & 1\\ -1 & & -1 & 4 & -1\\ \hline & 1 & -4 & 1 & \boxed{0}\end{array}\longrightarrow t^3-3t^2-3t+1=\boxed{(t+1)(t^2-4t+1)}$$

$$t^2-4t+1=0\longrightarrow t=\frac{4\pm\sqrt{16-4}}{2}=\frac{4\pm\sqrt{12}}{2}=\frac{4\pm2\sqrt{3}}{2}=2\pm\sqrt{3}$$

$\boxed{\text{Soluciones: } -1,\ 2\pm\sqrt{3}}$

b) $(1_\theta)^3=1_{3\theta}=\cos3\theta+i\sin3\theta$

$(1_\theta)^3=(\cos\theta+i\sin\theta)^3=\cos^3\theta+3\cos^2\theta\, i\sin\theta+3\cos\theta\, i^2\sin^2\theta+i^3\sin^3\theta=$

$=(\cos^3\theta-3\cos\theta\sin^2\theta)+i(3\cos^2\theta\sin\theta-\sin^3\theta)$

$\Rightarrow$ $\cos3\theta=\cos^3\theta-3\cos\theta\sin^2\theta$ ✓

$\boxed{\sin3\theta=3\cos^2\theta\sin\theta-\sin^3\theta}$

c) $$\operatorname{Tg}3\theta=\frac{3\cos^2\theta\sin\theta-\sin^3\theta}{\cos^3\theta-3\cos\theta\sin^2\theta}=\frac{\frac{3\cos^2\theta\sin\theta}{\cos^3\theta}-\frac{\sin^3\theta}{\cos^3\theta}}{\frac{\cos^3\theta}{\cos^3\theta}-\frac{3\cos\theta\sin^2\theta}{\cos^3\theta}}=\frac{3\operatorname{Tg}\theta-\operatorname{Tg}^3\theta}{1-3\operatorname{Tg}^2\theta}\ ✓$$

No es válida si se anula el denominador de $\operatorname{Tg}3\theta=\frac{\sin3\theta}{\cos3\theta}$

$\cos3\theta=0\rightarrow 3\theta=90^\circ+N\cdot360^\circ\Rightarrow\theta=30^\circ+N\cdot120^\circ$: $\boxed{\theta=\begin{cases}30^\circ\\150^\circ\end{cases}}$

Y tampoco sería válida si no existe $\operatorname{tg}\theta$: $\boxed{\theta=90^\circ}$ Siendo $0^\circ\le\theta\le180^\circ$

d) $\theta=15^\circ\rightarrow \operatorname{Tg}(3\cdot15^\circ)=\frac{3\operatorname{Tg}15^\circ-\operatorname{Tg}^3 15^\circ}{1-3\operatorname{Tg}^2 15^\circ}$; $\operatorname{Tg}15^\circ=t$ $\longrightarrow 1=\frac{3t-t^3}{1-3t^2}$;

$1-3t^2=3t-t^3$; $t^3-3t^2-3t+1=0\Rightarrow t=\begin{cases}-1\\2+\sqrt{3}\\\boxed{2-\sqrt{3}}\end{cases}\longleftarrow\boxed{\operatorname{Tg}15^\circ=2-\sqrt{3}}$

$$\operatorname{Tg}75^\circ=\operatorname{tg}(90-15^\circ)=\frac{1}{\operatorname{tg}15^\circ}=\frac{1}{2-\sqrt{3}}=\frac{2+\sqrt{3}}{(2-\sqrt{3})(2+\sqrt{3})}=\frac{2+\sqrt{3}}{4-3}=\boxed{2+\sqrt{3}}$$

M07 P1 TZ2 #11

$(z+1+i)(z+1-i)=(z+1)^2-i^2=z^2+2z+1+1=z^2+2z+2$

$P(z)=z^3+mz^2+nz-8=(z^2+2z+2)(z-r)$

Debe ser $r=4$, para que resulte -8

$(z^2+2z+2)(z-4)=z^3-4z^2+2z^2-8z+2z-8=z^3-2z^2-6z-8$

$\boxed{m=-2}$

$\boxed{n=-6}$

M07 P1 TZ1 #7

a) $z = 4\cdot\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) + 4\sqrt{3}\left(\frac{\sqrt{3}}{2} + i\cdot\frac{1}{2}\right) = (-2+6) + i\,(2\sqrt{3}+2\sqrt{3}) = \boxed{4+4i\sqrt{3}}$

$z = 4+4i\sqrt{3} = \left\{\begin{array}{l} m = \sqrt{16+48} = 8 \\ \operatorname{tg}\alpha = \frac{4\sqrt{3}}{4} \to \alpha = \frac{\pi}{3}\end{array}\right\} = 8_{\pi/3} = \boxed{8e^{i\pi/3}}$

b) $\sqrt[3]{8_{60^\circ}} = \left(\sqrt[3]{8}\right)_{20^\circ + 120k} = \left\{\begin{array}{l} 2_{20^\circ} = 2e^{i\pi/9} \\ 2_{140^\circ} = 2e^{i7\pi/9} \\ 2_{260^\circ} = 2e^{i13\pi/9}\end{array}\right.$

N07 P1 #19

$\sqrt{3}+i = \left\{\begin{array}{l} m = \sqrt{3+1} = 2 \\ \operatorname{tg}\alpha = \frac{1}{\sqrt{3}} \to \alpha = \pi/6\end{array}\right. \quad ; \quad \sqrt{3}+i = 2_{\pi/6}$

$\sqrt{3}-i = \left\{\begin{array}{l} m = \sqrt{3+1} = 2 \\ \operatorname{tg}\alpha = \frac{-1}{\sqrt{3}} \to \alpha = -\pi/6\end{array}\right. \quad ; \quad \sqrt{3}-i = 2_{-\pi/6}$

$(\sqrt{3}+i)^n + (\sqrt{3}-i)^n = (2_{\pi/6})^n + (2_{-\pi/6})^n = (2^n)_{n\pi/6} + (2^n)_{-n\pi/6} =$

$= 2^n\left(\cos\frac{n\pi}{6} + i\sin\frac{n\pi}{6}\right) + 2^n\left(\cos\frac{-n\pi}{6} + i\sin\frac{-n\pi}{6}\right) =$

$= 2^n\left(\cos\frac{n\pi}{6} + \cancel{i\sin\frac{n\pi}{6}}\right) + 2^n\left(\cos\frac{n\pi}{6} - \cancel{i\sin\frac{n\pi}{6}}\right) = 2\cdot 2^n\cos\frac{n\pi}{6} = 2^{n+1}\cos\frac{n\pi}{6} \in \mathbb{R}$ ✓

N07 P2 #2B

a) $1+i = \left\{\begin{array}{l} m = \sqrt{1+1} = \sqrt{2} \\ \operatorname{tg}\alpha = \frac{1}{1} \to \alpha = \pi/4\end{array}\right. \quad ; \quad 1+i = \boxed{\sqrt{2}\,e^{i\pi/4}} \quad \boxed{\begin{array}{l} a=2 \\ b=4\end{array}}$

b) $\left(\frac{1+i}{\sqrt{2}}\right)^n = \frac{(1+i)^n}{(\sqrt{2})^n} = \frac{\left(\sqrt{2}_{\pi/4}\right)^n}{(\sqrt{2})^n} = \frac{(\sqrt{2})^n_{n\pi/4}}{(\sqrt{2})^n} = 1_{n\pi/4} = \left\{\begin{array}{ll} 1_0 & \leftarrow n = 0, 8, 16, \ldots \\ 1_{\pi/4} & \leftarrow n = 1, 9, \ldots \\ 1_{\pi/2} & \leftarrow n = 2, 10, \ldots \\ 1_{3\pi/4} & \leftarrow n = 3, 11, \ldots \\ 1_{\pi} & \leftarrow n = 4, 12, \ldots \\ 1_{5\pi/4} & \leftarrow n = 5, 13, \ldots \\ 1_{3\pi/2} & \leftarrow n = 6, 14, \ldots \\ 1_{7\pi/4} & \leftarrow n = 7, 15, \ldots \end{array}\right.$

c) $z^8 - 1 = 0 \Rightarrow z = \sqrt[8]{1} = 1_{\frac{k2\pi}{8}} = 1_{\frac{k\pi}{4}} =$ Las 8 soluciones anteriores

Muestra 08 P1 #4

$(a+bi)^2 = 3+4i$

$a^2 + 2abi + b^2i^2 = 3+4i$

$a^2 - b^2 + 2abi = 3+4i \longrightarrow \left.\begin{array}{l} a^2 - b^2 = 3 \\ 2ab = 4 \to b = \frac{2}{a}\end{array}\right\} \longrightarrow a^2 - \frac{4}{a^2} = 3 \quad ; \quad a^4 - 4 = 3a^2 \quad ;$

$a^4 - 3a^2 - 4 = 0 \quad ; \quad a^2 = \frac{3 \pm \sqrt{9+16}}{2} = \frac{3 \pm 5}{2} \begin{array}{l} \nearrow 4 \to \boxed{a = \begin{array}{l} 2 \to b = 1 \\ -2 \to b = -1\end{array}} \begin{array}{l} \to 2+i \\ \to 2-i\end{array} \\ \searrow \cancel{-1}\end{array}$

$\sqrt{3+4i} = a+bi = \boxed{\begin{array}{l} 2+i \\ 2-i\end{array}}$

Muestra 08 P1 #5

$(z-(2+i))\,(z-(2-i)) = (z-2-i)(z-2+i) = (z-2)^2 - i^2 = z^2 - 4z + 5$

$z^3 - 6z^2 + 13z - 10 = (z^2 - 4z + 5)(z - a)$

tiene que ser $a = 2$, para que resulte -10

$(z^2 - 4z + 5)(z - 2) = z^3 - 2z^2 - 4z^2 + 8z + 5z - 10 = \boxed{z^3 - 6z^2 + 13z - 10}$ ✓

Las raíces son: $\boxed{2+i,\ 2-i,\ 2}$

Muestra 08
P1 #6

$|z| = \sqrt{10} \Rightarrow z \cdot z^* = 10$

$5z + \frac{10}{z^*} = 6 - 18i \; ; \; 5zz^* + 10 = (6-18i)z^* \; ; \; 50 + 10 = (6-18i)\cdot z^* \; ;$

$60 = (6-18i)z^* \; ; \; z^* = \frac{60}{6-18i} = \frac{10}{1-3i} = \frac{10(1+3i)}{(1-3i)(1+3i)} = \frac{10(1+3i)}{1+9} = 1+3i \Rightarrow$

$\Rightarrow \boxed{z = 1-3i}$

Muestra 08
P1 #7

$$\sqrt[3]{8i} = \sqrt[3]{8_{90^\circ}} = \left(\sqrt[3]{8}\right)_{\frac{90}{3}+\frac{k\cdot 360}{3}} = 2_{30+120k} = \begin{cases} 2_{30} = 2\cdot\left(\frac{\sqrt{3}}{2} + i\cdot\frac{1}{2}\right) = \boxed{\sqrt{3}+i} \\ 2_{150} = 2\cdot\left(\frac{\sqrt{3}}{2} - i\frac{1}{2}\right) = \boxed{\sqrt{3}-i} \\ 2_{270} = \boxed{-2i} \end{cases}$$

Muestra 08
P1 #8

$$\left.\begin{array}{l} iz_1 + 2z_2 = 3 \\ z_1 + (1-i)z_2 = 4 \end{array}\right\} \quad \left.\begin{array}{l} iz_1 + 2z_2 = 3 \\ -iz_1 - i(1-i)z_2 = -4i \end{array}\right\}$$

$(2 - i + i^2)z_2 = 3 - 4i$

$(1-i)z_2 = 3-4i \; ; \; z_2 = \frac{3-4i}{1-i} = \frac{(3-4i)(1+i)}{(1-i)(1+i)} = \frac{3+3i-4i+4}{1+1} =$

$= \frac{7-i}{2} = \boxed{\frac{7}{2} - i\frac{1}{2}}$

$z_1 = 4 - (1-i)z_2 = 4 - (1-i)\left(\frac{7}{2} - \frac{i}{2}\right) = 4 - \frac{7}{2} + \frac{i}{2} + \frac{7i}{2} - \frac{i^2}{2} = 4 - \frac{7}{2} + \frac{1}{2} + \frac{8i}{2} = \boxed{1+4i}$

Muestra 08
P1 #9

$\frac{2+bi}{1-bi} = \frac{-7+9i}{10} \; ; \; 20 + 10bi = -7 + 7bi + 9i - 9bi^2$

$20 = -7 + 9b \rightarrow \boxed{b=3}$

$10b = 7b + 9$

Muestra 06
P2 #5
Muestra 08
P1 #44B

a) $e^{i\theta} + \frac{1}{2}e^{2i\theta} + \frac{1}{4}e^{3i\theta} + \cdots \quad \boxed{\text{razón} = \frac{1}{2}e^{i\theta}}$

b) $\left|\frac{1}{2}e^{i\theta}\right| = \frac{1}{2} < 1$ ✓

c) $S_\infty = \frac{e^{i\theta}}{1 - \frac{1}{2}e^{i\theta}} = \boxed{\frac{2e^{i\theta}}{2 - e^{i\theta}}}$

d) $S_\infty = \frac{2(\cos\theta + i\sin\theta)}{2 - (\cos\theta + i\sin\theta)} = \frac{2\cos\theta + 2i\sin\theta}{(2-\cos\theta) - i\sin\theta} = \frac{(2\cos\theta + 2i\sin\theta)((2-\cos\theta) + i\sin\theta)}{((2-\cos\theta) - i\sin\theta)((2-\cos\theta)+i\sin\theta)} =$

$= \frac{4\cos\theta - 2\cos^2\theta + \cancel{2i\cos\theta\sin\theta} + 4i\sin\theta - \cancel{2i\sin\theta\cos\theta} + 2i^2\sin^2\theta}{(2-\cos\theta)^2 + \sin^2\theta} =$

$= \frac{(4\cos\theta - 2\cos^2\theta - 2\sin^2\theta) + 4i\sin\theta}{4 - 4\cos\theta + \cos^2\theta + \sin^2\theta} = \frac{(4\cos\theta - 2) + 4i\sin\theta}{5 - 4\cos\theta} = \boxed{\frac{4\cos\theta - 2}{5-4\cos\theta} + i\frac{4\sin\theta}{5-4\cos\theta}}$

$e^{i\theta} + \frac{1}{2}e^{2i\theta} + \frac{1}{4}e^{3i\theta} + \cdots = \left(\cos\theta + \frac{1}{2}\cos 2\theta + \frac{1}{4}\cos 3\theta + \cdots\right) + i\left(\sin\theta + \frac{1}{2}\sin 2\theta + \cdots\right)$

$\Rightarrow \boxed{\cos\theta + \frac{1}{2}\cos 2\theta + \frac{1}{4}\cos 3\theta + \cdots = \frac{4\cos\theta - 2}{5 - 4\cos\theta}}$ ✓

M08
P2 T21
#10

$(\text{sen}\,\theta + i(1-\cos\theta))^2 = \sin^2\theta + 2i\sin\theta(1-\cos\theta) - (1-\cos\theta)^2 = (\underbrace{\sin^2\theta - 1}_{=-\cos^2\theta} + 2\cos\theta - \cos^2\theta) + 2i\sin\theta(1-\cos\theta) =$

$= (2\cos\theta - 2\cos^2\theta) + 2i\sin\theta(1-\cos\theta) = 2\cos\theta(1-\cos\theta) + 2i\sin\theta(1-\cos\theta)$

Si llamamos α al argumento:

$\text{tg}\,\alpha = \frac{\cancel{2}\sin\theta\cancel{(1-\cos\theta)}}{\cancel{2}\cos\theta\cancel{(1-\cos\theta)}} = \text{tg}\,\theta \longrightarrow \boxed{\alpha = \theta}$

Muestra 08 P1 #45

a) $z^2+2z+4=0$

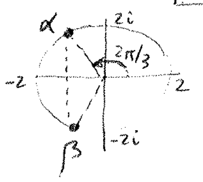
$$z=\frac{-2\pm\sqrt{4-16}}{2}=\frac{-2\pm 2\sqrt{3}i}{2}=-1\pm\sqrt{3}i=$$

$\nearrow -1+\sqrt{3}i \Big\{ \begin{array}{l} m=\sqrt{1+3}=2 \\ tg\,\alpha=-\sqrt{3} \rightarrow \alpha=120^\circ=\frac{2\pi}{3}\end{array}$

$\searrow -1-\sqrt{3}i \Big\{ \begin{array}{l} m=\sqrt{1+3}=2 \\ tg\,\alpha=+\sqrt{3} \rightarrow \alpha=240^\circ=\frac{4\pi}{3}\end{array}$

$$\boxed{\alpha=2e^{\frac{2\pi}{3}i}} \qquad \boxed{\beta=2e^{\frac{4\pi}{3}i}}$$

b)



c) (Demostrado en M03 P2 #3)

d) $$\frac{\alpha^3}{\beta^2}=\frac{\left(2e^{\frac{2\pi}{3}i}\right)^3}{\left(2e^{\frac{4\pi}{3}i}\right)^2}=\frac{8e^{2\pi i}}{4e^{\frac{8\pi}{3}i}}=\frac{2}{e^{8\pi/3\,i}}=2e^{-\frac{8\pi}{3}i}=$$

$$=2\left(-\frac{1}{2}-i\frac{\sqrt{3}}{2}\right)=\boxed{-1-i\sqrt{3}}$$

e) $$\alpha^3=\left(2e^{\frac{2\pi}{3}i}\right)^3=8e^{2\pi i}=8$$
$$\beta^3=\left(2e^{\frac{4\pi}{3}i}\right)^3=8e^{4\pi i}=8$$ ✓

f) $$\alpha\beta^*+\beta\alpha^*=(-1+\sqrt{3}i)(-1+\sqrt{3}i)+(-1-\sqrt{3}i)(-1-\sqrt{3}i)=$$
$$=(-1+i\sqrt{3})^2+(-1-i\sqrt{3})^2=1-\cancel{2i\sqrt{3}}+i^2 3+1+\cancel{2i\sqrt{3}}+i^2\cdot 3=2-6=\boxed{-4}$$

g) $$\alpha^m=\left(2_{2\pi/3}\right)^m=\left(2^m\right)_{\frac{2\pi m}{3}}$$

Si es real, el argumento debe ser $0+K\cdot\pi$ $(K=0,\pm1,\pm2,\dots)$

$$\frac{2\pi m}{3}=K\cancel{\pi} \Rightarrow \boxed{m=\frac{3K}{2}\quad (K=0,\pm1,\pm2,\dots)} \text{ Como } m\in\mathbb{Z}^+ \Rightarrow \boxed{m=3K,\ K\in\mathbb{Z}^+}$$

M08 P1 TZ2 #14

$w=1_{2\pi/5}$

a) $$\left(1_{2\pi/5}\right)^5-1=\left(1^5\right)_{2\pi}-1=1_{2\pi}-1=1-1=0$$ ✓

b)

$$\begin{array}{c|ccccc} & 1 & 0 & 0 & 0 & -1 \\ 1 & & 1 & 1 & 1 & 1 \\ \hline & 1 & 1 & 1 & 1 & \boxed{0} \end{array} \longrightarrow (z-1)(z^4+z^3+z^2+z+1)=z^5-1$$

$$w^5-1=0 \Rightarrow (w-1)(w^4+w^3+w^2+w+1)=0$$

$\nearrow w=1$ ~~$w=1$~~ porque $w\neq1$

$\searrow w^4+w^3+w^2+w+1=0$ ✓

c) $$\left(1_{2\pi/5}\right)^4+\left(1_{2\pi/5}\right)^3+\left(1_{2\pi/5}\right)^2+1_{2\pi/5}+1=0$$

$$1_{8\pi/5}+1_{6\pi/5}+1_{4\pi/5}+1_{2\pi/5}+1=0$$

$$\left(\cos\frac{8\pi}{5}+\cos\frac{6\pi}{5}+\cos\frac{4\pi}{5}+\cos\frac{2\pi}{5}+1\right)+i\left(\sin\frac{8\pi}{5}+\sin\frac{6\pi}{5}+\sin\frac{4\pi}{5}+\sin\frac{2\pi}{5}\right)=0$$

$$\cos\frac{8\pi}{5}+\cos\frac{6\pi}{5}+\cos\frac{4\pi}{5}+\cos\frac{2\pi}{5}+1=0$$

$$\cos\frac{2\pi}{5}+\cos\frac{4\pi}{5}+\cos\frac{4\pi}{5}+\cos\frac{2\pi}{5}=-1$$

$$2\left(\cos\frac{2\pi}{5}+\cos\frac{4\pi}{5}\right)=-1 \Rightarrow \cos\frac{2\pi}{5}+\cos\frac{4\pi}{5}=-\frac{1}{2}$$ ✓

M08 P1 TZ1 #1

$$1-i\sqrt{3} = \begin{cases} m=\sqrt{1+3}=2 \\ \operatorname{tg}\alpha = -\sqrt{3} \longrightarrow \alpha = -60^\circ \end{cases}$$

$$\frac{1}{(1-i\sqrt{3})^3} = \frac{1}{(2_{-60^\circ})^3} = \frac{1}{8_{-180^\circ}} = \left(\frac{1}{8}\right)_{180^\circ} = \boxed{-\frac{1}{8}}$$

M08 P2 TZ1 #14

$$1+i\sqrt{3} = \begin{cases} m=\sqrt{1+3}=2 \\ \operatorname{tg}\alpha=\sqrt{3} \longrightarrow \alpha = 60^\circ \end{cases} \qquad 1-i = \begin{cases} m=\sqrt{1+1}=\sqrt{2} \\ \operatorname{tg}\alpha=-1 \longrightarrow \alpha=-45^\circ = 315^\circ \end{cases}$$

$$(1+i\sqrt{3})^m = (2_{60^\circ})^m = \boxed{(2^m)_{60m}}$$

$$(1-i)^n = (\sqrt{2}_{315})^n = \boxed{(\sqrt{2^n})_{315n}}$$

$$z_1 = z_2 \Rightarrow (2^m)_{60m} = (\sqrt{2^n})_{315n} \longrightarrow 2^m = \sqrt{2^n} \Rightarrow m = \frac{n}{2}$$

$$\longrightarrow 60m - 315n = k\cdot 360 \quad (k=0,\pm1,\pm2,\dots)$$

$$n = 2m \longrightarrow 60m - 315\cdot 2m = k\cdot 360$$

$$-570m = k\cdot 360$$

$$-19m = 12k \longrightarrow \boxed{m=12} \longrightarrow \boxed{n=24}$$

con $k=-19$

M08 P2 TZ2 #9

$$w = \frac{z}{z^2+1} = \frac{x+iy}{(x+iy)^2+1} = \frac{x+iy}{x^2+2xiy+i^2y^2+1} = \frac{x+iy}{(x^2-y^2+1)+i\,2xy} =$$

$$= \frac{(x+iy)\left((x^2-y^2+1)-i\,2xy\right)}{\left[(x^2-y^2+1)+i\,2xy\right]\left[(x^2-y^2+1)-i\,2xy\right]} = \frac{x(x^2-y^2+1)-2x^2yi+iy(x^2-y^2+1)+2xy^2}{(x^2-y^2+1)^2+4x^2y^2} =$$

$$= \frac{x(x-y^2+1+2y^2)}{(x^2-y^2+1)^2+4x^2y^2} + i\,\frac{-2x^2y+yx^2-y^3+y}{(x^2-y^2+1)^2+4x^2y^2}$$

$$\operatorname{Im} w = 0 \Rightarrow -2x^2y+yx^2-y^3+y=0 \; ; \; y(-x^2-y^2+1)=0 \begin{cases} \nearrow \cancel{y=0} \\ \searrow x^2+y^2=1 \Rightarrow |z|=1 \checkmark \end{cases}$$

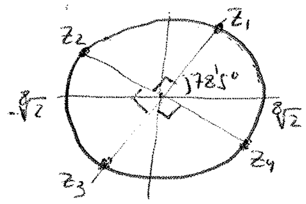
N08 P1 #13A

a) $z^4 = 1-i \longrightarrow z = \sqrt[4]{1-i}$

$$1-i = \begin{cases} m=\sqrt{1+1}=\sqrt{2} \\ \operatorname{tg}\alpha=-1 \Rightarrow \alpha = 315^\circ \end{cases}$$

$$z = \sqrt[4]{1-i} = \sqrt[4]{(\sqrt{2})_{315^\circ}} = \left(\sqrt[4]{\sqrt{2}}\right)_{\frac{315}{4}+\frac{k\cdot 360}{4}} = \left(\sqrt[8]{2}\right)_{78'75^\circ + 90k} = \boxed{\begin{cases} \sqrt[8]{2}_{\,78'75^\circ} \\ \sqrt[8]{2}_{\,168'75^\circ} \\ \sqrt[8]{2}_{\,258'75^\circ} \\ \sqrt[8]{2}_{\,348'75^\circ} \end{cases}}$$

b)



c) $z_1 = \sqrt[8]{2}_{\,78'75^\circ}$, $z_2 = \sqrt[8]{2}_{\,168'75}$

$$\frac{z_2}{z_1} = \frac{\sqrt[8]{2}_{\,168'75}}{\sqrt[8]{2}_{\,78'75}} = 1_{90} = \boxed{i}$$

N08 P1 #13B

a) $(x-1)(x^4+x^3+x^2+x+1) = x^5+\cancel{x^4}+\cancel{x^3}+\cancel{x^2}+\cancel{x}-\cancel{x^4}-\cancel{x^3}-\cancel{x^2}-\cancel{x}-1 = \boxed{x^5-1}$

b) $b^5-1=0 \Rightarrow (b-1)(b^4+b^3+b^2+b+1)=0 \to \cancel{b=1}$ porque $b\notin\mathbb{R}$

$\to b^4+b^3+b^2+b+1=0$ ✓

c) $u+v = b+b^4+b^2+b^3 = -1$ ✓

$u\cdot v = (b+b^4)(b^2+b^3) = b^3+b^4+b^6+b^7 =$

$= -b^5 = -1$ ✓

porque $b^5-1=0$

$b^4+b^3+b^2+b+1=0 \Rightarrow$

$\Rightarrow b^3(b^4+b^3+b^2+b+1)=0 \Rightarrow$

$\Rightarrow b^7+b^6+b^5+b^4+b^3=0 \Rightarrow$

$b^7+b^6+b^4+b^3=-b^5$

$(u+v)=-1 \Rightarrow (u+v)^2=1 \to u^2+2uv+v^2=1$; $u^2+v^2=1-2uv=1-2(-1)=3$

$(u-v)^2=u^2+v^2-2uv = 3-2\cdot(-1)=5 \Rightarrow u-v = \sqrt{5}$ porque $u-v>0$; $\cancel{-\sqrt{5}}$

M09 P1 TZ1 #1

a) $|w|=\sqrt{4+a^2}$; $|z|=\sqrt{1+4}=\sqrt{5}$

$|w|=2|z| \Rightarrow \sqrt{4+a^2}=2\sqrt{5}$; $4+a^2=4\cdot5$; $a^2=16$; $\boxed{a=\pm4}$

b) $z\cdot w = (1+2i)(2+ai) = 2+ai+4i+2ai^2 = (2-2a)+i(a+4)$

$Re(z\cdot w) = 2\,Im(z\cdot w) \Rightarrow 2-2a=2(a+4)$; $2-2a=2a+8$; $-4a=6$; $\boxed{a=-\frac{3}{2}}$

M09 P1 TZ1 #13

a) $z\in\mathbb{R}^+ \Rightarrow L(z)=\ln|z|+i\arg(z) = \ln z+i\cdot0=\ln z$ ✓

b) $L(-1)=L(1_{\pi})=\ln|1|+i\cdot\pi = \boxed{i\pi}$

$1-i = \begin{cases} m=\sqrt{1+1}=\sqrt{2} \\ \tan\alpha=-1 \to \alpha=\frac{7\pi}{4}\end{cases}$ $\quad L(1-i)=\ln\sqrt{2}+i\frac{7\pi}{4} = \boxed{\frac{1}{2}\ln2+i\frac{7\pi}{4}} = \boxed{\ln\sqrt{2}+i\frac{7\pi}{4}}$

$-1+i = \begin{cases} m=\sqrt{1+1}=\sqrt{2} \\ \tan\alpha=-1 \to \alpha=\frac{3\pi}{4}\end{cases}$ $\quad L(-1+i)=\ln\sqrt{2}+i\frac{3\pi}{4} = \boxed{\frac{1}{2}\ln2+i\frac{3\pi}{4}} = \boxed{\ln\sqrt{2}+i\frac{3\pi}{4}}$

c) $L(z_1)+\ln(z_2) = \ln|z_1|+i\arg(z_1)+\ln|z_2|+i\arg(z_2) =$

$= \ln|z_1|\cdot|z_2| + i\left(\arg(z_1)+\arg(z_2)\right) =$

$= \ln|z_1\cdot z_2| + i\left(\arg(z_1)+\arg(z_2)\right)$

$\ln(z_1\cdot z_2) = \ln|z_1 z_2| + i\arg(z_1\cdot z_2)$

La diferencia estará en aquellos casos en que $\arg(z_1)+\arg(z_2)$ sea mayor de 360°, ya que, por la definición de $L(z)$, se debe utilizar el argumento principal.

Por ejemplo:

No se cumple con: $z_1=2_{7\pi/4}$, $z_2=3_{5\pi/6}$ $\to$

$L(z_1)+L(z_2) = \ln2+i\frac{7\pi}{4}+\ln3+i\cdot\frac{5\pi}{6} =$

$= \ln6+i\frac{31\pi}{12}$

$L(z_1\cdot z_2) = L(2_{7\pi/4}\cdot3_{5\pi/6}) = L(6_{31\pi/12}) =$

$= L(6_{7\pi/12}) = \ln6+i\frac{7\pi}{12}$ ✓

Sí se cumple con: $z_1=2_{3\pi/4}$, $z_2=3_{5\pi/6}$ porque $\frac{3\pi}{4}+\frac{5\pi}{6}=\frac{19\pi}{12}<2\pi$.

M09 P1
TZ2 #7

a) $z_1 = 2$

$z_2 = 1+\sqrt{3}i \longrightarrow \boxed{z_3 = 1-\sqrt{3}i}$

b) $(z-2)(z-(1+\sqrt{3}i))(z-(1-\sqrt{3}i)) = (z-2)(z-1-\sqrt{3}i)(z-1+\sqrt{3}i) = (z-2)((z-1)^2+3) =$

$= (z-2)(z^2-2z+1+3) = (z-2)(z^2-2z+4) = z^3-2z^2+4z-2z^2+4z-8 =$

$= \boxed{z^3-4z^2+8z-8}$ $\quad \boxed{\begin{array}{l} b=-4 \\ c=8 \\ d=-8 \end{array}}$

c) $z_2 = 1+\sqrt{3}i : \begin{cases} m = \sqrt{1+3} = 2 \\ \text{tg}\,\alpha = \sqrt{3} \to \alpha = \pi/3 \end{cases}$ $\quad \boxed{z_2 = 2e^{i\pi/3}}$

$z_3 = 1-\sqrt{3}i : \begin{cases} m = \sqrt{1+3} = 2 \\ \text{tg}\,\alpha = -\sqrt{3} \to \alpha = \frac{5\pi}{3} \end{cases}$ $\quad \boxed{z_3 = 2e^{i5\pi/3}}$

M09 P2
TZ1 #2

$$\begin{array}{c|ccccc} & 1 & -5 & 7 & -5 & 6 \\ i & & i & -5i-1 & 5+6i & -6 \\ \hline & 1 & -5+i & -5i+6 & 6i & \lfloor 0 \\ -i & & -i & 5i & -6i & \\ \hline & 1 & -5 & 6 & \lfloor 0 & \end{array}$$

$z^2-5z+6=0 \quad ; \quad z = \frac{5\pm\sqrt{25-24}}{2} = \frac{5\pm 1}{2} \begin{cases} 3 \\ 2 \end{cases}$

$\boxed{\text{Raíces}: \ i, -i, 3, 2}$

N09
P1 #2

$(1+\sqrt{3}i)^n \in \mathbb{R}$

$1+\sqrt{3}i = \begin{cases} m = \sqrt{1+3} = 2 \\ \text{tg}\,\alpha = \sqrt{3} \to \alpha = \pi/3 \end{cases}$

$(1+\sqrt{3}i)^n = \left(2_{\pi/3}\right)^n = \left(2^n\right)_{n\pi/3} \in \mathbb{R} \Rightarrow \frac{n\pi}{3} = \pi k \quad (k = 0, \pm 1, \pm 2, \dots)$

$\boxed{n = 3k}$

N09
P1 #13

a) $\frac{1}{z} = \frac{1}{x+iy} = \frac{1\cdot(x-iy)}{(x+iy)(x-iy)} = \frac{x-iy}{x^2+y^2} = \frac{x}{x^2+y^2} - i\frac{y}{x^2+y^2}$

$z+\frac{1}{z} = x+iy+\frac{x}{x^2+y^2} - i\frac{y}{x^2+y^2} = \left(x+\frac{x}{x^2+y^2}\right) + iy\left(1-\frac{1}{x^2+y^2}\right)$

$z+\frac{1}{z} = k \in \mathbb{R} \Rightarrow y\left(1-\frac{1}{x^2+y^2}\right) = 0 \begin{cases} \boxed{y=0} \\ 1 = \frac{1}{x^2+y^2} \ ; \ \boxed{x^2+y^2=1} \end{cases}$ ✓

$x^2+y^2=1 \Rightarrow |k| = \left|z+\frac{1}{z}\right| \leq |z| + \left|\frac{1}{z}\right| = |z| + \frac{1}{|z|} = 1+\frac{1}{1} = 2$ ✓

b) $w^n + w^{-n} = (\cos\theta + i\sin\theta)^n + [\cos(-\theta)+i\sin(-\theta)]^n = \cos(n\theta) + i\sin(n\theta) + \cos(-n\theta) +$
$+ i\sin(-n\theta) = \cos n\theta + \cancel{i\sin n\theta} + \cos n\theta - \cancel{i\sin n\theta} = 2\cos n\theta$ ✓

$3w^2 - w + 2 - w^{-1} + 3w^{-2} = 0 \quad ; \quad 3(w^2+w^{-2}) - (w+w^{-1}) + 2 = 0 \quad ;$

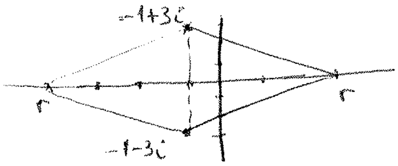
$3\cdot 2\cos 2\theta - 2\cos\theta + 2 = 0 \quad ; \quad 3\cos 2\theta - \cos\theta + 1 = 0 \quad ; \quad 3(\cos^2\theta - \sin^2\theta) - \cos\theta + 1 = 0 ;$

$3(2\cos^2\theta - 1) - \cos\theta + 1 = 0 \quad ; \quad 6\cos^2\theta - \cos\theta - 2 = 0$

$\cos\theta = \frac{1\pm\sqrt{1+48}}{12} = \frac{1\pm 7}{12} \begin{cases} \frac{2}{3} \to \sin\theta = \pm\frac{\sqrt{5}}{3} \Rightarrow \boxed{w = \frac{2}{3} \pm i\frac{\sqrt{5}}{3}} \\ -1/2 \to \cos\theta = \pm\frac{\sqrt{3}}{2} \Rightarrow \boxed{w = -\frac{1}{2} \pm i\frac{\sqrt{3}}{2}} \end{cases}$

N09 P2 #7

$z_1=-1+3i \;\longrightarrow \begin{cases} z_2=-1-3i \\ z_3=r\in\mathbb{R}\end{cases}$



Si $r>0 \to \text{Area} = \dfrac{(r+1)\cdot 6}{2} \Rightarrow 9=3(r+1) \Rightarrow \boxed{r=2}$

Si $r<0 \to \text{Area} = \dfrac{(-r-1)6}{2} \Rightarrow 9=-3(1+r) \Rightarrow \boxed{r=-4}$

$$(z-(-1+3i))\,(z-(-1-3i))\,(z-r) = (z+1-3i)(z+1+3i)(z-r) = ((z+1)^2+3^2)(z-r) =$$
$$= (z^2+2z+10)(z-r) = z^3-rz^2+2z^2-2rz+10z-10r = z^3+(2-r)z^2+(10-2r)z-10r$$

$r=2 \Rightarrow \boxed{a=0\;;\; b=6\;;\; c=-20}$

$r=-4 \Rightarrow \boxed{a=6\;;\; b=18\;;\; c=40}$

M10 P1 TZ2 #13

a) $w=1_{2\pi/3} \Rightarrow w^3=(1_{2\pi/3})^3 = 1_{6\pi/3} = 1_{2\pi} = 1$ ✓

$$1+w+w^2 = 1+1_{2\pi/3}+1_{4\pi/3} = \left(1+\cos\tfrac{2\pi}{3}+\cos\tfrac{4\pi}{3}\right)+i\left(\sin \tfrac{2\pi}{3}+\sin\tfrac{4\pi}{3}\right) =$$
$$= \left(1+\tfrac{-1}{2}+\tfrac{-1}{2}\right)+i\left(\tfrac{\sqrt3}{2}+\tfrac{-\sqrt3}{2}\right) = 0+i0 = 0 \; ✓$$

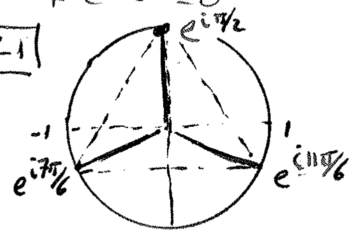
b) $1+w+w^2=0 \Rightarrow e^{i\theta}(1+w+w^2)=0\;;\; e^{i\theta}\left(1+e^{i2\pi/3}+e^{i\frac{4\pi}{3}}\right)=0\;;$

$e^{i\theta}+e^{i(\theta+2\pi/3)}+e^{i(\theta+4\pi/3)}=0$ ✓

$\theta=\frac{\pi}{2} \longrightarrow e^{i\pi/2}+e^{i\left(\frac{\pi}{2}+\frac{2\pi}{3}\right)}+e^{i\left(\pi/2+\frac{4\pi}{3}\right)}=0\;;\; e^{i\pi/2}+e^{i\frac{7\pi}{6}}+e^{i\frac{11\pi}{6}}=0$

c) $F(z)=(z-1)(z-w)(z-w^2) = z^3-z^2\underbrace{(w^2+w+1)}_{=0}+z\underbrace{(w^2+w+1)}_{=0}\cdot w-\underbrace{w^3}_{=1} = \boxed{z^3-1}$

$F(z)=7 \Rightarrow z^3-1=7\;;\; z^3=8\;;\; z=\sqrt[3]{8} = \begin{cases} z_0 = \boxed{2} \\ z_{2\pi/3} = \boxed{2w} \\ z_{4\pi/3} = \boxed{2w^2}\end{cases}$



M10 P2 TZ1 #4

a) $z^3=-2+2i\;;\quad z=\sqrt[3]{-2+2i}$

$-2+2i = \begin{cases} r=\sqrt{4+4}=\sqrt8 \\ tg\,\theta=\frac{2}{-2} \Rightarrow \theta=135^\circ \end{cases} = (\sqrt8)_{135^\circ}$

$$z=\sqrt[3]{(\sqrt8)_{135^\circ}} = \left(\sqrt[6]{8}\right)_{\frac{135^\circ+N\cdot 360^\circ}{3}} = \left(\sqrt[6]{8}\right)_{45^\circ+N\cdot120^\circ} = \boxed{\begin{cases}(\sqrt[6]{8})_{45^\circ}\\(\sqrt[6]{8})_{165^\circ}\\(\sqrt[6]{8})_{285^\circ}\end{cases}} = \boxed{\begin{cases}(\sqrt2)_{45^\circ}\\(\sqrt2)_{165^\circ}\\(\sqrt2)_{285^\circ}\end{cases}}$$

b) $(\sqrt2)_{45^\circ} = \sqrt2(\cos45^\circ+i\sin45^\circ) =$
$= \sqrt2\left(\frac{\sqrt2}{2}+i\cdot\frac{\sqrt2}{2}\right) = \frac{2}{2}+i\frac{2}{2} = 1+i$ ✓

M10 P2 TZ2 #9

a) $z=1_\theta \to z^n+\frac{1}{z^n} = 1_{n\theta}+1_{-n\theta} = \cos n\theta+i\sin n\theta+\cos(-n\theta)+i\sin(-n\theta) =$

$= \cos n\theta + \cancel{i\sin n\theta} + \cos n\theta - \cancel{i\sin n\theta} = 2\cos n\theta \Rightarrow \mathrm{Im}\left(z^n+\frac{1}{z^n}\right)=0$ ✓

b)
$$\frac{z-1}{z+1} = \frac{x+iy-1}{x+iy+1} = \frac{(x-1)+iy}{(x+1)+iy} = \frac{[(x-1)+iy][(x+1)-iy]}{[(x+1)+iy][(x+1)-iy]} = \frac{(x^2-1)-i(x-1)y+iy(x+1)+y^2}{(x+1)^2+y^2}$$

$$\mathrm{Re}\left(\frac{z-1}{z+1}\right) = \frac{(x^2-1)+y^2}{(x+1)^2+y^2} = \frac{x^2+y^2-1}{(x+1)^2+y^2} = \frac{0}{(x+1)^2+y^2} = 0 \; ✓$$

(Porque $x=\cos\theta$, $y=\sin\theta \Rightarrow x^2+y^2=1$)

N10 P1 #11

a) $w=i \Rightarrow \dfrac{z+i}{z+2}=i \;;\; z+i = zi+2i \;;\; z(1-i)=i \;;\; z=\dfrac{i}{1-i} \;;$

$$z=\frac{i(1+i)}{(1-i)(1+i)}=\frac{i-1}{1+1}=\boxed{-\frac{1}{2}+i\frac{1}{2}}=\left\{\begin{array}{l} m=\sqrt{\frac14+\frac14}=\frac{\sqrt2}{2} \\ Tg\,\alpha=-1 \rightarrow \alpha=3\pi/4\end{array}\right\}=\boxed{\frac{\sqrt2}{2}\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)}$$

b) $$w=\frac{x+iy+i}{x+iy+2}=\frac{x+i(y+1)}{(x+2)+iy}=\frac{[x+i(y+1)][(x+2)-iy]}{[(x+2)+iy][(x+2)-iy]}=$$

$$=\frac{x(x+2)-ixy+i(y+1)(x+2)+(y+1)y}{(x+2)^2+y^2}=\frac{(x^2+2x+y^2+y)+i(\cancel{-xy}+\cancel{yx}+2y+x+2)}{(x+2)^2+y^2} \checkmark$$

c) $$R(w)=1 \Rightarrow \frac{x^2+2x+y^2+y}{(x+2)^2+y^2}=1 \;;\; \cancel{x^2}+2x+\cancel{y^2}+y=\cancel{x^2}+4x+4+\cancel{y^2} \rightarrow \boxed{y=2x+4}$$

Pendiente = 2

d) $$\arg(w)=\frac{\pi}{4} \Rightarrow Tg\frac{\pi}{4}=\frac{\frac{x+2y+2}{(x+2)^2+y^2}}{\frac{x^2+2x+y^2+y}{\cancel{(x+2)^2+y^2}}} \Rightarrow 1\cdot(x^2+2x+y^2+y)=x+2y+2 \;;$$

$$x^2+y^2+x-y-2=0$$

$$\arg(z)=\frac{\pi}{4} \Rightarrow Tg\frac{\pi}{4}=\frac{y}{x} \Rightarrow 1=\frac{y}{x} \rightarrow y=x \rightarrow x^2+y^2+\cancel{x}-\cancel{y}=2 \Rightarrow \boxed{|z|=\sqrt2}$$

N10 P2 #6

Raíces: $1+i \rightarrow 1-i$

$1-2i \rightarrow 1+2i$

Factores: $(x-(1+i))(x-(1-i))(x-(1-2i))(x-(1+2i))=$

$=((x-1)-i)((x-1)+i)((x-1)+2i)((x-1)-2i)=((x-1)^2+1)((x-1)^2+4)=$

$=(x^2-2x+2)(x^2-2x+5)=x^4-2x^3+5x^2-2x^3+4x^2-10x+2x^2-4x+10=$

$=x^4-4x^3+11x^2-14x+10$ $\boxed{a=-4;\; b=11;\; c=-14;\; d=10}$

M11 P1 TZ1 #2

$\dfrac{z}{z+2}=2-i \;;\; z=2z-iz+4-2i \;;\; -z+iz=4-2i \;;\; z(-1+i)=4-2i \;;$

$$z=\frac{4-2i}{-1+i}=\frac{(4-2i)(-1-i)}{(-1-i)(-1+i)}=\frac{-4-4i+2i-2}{1+1}=\frac{-6-2i}{2}=\boxed{-3-i}$$

M11 P1 TZ1 #13

a) $(\cos\theta+i\sin\theta)^3=\cos^3\theta+3\cos^2\theta\, i\sin\theta+3\cos\theta\, i^2\sin^2\theta+i^3\sin^3\theta=$

$=\boxed{(\cos^3\theta-3\cos\theta\sin^2\theta)+i(3\cos^2\theta\sin\theta-\sin^3\theta)}$

b) $(\cos\theta+i\sin\theta)^3=\cos3\theta+i\sin3\theta \Rightarrow \cos3\theta=\cos^3\theta-3\cos\theta\sin^2\theta=\cos^3\theta-3\cos\theta(1-\cos^2\theta)=$

$=4\cos^3\theta-3\cos\theta$ ✓

c) $(\cos\theta+i\sin\theta)^5=\cos^5\theta+5\cos^4\theta\, i\sin\theta+10\cos^3\theta\, i^2\sin^2\theta+10\cos^2\theta\, i^3\sin^3\theta+5\cos\theta\, i^4\sin^4\theta+i^5\sin^5\theta=$

$=(\cos^5\theta-10\cos^3\theta\sin^2\theta+5\cos\theta\sin^4\theta)+i(5\cos^4\theta\sin\theta-10\cos^2\theta\sin^3\theta+\sin^5\theta)$

$\cos5\theta=\cos^5\theta-10\cos^3\theta\sin^2\theta+5\cos\theta\sin^4\theta=\cos^5\theta-10\cos^3\theta(1-\cos^2\theta)+5\cos\theta(1-\cos^2\theta)^2=$

$=\cos^5\theta-10\cos^3\theta+10\cos^5\theta+5\cos\theta(1-2\cos^2\theta+\cos^4\theta)=$

$=\cos^5\theta-10\cos^3\theta+10\cos^5\theta+5\cos\theta-10\cos^3\theta+5\cos^5\theta=16\cos^5\theta-20\cos^3\theta+5\cos\theta$ ✓

d) $\cos 5\theta + \cos 3\theta + \cos\theta = 0 \rightarrow 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta + 4\cos^3\theta - 3\cos\theta + \cos\theta = 0 \rightarrow$

$\rightarrow \cos\theta \cdot (16\cos^4\theta - 16\cos^2\theta + 3) = 0 \rightarrow \cos\theta = 0 \Rightarrow \boxed{\theta = \pm\pi/2}$; $\rightarrow 16\cos^4\theta - 16\cos^2\theta + 3 = 0$

Llamo $t = \cos^2\theta \rightarrow 16t^2 - 16t + 3 = 0$

$t = \frac{16 \pm \sqrt{256-192}}{32} = \frac{16\pm 8}{32}$; $\frac{24}{32} = \frac{3}{4} \rightarrow \cos^2\theta = \frac{3}{4}$; $\cos\theta = \pm\frac{\sqrt3}{2} \rightarrow \boxed{\theta = \pm\frac{\pi}{6}}$; $\frac{8}{32} = \frac{1}{4} \rightarrow \cos^2\theta = \frac{1}{4}$; $\cos\theta = \pm\frac{1}{2} \rightarrow \boxed{\theta = \pm\frac{\pi}{3}}$

M11 TZ2 P1 #12

a)

$$\begin{array}{c|cccc} & 1 & 0 & 0 & 1 \\ -1 & & -1 & 1 & -1 \\ \hline & 1 & -1 & 1 & 0 \end{array}$$

$z^3 + 1 = (z+1)(z^2 - z + 1)$

b) $\gamma^3 = \left[\frac{1}{2}(1+i\sqrt3)\right]^3 = \frac{1}{8}(1 + 3i\sqrt3 + 3i^2 \cdot 3 + i^3 \cdot 3\sqrt3) = \frac{1}{8}(1 + 3i\sqrt3 - 9 - 3i\sqrt3) = \frac{-8}{8} = -1$ $\checkmark$

$\gamma^2 = \left[\frac{1}{2}(1+i\sqrt3)\right]^2 = \frac{1}{4}(1 + 2i\sqrt3 + i^2 \cdot 3) = \frac{1}{4}(1 + 2i\sqrt3 - 3) = \frac{1}{4}(2 + 2i\sqrt3 - 4) = \frac{1+i\sqrt3}{2} - 1$ $\checkmark$

$(1-\gamma)^6 = (-\gamma^2)^6 = \gamma^{12} = (\gamma^3)^4 = (-1)^4 = \boxed{1}$

N11 P2#6

$$\left.\begin{array}{l} z_1 z_2 = -\sqrt3 + i \\ \frac{z_1}{z_2} = 2i \end{array}\right\}$$

$-\sqrt3 + i = \left\{\begin{array}{l} m = \sqrt{3+1} = 2 \\ \tan\alpha = \frac{1}{-\sqrt3} \Rightarrow \alpha = 150° \equiv 5\pi/6 \end{array}\right\} = 2_{5\pi/6}$ $\qquad 2i = \left\{\begin{array}{l} m = 2 \\ \alpha = 90° \equiv \pi/2 \end{array}\right\} = 2_{\pi/2}$

$z_1 = (m_1)_{\alpha_1}$, $z_2 = (m_2)_{\alpha_2}$ $\left(\begin{array}{l}\alpha_1 \in [0,\pi] \\ \alpha_2 \in [0,\pi]\end{array}\right)$

$z_1 z_2 = -\sqrt3 + i \longrightarrow (m_1)_{\alpha_1} \cdot (m_2)_{\alpha_2} = 2_{5\pi/6}$

$\frac{z_1}{z_2} = 2i \longrightarrow \frac{(m_1)_{\alpha_1}}{(m_2)_{\alpha_2}} = 2_{\pi/2}$

$\Rightarrow \left.\begin{array}{l} m_1 \cdot m_2 = 2 \\ \frac{m_1}{m_2} = 2 \end{array}\right\} \rightarrow$ $\qquad \left.\begin{array}{l} \alpha_1 + \alpha_2 = 5\pi/6 \\ \alpha_1 - \alpha_2 = \pi/2 \end{array}\right\} \rightarrow$

$\rightarrow \boxed{m_1 = 2} \rightarrow \boxed{m_2 = 1}$

$\rightarrow 2\alpha_1 = \frac{8\pi}{6} \Rightarrow \boxed{\alpha_1 = \frac{2\pi}{3}} \Rightarrow \boxed{\alpha_2 = \frac{\pi}{6}}$

N11 P2#10

$z = \frac{2-i}{1+i} - \frac{6+8i}{a+i} = \frac{(2-i)(1-i)}{(1+i)(1-i)} - \frac{(6+8i)(a-i)}{(a+i)(a-i)} = \frac{2-2i-i+i^2}{1-i^2} - \frac{6a-6i+8ai-8i^2}{a^2-i^2} =$

$= \frac{1-3i}{2} - \frac{(6a+8)+(8a-6)i}{a^2+1} = \left(\frac{1}{2} - \frac{6a+8}{a^2+1}\right) + \left(-\frac{3}{2} - \frac{8a-6}{a^2+1}\right)i$

$\mathrm{Re}\, z = \mathrm{Im}\, z \Rightarrow \frac{1}{2} - \frac{6a+8}{a^2+1} = -\frac{3}{2} - \frac{8a-6}{a^2+1}$; $(a^2+1) - 2(6a+8) = -3(a^2+1) - 2(8a-6)$;

$a^2 + 1 - 12a - 16 = -3a^2 - 3 - 16a + 12$; $4a^2 + 4a - 24 = 0$; $a^2 + a - 6 = 0$; $\boxed{a = 2,\ -3}$

N11 P2#14

$1 + \frac{1}{3}e^{i\theta} + \frac{1}{9}e^{2i\theta} + \dots$

$z = \frac{1}{3}e^{i\theta} \longrightarrow |z| = \left|\frac{1}{3}e^{i\theta}\right| = \frac{1}{3}|e^{i\theta}| = \frac{1}{3}|\cos\theta + i\sin\theta| = \frac{1}{3}\sqrt{\cos^2\theta + \sin^2\theta} = \frac{1}{3}$ $\checkmark$

$S_\infty = \frac{1}{1 - 1/3\, e^{i\theta}} = \frac{3}{3 - e^{i\theta}} = \frac{3}{3 - (\cos\theta + i\sin\theta)} = \frac{3}{(3-\cos\theta) - i\sin\theta} =$

$= \frac{3 \cdot [(3-\cos\theta) + i\sin\theta]}{[(3-\cos\theta) - i\sin\theta][(3-\cos\theta) + i\sin\theta]} = \frac{9 - 3\cos\theta + 3i\sin\theta}{(3-\cos\theta)^2 + \sin^2\theta} = \frac{9 - 3\cos\theta + 3i\sin\theta}{9 - 6\cos\theta + \cos^2\theta + \sin^2\theta} =$

$= \boxed{\frac{9 - 3\cos\theta}{10 - 6\cos\theta} + \frac{3\sin\theta}{10 - 6\cos\theta}i} \Rightarrow 0 + \frac{1}{3}\sin\theta + \frac{1}{9}\sin 2\theta + \frac{1}{27}\sin 3\theta + \dots = \frac{3\sin\theta}{10 - 6\cos\theta}$ $\checkmark$

M11 P1 #2

$$\sqrt[3]{i} = \sqrt[3]{1_{90^\circ}} = \begin{cases} 1_{30^\circ} = 1\cdot(\cos 30^\circ + i \operatorname{sen} 30^\circ) = \boxed{\frac{\sqrt{3}}{2}+\frac{1}{2} i} \\ 1_{150^\circ} = 1\cdot(\cos 150^\circ + i \operatorname{sen} 150^\circ) = \boxed{-\frac{\sqrt{3}}{2}+\frac{1}{2} i} \\ 1_{270^\circ} = \boxed{-i} \end{cases}$$

M12 P1 TZ2 #6

$$(4-5i)m + 4n = 16+15i$$

a) $m\in\mathbb{R}$, $n\in\mathbb{R}$

$$(4m+4n) - 5mi = 16+15i \Rightarrow \begin{cases} 4m+4n=16 \rightarrow \boxed{n=7} \\ -5m = 15 \rightarrow \boxed{m=-3} \end{cases}$$

b) $m = a+bi$, $n = a-bi$

$$(4-5i)(a+bi) + 4(a-bi) = 16+15i$$

$$4a+\cancel{4bi}-5ai+5b+4a-\cancel{4bi} = 16+15i$$

$$(8a+5b)-5ai = 16+15i \Rightarrow \begin{cases} 8a+5b=16 \rightarrow \boxed{b=8} \\ -5a = 15 \rightarrow \boxed{a=-3} \end{cases}$$

$\boxed{m=-3+8i}$ $\boxed{n=-3-8i}$

M12 P1 TZ2 #12a

a) $(x+iy)^2 = -5+12i$

$x^2+2xyi-y^2 = -5+12i$

$$(x^2-y^2)+2xyi = -5+12i \Rightarrow \begin{cases} x^2-y^2=-5 \\ 2xy=12 \rightarrow xy=6 \end{cases}$$

b) $\begin{cases} x^2-y^2=-5 \\ xy=6 \rightarrow y=\frac{6}{x} \end{cases} \rightarrow x^2-\left(\frac{6}{x}\right)^2=-5$; $x^4-36=-5x^2$; $x^4+5x^2-36=0$

$t=x^2 \rightarrow t^2+5t-36=0$; $t=\frac{-5\pm\sqrt{25+144}}{2} = \frac{-5\pm 13}{2} \begin{cases} 4 \\ -9 \end{cases}$

$x^2=4 \Rightarrow x = \begin{cases} 2 \Rightarrow y=3 \\ -2 \Rightarrow y=-3 \end{cases} \rightarrow \boxed{\sqrt{-5+12i} = \pm(2+3i)}$

$x^2=\cancel{-9}$ No nos interesa porque x, y son ambos reales.

c) $(z^*)^2 = (a-bi)^2 = a^2-2abi+b^2i^2 = (a^2-b^2)-2abi$

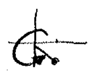
$(z^2)^* = [(a+bi)^2]^* = [a^2+2abi+b^2i^2]^* = [(a^2-b^2)+2abi]^* = (a^2-b^2)-2abi$

d) $\sqrt{-5-12i} = \sqrt{(-5+12i)^*} = \left[\sqrt{-5+12i}\right]^* = [\pm(2+3i)]^* = \boxed{\pm(2-3i)}$

M12 P1 TZ1 #3

$$z_1 = a + a\sqrt{3}i \begin{cases} |z_1| = \sqrt{a^2+(a\sqrt{3})^2} = \sqrt{a^2+3a^2} = 2|a| \\ \operatorname{tg}\alpha_1 = \frac{a\sqrt{3}}{a} = \sqrt{3} \Rightarrow \alpha_1 = \begin{cases} 60^\circ & (\text{si } a>0) \\ 240^\circ & (\text{si } a<0) \end{cases} \end{cases} \Rightarrow \boxed{\begin{array}{l} z_1 = 2|a|\cdot\left(\cos\frac{\pi}{3}+i\operatorname{sen}\frac{\pi}{3}\right) \\ \text{ó} \\ z_1 = 2|a|\left(\cos\frac{4\pi}{3}+i\operatorname{sen}\frac{4\pi}{3}\right) \end{array}}$$

$$z_2 = 1-i = \begin{cases} |z_2| = \sqrt{1+1} = \sqrt{2} \\ \operatorname{Tg}\alpha_2 = \frac{-1}{1} \Rightarrow \alpha_2 = -45^\circ \equiv 315^\circ \end{cases} \boxed{z_2 = \sqrt{2}\left(\cos\frac{7\pi}{4}+i\operatorname{sen}\frac{7\pi}{4}\right)}$$



$$\left(\frac{z_1}{z_2}\right)^6 = \left(\frac{2|a|_{\pi/3}}{\sqrt{2}_{7\pi/4}}\right)^6 = \left(\frac{2|a|}{\sqrt{2}}\right)^6_{6\left(\frac{\pi}{3}-\frac{7\pi}{4}\right)} = \left(\sqrt{2}|a|\right)^6_{6\cdot\frac{-17\pi}{12}} = (8a^6)_{-\frac{17\pi}{2}} \equiv (8a^6)_{3\pi/2} = \boxed{-8a^6 i}$$

ó

$$\left(\frac{z_1}{z_2}\right)^6 = \left(\frac{2|a|_{4\pi/3}}{\sqrt{2}_{7\pi/4}}\right)^6 = \left(\frac{2|a|}{\sqrt{2}}\right)^6_{6\left(\frac{4\pi}{3}-\frac{7\pi}{4}\right)} = (8a^6)_{-\frac{5\pi}{2}} = (8a^6)_{3\pi/2} =$$

M12 P1 TZ1 #7

$|z|+z=6-2i$

$\sqrt{x^2+y^2}+x+iy=6-2i \longrightarrow \begin{cases} x+\sqrt{x^2+y^2}=6 \\ \boxed{y=-2} \end{cases} \longrightarrow x+\sqrt{x^2+4}=6$; $\sqrt{x^2+4}=6-x$;

$x^2+4=(6-x)^2$; $\cancel{x^2}+4=36-12x+\cancel{x^2}$; $12x=32$; $x=\frac{32}{12}=\boxed{\frac{8}{3}}$

N12 P1 #10

a) $z_1=2\sqrt{3}\,\mathrm{cis}\,\frac{3\pi}{2} = 2\sqrt{3}\left(\cos\frac{3\pi}{2}+i\sin\frac{3\pi}{2}\right) = \boxed{-2\sqrt{3}i}$

$z_1+z_2 = -2\sqrt{3}i-1+\sqrt{3}i = -1-\sqrt{3}i$

$(z_1+z_2)^* = \boxed{-1+\sqrt{3}i}$

b) $z_2=-1+\sqrt{3}i \begin{cases} |z|=\sqrt{1+3}=2 \\ \mathrm{Tg}\,\alpha=\frac{\sqrt{3}}{-1}=-\sqrt{3} \longrightarrow \alpha=\frac{2\pi}{3} \end{cases} \longrightarrow \boxed{z_2=2_{2\pi/3}}=\boxed{2\,\mathrm{cis}\,\frac{2\pi}{3}}$

$z^3=z_2 \Rightarrow z^3=2_{2\pi/3}$; $(r_\theta)^3=2_{2\pi/3}$; $(r^3)_{3\theta}=2_{2\pi/3} \Rightarrow$

$\Rightarrow r^3=2 \longrightarrow \boxed{r=\sqrt[3]{2}}$

$3\theta=\frac{2\pi}{3}+N\cdot 2\pi \longrightarrow \theta=\frac{2\pi}{9}+N\cdot\frac{2\pi}{3} = \boxed{\begin{cases} 2\pi/9 \\ 8\pi/9 \\ 14\pi/9 \end{cases}} \longrightarrow z=\sqrt[3]{z_2}=\boxed{\begin{cases} (\sqrt[3]{2})_{2\pi/9} \\ (\sqrt[3]{2})_{8\pi/9} \\ (\sqrt[3]{2})_{14\pi/9} \end{cases}}$

c) $(r\,\mathrm{cis}\,\theta)^2=(\cancel{1}\cancel{-1}+\sqrt{3}i)^2$; $r^2_{2\theta}=3i^2$; $r^2_{2\theta}=-3$; $r^2_{2\theta}=3_\pi \longrightarrow$

$r^2=3 \longrightarrow \boxed{r=\sqrt{3}}$

$2\theta=\pi+N\cdot 2\pi \longrightarrow \theta=\frac{\pi}{2}+N\pi = \boxed{\begin{cases} \pi/2 \\ 3\pi/2 \end{cases}}$

También

$z^2=(1+z_2)^2 \Rightarrow z=\pm(1+z_2)=\pm(\cancel{1}\cancel{-1}+\sqrt{3}i)=\pm\sqrt{3}i = \begin{cases} (\sqrt{3})_{\pi/2} \\ (\sqrt{3})_{3\pi/2} \end{cases}$ ✓

$z=-\frac{1}{z_2}=-\frac{1}{2_{\pi/3}}=-\left(\frac{1}{2}\right)_{-\pi/3}=\left(\frac{1}{2}\right)_{\pi-\pi/3}=\boxed{\left(\frac{1}{2}\right)_{2\pi/3}}$

d) $\left(\frac{z_1}{z_2}\right)^n=\left[\frac{(2\sqrt{3})_{3\pi/2}}{2_{2\pi/3}}\right]^n=\left[\left(\frac{2\sqrt{3}}{2}\right)_{\frac{3\pi}{2}-\frac{2\pi}{3}}\right]^n=\left[\left(\frac{2\sqrt{3}}{2}\right)_{\frac{5\pi}{6}}\right]^n=\left(\frac{2\sqrt{3}}{2}\right)^n_{\frac{5\pi n}{6}}$

$\left(\frac{z_1}{z_2}\right)^n\in\mathbb{R}^+ \Rightarrow \frac{5\pi n}{6}=0+k\cdot 2\pi$; $n=\frac{12k}{5}$

para $k=5 \longrightarrow \boxed{n=12}$

N12 P2#10

$$w=\cos\theta+i\sin\theta \longrightarrow w^2=(\cos\theta+i\sin\theta)^2=\cos^2\theta+2\cos\theta\sin\theta\, i+i^2\sin^2\theta=$$
$$=(\cos^2\theta-\sin^2\theta)+2\sin\theta\cos\theta\, i$$

$$1-w^2=(1-\cos^2\theta+\sin^2\theta)-2\sin\theta\cos\theta\, i=2\sin^2\theta-2\sin\theta\cos\theta\, i$$

$$(1-w^2)^*=2\sin^2\theta+2\sin\theta\cos\theta\, i$$

$$|(1-w^2)^*|=\sqrt{(2\sin^2\theta)^2+(2\sin\theta\cos\theta)^2}=\sqrt{4\sin^4\theta+4\sin^2\theta\cos^2\theta}=$$
$$=\sqrt{4\sin^4\theta+4\sin^2\theta\cdot(1-\sin^2\theta)}=\sqrt{\cancel{4\sin^4\theta}+4\sin^2\theta-\cancel{4\sin^4\theta}}=\boxed{2\sin\theta}$$

$$\operatorname{tg}\alpha=\frac{\cancel{2}\sin\theta\cos\theta}{\cancel{2}\sin^2\theta}=\frac{1}{\operatorname{tg}\theta}\Rightarrow \alpha=\boxed{\frac{\pi}{2}-\theta}$$

M13 TZ1 P1#1

a) $w=2+2i \left\{\begin{array}{l}|w|=\sqrt{4+4}=\sqrt{8}=2\sqrt{2}\\ \operatorname{tg}\alpha=\frac{2}{2}=1\rightarrow \alpha=\pi/4\end{array}\right. \Rightarrow \boxed{w=(2\sqrt{2})_{\pi/4}}$

b) $z=\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}=1_{5\pi/6}$

$$w^4\cdot z^6=(2\sqrt{2})^4_{\frac{4\pi}{4}}\cdot(1^6)_{6\cdot 5\pi/6}=64_{\pi}\cdot 1_{5\pi}=64_{6\pi}=64_0=\boxed{64}$$

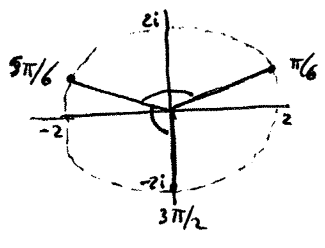
M13 TZ2 P1#13

a) $z_1=\sqrt{3}+i \left\{\begin{array}{l}|z_1|=\sqrt{3+1}=2\\ \operatorname{tg}\alpha=\frac{1}{\sqrt{3}}\rightarrow \alpha=\pi/6\end{array}\right. \Rightarrow \boxed{z_1=2_{\pi/6}}$

$z_2=-\sqrt{3}+i \left\{\begin{array}{l}|z_2|=\sqrt{3+1}=2\\ \operatorname{tg}\alpha=\frac{1}{-\sqrt{3}}\rightarrow \alpha=5\pi/6\end{array}\right. \Rightarrow \boxed{z_2=2_{5\pi/6}}$

$z_3=-2i \Rightarrow z_3=\boxed{2_{3\pi/2}}$

$$\frac{3\pi}{2}-\frac{5\pi}{6}=\frac{9\pi-5\pi}{6}=\frac{4\pi}{6}=\frac{2\pi}{3}$$
$$\frac{5\pi}{6}-\frac{\pi}{6}=\frac{4\pi}{6}=\frac{2\pi}{3}$$
$$2\pi-\frac{2\pi}{3}-\frac{2\pi}{3}=\frac{6\pi-2\pi-2\pi}{3}=\frac{2\pi}{3}$$



Como los Tres ángulos son iguales, también lo son los módulos de los Tres nº complejos ⇒ Triángulo Equilátero

$$z_1^{3n}+z_2^{3n}=(2^{3n})_{\frac{3n\pi}{6}}+(2^{3n})_{3n\frac{5\pi}{6}}=(2^{3n})_{\frac{\pi n}{2}}+(2^{3n})_{\frac{5\pi n}{2}}=$$
$$=(2^{3n})_{\frac{\pi n}{2}}+(2^{3n})_{\frac{\pi n}{2}}=2\cdot(2^{3n})_{\frac{\pi n}{2}}$$

$$2\cdot z_3^{3n}=2\cdot(2^{3n})_{3n\cdot\frac{3\pi}{2}}=2(2^{3n})_{\frac{9\pi n}{2}}=2(2^{3n})_{\frac{\pi n}{2}}$$

Nota: $\frac{5\pi n}{2}=\frac{(4\pi+\pi)n}{2}=2\pi n+\frac{\pi n}{2}\equiv\frac{\pi n}{2}$

$\frac{9\pi n}{2}=\frac{(8\pi+\pi)n}{2}=4\pi n+\frac{\pi n}{2}\equiv\frac{\pi n}{2}$

b) $z^7=1 \rightarrow z=\sqrt[7]{1} = \left(\sqrt[7]{1}\right)_{\frac{0+2\pi N}{7}} = 1_{\frac{2\pi N}{7}} = \left\{ \begin{array}{l} 1_0 \\ 1_{2\pi/7} \leftarrow w \\ 1_{4\pi/7} \\ 1_{6\pi/7} \\ 1_{8\pi/7} = 1_{-6\pi/7} \\ 1_{10\pi/7} = 1_{-4\pi/7} \\ 1_{12\pi/7} = 1_{-2\pi/7} \end{array} \right.$

$$1+w = 1+1_{2\pi/7} = 1+\cos\frac{2\pi}{7} + i\,\text{sen}\,\frac{2\pi}{7}$$

$$\text{tg}\,\alpha = \frac{\text{sen}\,2\pi/7}{1+\cos 2\pi/7} = \frac{2\,\text{sen}\,\pi/7\,\cos\pi/7}{1+\cos^2\pi/7-\text{sen}^2\pi/7} = \frac{\cancel{2}\,\text{sen}\,\pi/7\,\cancel{\cos\pi/7}}{\cancel{2}\cos^{\cancel{2}}\pi/7} = \text{tg}\,\frac{\pi}{7} \Rightarrow \boxed{\alpha=\frac{\pi}{7}}$$

$$z^7-1 = (z-1)\cdot(z-1_{2\pi/7})(z-1_{4\pi/7})(z-1_{6\pi/7})(z-1_{8\pi/7})(z-1_{10\pi/7})(z-1_{12\pi/7})$$

$$(z-1_{2\pi/7})\cdot(z-1_{12\pi/7}) = z^2 - z\cdot 1_{12\pi/7} - z\cdot 1_{2\pi/7} + 1_{\frac{14\pi}{7}} =$$
$$= z^2 - z\cdot(1_{-2\pi/7}+1_{2\pi/7}) + 1_{2\pi} =$$
$$= z^2 - z\left(\cos(-2\pi/7) + i\,\text{sen}\,\cancel{(-2\pi/7)} + \cos 2\pi/7 + i\,\text{sen}\,\cancel{2\pi/7}\right) + 1 =$$
$$= z^2 - 2z\cos\left(\frac{2\pi}{7}\right) + 1 \quad ✓$$

$$(z-1_{4\pi/7})(z-1_{10\pi/4}) = z^2 - z(1_{-4\pi/7}+1_{4\pi/7}) + 1_{14\pi/7} =$$
$$= \boxed{z^2 - 2z\cos\left(\frac{4\pi}{7}\right)+1}$$

$$(z-1_{6\pi/7})(z-1_{8\pi/7}) = \,---\, = \boxed{z^2 - 2z\cos\left(\frac{6\pi}{7}\right)+1}$$

N13 P1#12

a) $z^n + z^{-n} = (1^n)_{n\theta} + (1^{-n})_{-n\theta} = 1_{n\theta} + 1_{-n\theta} =$

$= \cos n\theta + i\,\text{sen}\,\cancel{n\theta} + \cos(-n\theta) + i\,\text{sen}\,\cancel{(-n\theta)} = 2\cos(n\theta)$ ✓

NOTA: los cosenos de ángulos opuestos son iguales y los senos son opuestos.

b) $(z+z^{-1})^4 = (2\cos(1\cdot\theta))^4 = (2\cos\theta)^4 = 16\cdot\cos^4\theta$

c) $(z+z^{-1})^4 = \binom{4}{0}z^4 + \binom{4}{1}z^3\cdot z^{-1} + \binom{4}{2}z^2\cdot z^{-2} + \binom{4}{3}z\cdot z^{-3} + \binom{4}{4}z^{-4} =$

$= z^4 + 4z^2 + 6 + 4z^{-2} + z^{-4} =$

$= (z^4+z^{-4}) + 4(z^2+z^{-2}) + 6 = 2\cos 4\theta + 4\cdot 2\cos 2\theta + 6 =$

$= 2\cos 4\theta + 8\cos 2\theta + 6$

$16\cos^4\theta = 2\cos 4\theta + 8\cos 2\theta + 6 \rightarrow \boxed{\cos^4\theta = \frac{1}{8}\cos 4\theta + \frac{1}{2}\cos 2\theta + \frac{3}{8}}$ $\quad p=1/8$, $q=1/2$, $r=3/8$

d) $(z+z^{-1})^6 = (2\cos\theta)^6 = 64\cos^6\theta$

$(z+z^{-1})^6 = z^6 + 6z^4 + 15z^2 + 20 + 15z^{-2} + 6z^{-4} + z^{-6} =$

$= 2\cos 6\theta + 6\cdot 2\cos 4\theta + 15\cdot 2\cos 2\theta + 20 = 2\cos 6\theta + 12\cos 4\theta + 30\cos 2\theta + 20$

$\cos^6\theta = \frac{2}{64}\cos 6\theta + \frac{12}{64}\cos 4\theta + \frac{30}{64}\cos 2\theta + \frac{20}{64} = \frac{1}{32}\cos 6\theta + \frac{3}{16}\cos 4\theta + \frac{15}{32}\cos 2\theta + \frac{5}{16}$ ✓

N13 P2#6

$$z=\frac{a+i}{a-i}=\frac{(a+i)^2}{(a-i)(a+i)}=\frac{a^2+2ai+i^2}{a^2-i^2}=\frac{a^2-1+2ai}{a^2+1}=\frac{a^2-1}{a^2+1}+\frac{2a}{a^2+1}i$$

a] $z\in\mathbb{R} \Rightarrow \frac{2a}{a^2+1}=0 \Rightarrow \boxed{a=0}$

$z\in\mathbb{I} \Rightarrow \frac{a^2-1}{a^2+1}=0\ ;\ a^2-1=0 \Rightarrow \boxed{a=\pm 1}$

c] $|z|=\left|\frac{a+i}{a-i}\right|=\frac{|a+i|}{|a-i|}=\frac{\sqrt{a^2+1}}{\sqrt{a^2+1}}=\boxed{1}$ ✓

Muestra 14 P1#10

$z_1=2\operatorname{cis}150^\circ\ ;\ z_2=-1+i$

a] $z_2=-1+i=\left\{\begin{array}{l} r=\sqrt{1+1}=\sqrt{2}\\ \operatorname{tg}\alpha=\frac{1}{-1}\Rightarrow \alpha=315^\circ\end{array}\right\}=(\sqrt{2})_{135^\circ}$

$$\frac{z_1}{z_2}=\frac{2_{150^\circ}}{(\sqrt{2})_{135^\circ}}=\left(\frac{2}{\sqrt{2}}\right)_{15^\circ}=\boxed{(\sqrt{2})_{15^\circ}}$$

b] $i\Big/\frac{z_1}{z_2}=\frac{1_{90^\circ}}{(\sqrt{2})_{15^\circ}}=\left(\frac{1}{\sqrt{2}}\right)_{75^\circ}$

$$z_1=2\operatorname{cis}150^\circ=2(\cos150^\circ+i\operatorname{sen}150^\circ)=2\cdot\left(\frac{-\sqrt{3}}{2}+i\cdot\frac{1}{2}\right)=-\sqrt{3}+i$$

$$\frac{i}{z_1/z_2}=\frac{i\cdot z_2}{z_1}=\frac{i\cdot(-1+i)}{-\sqrt{3}+i}=\frac{-1-i}{-\sqrt{3}+i}=\frac{1+i}{\sqrt{3}-i}=\frac{(1+i)(\sqrt{3}+i)}{(\sqrt{3}-i)(\sqrt{3}+i)}=$$

$$=\frac{\sqrt{3}+i+i\sqrt{3}+i^2}{(\sqrt{3})^2-i^2}=\frac{(\sqrt{3}-1)+(1+\sqrt{3})i}{3+1}=\frac{\sqrt{3}-1}{4}+\frac{1+\sqrt{3}}{4}i=$$

$$=\left\{\begin{array}{l} r=\sqrt{\left(\frac{\sqrt{3}-1}{4}\right)^2+\left(\frac{1+\sqrt{3}}{4}\right)^2}=\sqrt{\frac{3-2\cancel{\sqrt{3}}+1}{16}+\frac{1+2\cancel{\sqrt{3}}+3}{16}}=\sqrt{\frac{8}{16}}=\sqrt{\frac{1}{2}}=\frac{1}{\sqrt{2}}\ ✓\\ \operatorname{tg}\alpha=\frac{(1+\sqrt{3})/\cancel{4}}{(\sqrt{3}-1)/\cancel{4}}=\frac{1+\sqrt{3}}{\sqrt{3}-1}=\boxed{\frac{\sqrt{3}+1}{\sqrt{3}-1}=\operatorname{tg}75^\circ}\end{array}\right.$$

Muestra 14 P2#4

a] $z=-\sqrt{3}+i=\left\{\begin{array}{l} r=\sqrt{3+1}=2\\ \operatorname{tg}\alpha=\frac{1}{-\sqrt{3}}\rightarrow \alpha=150^\circ\end{array}\right\}=\boxed{2_{150^\circ}}$

b] $\sqrt[3]{z}=\sqrt[3]{2_{150^\circ}}=(\sqrt[3]{2})_{\frac{150^\circ+N\cdot 360^\circ}{3}}=(\sqrt[3]{2})_{50^\circ+120^\circ N}=\left\{\begin{array}{l}(\sqrt[3]{2})_{50^\circ}=\boxed{0'810+i0'965}\\ (\sqrt[3]{2})_{170^\circ}\\ (\sqrt[3]{2})_{290^\circ}\end{array}\right.$

c] $z^m=(2_{150^\circ})^m=(2^m)_{m\cdot150^\circ}$

$z^m\in\mathbb{R}^+ \Rightarrow m\cdot150^\circ=360^\circ K \Rightarrow m=\frac{360K}{150}=\frac{12K}{5}\Rightarrow \boxed{m=12}$ con $K=5$.

M14 TZ2 P1#7

a] $\frac{1}{u}+\frac{1}{v}=\frac{10}{w}\ ;\ \frac{v+u}{uv}=\frac{10}{w}\rightarrow w=\frac{10uv}{v+u}=\frac{10(2+3i)(3+2i)}{2+3i+3+2i}=$

$$=\frac{10\cdot(6+4i+9i-6)}{5+5i}=\frac{130i}{5(1+i)}=\frac{130i(1-i)}{5(1+i)(1-i)}=$$

$$=\frac{130i+130}{5(1+1)}=\frac{130+130i}{10}=\boxed{13+13i}\quad a=b=13$$

b] $w^*=13-13i=\left\{\begin{array}{l} r=\sqrt{13^2+13^2}=13\sqrt{2}\\ \operatorname{tg}\theta=\frac{-13}{13}=-1\Rightarrow\theta=\frac{7\pi}{4}\end{array}\right\}\Rightarrow \boxed{w^*=13\sqrt{2}\,e^{i7\pi/4}}$

M14 TZ1 P1#13

$u_1 = 3$, razón $= 1+i$ $\Big\}$ $u_n = 3\cdot(1+i)^{n-1}$

a) $u_4 = 3(1+i)^3 = 3(1+3i+3i^2+i^3) = 3(1+3i-3-i) = 3(-2+2i) = \boxed{-6+6i}$

b) $S_{20} = \dfrac{u_{20}\cdot(1+i)-u_1}{1+i-1} = \dfrac{3(1+i)^{19}(1+i)-3}{i} = \dfrac{3(1+i)^{20}-3}{i}$

$1+i = \begin{cases} |1+i| = \sqrt{1+1} = \sqrt{2} \\ \operatorname{tg}\alpha = \frac{1}{1} = 1 \Rightarrow \alpha = 45^\circ \end{cases} = (\sqrt{2})_{45^\circ}$

$(1+i)^{20} = (\sqrt{2})^{20}_{20\cdot 45^\circ} = (2^{10})_{900^\circ} = (2^{10})_{180^\circ} = -(2^{10})$ $\quad$ $900 \,\lfloor 360$, 180, 2

$S_{20} = \dfrac{-3\cdot 2^{10}-3}{i} = \dfrac{(-3\cdot 2^{10}-3)\cdot(-i)}{i\cdot(-i)} = \dfrac{(3\cdot 2^{10}+3)i}{1} = \boxed{3i\cdot(1+2^{10})}$ $\quad$ $a = 3i$, $n = 10$

c) $v_n = u_n\cdot u_{n+k} = 3(1+i)^{n-1}\cdot 3(1+i)^{n+k-1} = 9(1+i)^{2n+k-2} =$

$= 9(1+i)^k\cdot[(1+i)^2]^{n-1} =$

$= 9(1+i)^k\cdot(2i)^{n-1}$ $\quad$ Progresión Geométrica con: $\boxed{v_1 = 9(1+i)^k}$ y $\boxed{\text{razón} = 2i}$

d) $w_n = |u_n - u_{n+1}| = |3(1+i)^{n-1} - 3(1+i)^n| = |3(1+i)^{n-1}\cdot(1-(1+i))| =$

$= |3(1+i)^{n-1}\cdot(-i)| = |3|\cdot|(1+i)^{n-1}|\cdot|-i| = 3\cdot|1+i|^{n-1}\cdot 1 =$

$= 3\cdot(\sqrt{2})^{n-1} \longrightarrow$ Progresión Geométrica con: $\boxed{w_1 = 3}$ $\boxed{\text{razón} = \sqrt{2}}$

NOTA: $|1+i| = \sqrt{1+1} = \sqrt{2}$

Como $|u_n - u_{n+1}|$ sería la distancia entre los puntos afijos de u_n y u_{n+1}, que formen progresión geométrica de razón $\sqrt{2}$ significa que están cada vez más alejados.

M14 TZ2 P2#13

$z = r(\cos\theta + i\operatorname{sen}\theta) \longrightarrow z^n = r^n(\cos n\theta + i\operatorname{sen} n\theta)$

a) $n=1$ $\quad$ $z^1 = r^1(\cos 1\theta + i\operatorname{sen}1\theta) = r(\cos\theta + i\operatorname{sen}\theta) = z$ ✓

$n=k$ $\quad$ Suponiendo cierto que $z^k = r^k(\cos k\theta + i\operatorname{sen}k\theta)$ demostraremos que $z^{k+1} = r^{k+1}(\cos(k+1)\theta + i\operatorname{sen}(k+1)\theta)$

$z^{k+1} = z\cdot z^k = r(\cos\theta + i\operatorname{sen}\theta)\cdot r^k(\cos k\theta + i\operatorname{sen}k\theta) =$

$= r^{k+1}(\cos\theta\cos k\theta + i\cos\theta\operatorname{sen}k\theta + i\operatorname{sen}\theta\cos k\theta - \operatorname{sen}\theta\operatorname{sen}k\theta)$

$= r^{k+1}[(\cos\theta\cos k\theta - \operatorname{sen}\theta\operatorname{sen}k\theta) + i(\cos\theta\operatorname{sen}k\theta + \operatorname{sen}\theta\cos k\theta)] =$

$= r^{k+1}[\cos(\theta+k\theta) + i\operatorname{sen}(\theta+k\theta)] =$

$= r^{k+1}\cdot(\cos(k+1)\theta + i\operatorname{sen}(k+1)\theta)$ ✓

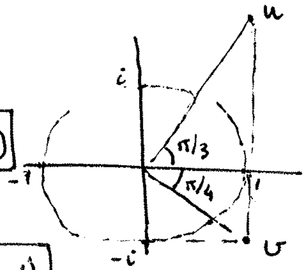
b) $u = 1+\sqrt{3}i \quad ; \quad v = 1-i$

$$u = 1+\sqrt{3}i = \left\{ \begin{array}{l} |u| = \sqrt{1+3} = 2 \\ \operatorname{tg}\alpha = \frac{\sqrt{3}}{1} \rightarrow \alpha = \pi/3 \end{array} \right\} = \boxed{2_{\pi/3}}$$

$$v = 1-i = \left\{ \begin{array}{l} |v| = \sqrt{1+1} = \sqrt{2} \\ \operatorname{tg}\alpha = \frac{-1}{1} = -1 \rightarrow \alpha = 7\pi/4 \end{array} \right\} = (\sqrt{2})_{7\pi/4}$$

$$u^3 \cdot v^4 = (2_{\pi/3})^3 \cdot \left[(\sqrt{2})_{7\pi/4}\right]^4 = 8_{\pi} \cdot 4_{7\pi} = 32_{8\pi} = \boxed{32}$$

c)



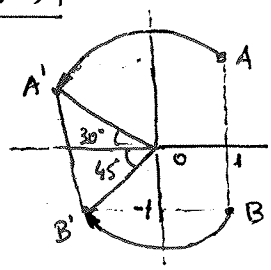
$A(1, \sqrt{3})$

$$u \cdot 1_{\pi/2} = (1+\sqrt{3}i)i = -\sqrt{3}+i \rightarrow \boxed{A'(-\sqrt{3}, 1)}$$

$B(1, -1)$

$$v \cdot 1_{-\pi/2} = (1-i)(-i) = -1-i \rightarrow \boxed{B'(-1,-1)}$$

d)



$$OA' = \sqrt{1+(\sqrt{3})^2} = 2$$

$$OB' = \sqrt{1+1} = \sqrt{2}$$

$$\text{Area} = \frac{2 \cdot \sqrt{2} \operatorname{sen}(30^\circ + 45^\circ)}{2} =$$

$$= \sqrt{2} \cdot (\operatorname{sen} 30^\circ \cos 45^\circ + \cos 30^\circ \operatorname{sen} 45^\circ) = \sqrt{2} \cdot \left(\frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} \right) =$$

$$= \frac{2}{4} + \frac{2\sqrt{3}}{4} = \frac{2(\sqrt{3}+1)}{4} = \boxed{\frac{\sqrt{3}+1}{2}}$$

e)

$1+\sqrt{3}i$ raíz de la ecuación $\Rightarrow$ $1-\sqrt{3}i$ También es raíz.
$1-i$ " " " " $\Rightarrow$ $1+i$ " " "

$$(z-(1+\sqrt{3}i))(z-(1-\sqrt{3}i))(z-(1-i))(z-(1+i)) =$$

$$= ((z-1)-\sqrt{3}i)((z-1)+\sqrt{3}i)((z-1)+i)((z-1)-i) =$$

$$= \left[(z-1)^2 - (\sqrt{3}i)^2\right]\left[(z-1)^2 - i^2\right] =$$

$$= (z^2-2z+1+3)(z^2-2z+1+1) = (z^2-2z+4)(z^2-2z+2) =$$

$$= z^4 - 2z^3 + 2z^2 - 2z^3 + 4z^2 - 4z + 4z^2 - 8z + 8 =$$

$$= \boxed{z^4 - 4z^3 + 10z^2 - 12z + 8}$$

$b = -4$
$c = 10$
$d = -12$
$e = 8$

Nov 14 P1#13a

a)(i) $1+i\,tg\,\theta = 1 + i\,\frac{sen\,\theta}{cos\,\theta} = \frac{cos\,\theta + i\,sen\,\theta}{cos\,\theta} = \frac{1}{cos\,\theta}\cdot 1_\theta$

$1-i\,tg\,\theta = 1 - i\,\frac{sen\,\theta}{cos\,\theta} = \frac{cos\,\theta - i\,sen\,\theta}{cos\,\theta} = \frac{cos(-\theta) + i\,sen(-\theta)}{cos\,\theta} = \frac{1}{cos\,\theta}\cdot 1_{-\theta}$

$(1+i\,tg\,\theta)^n + (1-i\,tg\,\theta)^n = \left[\frac{1}{cos\,\theta}\cdot 1_\theta\right]^n + \left[\frac{1}{cos\,\theta}\, 1_{-\theta}\right]^n =$

$= \frac{1}{cos^n\theta}\cdot 1_{n\theta} + \frac{1}{cos^n\theta}\, 1_{-n\theta} = \frac{1}{cos^n\theta}\left[1_{n\theta} + 1_{-n\theta}\right] =$

$= \frac{1}{cos^n\theta}\left(cos\,n\theta + i\,sen\,n\theta + cos(-n\theta) + i\,sen(-n\theta)\right) =$

$= \frac{1}{cos^n\theta}\left(cos\,n\theta + \cancel{i\,sen\,n\theta} + cos\,n\theta - \cancel{i\,sen\,n\theta}\right) = \frac{2cos\,n\theta}{cos^n\theta}$ ✓

(ii) $z = i\,tg\,\frac{3\pi}{8}$

$(1+z)^4 + (1-z)^4 = \frac{2\cos 4\frac{3\pi}{8}}{cos^4\frac{3\pi}{8}} = \frac{2\cos\frac{3\pi}{2}}{cos^4\frac{3\pi}{8}} = \frac{0}{cos^4\frac{3\pi}{8}} = 0$ ✓

(iii) Debe anular $\frac{2\cos 4\theta}{cos^4\theta} \Rightarrow 4\theta = \begin{cases}\pi/2 \longrightarrow \theta = \pi/8 \Rightarrow \boxed{z = i\,tg\,\frac{\pi}{8}} \\ 3\pi/2\ ✓\end{cases}$

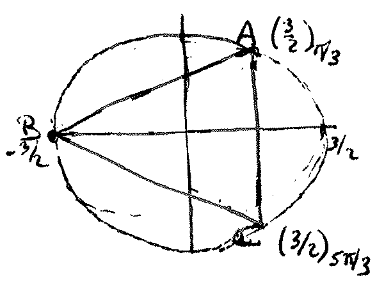
M15 TZ2 P1#7

a) $8z^3 + 27 = 0$; $8z^3 = -27$; $z^3 = -\frac{27}{8}$; $z = \sqrt[3]{-\frac{27}{8}} = \frac{3}{2}\sqrt[3]{-1}$

$z = \frac{3}{2}\sqrt[3]{-1} = \frac{3}{2}\sqrt[3]{1_\pi} = \frac{3}{2}\cdot 1_{\frac{\pi+2\pi N}{3}} = \frac{3}{2}\cdot 1_{\frac{\pi}{3}+\frac{2\pi N}{3}} =$

$= \begin{cases}\frac{3}{2}\cdot 1_{\pi/3} = \left(\frac{3}{2}\right)_{\pi/3} \\ \frac{3}{2}\cdot 1_\pi = \left(\frac{3}{2}\right)_\pi = -\frac{3}{2} \\ \frac{3}{2}\cdot 1_{5\pi/3} = \left(\frac{3}{2}\right)_{5\pi/3}\end{cases}$

b) Se trata de un triángulo equilátero, cuyos lados miden:



$\left(\frac{3}{2}\right)_{\pi/3} \longrightarrow A\left(\frac{3}{2}cos\,\pi/3\ ,\ \frac{3}{2}sen\,\frac{\pi}{3}\right) = \left(\frac{3}{2}\cdot\frac{1}{2}\ ,\ \frac{3}{2}\cdot\frac{\sqrt{3}}{2}\right) = \left(\frac{3}{4}\ ,\ \frac{3\sqrt{3}}{4}\right)$

$\left(\frac{3}{2}\right)_\pi \longrightarrow B(-3/2\ ,\ 0)$

$AB = \sqrt{\left(\frac{3}{4}+\frac{3}{2}\right)^2 + \left(\frac{3\sqrt{3}}{4} - 0\right)^2} = \sqrt{\left(\frac{9}{4}\right)^2 + \left(\frac{3\sqrt{3}}{4}\right)^2} = \sqrt{\frac{81+27}{16}} = \sqrt{\frac{108}{16}} =$

$= \sqrt{\frac{4\cdot 3^3}{16}} = \sqrt{\frac{3^3}{4}} = \frac{3}{2}\sqrt{3}$

$\text{Área} = \frac{\frac{3}{2}\sqrt{3}\cdot\frac{3}{2}\sqrt{3}\cdot sen\,60^\circ}{2} = \frac{\left(\frac{3}{2}\sqrt{3}\right)^2\cdot\frac{\sqrt{3}}{2}}{2} = \frac{\frac{27}{4}\cdot\frac{\sqrt{3}}{2}}{2} = \frac{27\sqrt{3}}{16}$ ✓