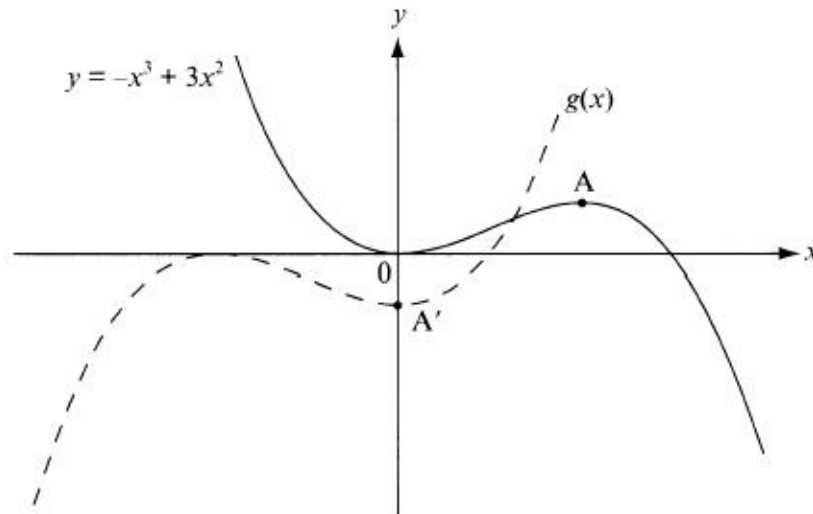


**Funciones: Cuestiones básicas en exámenes BI - NS [EXTRACTO]**

May 00  
P2 #2

The diagram below shows the graphs of  $y = -x^3 + 3x^2$  and  $g(x) = x^3 + 3x^2 - 4$



The graph of  $y = -x^3 + 3x^2$  is reflected in the  $y$ -axis, then translated using the vector  $\begin{pmatrix} -1 \\ -1 \end{pmatrix}$  to give the graph of  $y = h(x)$ .

(a) Write  $h(x)$  in the form  $h(x) = ax^3 + bx^2 + cx + d$ .

The graph of  $y = x^3 + 3x^2 - 4$  is obtained by applying a composition of two transformations to the graph of  $y = -x^3 + 3x^2$ .

(b) State the two transformations whose composition maps the graph of  $y = -x^3 + 3x^2$  onto the graph of  $y = x^3 + 3x^2 - 4$  and also maps point A onto point A'.

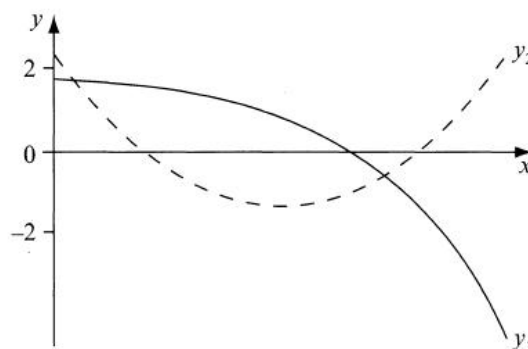
Mayo 01  
P1 #5

Let  $f : x \mapsto \sqrt{\frac{1}{x^2} - 2}$ . Find

- (a) the set of real values of  $x$  for which  $f$  is real and finite ;
- (b) the range of  $f$ .

Mayo 01  
P1 #19

The diagram shows the graph of the functions  $y_1$  and  $y_2$ .



On the same axes sketch the graph of  $\frac{y_1}{y_2}$ . Indicate clearly where the  $x$ -intercepts and asymptotes occur.

May 02  
P1 #15

La función uno a uno  $f$  está definida en el dominio  $x > 0$  por  $f(x) = \frac{2x-1}{x+2}$

- (a) Indique el rango  $A$  de  $f$ .
- (b) Halle una expresión de  $f^{-1}(x)$  para  $x \in A$ .

Nov 02  
P1 #4

Halle las ecuaciones de las asíntotas de la gráfica de  $y = \frac{x^2 - 5x - 4}{x^2 - 5x + 4}$ .

Nov 02  
P1 #7

Las funciones  $f(x)$  y  $g(x)$  están dadas por  $f(x) = \sqrt{x-2}$  y  $g(x) = x^2 + x$ .  
La función  $(f \circ g)(x)$  está definida para  $x \in \mathbb{R}$ , a excepción del intervalo  $]a, b[$ .

- (a) Halle el valor de  $a$  y de  $b$ .
- (b) Halle el recorrido de  $f \circ g$ .

May 03  
P1 #17

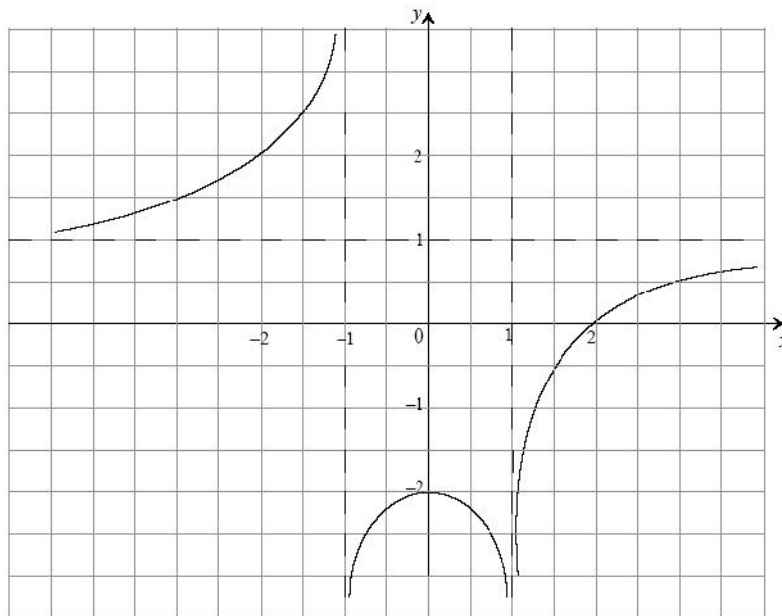
La función  $f$  está definida para  $x \leq 0$  por  $f(x) = \frac{x^2 - 1}{x^2 + 1}$

Halle una expresión de  $f^{-1}(x)$ .

Nov 03  
P1 #6

En el diagrama aparece la gráfica de  $f(x)$ .

- (a) Sobre el mismo diagrama dibuje aproximadamente la gráfica de  $\frac{1}{f(x)}$ , indicando claramente todas las asíntotas.



- (b) Sobre el mismo diagrama, escriba las coordenadas del máximo local, el mínimo local y las intersecciones con el eje  $x$  y con el eje  $y$  de  $\frac{1}{f(x)}$ .

May 04  
P1 TZ2  
#11

La función  $f$  viene definida por  $f : x \mapsto x^3$ .

Halle la expresión de  $g(x)$  en función de  $x$  en cada uno de los siguientes casos

- (a)  $(f \circ g)(x) = x + 1$ ;
- (b)  $(g \circ f)(x) = x + 1$ .

May 05  
P1 TZ1  
#14



Consider the functions  $f(x) = e^{2x}$  and  $g(x) = \sin \frac{\pi x}{2}$ .

- (a) Find the period of the function  $f \circ g$ .
- (b) Find the intervals for which  $(f \circ g(x)) > 4$ .

May 05  
P1 TZ1  
#16



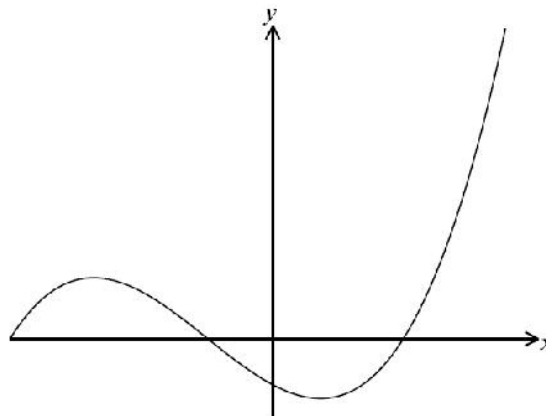
(a) Sketch the graph of  $y = \frac{x-12}{\sqrt{x^2-4}}$ .

- (b) Write down
  - (i) the  $x$ -intercept;
  - (ii) the equations of all asymptotes.

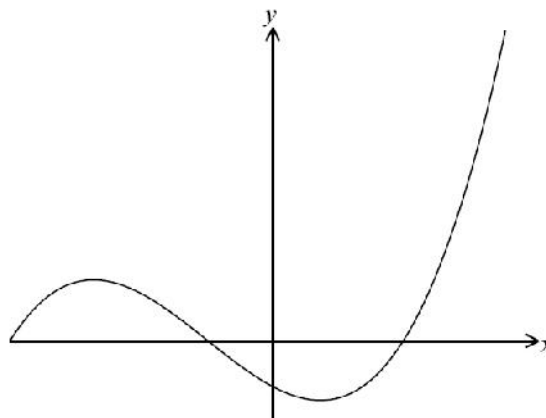
Mayo 06  
P1 #20

Each of the diagrams below shows the graph of a function  $f$ . Sketch on the given axes the graph of

- (a)  $f(-x)$ ;



- (b)  $\frac{1}{f(x)}$



Nov 09  
P1 #4



Consider the function  $f$ , where  $f(x) = \arcsin(\ln x)$ .

- (a) Find the domain of  $f$ .
- (b) Find  $f^{-1}(x)$ .

Nov 09

P2 #1



Find the values of  $k$  such that the equation  $x^3 + x^2 - x + 2 = k$  has three distinct real solutions.

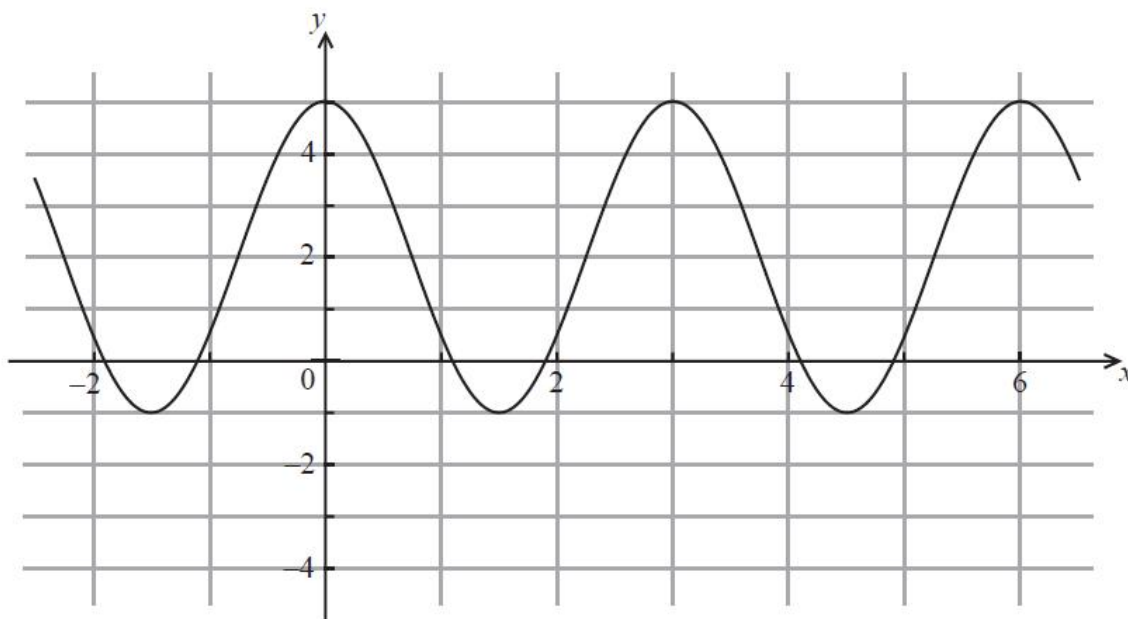
Mayo 10

P2 TZ1

#1



The graph below shows  $y = a \cos(bx) + c$ .



Find the value of  $a$ , the value of  $b$  and the value of  $c$ .

Mayo 10

P2 TZ1

#9



Let  $f(x) = \frac{4 - x^2}{4 - \sqrt{x}}$ .

(a) State the largest possible domain for  $f$ .

(b) Solve the inequality  $f(x) \geq 1$ .

Nov 10

P1 #9



Considere la función  $f : x \rightarrow \sqrt{\frac{\pi}{4} - \arccos x}$ .

(a) Halle el mayor dominio posible de  $f$ .

(b) Determine una expresión para la función inversa,  $f^{-1}$ , y escriba su dominio.

Nov 11

P2 #8



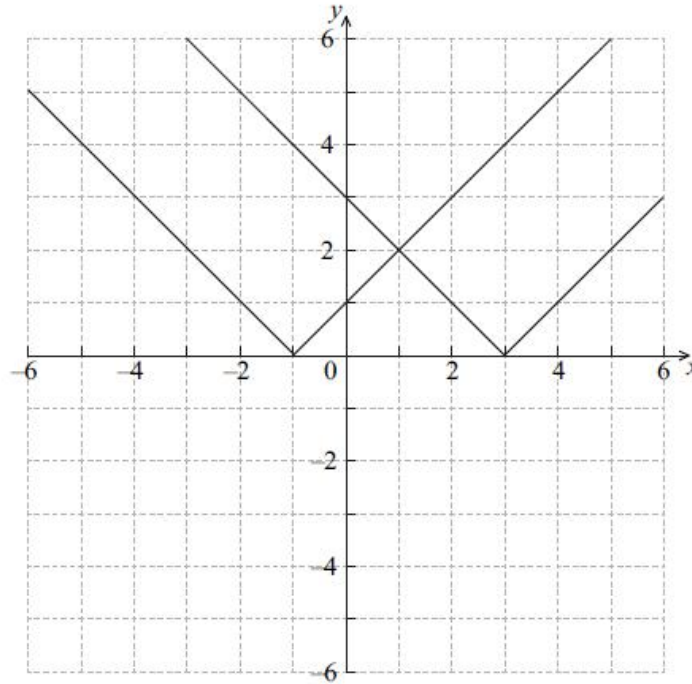
Given that  $f(x) = \frac{1}{1 + e^{-x}}$ ,

(a) find  $f^{-1}(x)$ , stating its domain;

(b) find the value of  $x$  such that  $f(x) = f^{-1}(x)$

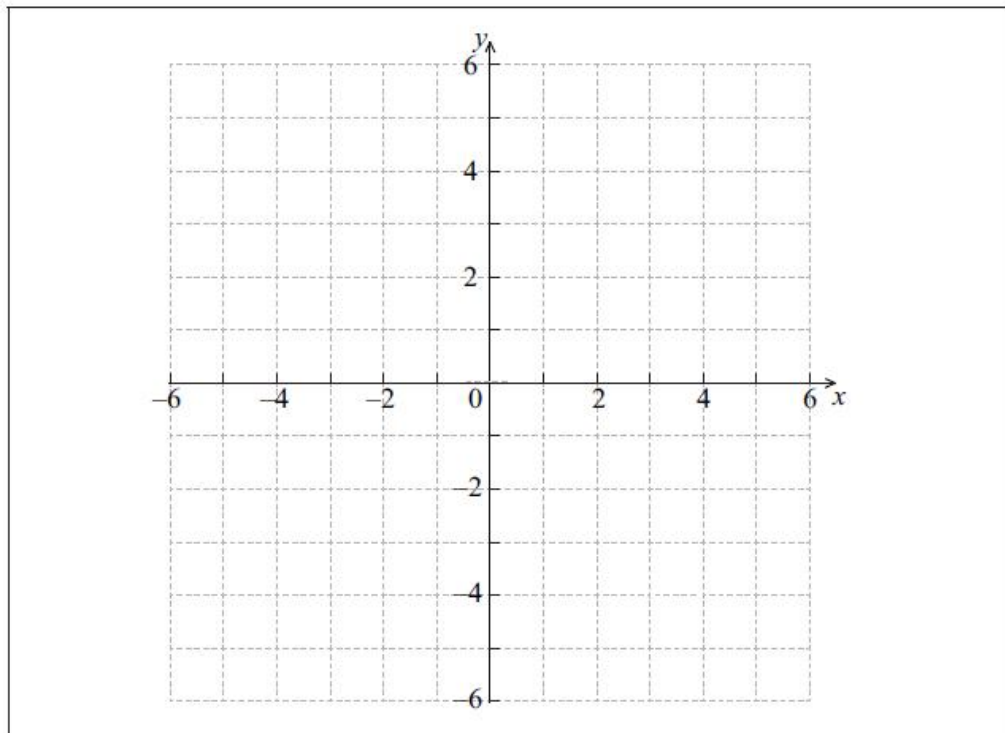
Mayo 12  
P1 TZ1  
#2

The graphs of  $y = |x + 1|$  and  $y = |x - 3|$  are shown below.



Let  $f(x) = |x + 1| - |x - 3|$ .

(a) Draw the graph of  $y = f(x)$  on the blank grid below.



Mayo 12  
P2 TZ1  
#6



Let  $f(x) = \ln x$ . The graph of  $f$  is transformed into the graph of the function  $g$  by a translation of  $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ , followed by a reflection in the  $x$ -axis. Find an expression for  $g(x)$ , giving your answer as a single logarithm.

Nov 13

P2 #3



Consider the function  $f(x) = \ln x - e^{\cos x}$ ,  $0 < x \leq 10$ .

(a) Draw approximately the graph of  $y = f(x)$ , indicating the coordinates of all the maximums and minimums and of the points of intersection with the  $x$ -axis.

(b) Solve the inequality  $\ln x \leq e^{\cos x}$ ,  $0 < x \leq 10$ .

Mayo 14

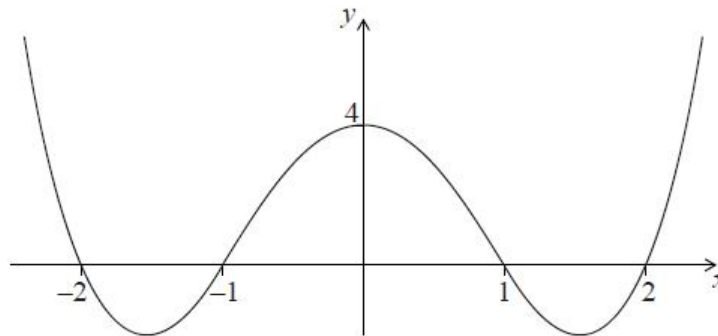
P2 TZ1

#12



Let  $f(x) = |x| - 1$ .

(a) The graph of  $y = g(x)$  is drawn below.



(i) Find the value of  $(f \circ g)(1)$ .

(ii) Find the value of  $(f \circ g \circ g)(1)$ .

(iii) Sketch the graph of  $y = (f \circ g)(x)$ .

(b) (i) Sketch the graph of  $y = f(x)$ .

(ii) State the zeros of  $f$ .

(c) (i) Sketch the graph of  $y = (f \circ f)(x)$ .

(ii) State the zeros of  $f \circ f$ .

(d) Given that we can denote  $\underbrace{f \circ f \circ f \circ \dots \circ f}_{n \text{ times}}$  as  $f^n$ ,

(i) find the zeros of  $f^3$ ;

(ii) find the zeros of  $f^4$ ;

(iii) deduce the zeros of  $f^8$ .

(e) The zeros of  $f^{2^n}$  are  $a_1, a_2, a_3, \dots, a_N$ .

(i) State the relation between  $n$  and  $N$ ;

(ii) Find, and simplify, an expression for  $\sum_{r=1}^N |a_r|$  in terms of  $n$ .