M00 P1 #3

$$z_1 = a_{\pi/4} \qquad z_2 = b_{\pi/3}$$

$$\left(\frac{z_1}{z_2}\right)^3 = \left(\frac{a_{\pi/4}}{b_{\pi/3}}\right)^3 = \left[\left(\frac{a}{b}\right)_{\pi/4-\pi/3}\right]^3 = \left[\left(\frac{a}{b}\right)_{-\pi/12}\right]^3 = \left(\frac{a}{b}\right)^3_{-3\pi/12} =$$

$$= \left(\frac{a^3}{b^3}\right)_{-\pi/4} = \frac{a^3}{b^3}\left(\cos\frac{-\pi}{4} + i\sin\frac{-\pi}{4}\right) = \frac{a^3}{b^3}\left(\frac{\sqrt2}{2} - i\cdot\frac{\sqrt2}{2}\right) = \boxed{\frac{\sqrt2 a^3}{2b^3} - i\,\frac{\sqrt2 a^3}{2b^3}}$$

N00 P1 #10

$$(1+ki)^2 + k(1+ki) + 5 = 0$$

$$1 + 2ki - k^2 + k + k^2 i + 5 = 0$$

$$(-k^2+k+6) + i(k^2+2k) = 0$$

$$-k^2+k+6 = 0 \implies k = \cancel{3},\ -2$$

$$k^2+2k = 0 \implies k = -2,\ \cancel{0} \implies \boxed{k=-2}$$

M01 P1 #10

Si tiene el factor $z+2i$, tendrá también el factor $z-2i$. La tercera raíz deberá ser un nº real 'r', es decir, el factor $z-r$. Además dispondrá del factor 2, ya que el monomio de mayor exponente lo tiene:

$$2\cdot(z-r)(z+2i)(z-2i) = (2z-2r)(z^2-4i^2) = (2z-2r)(z^2+4) =$$

$$= 2z^3 + 8z - 2rz^2 - 8r = 2z^3 \underbrace{-2rz^2}_{-3} + 8z \underbrace{-8r}_{-12}$$

$$\left.\begin{array}{l} -2r = -3 \\ -8r = -12 \end{array}\right\} r = 3/2 \implies \quad 2z^3 - 3z^2 + 8z - 12 = \boxed{2\left(z-\tfrac{3}{2}\right)(z+2i)(z-2i)}$$

<u>otra forma</u>:

	2	−3	8	−12
2i		4i	−8−6i	12
	2	−3+4i	−6i	0
−2i		−4i	6i	
	2	−3	0	
3/2		3		
	2	0		

N00 P1 #18

$$|z+16| = 4|z+1|$$

$$z = x+iy \to \begin{cases} z+16 = x+16+iy \implies |z+16| = \sqrt{(x+16)^2+y^2} \\ z+1 = x+1+iy \implies |z+1| = \sqrt{(x+1)^2+y^2} \end{cases}$$

$$\sqrt{(x+16)^2+y^2} = 4\sqrt{(x+1)^2+y^2} \ ; \quad (x+16)^2+y^2 = 16\cdot\left((x+1)^2+y^2\right);$$

$$x^2 + \cancel{32x} + 256 + y^2 = 16x^2 + \cancel{32x} + 16 + 16y^2$$

$$240 = 15x^2 + 15y^2$$

$$240 = 15(x^2+y^2)$$

$$16 = x^2+y^2 \implies |z| = \sqrt{x^2+y^2} = \boxed{4}$$

M01
P1 #14

$$z = (b+i)^2 = b^2 + 2bi + i^2 = (b^2-1) + 2bi \Rightarrow \operatorname{tg}\alpha = \frac{2b}{b^2-1}$$

$$\arg z = 60^\circ \Rightarrow \operatorname{tg} 60^\circ = \frac{2b}{b^2-1} \; ; \; \sqrt{3} = \frac{2b}{b^2-1} \; ; \; \sqrt{3}b^2 - \sqrt{3} = 2b \; ;$$

$$\sqrt{3}\, b^2 - 2b - \sqrt{3} = 0 \; ; \quad b = \frac{2 \pm \sqrt{4+12}}{2\sqrt{3}} = \frac{2 \pm 4}{2\sqrt{3}} = \begin{cases} \dfrac{6}{2\sqrt{3}} = \dfrac{3}{\sqrt{3}} = \boxed{\sqrt{3}} \\ \cancel{\dfrac{-2}{2\sqrt{3}}} \text{ No vale porque } b>0 \end{cases}$$

N01
P1#2

$$i(z+2) = 1 - 2z$$

$$iz + 2i = 1 - 2z \; ; \quad z(2+i) = 1 - 2i \; ; \quad z = \frac{1-2i}{2+i} = \frac{(1-2i)(2-i)}{(2+i)(2-i)} = \frac{2 - i - 4i + 2i^2}{4 - \cancel{2i} + \cancel{2i} - i^2} =$$

$$= \frac{\cancel{2} - 5i - \cancel{2}}{4+1} = \boxed{-i}$$

N01
P2#4

a)

	1	0	0	0	0	−1
1		1	1	1	1	1
	1	1	1	1	1	0

$$z^5 - 1 = \boxed{(z-1)(z^4 + z^3 + z^2 + z + 1)}$$

b)

$$z^5 - 1 = 0 \Rightarrow z = \sqrt[5]{1_0} = \left(\sqrt[5]{1}\right)_{\frac{0}{5} + \frac{2\pi k}{5}} = 1_{\frac{2\pi k}{5}} = \boxed{\begin{cases} 1_0 = 1 \\ 1_{2\pi/5} = \cos\frac{2\pi}{5} + i \sin\frac{2\pi}{5} \\ 1_{4\pi/5} = \cos\frac{4\pi}{5} + i \sin\frac{4\pi}{5} \\ 1_{6\pi/5} = 1_{-4\pi/5} = \cos\frac{-4\pi}{5} + i \sin\frac{-4\pi}{5} \\ 1_{8\pi/5} = 1_{-2\pi/5} = \cos\frac{-2\pi}{5} + i \sin\frac{-2\pi}{5} \end{cases}}$$

c)

$$z^5 - 1 = (z-1)\left(z - \cos\frac{2\pi}{5} - i\sin\frac{2\pi}{5}\right)\left(z - \cos\frac{4\pi}{5} - i\sin\frac{4\pi}{5}\right)\left(z - \cos\frac{-4\pi}{5} - i\sin\frac{-4\pi}{5}\right)\left(z - \cos\frac{-2\pi}{5} - i\sin\frac{-2\pi}{5}\right)$$

El desarrollo de $z^4 + z^3 + z^2 + z + 1$ es el producto de los últimos 4 factores:

$$\left(z - \cos\frac{2\pi}{5} - i\sin\frac{2\pi}{5}\right)\left(z - \cos\frac{-2\pi}{5} - i\sin\frac{-2\pi}{5}\right) = \left(z - \cos\frac{2\pi}{5} - i\sin\frac{2\pi}{5}\right)\left(z - \cos\frac{2\pi}{5} + i\sin\frac{2\pi}{5}\right) =$$

$$= \left(z - \cos\frac{2\pi}{5}\right)^2 - \left(i\sin\frac{2\pi}{5}\right)^2 = z^2 - 2z\cos\frac{2\pi}{5} + \cos^2\frac{2\pi}{5} + \sin^2\frac{2\pi}{5} = z^2 - 2z\cos\frac{2\pi}{5} + 1$$

Igualmente, los otros dos factores resultarían: $z^2 - 2z\cos\frac{4\pi}{5} + 1$

Por lo tanto: $z^4 + z^3 + z^2 + z + 1 = \boxed{\left(z^2 - 2z\cos\frac{2\pi}{5} + 1\right)\left(z^2 - 2z\cos\frac{4\pi}{5} + 1\right)} \simeq$

$$\simeq (z^2 - 0'6180z + 1)(z^2 + 1'6180z + 1)$$

M02
P1#3

a) $8i = \boxed{8_{\pi/2}}$

b)

$$z = \sqrt[3]{8i} = \sqrt[3]{8_{\pi/2}} = \left(\sqrt[3]{8}\right)_{\frac{\pi}{6} + \frac{2\pi k}{3}} = \begin{cases} \boxed{2_{\pi/6}} \longrightarrow 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right) = 2\left(\frac{\sqrt{3}}{2} + i\cdot\frac{1}{2}\right) = \boxed{\sqrt{3} + i} \\ 2_{5\pi/6} \\ 2_{7\pi/6} \end{cases}$$

N02
P2#2

a) $\cos\frac{\pi}{4} - i\sin\frac{\pi}{4} = \cos\frac{-\pi}{4} + i\sin\frac{-\pi}{4} = 1_{-\pi/4}$ de la misma forma se obtiene $1_{-\pi/24}$

$$z = \frac{\left(1_{-\pi/4}\right)^2 \left(1_{\pi/3}\right)^3}{\left(1_{-\pi/24}\right)^4} = \frac{1_{-\pi/2} \cdot 1_{\pi}}{1_{-\pi/6}} = 1_{-\pi/2 + \pi + \pi/6} = \boxed{1_{2\pi/3}}$$

b) $\left(1_{2\pi/3}\right)^3 = \left(1^3\right)_{2\pi} = 1_{2\pi} = 1_0 = 1$ ✓

c)

$$(1+2z)(z+z^2) = z + z^2 + 4z + 2z^3 = 2 + \left(1_{2\pi/3}\right)^2 + 4\cdot 1_{2\pi/3} + 2\cdot\left(1_{2\pi/3}\right)^3 = 2 + 1_{4\pi/3} + 4\cdot 1_{2\pi/3} + 2\cdot 1 =$$

$$= 2 + \cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3} + 4\cos\frac{2\pi}{3} + 4i\sin\frac{2\pi}{3} + 2 = \boxed{\frac{3}{2} + i\,\frac{3\sqrt{3}}{2}}$$

M03 P2 #3 / N03 P2#2

$$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta \quad , \quad n\in\mathbb{Z}^+$$

$\underline{n=1}$: $(\cos\theta + i\sin\theta)^1 = \cos(1\theta) + i\sin(1\theta)$

$\cos\theta + i\sin\theta = \cos\theta + i\sin\theta$

$\underline{n=K}$: Suponiendo cierto que: $(\cos\theta+i\sin\theta)^K = \cos K\theta + i\sin K\theta$, Tengo que demostrar que: $(\cos\theta+i\sin\theta)^{K+1} = \cos(K+1)\theta + i\sin(K+1)\theta$

$$(\cos\theta+i\sin\theta)^{K+1} = (\cos\theta+i\sin\theta)^K\cdot(\cos\theta+i\sin\theta) = (\cos K\theta + i\sin K\theta)(\cos\theta+i\sin\theta) =$$

$$= \cos K\theta\cos\theta + i\cos K\theta\sin\theta + i\sin K\theta\cos\theta + i^2\sin K\theta\sin\theta =$$

$$= (\cos K\theta\cos\theta - \sin K\theta\sin\theta) + i(\cos K\theta\sin\theta + \sin K\theta\cos\theta) =$$

$$= \cos(K\theta+\theta) + i\sin(K\theta+\theta) = \cos(K+1)\theta + i\sin(K+1)\theta$$

M03 P1 #11

$$\sqrt{z} = \frac{2}{1-i} + 1-4i = \frac{2(1+i)}{(1-i)(1+i)} + 1-4i = \frac{\cancel{2}(1+i)}{\cancel{2}} + 1-4i = 2-3i$$

$$z = (2-3i)^2 = 4-12i+9i^2 = \boxed{-5-12i}$$

M03 P2#3

i)
$$\frac{1}{z} = \frac{1}{\cos\theta+i\sin\theta} = \frac{1\cdot(\cos\theta-i\sin\theta)}{(\cos\theta+i\sin\theta)(\cos\theta-i\sin\theta)} = \frac{\cos\theta-i\sin\theta}{\cos^2\theta - \cancel{i\cos\theta\sin\theta} + \cancel{i\cos\theta\sin\theta} - i^2\sin^2\theta} =$$

$$= \frac{\cos\theta - i\sin\theta}{\cos^2\theta+\sin^2\theta} = \frac{\cos(-\theta)+i\sin(-\theta)}{1} = \cos(-\theta)+i\sin(-\theta)$$

ii)
$$z^n + z^{-n} = z^n + \left(\frac{1}{z}\right)^n = (1_\theta)^n + (1_{-\theta})^n = 1_{n\theta} + 1_{-n\theta} =$$

$$= \cos n\theta + i\sin n\theta + \cos(-n\theta) + i\sin(-n\theta) =$$

$$= \cos n\theta + \cancel{i\sin n\theta} + \cos n\theta - \cancel{i\sin n\theta} = 2\cos n\theta$$

iii)
$$(z+z^{-1})^5 = z^5 + 5z^4z^{-1} + 10z^3z^{-2} + 10z^2z^{-3} + 5zz^{-4} + z^{-5} =$$

$$= \boxed{z^5 + 5z^3 + 10z + 10z^{-1} + 5z^{-3} + z^{-5}}$$

iv)
$z+z^{-1} = 2\cos\theta$; $z^3+z^{-3} = 2\cos3\theta$; $z^5+z^{-5} = 2\cos5\theta$

$$(2\cos\theta)^5 = 2\cos5\theta + 5\cdot2\cos3\theta + 10\cdot2\cos\theta$$

$$32\cos^5\theta = 2\cos5\theta + 10\cos3\theta + 20\cos\theta$$

$$\cos^5\theta = \frac{1}{32}(2\cos5\theta + 10\cos3\theta + 20\cos\theta) = \boxed{\frac{1}{16}(\cos5\theta + 5\cos3\theta + 10\cos\theta)}$$

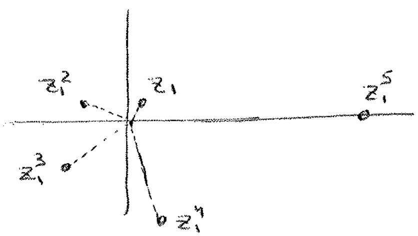
$\boxed{a=1}$, $\boxed{b=5}$, $\boxed{c=10}$

N03 P2#2

i) $z_1 = 2_{2\pi/5}$ $\qquad (2_{2\pi/5})^5 - 32 = (2^5)_{2\pi} - 32 = 32_{2\pi} - 32 = 32-32 = 0$

ii) $z_1^2 = \boxed{4_{4\pi/5}}$; $z_1^3 = \boxed{8_{6\pi/5}}$; $z_1^4 = \boxed{16_{8\pi/5}}$; $z_1^5 = \boxed{32}$

iii)



M04 P1 TZ2 #6

$$z = 1 + \frac{i}{i-\sqrt{3}} = 1 + \frac{i(i+\sqrt{3})}{(i-\sqrt{3})(i+\sqrt{3})} = 1 + \frac{i^2 + i\sqrt{3}}{i^2 - (\sqrt{3})^2} = 1 + \frac{-1+i\sqrt{3}}{-4} =$$

$$= \frac{-4-1+i\sqrt{3}}{-4} = \frac{-5+i\sqrt{3}}{-4} = \boxed{\frac{5}{4} - i\frac{\sqrt{3}}{4}}$$

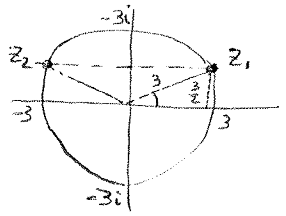
M04 P2 TZ1 #3

a) $|z| = |z-3i| \Rightarrow \sqrt{x^2+y^2} = \sqrt{x^2+(y-3)^2}$; $\cancel{x^2}+\cancel{y^2} = \cancel{x^2}+\cancel{y^2}-6y+9 \Rightarrow y = \frac{9}{6} = \frac{3}{2}$ ✓

b) $z = x + \frac{3}{2}i$

$|z| = 3 \Rightarrow \sqrt{x^2+\frac{9}{4}} = 3$; $x^2+\frac{9}{4} = 9$; $x^2 = \frac{27}{4} \Rightarrow x = \pm\frac{3\sqrt{3}}{2}$ $\boxed{\begin{array}{l} z_1 = \frac{3\sqrt{3}}{2} + \frac{3}{2}i \\ z_2 = -\frac{3\sqrt{3}}{2} + \frac{3}{2}i \end{array}}$

Esta es otra forma de hallarlo.



$\operatorname{sen}\alpha = \frac{3/2}{3} = \frac{1}{2} \Rightarrow \arg z_1 = \frac{\pi}{6}$ ✓

$\arg z_2 = \pi - \frac{\pi}{6} = \boxed{\frac{5\pi}{6}}$

c) $\arg\left(\frac{z_1^k z_2}{2i}\right) = \arg\frac{(3_{\pi/6})^k \cdot 3_{5\pi/6}}{2_{\pi/2}} = \arg\frac{(3^k)_{k\pi/6} \cdot 3_{5\pi/6}}{2_{\pi/2}} = \frac{k\pi}{6} + \frac{5\pi}{6} - \frac{\pi}{2} = \frac{k\pi+2\pi}{6} = \frac{k+2}{6}\pi$

$\frac{k+2}{6}\pi = \pi \Rightarrow \frac{k+2}{6} = 1 \Rightarrow \boxed{k=4}$

N04 P1 #4

$(a+i)(2-bi) = 7-i$

$2a - abi + 2i - bi^2 = 7-i \;\to\; \left.\begin{array}{l} 2a+b=7 \\ 2-ab=-1 \end{array}\right\}$; $\left.\begin{array}{l} 2a+b=7 \\ ab=3 \end{array}\right\} \to b = 7-2a$

$a(7-2a) = 3$; $7a - 2a^2 = 3$; $0 = 2a^2 - 7a + 3$; $a = \frac{7\pm\sqrt{49-24}}{4} = \frac{7\pm5}{4}$ $\begin{array}{l} \boxed{3} \Rightarrow \boxed{b=1} \\ \cancel{1/2} \text{ porque } a \in \mathbb{Z} \end{array}$

N04 P1 #13

$z^3 - 8i = 0 \Rightarrow z = \sqrt[3]{8i} = \sqrt[3]{8_{90°}} = \left(\sqrt[3]{8}\right)_{\frac{90}{3}+\frac{360k}{3}} = 2_{30+120k} =$

$$= \begin{cases} 2_{30} = \boxed{2(\cos 30 + i\sin 30)} = \boxed{\sqrt{3}+i} \\ 2_{150} = \boxed{2(\cos 150 + i\sin 150)} = \boxed{-\sqrt{3}+i} \\ 2_{270} = \boxed{2(\cos 270° + i\sin 270°)} = \boxed{-2i} \end{cases}$$

N04 P2 #1

a) $z^n + \frac{1}{z^n} = z^n + z^{-n} = \cos n\theta + i\sin n\theta + \cos(-n\theta) + i\sin(-n\theta) =$

$= \cos n\theta + \cancel{i\sin n\theta} + \cos n\theta - \cancel{i\sin n\theta} = 2\cos n\theta$ ✓

b) $\left(z+\frac{1}{z}\right)^4 = z^4 + 4z^3\frac{1}{z} + 6z^2\frac{1}{z^2} + 4z\frac{1}{z^3} + \frac{1}{z^4} = z^4 + 4z^2 + 6 + 4\cdot\frac{1}{z^2} + \frac{1}{z^4} =$

$= \left(z^4 + \frac{1}{z^4}\right) + 4\left(z^2 + \frac{1}{z^2}\right) + 6$

$(2\cos\theta)^4 = 2\cos 4\theta + 4\cdot 2\cos 2\theta + 6 \Rightarrow \cos^4\theta = \frac{2\cos 4\theta + 8\cos 2\theta + 6}{16} = \frac{1}{8}(\cos 4\theta + 4\cos 2\theta + 3)$ ✓

M05 P1 TZ2 #7

$(z+2)(z-(-3+2i))(z-(-3-2i)) = (z+2)(z+3-2i)(z+3+2i) = (z+2)((z+3)^2 - 4i^2) =$

$= (z+2)(z^2+6z+9+4) = (z+2)(z^2+6z+13) = z^3 + 6z^2 + 13z + 2z^2 + 12z + 26 =$

$= \boxed{z^3 + 8z^2 + 25z + 26} \to \boxed{a=8,\ b=25,\ c=26}$

M05 P1 TZ2 #11

$|z| = 2\sqrt{5} \Rightarrow z \cdot z^* = (2\sqrt{5})^2 = 20$

$\frac{25}{z} - \frac{15}{z^*} = 1-8i$; $\frac{25z^* - 15z}{z \cdot z^*} = 1-8i$; $\frac{25z^* - 15z}{20} = 1-8i$; $\frac{5z^* - 3z}{4} = 1-8i$;

$5z^* - 3z = 4-32i \longrightarrow 5a - 5bi - 3a - 3bi = 4-32i$; $2a - 8bi = 4-32i$;

$a - 4bi = 2-16i \Rightarrow \boxed{\begin{matrix} a=2 \\ b=4 \end{matrix}}$; $z = 2+4i$

N05 P1 TZ1 #11

$\frac{z}{3+4i} + \frac{z-1}{5i} = \frac{5}{3-4i}$

$\frac{1}{3+4i} = \frac{3-4i}{(3+4i)(3-4i)} = \frac{3-4i}{9+16} = \frac{3-4i}{25}$

$\frac{1}{5i} = \frac{1 \cdot i}{5i \cdot i} = -\frac{i}{5}$

$\frac{5}{3-4i} = \frac{5(3+4i)}{(3-4i)(3+4i)} = \frac{15+20i}{9+16} = \frac{15+20i}{25}$

$\frac{z}{3+4i} + \frac{z-1}{5i} = \frac{5}{3-4i} \Rightarrow \frac{z(3-4i)}{25} - \frac{(z-1)i}{5} = \frac{15+20i}{25}$; $z(3-4i) - 5(z-1)i = 15+20i$;

$z(3-4i) - 5zi + 5i = 15+20i$; $z(3-4i-5i) = 15+15i$; $z = \frac{15+15i}{3-9i}$;

$z = \frac{5+5i}{1-3i} = \frac{(5+5i)(1+3i)}{(1-3i)(1+3i)} = \frac{5+15i+5i-15}{1+9} = \frac{-10+20i}{10} = \boxed{-1+2i}$

N05 P1 #6

$z_1 + z_2 = 3 \longrightarrow \frac{a}{1+i} + \frac{b}{1-2i} = 3$; $a(1-2i) + b(1+i) = 3(1+i)(1-2i)$;

$a - 2ai + b + bi = 3(1-2i+i+2)$; $(a+b) + (b-2a)i = 9-3i \Rightarrow$

$\Rightarrow \left.\begin{matrix} a+b=9 \\ b-2a=-3 \end{matrix}\right\} \longrightarrow \left.\begin{matrix} 2a+2b=18 \\ b-2a=-3 \end{matrix}\right\}$

$3b = 15 \Rightarrow \boxed{b=5} \longrightarrow \boxed{a=4}$

N05 P2 #5

$z_1 \cdot z_2 = (2+i)(3+i) = 6 + 2i + 3i + i^2 = \boxed{5+5i}$

$z_1 = 2+i = (\sqrt{5})_{\text{arcTg}\frac{1}{2}}$

$z_2 = 3+i = (\sqrt{10})_{\text{arcTg}\frac{1}{3}}$

$z_1 \cdot z_2 = 5+5i = (\sqrt{50})_{\text{arcTg}\,1} = (\sqrt{50})_{\pi/4}$

$\Rightarrow \frac{\pi}{4} = \text{arcTg}\frac{1}{2} + \text{arcTg}\frac{1}{3}$ ✓

M06 P1 #2

a) $z_2 = 1+\sqrt{3}i = \boxed{2_{\pi/3}}$

b) $|z_1 \cdot z_2^3| = |r_{\pi/4} \cdot (2_{\pi/3})^3| = |r_{\pi/4} \cdot 8_{\pi}| = |8r_{\pi/4+\pi}| = |8r_{5\pi/4}| = 8r$

$8r = 2 \Rightarrow \boxed{r = \frac{1}{4}}$

M06
P2#2

a) $z^3 = (\cos\theta + i\operatorname{sen}\theta)^3 = \cos^3\theta + 3\cos^2\theta\, i\operatorname{sen}\theta + 3\cos\theta\, i^2\operatorname{sen}^2\theta + i^3\operatorname{sen}^3\theta =$

$= \boxed{(\cos^3\theta - 3\operatorname{sen}^2\theta\cos\theta) + i\,(3\cos^2\theta\operatorname{sen}\theta - \operatorname{sen}^3\theta)}$

$z^3 = (1_\theta)^3 = (1^3)_{3\theta} = \cos 3\theta + i\operatorname{sen} 3\theta$

$\cos 3\theta = \cos^3\theta - 3\operatorname{sen}^2\theta\cos\theta = \cos^3\theta - 3(1-\cos^2\theta)\cos\theta = \cos^3\theta - 3\cos\theta + 3\cos^3\theta =$
$= 4\cos^3\theta - 3\cos\theta$ ✓

$\operatorname{sen} 3\theta = 3\cos^2\theta\operatorname{sen}\theta - \operatorname{sen}^3\theta = 3(1-\operatorname{sen}^2\theta)\operatorname{sen}\theta - \operatorname{sen}^3\theta = 3\operatorname{sen}\theta - 3\operatorname{sen}^3\theta - \operatorname{sen}^3\theta =$
$= 3\operatorname{sen}\theta - 4\operatorname{sen}^3\theta$ ✓

b)

$$\frac{\operatorname{sen}3\theta - \operatorname{sen}\theta}{\cos 3\theta + \cos\theta} = \frac{3\operatorname{sen}\theta - 4\operatorname{sen}^3\theta - \operatorname{sen}\theta}{4\cos^3\theta - 3\cos\theta + \cos\theta} = \frac{2\operatorname{sen}\theta - 4\operatorname{sen}^3\theta}{4\cos^3\theta - 2\cos\theta} = \frac{\operatorname{sen}\theta - 2\operatorname{sen}^3\theta}{2\cos^3\theta - \cos\theta} =$$

$$= \frac{\operatorname{sen}\theta\,(1 - 2\cdot\operatorname{sen}^2\theta)}{\cos\theta\,(2\cos^2\theta - 1)} = tg\,\theta\cdot\frac{1-2(1-\cos^2\theta)}{2\cos^2\theta - 1} = tg\,\theta\cdot\frac{1-2+2\cos^2\theta}{2\cos^2\theta-1} =$$

$$= tg\,\theta\cdot\frac{\cancel{2\cos^2\theta-1}}{\cancel{2\cos^2\theta-1}} = tg\,\theta$$ ✓

c) $\operatorname{sen}\theta = \frac{1}{3} \Rightarrow \cos\theta = \pm\sqrt{1-\frac{1}{9}} = \pm\frac{2\sqrt2}{3} =$ $\frac{2\sqrt2}{3}$; $\cancel{-\frac{2\sqrt2}{3}}$ Porque $-\frac{\pi}{4} < \theta < \pi/4$

$$tg\,3\theta = \frac{\operatorname{sen}3\theta}{\cos 3\theta} = \frac{3\cdot\frac13 - 4\cdot\left(\frac13\right)^3}{4\left(\frac{2\sqrt2}{3}\right)^3 - 3\cdot\frac{2\sqrt2}{3}} = \frac{1-\frac{4}{27}}{\frac{64\sqrt2}{27} - 2\sqrt2} = \frac{27-4}{64\sqrt2 - 54\sqrt2} = \boxed{\frac{23}{10\sqrt2}} = \frac{23\sqrt2}{20}$$

N06
P1#10

$$\begin{cases} 2z_1 + 3z_2 = 7 \\ z_1 + i z_2 = 4+4i \end{cases} \qquad \begin{cases} 2z_1 + 3z_2 = 7 \\ -2z_1 - 2iz_2 = -8-8i \end{cases}$$

$(3-2i)z_2 = -1-8i$

$$z_2 = \frac{-1-8i}{3-2i} = \frac{(-1-8i)(3+2i)}{(3-2i)(3+2i)} = \frac{-3-2i-24i+16}{9+4} = \boxed{1-2i}$$

$2z_1 + 3(1-2i) = 7$; $2z_1 = 7-3+6i$; $z_1 = \boxed{2+3i}$

M07
P2 TZ2
#5

a) $$\frac{u}{v} = \frac{1+\sqrt3 i}{1+i} = \frac{(1+\sqrt3 i)(1-i)}{(1+i)(1-i)} = \frac{1-i+\sqrt3 i+\sqrt3}{1+1} = \frac{\sqrt3+1}{2} + i\,\frac{\sqrt3-1}{2}$$ ✓

$u = 1+\sqrt3 i$ $\begin{cases} m = \sqrt{1+3} = 2 \\ tg\,\alpha = \sqrt3 \to \alpha = \pi/3 \end{cases}$ $u = 2_{\pi/3}$

$v = 1+i$ $\begin{cases} m = \sqrt{1+1} = \sqrt2 \\ tg\,\alpha = \frac{1}{1} \to \alpha = \pi/4 \end{cases}$ $v = (\sqrt2)_{\pi/4}$

$\Rightarrow \frac{u}{v} = \frac{2_{\pi/3}}{(\sqrt2)_{\pi/4}} = \left(\frac{2}{\sqrt2}\right)_{\pi/3-\pi/4} = (\sqrt2)_{\pi/12}$ ✓

$$\frac{u}{v} = \frac{\sqrt3+1}{2} + i\,\frac{\sqrt3-1}{2} = \sqrt2\left(\cos\frac{\pi}{12} + i\operatorname{sen}\frac{\pi}{12}\right) \Rightarrow \begin{cases} \operatorname{sen}\pi/12 = \frac{\sqrt3-1}{2\sqrt2} \\ \cos\pi/12 = \frac{\sqrt3+1}{2\sqrt2} \end{cases} \Rightarrow tg\,\pi/12 = \frac{\sqrt3-1}{\sqrt3+1} = \boxed{2-\sqrt3}$$

b) $\underline{n=1}$ $(1+\sqrt3 i)^1 = 2^1\left(\cos\frac{\pi}{3} + i\operatorname{sen}\pi/3\right)$ ✓

$\underline{n=K}$ Suponiendo cierto que $(1+\sqrt3 i)^K = 2^K\left(\cos\frac{K\pi}{3} + i\operatorname{sen}\frac{K\pi}{3}\right)$, vamos a demostrar que $(1+\sqrt3 i)^{K+1} = 2^{K+1}\left(\cos\frac{(K+1)\pi}{3} + i\operatorname{sen}\frac{(K+1)\pi}{3}\right)$

$(1+\sqrt3 i)^{K+1} = (1+\sqrt3 i)^K(1+\sqrt3 i) = 2^K\left(\cos\frac{K\pi}{3} + i\operatorname{sen}\frac{K\pi}{3}\right)\cdot 2\left(\cos\frac{\pi}{3} + i\operatorname{sen}\frac{\pi}{3}\right) =$

$= 2^{K+1}\left(\cos\frac{K\pi}{3}\cos\frac{\pi}{3} + \cos\frac{K\pi}{3}\, i\operatorname{sen}\frac{\pi}{3} + i\operatorname{sen}\frac{K\pi}{3}\cos\frac{\pi}{3} - \operatorname{sen}\frac{K\pi}{3}\operatorname{sen}\frac{\pi}{3}\right) =$

$= 2^{K+1}\left(\cos\left(\frac{K\pi}{3}+\frac{\pi}{3}\right) + i\operatorname{sen}\left(\frac{K\pi}{3}+\frac{\pi}{3}\right)\right) = 2^{K+1}\left(\cos\frac{(K+1)\pi}{3} + i\operatorname{sen}\frac{(K+1)\pi}{3}\right)$ ✓

c) $z = \dfrac{\sqrt{2}\,v+u}{\sqrt{2}\,v-u} = \dfrac{\sqrt{2}\,(1+i)+1+\sqrt{3}i}{\sqrt{2}\,(1+i)-1-\sqrt{3}i} = \dfrac{(\sqrt{2}+1)+i\,(\sqrt{2}+\sqrt{3})}{(\sqrt{2}-1)+i\,(\sqrt{2}-\sqrt{3})} = \dfrac{\left[(\sqrt{2}+1)+i\,(\sqrt{2}+\sqrt{3})\right]\left[(\sqrt{2}-1)-i\,(\sqrt{2}-\sqrt{3})\right]}{\left[(\sqrt{2}-1)+i\,(\sqrt{2}-\sqrt{3})\right]\left[(\sqrt{2}-1)-i\,(\sqrt{2}-\sqrt{3})\right]} =$

$$= \frac{(\sqrt{2}+1)(\sqrt{2}-1) - i\,(\sqrt{2}+1)(\sqrt{2}-\sqrt{3}) + i\,(\sqrt{2}+\sqrt{3})(\sqrt{2}-1) + (\sqrt{2}+\sqrt{3})(\sqrt{2}-\sqrt{3})}{(\sqrt{2}-1)^2+(\sqrt{2}-\sqrt{3})^2} \Rightarrow$$

$$\Rightarrow \operatorname{Re}[z] = \frac{(\sqrt{2}+1)(\sqrt{2}-1)+(\sqrt{2}+\sqrt{3})(\sqrt{2}-\sqrt{3})}{(\sqrt{2}-1)^2+(\sqrt{2}-\sqrt{3})^2} = \frac{2-1+2-3}{2-2\sqrt{2}+1+2-2\sqrt{6}+3} = \frac{0}{8-2\sqrt{2}-2\sqrt{6}} = 0\ ✓$$

M07
P2 TZ1
#5

a)
$$\begin{array}{r|rrrr} & 1 & -3 & -3 & 1 \\ -1 & & -1 & 4 & -1 \\ \hline & 1 & -4 & 1 & 0 \end{array} \longrightarrow t^3-3t^2-3t+1 = \boxed{(t+1)(t^2-4t+1)}$$

$$t^2-4t+1=0 \longrightarrow t = \frac{4\pm\sqrt{16-4}}{2} = \frac{4\pm\sqrt{12}}{2} = \frac{4\pm2\sqrt{3}}{2} = 2\pm\sqrt{3}$$

$\boxed{\text{Soluciones}:\ -1,\ 2\pm\sqrt{3}}$

b)
$$(1_\theta)^3 = 1_{3\theta} = \cos 3\theta + i\sin 3\theta$$
$$(1_\theta)^3 = (\cos\theta+i\sin\theta)^3 = \cos^3\theta+3\cos^2\theta\, i\sin\theta+3\cos\theta\, i^2\sin^2\theta+i^3\sin^3\theta =$$
$$= (\cos^3\theta-3\cos\theta\sin^2\theta)+i\,(3\cos^2\theta\sin\theta-\sin^3\theta)$$

$$\Rightarrow \quad \cos 3\theta = \cos^3\theta - 3\cos\theta\sin^2\theta \quad ✓$$
$$\boxed{\sin 3\theta = 3\cos^2\theta\sin\theta - \sin^3\theta}$$

c)
$$\operatorname{Tg} 3\theta = \frac{3\cos^2\theta\sin\theta-\sin^3\theta}{\cos^3\theta-3\cos\theta\sin^2\theta} = \frac{\dfrac{3\cos^2\theta\sin\theta}{\cos^3\theta}-\dfrac{\sin^3\theta}{\cos^3\theta}}{\dfrac{\cos^3\theta}{\cos^3\theta}-\dfrac{3\cos\theta\sin^2\theta}{\cos^3\theta}} = \frac{3\operatorname{Tg}\theta-\operatorname{Tg}^3\theta}{1-3\operatorname{Tg}^2\theta} \quad ✓$$

No es válida si se anula el denominador de $\operatorname{Tg}3\theta = \dfrac{\sin 3\theta}{\cos 3\theta}$

$\cos 3\theta = 0 \longrightarrow 3\theta = 90° + N\cdot 360° \Rightarrow \theta = 30° + N\cdot 120°$: $\boxed{\theta = \begin{cases} 30° \\ 150° \end{cases}}$

Y tampoco sería válida si no existe $\operatorname{tg}\theta$: $\boxed{\theta = 90°}$ Siendo $0° \le \theta \le 180°$

d)
$$\theta = 15° \longrightarrow \operatorname{Tg}(3\cdot15°) = \frac{3\operatorname{Tg}15° - \operatorname{Tg}^3 15°}{1-3\operatorname{Tg}^2 15°}\ ;\ \operatorname{Tg}15° = t \longrightarrow 1 = \frac{3t-t^3}{1-3t^2}\ ;$$

$$1-3t^2 = 3t-t^3\ ;\quad t^3-3t^2-3t+1=0 \Rightarrow t = \begin{cases} -1 \\ 2+\sqrt{3} \\ \boxed{2-\sqrt{3}} \end{cases} \longleftarrow \boxed{\operatorname{Tg}15° = 2-\sqrt{3}}$$

$$\operatorname{Tg}75° = \operatorname{tg}(90-15°) = \frac{1}{\operatorname{tg}15°} = \frac{1}{2-\sqrt{3}} = \frac{2+\sqrt{3}}{(2-\sqrt{3})(2+\sqrt{3})} = \frac{2+\sqrt{3}}{4-3} = \boxed{2+\sqrt{3}}$$

M07
P1 TZ2
#11

$$(z+1+i)(z+1-i) = (z+1)^2 - i^2 = z^2+2z+1+1 = z^2+2z+2$$

$$P(z) = z^3+mz^2+nz-8 = (z^2+2z+2)(z-r)$$

Debe ser $r=4$, para que resulte -8

$$(z^2+2z+2)(z-4) = z^3-4z^2+2z^2-8z+2z-8 = z^3-2z^2-6z-8$$

$\boxed{m=-2}$

$\boxed{n=-6}$

M07 P1 T21 #7

a) $z = 4\cdot\left(-\frac{1}{2} + i\frac{\sqrt3}{2}\right) + 4\sqrt3\left(\frac{\sqrt3}{2} + i\cdot\frac{1}{2}\right) = (-2+6) + i\,(2\sqrt3+2\sqrt3) = \boxed{4+4i\sqrt3}$

$z = 4+4i\sqrt3 = \begin{cases} m=\sqrt{16+48}=8 \\ \operatorname{tg}\alpha = \frac{4\sqrt3}{4} \to \alpha=\frac{\pi}{3}\end{cases} = 8_{\pi/3} = \boxed{8e^{i\pi/3}}$

b) $\sqrt[3]{8_{60^\circ}} = \left(\sqrt[3]{8}\right)_{20^\circ+120k} = \begin{cases} 2_{20^\circ} = \boxed{2e^{i\pi/9}}\\ 2_{140^\circ} = \boxed{2e^{i7\pi/9}} \\ 2_{260^\circ} = \boxed{2e^{i13\pi/9}}\end{cases}$

N07 P1 #19

$\sqrt3+i = \begin{cases} m=\sqrt{3+1}=2\\ \operatorname{tg}\alpha=\frac{1}{\sqrt3}\to\alpha=\pi/6\end{cases}$; $\sqrt3+i = 2_{\pi/6}$

$\sqrt3-i = \begin{cases} m=\sqrt{3+1}=2 \\ \operatorname{tg}\alpha = \frac{-1}{\sqrt3}\to \alpha=-\pi/6\end{cases}$; $\sqrt3-i=2_{-\pi/6}$

$(\sqrt3+i)^n + (\sqrt3-i)^n = (2_{\pi/6})^n + (2_{-\pi/6})^n = (2^n)_{n\pi/6} + (2^n)_{-n\pi/6} =$

$= 2^n\left(\cos\frac{n\pi}{6} + i\sin\frac{n\pi}{6}\right) + 2^n\left(\cos\frac{-n\pi}{6} + i \sin\frac{-n\pi}{6}\right) =$

$= 2^n\left(\cos\frac{n\pi}{6} + \cancel{i\sin\frac{n\pi}{6}}\right) + 2^n\left(\cos\frac{n\pi}{6} - \cancel{i\sin\frac{n\pi}{6}}\right) = 2\cdot2^n\cos\frac{n\pi}{6} = 2^{n+1}\cos\frac{n\pi}{6}\in\mathbb{R}$ ✓

N07 P2 #28

a) $1+i = \begin{cases} m=\sqrt{1+1}=\sqrt2\\ \operatorname{tg}\alpha=\frac{1}{1}\to\alpha=\pi/4\end{cases}$; $1+i=\boxed{\sqrt2\, e^{i\pi/4}}$ $\boxed{\begin{matrix}a=2\\ b=4\end{matrix}}$

b) $\left(\frac{1+i}{\sqrt2}\right)^n = \frac{(1+i)^n}{(\sqrt2)^n} = \frac{(\sqrt2_{\pi/4})^n}{(\sqrt2)^n} = \frac{(\sqrt2)^n_{n\pi/4}}{(\sqrt2)^n} = 1_{n\pi/4} = \begin{cases} 1_0 \longleftarrow n=0,8,16,\dots \\ 1_{\pi/4} \longleftarrow n=1,9,\dots \\ 1_{\pi/2} \longleftarrow n=2,10,\dots \\ 1_{3\pi/4} \longleftarrow n=3,11,\dots \\ 1_{\pi} \longleftarrow n=4,12,\dots \\ 1_{5\pi/4} \longleftarrow n=5,13,\dots \\ 1_{3\pi/2} \longleftarrow n=6,14,\dots \\ 1_{7\pi/4} \longleftarrow n=7,15,\dots \end{cases}$

c) $z^8-1=0 \Rightarrow z=\sqrt[8]{1} = 1_{\frac{k2\pi}{8}} = 1_{\frac{k\pi}{4}} =$ Las 8 soluciones anteriores

Muestra 08 P1 #4

$(a+bi)^2 = 3+4i$

$a^2+2abi+b^2i^2 = 3+4i$

$a^2-b^2+2abi = 3+4i \longrightarrow \begin{cases}a^2-b^2=3\\ 2ab=4\to b=\frac{2}{a}\end{cases} \longrightarrow a^2-\frac{4}{a^2}=3$; $a^4-4=3a^2$;

$a^4-3a^2-4=0$; $a^2=\frac{3\pm\sqrt{9+16}}{2}=\frac{3\pm5}{2}$ $\begin{cases} 4 \to \boxed{a = \begin{cases} 2 \to b=1 \\ -2 \to b=-1\end{cases}} \begin{matrix}\to 2+i \\ \to 2-i\end{matrix} \\ \cancel{-1}\end{cases}$

$\sqrt{3+4i} = a+bi = \boxed{\begin{cases}2+i\\ 2-i\end{cases}}$

Muestra 08 P1 #5

$(z-(2+i))\,(z-(2-i)) = (z-2-i)(z-2+i) = (z-2)^2-i^2 = z^2-4z+5$

$z^3-6z^2+13z-10 = (z^2-4z+5)(z-a)$

Tiene que ser $a=2$, para que resulte -10

$(z^2-4z+5)(z-2) = z^3-2z^2-4z^2+8z+5z-10 = \boxed{z^3-6z^2+13z-10}$ ✓

Las raíces son: $\boxed{2+i,\ 2-i,\ 2}$

Muestra 08
P1 #6

$|z| = \sqrt{10} \Rightarrow z \cdot z^* = 10$

$5z + \frac{10}{z^*} = 6 - 18i \; ; \quad 5zz^* + 10 = (6-18i)z^* \; ; \quad 50 + 10 = (6-18i) \cdot z^* \; ;$

$60 = (6-18i)z^* \; ; \quad z^* = \frac{60}{6-18i} = \frac{10}{1-3i} = \frac{10(1+3i)}{(1-3i)(1+3i)} = \frac{10(1+3i)}{1+9} = 1+3i \Rightarrow$

$\Rightarrow \boxed{z = 1-3i}$

Muestra 08
P1 #7

$$\sqrt[3]{8i} = \sqrt[3]{8_{90^\circ}} = \left(\sqrt[3]{8}\right)_{\frac{90}{3} + \frac{k \cdot 360}{3}} = 2_{30 + 120k} = \begin{cases} 2_{30} = 2 \cdot \left(\frac{\sqrt{3}}{2} + i \cdot \frac{1}{2}\right) = \boxed{\sqrt{3} + i} \\ 2_{150} = 2 \cdot \left(\frac{\sqrt{3}}{2} - i \frac{1}{2}\right) = \boxed{\sqrt{3} - i} \\ 2_{270} = \boxed{-2i} \end{cases}$$

Muestra 08
P1 #8

$$\left.\begin{array}{l} iz_1 + 2z_2 = 3 \\ z_1 + (1-i)z_2 = 4 \end{array}\right\} \quad \left.\begin{array}{l} iz_1 + 2z_2 = 3 \\ -iz_1 - i(1-i)z_2 = -4i \end{array}\right\}$$

$(2 - i + i^2)z_2 = 3 - 4i$

$(1-i)z_2 = 3 - 4i \; ; \quad z_2 = \frac{3-4i}{1-i} = \frac{(3-4i)(1+i)}{(1-i)(1+i)} = \frac{3+3i-4i+4}{1+1} =$

$= \frac{7-i}{2} = \boxed{\frac{7}{2} - i\frac{1}{2}}$

$z_1 = 4 - (1-i)z_2 = 4 - (1-i)\left(\frac{7}{2} - \frac{i}{2}\right) = 4 - \frac{7}{2} + \frac{i}{2} + \frac{7i}{2} - \frac{i^2}{2} = 4 - \frac{7}{2} + \frac{1}{2} + \frac{8i}{2} = \boxed{1+4i}$

Muestra 08
P1 #9

$\frac{2+bi}{1-bi} = \frac{-7+9i}{10} \; ; \quad 20 + 10bi = -7 + 7bi + 9i - 9bi^2$

$\left.\begin{array}{l} 20 = -7 + 9b \\ 10b = 7b + 9 \end{array}\right\} \rightarrow \boxed{b = 3}$

Muestra 06
P2 #5
Muestra 08
P1 #44B

a) $e^{i\theta} + \frac{1}{2}e^{2i\theta} + \frac{1}{4}e^{3i\theta} + \cdots \qquad \boxed{\text{razón} = \frac{1}{2}e^{i\theta}}$

b) $\left|\frac{1}{2}e^{i\theta}\right| = \frac{1}{2} < 1$ ✓

c) $S_\infty = \frac{e^{i\theta}}{1 - \frac{1}{2}e^{i\theta}} = \boxed{\frac{2e^{i\theta}}{2 - e^{i\theta}}}$

d) $S_\infty = \frac{2(\cos\theta + i\sin\theta)}{2 - (\cos\theta + i\sin\theta)} = \frac{2\cos\theta + 2i\sin\theta}{(2-\cos\theta) - i\sin\theta} = \frac{(2\cos\theta + 2i\sin\theta)((2-\cos\theta) + i\sin\theta)}{((2-\cos\theta) - i\sin\theta)((2-\cos\theta) + i\sin\theta)} =$

$= \frac{4\cos\theta - 2\cos^2\theta + \cancel{2i\cos\theta\sin\theta} + 4i\sin\theta - \cancel{2i\sin\theta\cos\theta} + 2i^2\sin^2\theta}{(2-\cos\theta)^2 + \sin^2\theta} =$

$= \frac{(4\cos\theta - 2\cos^2\theta - 2\sin^2\theta) + 4i\sin\theta}{4 - 4\cos\theta + \cos^2\theta + \sin^2\theta} = \frac{(4\cos\theta - 2) + 4i\sin\theta}{5 - 4\cos\theta} = \boxed{\frac{4\cos\theta - 2}{5 - 4\cos\theta} + i\frac{4\sin\theta}{5-4\cos\theta}}$

$e^{i\theta} + \frac{1}{2}e^{2i\theta} + \frac{1}{4}e^{3i\theta} + \cdots = \left(\cos\theta + \frac{1}{2}\cos 2\theta + \frac{1}{4}\cos 3\theta + \cdots\right) + i\left(\sin\theta + \frac{1}{2}\sin 2\theta + \cdots\right)$

$\Rightarrow \boxed{\cos\theta + \frac{1}{2}\cos 2\theta + \frac{1}{4}\cos 3\theta + \cdots = \frac{4\cos\theta - 2}{5 - 4\cos\theta}}$ ✓

M08
P2 T21
#10

$(\operatorname{sen}\theta + i(1-\cos\theta))^2 = \sin^2\theta + 2i\sin\theta(1-\cos\theta) - (1-\cos\theta)^2 = (\underbrace{\sin^2\theta - 1}_{-\cos^2\theta} + 2\cos\theta - \cos^2\theta) + 2i\sin\theta(1-\cos\theta) =$

$= (2\cos\theta - 2\cos^2\theta) + 2i\sin\theta(1-\cos\theta) = 2\cos\theta(1-\cos\theta) + 2i\sin\theta(1-\cos\theta)$

Si llamamos α al argumento:

$\operatorname{tg}\alpha = \frac{\cancel{2}\sin\theta\,(1-\cancel{\cos\theta})}{\cancel{2}\cos\theta\,(1-\cancel{\cos\theta})} = \operatorname{tg}\theta \rightarrow \boxed{\alpha = \theta}$

Muestra 08 P1 #45

a) $z^2+2z+4=0$

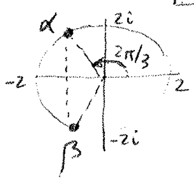
$$z = \frac{-2 \pm \sqrt{4-16}}{2} = \frac{-2 \pm 2\sqrt{3}i}{2} = -1 \pm \sqrt{3}i =$$

$\nearrow -1+\sqrt{3}i \left\{ \begin{array}{l} m = \sqrt{1+3} = 2 \\ tg\,\alpha = -\sqrt{3} \rightarrow \alpha = 120° = \frac{2\pi}{3} \end{array} \right.$

$\searrow -1-\sqrt{3}i \left\{ \begin{array}{l} m = \sqrt{1+3} = 2 \\ tg\,\alpha = +\sqrt{3} \rightarrow \alpha = 240° = \frac{4\pi}{3} \end{array} \right.$

$$\boxed{\alpha = 2e^{\frac{2\pi}{3}i}} \qquad \boxed{\beta = 2e^{\frac{4\pi}{3}i}}$$

b)



c) (Demostrado en M03 P2 #3)

d)
$$\frac{\alpha^3}{\beta^2} = \frac{\left(2e^{\frac{2\pi}{3}i}\right)^3}{\left(2e^{\frac{4\pi}{3}i}\right)^2} = \frac{8e^{2\pi i}}{4e^{\frac{8\pi}{3}i}} = \frac{2}{e^{\frac{8\pi}{3}i}} = 2e^{-\frac{8\pi}{3}i} =$$

$$= 2\left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right) = \boxed{-1-i\sqrt{3}}$$

e)
$$\alpha^3 = \left(2e^{\frac{2\pi}{3}i}\right)^3 = 8e^{2\pi i} = 8 \quad ✓$$
$$\beta^3 = \left(2e^{\frac{4\pi}{3}i}\right)^3 = 8e^{4\pi i} = 8$$

f)
$$\alpha\beta^* + \beta\alpha^* = (-1+\sqrt{3}i)(-1+\sqrt{3}i) + (-1-\sqrt{3}i)(-1-\sqrt{3}i) =$$
$$= (-1+i\sqrt{3})^2 + (-1-i\sqrt{3})^2 = 1 \cancel{-2i\sqrt{3}} + i^2 3 + 1 \cancel{+2i\sqrt{3}} + i^2 \cdot 3 = 2-6 = \boxed{-4}$$

g)
$$\alpha^n = \left(2_{2\pi/3}\right)^n = \left(2^n\right)_{\frac{2\pi n}{3}}$$

Si es real, el argumento debe ser $0 + K\cdot\pi$ $(K=0,\pm1,\pm2,\dots)$

$$\frac{2\pi n}{3} = K\cancel{\pi} \Rightarrow \boxed{n = \frac{3K}{2} \quad (K=0,\pm1,\pm2,\dots)}$$ Como $n \in \mathbb{Z}^+ \Rightarrow \boxed{n = 3K,\ K \in \mathbb{Z}^+}$

M08 P1 TZ2 #14

$w = 1_{2\pi/5}$

a) $\left(1_{2\pi/5}\right)^5 - 1 = \left(1^5\right)_{2\pi} - 1 = 1_{2\pi} - 1 = 1-1 = 0$ ✓

b)

	1	0	0	0	-1
1		1	1	1	1
	1	1	1	1	0

$\longrightarrow (z-1)(z^4+z^3+z^2+z+1) = z^5-1$

$w^5-1=0 \Rightarrow (w-1)(w^4+w^3+w^2+w+1)=0$ $\nearrow$ $\cancel{w=1}$ porque $w\neq 1$

$\searrow$ $w^4+w^3+w^2+w+1=0$ ✓

c)
$$\left(1_{2\pi/5}\right)^4 + \left(1_{2\pi/5}\right)^3 + \left(1_{2\pi/5}\right)^2 + 1_{2\pi/5} + 1 = 0$$
$$1_{8\pi/5} + 1_{6\pi/5} + 1_{4\pi/5} + 1_{2\pi/5} + 1 = 0$$
$$\left(\cos\frac{8\pi}{5} + \cos\frac{6\pi}{5} + \cos\frac{4\pi}{5} + \cos\frac{2\pi}{5} + 1\right) + i\left(\sin\frac{8\pi}{5} + \sin\frac{6\pi}{5} + \sin\frac{4\pi}{5} + \sin\frac{2\pi}{5}\right) = 0$$
$$\cos\frac{8\pi}{5} + \cos\frac{6\pi}{5} + \cos\frac{4\pi}{5} + \cos\frac{2\pi}{5} + 1 = 0$$
$$\cos\frac{2\pi}{5} + \cos\frac{4\pi}{5} + \cos\frac{4\pi}{5} + \cos\frac{2\pi}{5} = -1$$
$$2\left(\cos\frac{2\pi}{5} + \cos\frac{4\pi}{5}\right) = -1 \Rightarrow \cos\frac{2\pi}{5} + \cos\frac{4\pi}{5} = -\frac{1}{2} \quad ✓$$

M08 P1 TZ1 #1

$$1-i\sqrt{3} = \begin{cases} m=\sqrt{1+3}=2 \\ tg\,\alpha = -\sqrt{3} \longrightarrow \alpha = -60^\circ \end{cases}$$

$$\frac{1}{(1-i\sqrt{3})^3} = \frac{1}{\left(2_{-60^\circ}\right)^3} = \frac{1}{8_{-180^\circ}} = \left(\frac{1}{8}\right)_{180^\circ} = \boxed{-\frac{1}{8}}$$

M08 P2 TZ1 #14

$$1+i\sqrt{3} = \begin{cases} m=\sqrt{1+3}=2 \\ tg\,\alpha=\sqrt{3} \longrightarrow \alpha = 60^\circ \end{cases} \qquad 1-i = \begin{cases} m=\sqrt{1+1}=\sqrt{2} \\ tg\,\alpha=-1 \longrightarrow \alpha=-45^\circ = 315^\circ \end{cases}$$

$$(1+i\sqrt{3})^m = \left(2_{60^\circ}\right)^m = \boxed{\left(2^m\right)_{60m}}$$

$$(1-i)^n = \left(\sqrt{2}_{315}\right)^n = \boxed{\left(\sqrt{2^n}\right)_{315n}}$$

$$z_1 = z_2 \Rightarrow \left(2^m\right)_{60m} = \left(\sqrt{2^n}\right)_{315n} \longrightarrow 2^m = \sqrt{2^n} \Rightarrow m = \frac{n}{2}$$

$$\longrightarrow 60m - 315n = K\cdot 360 \quad (K = 0, \pm1, \pm2, \ldots)$$

$$n = 2m \longrightarrow 60m - 315\cdot 2m = K\cdot 360$$

$$-570m = K\cdot 360$$

$$-19m = 12K \longrightarrow \boxed{m=12} \text{ con } K=-19 \longrightarrow \boxed{n=24}$$

M08 P2 TZ2 #9

$$w = \frac{z}{z^2+1} = \frac{x+iy}{(x+iy)^2+1} = \frac{x+iy}{x^2+2xiy+i^2y^2+1} = \frac{x+iy}{(x^2-y^2+1)+i\,2xy} =$$

$$= \frac{(x+iy)\left((x^2-y^2+1)-i\,2xy\right)}{\left[(x^2-y^2+1)+i\,2xy\right]\left[(x^2-y^2+1)-i\,2xy\right]} = \frac{x(x^2-y^2+1)-2x^2yi+iy(x^2-y^2+1)+2xy^2}{(x^2-y^2+1)^2+4x^2y^2} =$$

$$= \frac{x(x-y^2+1+2y^2)}{(x^2-y^2+1)^2+4x^2y^2} + i\,\frac{-2x^2y+yx^2-y^3+y}{(x^2-y^2+1)^2+4x^2y^2}$$

$$\mathrm{Im}\, w = 0 \Rightarrow -2x^2y+yx^2-y^3+y=0 \;;\; y(-x^2-y^2+1)=0 \to \cancel{y=0}$$

$$\to x^2+y^2=1 \Rightarrow |z|=1 \;\checkmark$$

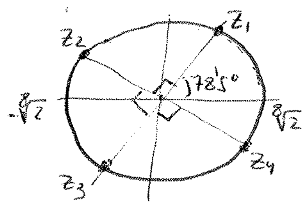
N08 P1 #13A

a) $z^4 = 1-i \longrightarrow z = \sqrt[4]{1-i}$

$$1-i = \begin{cases} m=\sqrt{1+1}=\sqrt{2} \\ tg\,\alpha=-1 \Rightarrow \alpha=315^\circ \end{cases}$$

$$z = \sqrt[4]{1-i} = \sqrt[4]{(\sqrt{2})_{315^\circ}} = \left(\sqrt[4]{\sqrt{2}}\right)_{\frac{315}{4}+\frac{K\cdot 360}{4}} = \left(\sqrt[8]{2}\right)_{78'75^\circ+90K} = \boxed{\begin{cases} \sqrt[8]{2}_{78'75^\circ} \\ \sqrt[8]{2}_{168'75^\circ} \\ \sqrt[8]{2}_{258'75^\circ} \\ \sqrt[8]{2}_{348'75^\circ} \end{cases}}$$

b)



c) $z_1 = \sqrt[8]{2}_{78'75^\circ}$, $z_2 = \sqrt[8]{2}_{168'75}$

$$\frac{z_2}{z_1} = \frac{\sqrt[8]{2}_{168'75}}{\sqrt[8]{2}_{78'75}} = 1_{90} = \boxed{i}$$

N08
P1 #13B

a) $(x-1)(x^4+x^3+x^2+x+1) = x^5+\cancel{x^4}+\cancel{x^3}+\cancel{x^2}+\cancel{x}-\cancel{x^4}-\cancel{x^3}-\cancel{x^2}-\cancel{x}-1 = \boxed{x^5-1}$

b) $b^5-1=0 \Rightarrow (b-1)(b^4+b^3+b^2+b+1)=0 \to \cancel{b=1}$ porque $b\notin\mathbb{R}$

$\to b^4+b^3+b^2+b+1=0$ ✓

c) $u+v = b+b^4+b^2+b^3 = -1$ ✓

$u\cdot v = (b+b^4)(b^2+b^3) = b^3+b^4+b^6+b^7 =$

$= -b^5 = -1$ ✓

porque $b^5-1=0$

$b^4+b^3+b^2+b+1=0 \Rightarrow$

$\Rightarrow b^3(b^4+b^3+b^2+b+1)=0 \Rightarrow$

$\Rightarrow b^7+b^6+b^5+b^4+b^3=0 \Rightarrow$

$b^7+b^6+b^4+b^3=-b^5$

$(u+v)=-1 \Rightarrow (u+v)^2=1 \to u^2+2uv+v^2=1$; $u^2+v^2=1-2uv=1-2\cdot(-1)=3$

$(u-v)^2=u^2+v^2-2uv=3-2\cdot(-1)=5 \Rightarrow u-v = \sqrt{5}$ porque $u-v>0$; $\cancel{-\sqrt{5}}$

M09
P1 TZ1
#1

a) $|w|=\sqrt{4+a^2}$; $|z|=\sqrt{1+4}=\sqrt{5}$

$|w|=2|z| \Rightarrow \sqrt{4+a^2}=2\sqrt{5}$; $4+a^2=4\cdot5$; $a^2=16$; $\boxed{a=\pm4}$

b) $z\cdot w=(1+2i)(2+ai)=2+ai+4i+2ai^2=(2-2a)+i(a+4)$

$Re(z\cdot w)=2\,Im(z\cdot w) \Rightarrow 2-2a=2(a+4)$; $2-2a=2a+8$; $-4a=6$; $\boxed{a=-\frac{3}{2}}$

M09
P1 TZ1
#13

a) $z\in\mathbb{R}^+ \Rightarrow L(z)=\ln|z|+i\arg(z)=\ln z+i\cdot0=\ln z$ ✓

b) $L(-1)=L(1_{\pi})=\ln|1|+i\cdot\pi=\boxed{i\pi}$

$1-i = \begin{cases} m=\sqrt{1+1}=\sqrt{2} \\ \tan\alpha=-1 \to \alpha=\frac{7\pi}{4} \end{cases}$ $\quad L(1-i)=\ln\sqrt{2}+i\frac{7\pi}{4}=\boxed{\frac{1}{2}\ln2+i\frac{7\pi}{4}}=\boxed{\ln\sqrt{2}+i\frac{7\pi}{4}}$

$-1+i = \begin{cases} m=\sqrt{1+1}=\sqrt{2} \\ \tan\alpha=-1 \to \alpha=\frac{3\pi}{4} \end{cases}$ $\quad L(-1+i)=\ln\sqrt{2}+i\frac{3\pi}{4}=\boxed{\frac{1}{2}\ln2+i\frac{3\pi}{4}}=\boxed{\ln\sqrt{2}+i\frac{3\pi}{4}}$

c) $L(z_1)+\ln(z_2)=\ln|z_1|+i\arg(z_1)+\ln|z_2|+i\arg(z_2)=$

$=\ln|z_1|\cdot|z_2|+i(\arg(z_1)+\arg(z_2))=$

$=\ln|z_1\cdot z_2|+i(\arg(z_1)+\arg(z_2))$

$\ln(z_1\cdot z_2)=\ln|z_1 z_2|+i\arg(z_1\cdot z_2)$

La diferencia estará en aquellos casos en que $\arg(z_1)+\arg(z_2)$ sea mayor de 360°, ya que, por la definición de $L(z)$, se debe utilizar el argumento principal.

Por ejemplo:

No se cumple con: $z_1=2_{7\pi/4}$, $z_2=3_{5\pi/6}$ →

$L(z_1)+L(z_2)=\ln2+i\frac{7\pi}{4}+\ln3+i\cdot\frac{5\pi}{6}=$
$=\ln6+i\frac{31\pi}{12}$

$L(z_1\cdot z_2)=L(2_{7\pi/4}\cdot3_{5\pi/6})=L(6_{31\pi/12})=$
$=L(6_{7\pi/12})=\ln6+i\frac{7\pi}{12}$ ✓

Sí se cumple con: $z_1=2_{3\pi/4}$, $z_2=3_{5\pi/6}$ porque $\frac{3\pi}{4}+\frac{5\pi}{6}=\frac{19\pi}{12}<2\pi$.

M09 P1
TZ2 #7

a) $z_1 = 2$

$z_2 = 1+\sqrt{3}i \longrightarrow \boxed{z_3 = 1-\sqrt{3}i}$

b) $(z-2)\left(z-(1+\sqrt{3}i)\right)\left(z-(1-\sqrt{3}i)\right) = (z-2)(z-1-\sqrt{3}i)(z-1+\sqrt{3}i) = (z-2)\left((z-1)^2+3\right) =$

$= (z-2)(z^2-2z+1+3) = (z-2)(z^2-2z+4) = z^3-2z^2+4z-2z^2+4z-8 =$

$= \boxed{z^3-4z^2+8z-8}$ $\qquad \boxed{\begin{array}{l} b=-4 \\ c=8 \\ d=-8 \end{array}}$

c) $z_2 = 1+\sqrt{3}i : \begin{cases} m = \sqrt{1+3} = 2 \\ \operatorname{tg}\alpha = \sqrt{3} \rightarrow \alpha = \pi/3 \end{cases}$ $\qquad \boxed{z_2 = 2e^{i\pi/3}}$

$z_3 = 1-\sqrt{3}i : \begin{cases} m = \sqrt{1+3} = 2 \\ \operatorname{tg}\alpha = -\sqrt{3} \rightarrow \alpha = \frac{5\pi}{3} \end{cases}$ $\qquad \boxed{z_3 = 2e^{i5\pi/3}}$

M09 P2
TZ1 #2

	1	−5	7	−5	6
i		i	$-5i-1$	$5+6i$	-6
	1	$-5+i$	$-5i+6$	$6i$	0
$-i$		$-i$	$5i$	$-6i$	
	1	−5	6	0	

$z^2-5z+6=0 \; ; \quad z = \frac{5\pm\sqrt{25-24}}{2} = \frac{5\pm1}{2} \begin{cases} 3 \\ 2 \end{cases}$

$\boxed{\text{Raíces}: \; i, -i, 3, 2}$

N09
P1 #2

$(1+\sqrt{3}i)^n \in \mathbb{R}$

$1+\sqrt{3}i = \begin{cases} m = \sqrt{1+3} = 2 \\ \operatorname{tg}\alpha = \sqrt{3} \rightarrow \alpha = \pi/3 \end{cases}$

$(1+\sqrt{3}i)^n = \left(2_{\pi/3}\right)^n = \left(2^n\right)_{n\pi/3} \in \mathbb{R} \Rightarrow \frac{n\pi}{3} = \pi k \quad (k = 0, \pm1, \pm2, \ldots)$

$\boxed{n = 3k}$

N09
P1 #13

a) $\frac{1}{z} = \frac{1}{x+iy} = \frac{1\cdot(x-iy)}{(x+iy)(x-iy)} = \frac{x-iy}{x^2+y^2} = \frac{x}{x^2+y^2} - i\frac{y}{x^2+y^2}$

$z+\frac{1}{z} = x+iy+\frac{x}{x^2+y^2} - i\frac{y}{x^2+y^2} = \left(x+\frac{x}{x^2+y^2}\right) + iy\left(1-\frac{1}{x^2+y^2}\right)$

$z+\frac{1}{z} = k \in \mathbb{R} \Rightarrow y\left(1-\frac{1}{x^2+y^2}\right) = 0 \begin{cases} \boxed{y=0} \\ 1 = \frac{1}{x^2+y^2} \; ; \; \boxed{x^2+y^2=1} \end{cases}$ ✓

$x^2+y^2=1 \Rightarrow |k| = \left|z+\frac{1}{z}\right| \leq |z| + \left|\frac{1}{z}\right| = |z| + \frac{1}{|z|} = 1 + \frac{1}{1} = 2$ ✓

b) $w^n + w^{-n} = (\cos\theta + i\sin\theta)^n + [\cos(-\theta) + i\sin(-\theta)]^n = \cos(n\theta) + i\sin(n\theta) + \cos(-n\theta) +$

$+ i\sin(-n\theta) = \cos n\theta + \cancel{i\sin n\theta} + \cos n\theta - \cancel{i\sin n\theta} = 2\cos n\theta$ ✓

$3w^2 - w + 2 - w^{-1} + 3w^{-2} = 0 \; ; \quad 3(w^2+w^{-2}) - (w+w^{-1}) + 2 = 0 \; ;$

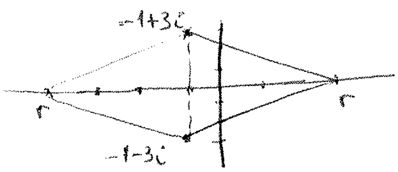
$3\cdot2\cos2\theta - 2\cos\theta + 2 = 0 \; ; \quad 3\cos2\theta - \cos\theta + 1 = 0 \; ; \quad 3(\cos^2\theta - \sin^2\theta) - \cos\theta + 1 = 0 \; ;$

$3(2\cos^2\theta - 1) - \cos\theta + 1 = 0 \; ; \quad 6\cos^2\theta - \cos\theta - 2 = 0$

$\cos\theta = \frac{1\pm\sqrt{1+48}}{12} = \frac{1\pm7}{12} \begin{cases} \frac{2}{3} \rightarrow \sin\theta = \pm\frac{\sqrt{5}}{3} \Rightarrow \boxed{w = \frac{2}{3} \pm i\frac{\sqrt{5}}{3}} \\ -1/2 \rightarrow \sin\theta = \pm\frac{\sqrt{3}}{2} \Rightarrow \boxed{w = -\frac{1}{2} \pm i\frac{\sqrt{3}}{2}} \end{cases}$

N09 P2 #7

$z_1 = -1+3i \longrightarrow \begin{cases} z_2 = -1-3i \\ z_3 = r \in \mathbb{R} \end{cases}$



Si $r>0 \to$ Area $= \dfrac{(r+1)\cdot 6}{2} \Rightarrow 9 = 3(r+1) \Rightarrow \boxed{r=2}$

Si $r<0 \to$ Area $= \dfrac{(-r-1)6}{2} \Rightarrow 9 = -3(1+r) \Rightarrow \boxed{r=-4}$

$$(z-(-1+3i))\,(z-(-1-3i))\,(z-r) = (z+1-3i)(z+1+3i)(z-r) = \left((z+1)^2+3^2\right)(z-r) =$$

$$= (z^2+2z+10)(z-r) = z^3 - rz^2 + 2z^2 - 2rz + 10z - 10r = z^3 + (2-r)z^2 + (10-2r)z - 10r$$

$r=2 \Rightarrow$ $a=0$; $b=6$; $c=-20$

$r=-4 \Rightarrow$ $a=6$; $b=18$; $c=40$

M10 P1 TZ2 #13

a) $w = 1_{2\pi/3} \Rightarrow w^3 = \left(1_{2\pi/3}\right)^3 = 1_{6\pi/3} = 1_{2\pi} = 1$ ✓

$$1 + w + w^2 = 1 + 1_{2\pi/3} + 1_{4\pi/3} = \left(1 + \cos\tfrac{2\pi}{3} + \cos\tfrac{4\pi}{3}\right) + i\left(\sin \tfrac{2\pi}{3} + \sin\tfrac{4\pi}{3}\right) =$$

$$= \left(1 + \tfrac{-1}{2} + \tfrac{-1}{2}\right) + i\left(\tfrac{\sqrt{3}}{2} + \tfrac{-\sqrt{3}}{2}\right) = 0 + i0 = 0 \ ✓$$

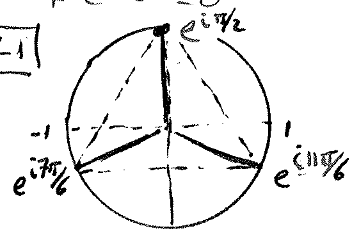
b) $1+w+w^2=0 \Rightarrow e^{i\theta}(1+w+w^2)=0$; $e^{i\theta}\left(1+e^{i2\pi/3}+e^{i\frac{4\pi}{3}}\right)=0$;

$e^{i\theta} + e^{i(\theta+2\pi/3)} + e^{i(\theta+4\pi/3)} = 0$ ✓

$\theta = \frac{\pi}{2} \longrightarrow e^{i\pi/2} + e^{i\left(\frac{\pi}{2}+\frac{2\pi}{3}\right)} + e^{i\left(\pi/2+\frac{4\pi}{3}\right)} = 0$; $e^{i\pi/2} + e^{i\frac{7\pi}{6}} + e^{i\frac{11\pi}{6}} = 0$

c) $F(z) = (z-1)(z-w)(z-w^2) = z^3 - z^2(\underbrace{w^2+w+1}_{=0}) + z(w^2+w+1)\cdot w - \underbrace{w^3}_{=1} = \boxed{z^3-1}$

$F(z)=7 \Rightarrow z^3-1=7$; $z^3=8$; $z=\sqrt[3]{8} = \begin{cases} z_0 = \boxed{2} \\ z_{\pi/3} = \boxed{2w} \\ z_{4\pi/3} = \boxed{2w^2} \end{cases}$



M10 P2 TZ1 #4

a) $z^3 = -2+2i$; $z = \sqrt[3]{-2+2i}$

$-2+2i = \begin{cases} r = \sqrt{4+4} = \sqrt{8} \\ \operatorname{tg}\theta = \frac{2}{-2} \Rightarrow \theta = 135° \end{cases} = (\sqrt{8})_{135°}$

$$z = \sqrt[3]{(\sqrt{8})_{135°}} = \left(\sqrt[6]{8}\right)_{\frac{135+N\cdot 360°}{3}} = \left(\sqrt[6]{8}\right)_{45°+N\cdot 120°} = \boxed{\begin{cases} (\sqrt[6]{8})_{45°} \\ (\sqrt[6]{8})_{165°} \\ (\sqrt[6]{8})_{285°} \end{cases}} = \boxed{\begin{cases} (\sqrt{2})_{45°} \\ (\sqrt{2})_{165°} \\ (\sqrt{2})_{285°} \end{cases}}$$

b) $(\sqrt{2})_{45°} = \sqrt{2}(\cos 45° + i\sin 45°) =$
$= \sqrt{2}\left(\frac{\sqrt{2}}{2} + i\cdot\frac{\sqrt{2}}{2}\right) = \frac{2}{2} + i\frac{2}{2} = 1+i$ ✓

M10 P2 TZ2 #9

a) $z = 1_\theta \rightarrow z^n + \frac{1}{z^n} = 1_{n\theta} + 1_{-n\theta} = \cos n\theta + i\sin n\theta + \cos(-n\theta) + i\sin(-n\theta) =$

$= \cos n\theta + i\cancel{\sin n\theta} + \cos n\theta - i\cancel{\sin n\theta} = 2\cos n\theta \Rightarrow \operatorname{Im}\left(z^n + \frac{1}{z^n}\right) = 0$ ✓

b)
$$\frac{z-1}{z+1} = \frac{x+iy-1}{x+iy+1} = \frac{(x-1)+iy}{(x+1)+iy} = \frac{[(x-1)+iy][(x+1)-iy]}{[(x+1)+iy][(x+1)-iy]} = \frac{(x^2-1) - i(x-1)y + iy(x+1) + y^2}{(x+1)^2+y^2}$$

$$\operatorname{Re}\left(\frac{z-1}{z+1}\right) = \frac{(x^2-1)+y^2}{(x+1)^2+y^2} = \frac{x^2+y^2-1}{(x+1)^2+y^2} = \frac{0}{(x+1)^2+y^2} = 0 \ ✓$$

(Porque $x = \cos\theta$, $y = \sin\theta \Rightarrow$
$\Rightarrow x^2+y^2=1$)

N10 P1 #11

a) $w=i \Rightarrow \dfrac{z+i}{z+2}=i \;;\; z+i=zi+2i \;;\; z(1-i)=i \;;\; z=\dfrac{i}{1-i}\;;$

$z=\dfrac{i(1+i)}{(1-i)(1+i)}=\dfrac{i-1}{1+1}=\boxed{-\frac{1}{2}+i\frac{1}{2}}=\left\{\begin{array}{l} m=\sqrt{\frac14+\frac14}=\frac{\sqrt2}{2}\\ Tg\alpha=-1\rightarrow \alpha=3\pi/4\end{array}\right\}=\boxed{\frac{\sqrt2}{2}\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)}$

b) $w=\dfrac{x+iy+i}{x+iy+2}=\dfrac{x+i(y+1)}{(x+2)+iy}=\dfrac{[x+i(y+1)][(x+2)-iy]}{[(x+2)+iy][(x+2)-iy]}=$

$=\dfrac{x(x+2)-ixy+i(y+1)(x+2)+(y+1)y}{(x+2)^2+y^2}=\dfrac{(x^2+2x+y^2+y)+i(-\cancel{xy}+\cancel{yx}+2y+x+2)}{(x+2)^2+y^2}$ ✓

c) $R(w)=1 \Rightarrow \dfrac{x^2+2x+y^2+y}{(x+2)^2+y^2}=1 \;;\; \cancel{x^2}+2x+\cancel{y^2}+y=\cancel{x^2}+4x+4+\cancel{y^2}\rightarrow \boxed{y=2x+4}$

Pendiente = 2

d) $\arg(w)=\frac{\pi}{4} \Rightarrow Tg\frac{\pi}{4}=\dfrac{\frac{x+2y+2}{(x+2)^2+y^2}}{\frac{x^2+2x+y^2+y}{(x+2)^2+y^2}} \Rightarrow 1\cdot(x^2+2x+y^2+y)=x+2y+2\;;$

$x^2+y^2+x-y-2=0$

$\arg(z)=\frac{\pi}{4}\Rightarrow Tg\frac{\pi}{4}=\frac{y}{x}\Rightarrow 1=\frac{y}{x}\rightarrow y=x \rightarrow x^2+y^2+\cancel{x}-\cancel{y}=2 \Rightarrow \boxed{|z|=\sqrt2}$

N10 P2 #6

Raíces: $1+i\rightarrow 1-i$

$1-2i\rightarrow 1+2i$

Factores: $(x-(1+i))(x-(1-i))(x-(1-2i))(x-(1+2i))=$

$=((x-1)-i)((x-1)+i)((x-1)+2i)((x-1)-2i)=((x-1)^2+1)((x-1)^2+4)=$

$=(x^2-2x+2)(x^2-2x+5)=x^4-2x^3+5x^2-2x^3+4x^2-10x+2x^2-4x+10=$

$=x^4-4x^3+11x^2-14x+10 \quad \boxed{a=-4;\; b=11;\; c=-14;\; d=10}$

M11 P1 TZ1 #2

$\dfrac{z}{z+2}=2-i \;;\; z=2z-iz+4-2i \;;\; -z+iz=4-2i \;;\; z(-1+i)=4-2i\;;$

$z=\dfrac{4-2i}{-1+i}=\dfrac{(4-2i)(-1-i)}{(-1-i)(-1+i)}=\dfrac{-4-4i+2i-2}{1+1}=\dfrac{-6-2i}{2}=\boxed{-3-i}$

M11 P1 TZ1 #13

a) $(\cos\theta+i\sin\theta)^3=\cos^3\theta+3\cos^2\theta\, i\sin\theta+3\cos\theta\, i^2\sin^2\theta+i^3\sin^3\theta=$

$=\boxed{(\cos^3\theta-3\cos\theta\sin^2\theta)+i(3\cos^2\theta\sin\theta-\sin^3\theta)}$

b) $(\cos\theta+i\sin\theta)^3=\cos3\theta+i\sin3\theta \Rightarrow \cos3\theta=\cos^3\theta-3\cos\theta\sin^2\theta=\cos^3\theta-3\cos\theta(1-\cos^2\theta)=$

$=4\cos^3\theta-3\cos\theta$ ✓

c) $(\cos\theta+i\sin\theta)^5=\cos^5\theta+5\cos^4\theta\, i\sin\theta+10\cos^3\theta\, i^2\sin^2\theta+10\cos^2\theta\, i^3\sin^3\theta+5\cos\theta\, i^4\sin^4\theta+i^5\sin^5\theta=$

$=(\cos^5\theta-10\cos^3\theta\sin^2\theta+5\cos\theta\sin^4\theta)+i(5\cos^4\theta\sin\theta-10\cos^2\theta\sin^3\theta+\sin^5\theta)$

$\cos5\theta=\cos^5\theta-10\cos^3\theta\sin^2\theta+5\cos\theta\sin^4\theta=\cos^5\theta-10\cos^3\theta(1-\cos^2\theta)+5\cos\theta(1-\cos^2\theta)^2=$

$=\cos^5\theta-10\cos^3\theta+10\cos^5\theta+5\cos\theta(1-2\cos^2\theta+\cos^4\theta)=$

$=\cos^5\theta-10\cos^3\theta+10\cos^5\theta+5\cos\theta-10\cos^3\theta+5\cos^5\theta=16\cos^5\theta-20\cos^3\theta+5\cos\theta$ ✓

d) $\cos 5\theta + \cos 3\theta + \cos\theta = 0 \rightarrow 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta + 4\cos^3\theta - 3\cos\theta + \cos\theta = 0 \rightarrow$

$\rightarrow \cos\theta \cdot (16\cos^4\theta - 16\cos^2\theta + 3) = 0 \begin{cases} \rightarrow \cos\theta = 0 \Rightarrow \boxed{\theta = \pm \pi/2} \\ \rightarrow 16\cos^4\theta - 16\cos^2\theta + 3 = 0 \end{cases}$

Llamo $t = \cos^2\theta \rightarrow 16t^2 - 16t + 3 = 0$

$t = \dfrac{16 \pm \sqrt{256 - 192}}{32} = \dfrac{16 \pm 8}{32} \begin{cases} \frac{24}{32} = \frac{3}{4} \rightarrow \cos^2\theta = \frac{3}{4}\ ; \cos\theta = \pm\frac{\sqrt{3}}{2} \rightarrow \boxed{\theta = \pm\frac{\pi}{6}} \\ \frac{8}{32} = \frac{1}{4} \rightarrow \cos^2\theta = \frac{1}{4}\ ; \cos\theta = \pm\frac{1}{2} \rightarrow \boxed{\theta = \pm\frac{\pi}{3}} \end{cases}$

M11 TZ2 P1 #12

a)

$$\begin{array}{r|rrrr} & 1 & 0 & 0 & 1 \\ -1 & & -1 & 1 & -1 \\ \hline & 1 & -1 & 1 & \boxed{0} \end{array} \qquad z^3 + 1 = (z+1)(z^2 - z + 1)$$

b) $\gamma^3 = \left[\frac{1}{2}(1+i\sqrt{3})\right]^3 = \frac{1}{8}(1 + 3i\sqrt{3} + 3i^2\cdot 3 + i^3\cdot 3\sqrt{3}) = \frac{1}{8}(1 + \cancel{3i\sqrt{3}} - 9 - \cancel{3i\sqrt{3}}) = \frac{-8}{8} = -1$ ✓

$\gamma^2 = \left[\frac{1}{2}(1+i\sqrt{3})\right]^2 = \frac{1}{4}(1 + 2i\sqrt{3} + i^2\cdot 3) = \frac{1}{4}(1 + 2i\sqrt{3} - 3) = \frac{1}{4}(2 + 2i\sqrt{3} - 4) = \frac{1+i\sqrt{3}}{2} - 1$ ✓

$(1-\gamma)^6 = (-\gamma^2)^6 = \gamma^{12} = (\gamma^3)^4 = (-1)^4 = \boxed{1}$

N11 P2#6

$\left.\begin{array}{l} z_1 z_2 = -\sqrt{3} + i \\ \dfrac{z_1}{z_2} = 2i \end{array}\right\}$

$-\sqrt{3} + i = \left\{\begin{array}{l} m = \sqrt{3+1} = 2 \\ \operatorname{tg}\alpha = \frac{1}{-\sqrt{3}} \Rightarrow \alpha = 150° \equiv 5\pi/6 \end{array}\right\} = 2_{5\pi/6} \qquad 2i = \left\{\begin{array}{l} m = 2 \\ \alpha = 90° \equiv \pi/2 \end{array}\right\} = 2_{\pi/2}$

$z_1 = (m_1)_{\alpha_1}$, $z_2 = (m_2)_{\alpha_2}$ $\quad \left(\begin{array}{l}\alpha_1 \in [0, \pi] \\ \alpha_2 \in [0, \pi]\end{array}\right)$

$\left.\begin{array}{l} z_1 z_2 = -\sqrt{3} + i \longrightarrow (m_1)_{\alpha_1} \cdot (m_2)_{\alpha_2} = 2_{5\pi/6} \\ \dfrac{z_1}{z_2} = 2i \longrightarrow \dfrac{(m_1)_{\alpha_1}}{(m_2)_{\alpha_2}} = 2_{\pi/2} \end{array}\right\} \Rightarrow \begin{array}{l} \left.\begin{array}{l} m_1 \cdot m_2 = 2 \\ \frac{m_1}{m_2} = 2 \end{array}\right\} \rightarrow \\ \left.\begin{array}{l} \alpha_1 + \alpha_2 = 5\pi/6 \\ \alpha_1 - \alpha_2 = \pi/2 \end{array}\right\} \rightarrow \end{array}$

$\rightarrow \boxed{m_1 = 2} \rightarrow \boxed{m_2 = 1}$

$\rightarrow 2\alpha_1 = \frac{8\pi}{6} \rightarrow \boxed{\alpha_1 = \frac{2\pi}{3}} \Rightarrow \boxed{\alpha_2 = \frac{\pi}{6}}$

N11 P2#10

$z = \dfrac{2-i}{1+i} - \dfrac{6+8i}{a+i} = \dfrac{(2-i)(1-i)}{(1+i)(1-i)} - \dfrac{(6+8i)(a-i)}{(a+i)(a-i)} = \dfrac{2-2i-i+i^2}{1-i^2} - \dfrac{6a-6i+8ai-8i^2}{a^2-i^2} =$

$= \dfrac{1-3i}{2} - \dfrac{(6a+8)+(8a-6)i}{a^2+1} = \left(\dfrac{1}{2} - \dfrac{6a+8}{a^2+1}\right) + \left(-\dfrac{3}{2} - \dfrac{8a-6}{a^2+1}\right)i$

$\operatorname{Re} z = \operatorname{Im} z \Rightarrow \dfrac{1}{2} - \dfrac{6a+8}{a^2+1} = -\dfrac{3}{2} - \dfrac{8a-6}{a^2+1}\ ; \quad (a^2+1) - 2(6a+8) = -3(a^2+1) - 2(8a-6);$

$a^2 + 1 - 12a - 16 = -3a^2 - 3 - 16a + 12\ ; \quad 4a^2 + 4a - 24 = 0;\ \ a^2 + a - 6 = 0\ ; \quad \boxed{a = \begin{cases} 2 \\ -3 \end{cases}}$

N11 P2#14

$1 + \frac{1}{3}e^{i\theta} + \frac{1}{9}e^{2i\theta} + \cdots$

$z = \frac{1}{3}e^{i\theta} \longrightarrow |z| = \left|\frac{1}{3}e^{i\theta}\right| = \frac{1}{3}\left|e^{i\theta}\right| = \frac{1}{3}\left|\cos\theta + i\sin\theta\right| = \frac{1}{3}\sqrt{\cos^2\theta + \sin^2\theta} = \frac{1}{3}$ ✓

$S_\infty = \dfrac{1}{1 - 1/3\, e^{i\theta}} = \dfrac{3}{3 - e^{i\theta}} = \dfrac{3}{3 - (\cos\theta + i\sin\theta)} = \dfrac{3}{(3-\cos\theta) - i\sin\theta} =$

$= \dfrac{3\cdot[(3-\cos\theta) + i\sin\theta]}{[(3-\cos\theta) - i\sin\theta][(3-\cos\theta) + i\sin\theta]} = \dfrac{9 - 3\cos\theta + 3i\sin\theta}{(3-\cos\theta)^2 + \sin^2\theta} = \dfrac{9 - 3\cos\theta + 3i\sin\theta}{9 - 6\cos\theta + \cos^2\theta + \sin^2\theta} =$

$= \boxed{\dfrac{9 - 3\cos\theta}{10 - 6\cos\theta} + \dfrac{3\sin\theta}{10 - 6\cos\theta}i} \Rightarrow 0 + \frac{1}{3}\sin\theta + \frac{1}{9}\sin 2\theta + \frac{1}{27}\sin 3\theta + \cdots = \dfrac{3\sin\theta}{10 - 6\cos\theta}$ ✓