Even and Odd Functions (Funciones pares e impares)

Even functions are functions for which the left half of the plane looks like the mirror image of the right half of the plane. Odd functions are functions where the left half of the plane looks like the mirror image of the right half of the plane, only upside-down.

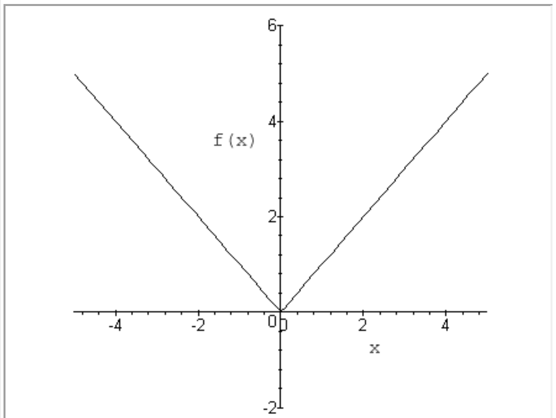
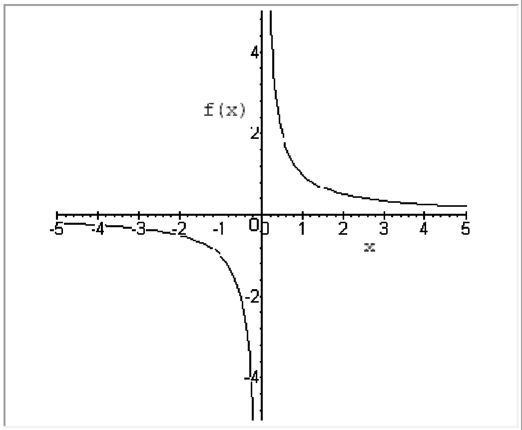
Mathematically, we say that a function f(x) is even if

$$f(x) = f(-x)$$

and is odd if

$$f(-x) = -f(x)$$

Some examples:

<table>
<tr><th>Some even functions</th><th>Some odd functions</th></tr>
<tr><td>
</td><td>
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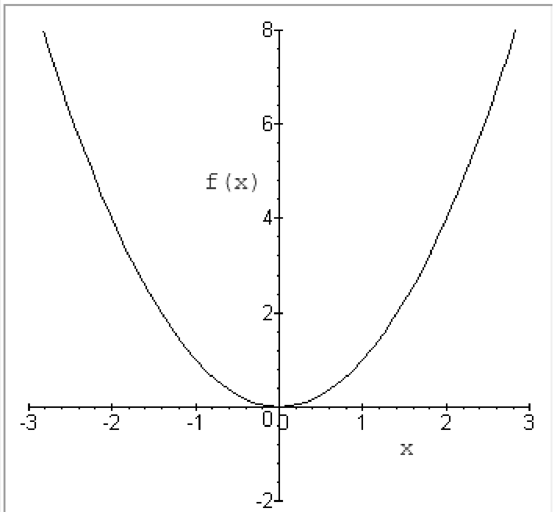
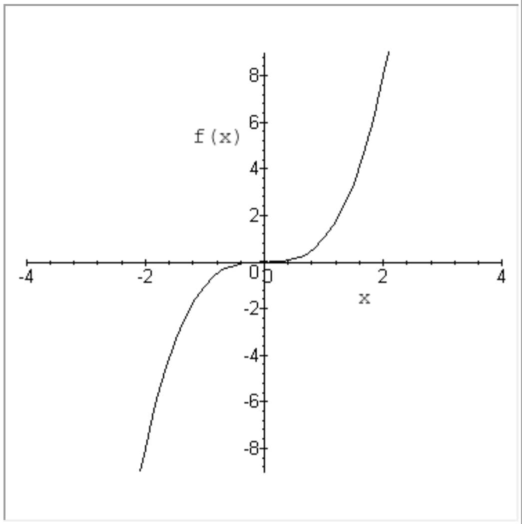
<tr><td>f(x) = |x|</td><td>f(x) = 1/x</td></tr>
<tr><td>
</td><td>
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<tr><td>f(x) = x^2</td><td>f(x) = x^3</td></tr>
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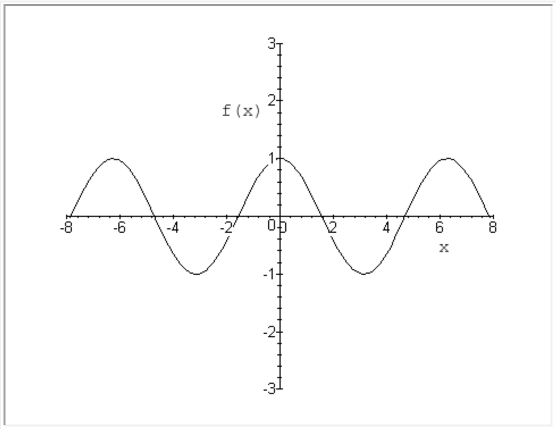
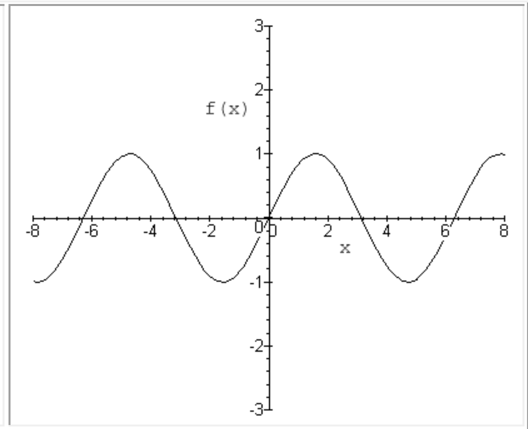
	
f(x) = cos(x)	f(x) = sin(x)

Exercises:

Check the symmetry of the following functions:

a) $f(x) = 2x^2 + x - 3$

b) $f(x) = 6x^2 - 2x$

c) $f(x) = 3x^2 - 1$

d) $f(x) = \frac{2x^2-1}{5x^2+3}$

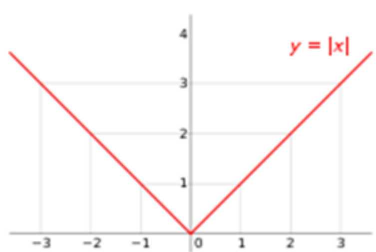
e) $f(x) = \sqrt{\frac{6x^2+1}{5x-3}}$

Piecewise functions (Funciones definidas a trozos)

In mathematics, a piecewise-defined function (also called a piecewise function) is a function whose definition changes depending on the value of the independent variable. Mathematically, a real-valued function *f* of a real variable *x* is a relationship whose definition is given differently on disjoint subsets of its domain (known as subdomains).

Notation and interpretation

Piecewise functions are defined using the common functional notation, where the body of the function is an array of functions and associated subdomains. For example, consider the piecewise definition of the absolute value function:



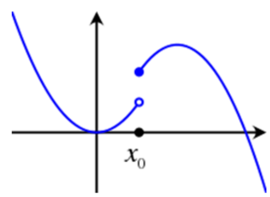
$$|x| \equiv f(x) = \begin{cases} -x, & \text{if } x < 0 \\ x, & \text{if } x \geq 0 \end{cases}$$

Graph of the absolute value function, $y = |x|$.

For all values of x less than zero, the first function ($-x$) is used, which negates the sign of the input value, making negative numbers positive. For all values of x greater than or equal to zero, the second function (x) is used, which evaluates trivially to the input value itself.

Continuity

Consider the piecewise function $f(x)$ evaluated at certain values of x:



A piecewise function comprising different quadratic functions on either side of x_0.

A piecewise function is continuous on a given interval if the following conditions are met:

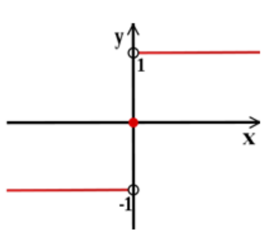
Consider the piecewise function $f(x)$ evaluated at certain values of x:

- it is defined throughout that interval
- its appropriate constituent functions are continuous on that interval
- there is no discontinuity at each endpoint of the subdomains within that interval.

The pictured function, for example, is piecewise continuous throughout its subdomains, but is not continuous on the entire domain. The pictured function contains a jump discontinuity at x_0.

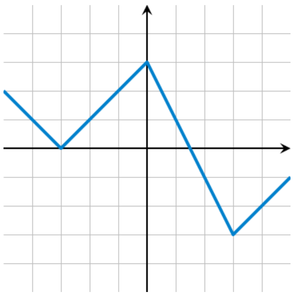
More examples of piecewise functions:

1)



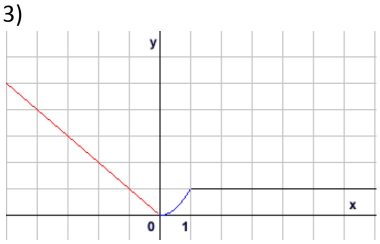
$$\operatorname{sgn}(x) = \begin{cases} -1 & \text{if } x < 0, \\ 0 & \text{if } x = 0, \\ 1 & \text{if } x > 0. \end{cases}$$

2)



$$f(x) = \begin{cases} -x-3 & \text{if } x \leq -3 \\ x+3 & \text{if } -3 < x < 0 \\ -2x+3 & \text{if } 0 \leq x < 3 \\ x-6 & \text{if } x \geq 3 \end{cases}$$

3)

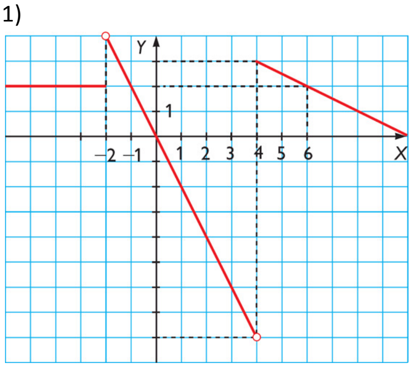


$$f(x) = \begin{cases} -x, & \text{si } x < 0 \\ x^2, & \text{si } 0 \leq x \leq 1 \\ 1, & \text{si } x > 1 \end{cases}$$

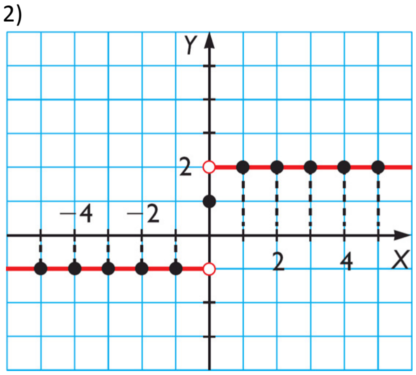
Exercises:

Find the algebraic expresion for the following piecewise functions:

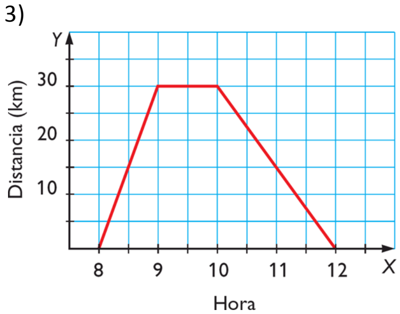
1)



2)



3)


Y
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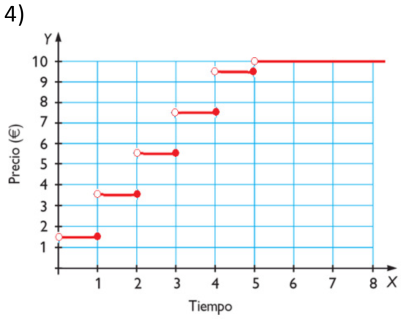
20
10
Distancia (km)
8
9
10
11
12
X
Hora

4)


Y
10
9
8
7
6
5
4
3
2
1
Precio (€)
1
2
3
4
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8
X
Tiempo