① a) $\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 3 & 4 \\ 0 & 4 & 2 \end{vmatrix} = (-10,-4,8) \longrightarrow \boxed{\vec{w} = (-10,-4,8)}$

b) $|k\cdot\vec{v}| = |k|\cdot|\vec{v}|$

Demostración:

$\vec{v} = (a,b,c) \longrightarrow k\cdot\vec{v} = (ka, kb, kc)\ ;\quad |k\cdot\vec{v}| = \sqrt{(ka)^2+(kb)^2+(kc)^2} =$

$= \sqrt{k^2(a^2+b^2+c^2)} = \sqrt{k^2}\,\sqrt{a^2+b^2+c^2} = |k|\cdot|\vec{v}|$ ✓

c) $\vec{u}\cdot(\vec{v}\times\vec{w}) = |\vec{u}|\cdot|\vec{v}\times\vec{w}|\cdot\cos\alpha = |\vec{u}|\cdot|\vec{v}||\vec{w}|\operatorname{sen}\beta\cos\alpha$

Siendo α el ángulo determinado por $\vec{u}$ y $\vec{v}\times\vec{w}$

Siendo β " " " " $\vec{v}$ y $\vec{w}$

Como $-1 \le \operatorname{sen}\beta \le 1$ y $-1 \le \cos\alpha \le 1 \Longrightarrow$

$\Longrightarrow -|\vec{u}||\vec{v}||\vec{w}| \le \vec{u}\cdot(\vec{v}\times\vec{w}) \le |\vec{u}||\vec{v}||\vec{w}|$

$\boxed{-60 \le \vec{u}\cdot(\vec{v}\times\vec{w}) \le 60}$

② $|\vec{u}\times\vec{v}| = \sqrt{4+9+36} = 7$

$|\vec{u}\times\vec{v}| = |\vec{u}||\vec{v}|\operatorname{sen}\alpha \Longrightarrow \operatorname{sen}\alpha = \frac{7}{16} \Longrightarrow \cos\alpha = \pm\sqrt{1-\frac{49}{256}} = \pm\sqrt{\frac{207}{256}} = \frac{\pm\sqrt{207}}{16} \Longrightarrow$

$\Longrightarrow \vec{u}\cdot\vec{v} = 2\cdot 8\cdot\frac{\pm\sqrt{207}}{16}\ ;\quad \boxed{\vec{u}\cdot\vec{v} = \pm\sqrt{207}}$

③ $\left.\begin{array}{l} \vec{AB} = (a-1, 1, b-1) \\ \vec{AC} = (0,-1,-1) \end{array}\right| \quad \vec{AB} \parallel \vec{AC} \Longrightarrow \frac{a-1}{0} = \frac{1}{-1} = \frac{b-1}{-1} \Longrightarrow \boxed{\begin{array}{l} a=1 \\ b=1 \end{array}}$

④ $\frac{x}{2} = y = \frac{z}{2-a} \longrightarrow \vec{v_1} = (2,1,2-a)$

$\left.\begin{array}{l} x-bz=0 \\ 2x-y-z+1=0 \end{array}\right\} \longrightarrow \vec{v_2} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & -b \\ 2 & -1 & -1 \end{vmatrix} = (-b, 1-2b, -1)$

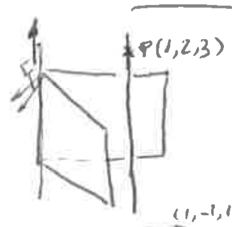
$\vec{v_1} \parallel \vec{v_2} \Longrightarrow \frac{-b}{2} = \frac{1-2b}{1} = \frac{-1}{2-a}$

$-b = 2-4b$

$3b = 2$

$\boxed{b = 2/3}$

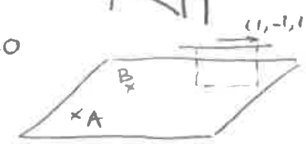
$\frac{1-4/3}{1} = \frac{-1}{2-a}\ ;\quad -\frac{1}{3}(2-a) = -1\ ;\quad 2-a = 3\ ;\quad \boxed{a=-1}$



⑤ a) $\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -1 & -2 \\ 1 & -2 & 4 \end{vmatrix} = (-8,-10,-3) \longrightarrow \boxed{r \equiv \frac{x-1}{-8} = \frac{y-2}{-10} = \frac{z-3}{-3}}$

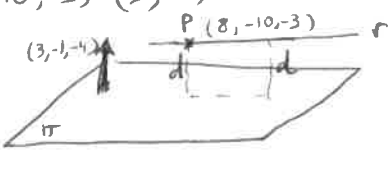
b) $\vec{AB} = (1,3,0)\ ;\quad \begin{vmatrix} x-1 & y-1 & z+1 \\ 1 & 3 & 0 \\ 1 & -1 & 1 \end{vmatrix} = 0 \qquad 3(x-1)-(y-1)-4(z+1) = 0$

$x = 1-y = z \longrightarrow \frac{x}{1} = \frac{y-1}{-1} = \frac{z}{1} \qquad \boxed{3x-y-4z-6=0}$

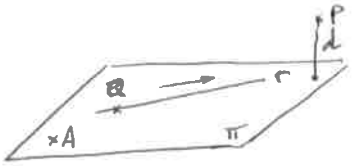


c) La recta es paralela al plano porque $(-8,-10,-3)\cdot(3,-1,-4) = 0$

$d(r,\pi) = d(P,\pi) = \frac{|3\cdot 0-1-4\cdot 0-6|}{\sqrt{9+1+16}} = \boxed{\frac{7}{\sqrt{26}}}$



(6)



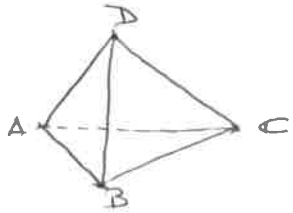
$A(1,1,0)$
$Q(-2,1,1)$ $\quad | \quad \vec{AQ} = (-3,0,1)$

$$\begin{vmatrix} x-1 & y-1 & z \\ -3 & 0 & 1 \\ 1 & 2 & -2 \end{vmatrix} = 0 \quad ; \quad -2(x-1) - 5(y-1) - 6z = 0$$

$$\boxed{-2x - 5y - 6z + 7 = 0}$$

$$d(P,\pi) = \frac{|-2\cdot 0 - 5\cdot 2 - 6\cdot(-1) + 7|}{\sqrt{4+25+36}} = \boxed{\frac{3}{\sqrt{65}}}$$

(7) a) $\vec{AB} = (1,2,0)$
$\vec{AC} = (5,1,0)$



$$\begin{vmatrix} x-1 & y-2 & z+1 \\ 1 & 2 & 0 \\ 5 & 1 & 0 \end{vmatrix} = 0$$

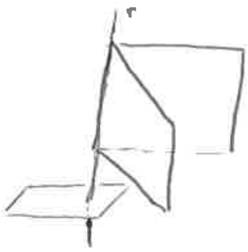
$$0\cdot(x-1) - 0(y-2) - 9(z+1) = 0$$

$$-9(z+1) = 0 \quad ; \quad \boxed{z+1=0}$$

b) $\vec{AD} = (2,1,6)$

$$\begin{vmatrix} 1 & 2 & 0 \\ 5 & 1 & 0 \\ 2 & 1 & 6 \end{vmatrix} = -54 \quad \Rightarrow \quad \boxed{V = 54/6 = 9}$$

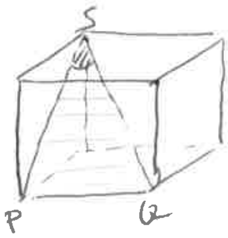
(8)
$$\left.\begin{array}{l} 2x-3y-7=0 \\ x+3z-8=0 \end{array}\right\} \rightarrow \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -3 & 0 \\ 1 & 0 & 3 \end{vmatrix} = (-9,-6,3)$$ Es el vector director de la recta.



$$-9x - 6y + 3z + D = 0$$

$$-9\cdot 0 - 6\cdot 4 + 3\cdot 5 + D = 0 \quad ; \quad D = 9 \qquad \boxed{-9x - 6y + 3z + 9 = 0}$$

(9) $Q = P + \vec{OR} = (1,0,0) + (0,3,0) = (1,3,0)$

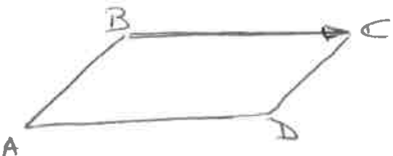


$\vec{SQ} = (1-0, 3-0, 0-4) = (1,3,-4)$

$\vec{SP} = (1-0, 0-0, 0-4) = (1,0,-4)$

$$\cos P\hat{S}Q = \frac{\vec{SP}\cdot\vec{SQ}}{|\vec{SP}|\,|\vec{SQ}|} = \frac{1+16}{\sqrt{26}\,\sqrt{17}} \Rightarrow \boxed{P\hat{S}Q = 36^\circ}$$

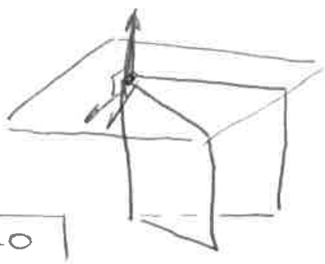
(10) $D = A + \vec{BC} = (0,1,3) + (1+2, 1-1, 3-2) =$
$= (0,1,3) + (3,0,1) = \boxed{(3,1,4)}$



$\vec{AB} = (-2,0,-1)$
$\vec{AC} = (1,0,0)$ $\quad \left| \quad \vec{AB}\times\vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & 0 & -1 \\ 1 & 0 & 0 \end{vmatrix} = (0,-1,0) \right.$

$$\text{Area} = |\vec{AB}\times\vec{AC}| = \sqrt{0+1+0} = \boxed{1}$$

(11)
$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & 1 \\ 2 & -2 & -1 \end{vmatrix} = (3,3,0)$$

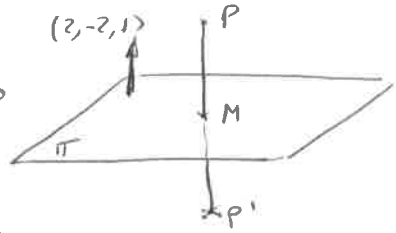


$$3x + 3y + D = 0$$

$$3\cdot 1 + 3\cdot 1 + D = 0 \quad ; \quad D = -6 \qquad \boxed{3x + 3y - 6 = 0}$$

(12)

$$\left.\begin{array}{l} x=3+2r \\ y=-2-2r \\ z=2+r \\ 2x-2y+z+15=0 \end{array}\right\} \quad \begin{array}{l} 2(3+2r)-2(-2-2r)+(2+r)+15=0 \\ 6+4r+4+4r+2+r+15=0 \\ 9r=-27 \\ r=-3 \Rightarrow M(-3,4,-1) \end{array}$$



$$\vec{PM}=(-3-3,\ 4+2,\ -1-2)=(-6,6,-3)$$

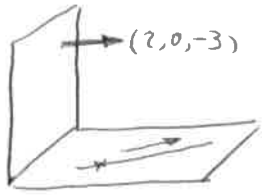
$$P'=P+2\cdot\vec{PM}=(3,-2,2)+2(-6,6,-3)=\boxed{(-9,10,-4)}$$

(13)

$$\begin{vmatrix} x+2 & y-1 & z-1 \\ 1 & 2 & -2 \\ 2 & 0 & -3 \end{vmatrix}=0$$

$$-6(x+2)-1(y-1)-4(z-1)=0$$

$$\boxed{-6x-y-4z-7=0}$$

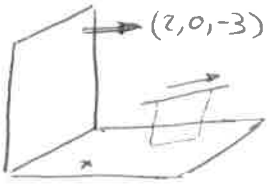


(14)

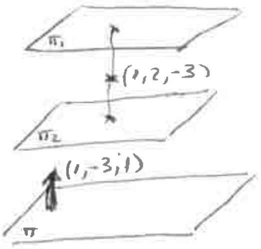
$$\begin{vmatrix} x-1 & y-1 & z-3 \\ 1 & 2 & -2 \\ 2 & 0 & -3 \end{vmatrix}=0$$

$$-6(x-1)-1(y-1)-4(z-3)=0$$

$$\boxed{-6x-y-4z+19=0}$$



(15) π_1 y π_2 tienen de ecuación: $x-3y+z+D=0$



$$d(P(1,2,-3);\ x-3y+z+D=0)=\sqrt{44}$$

$$\frac{|1-6-3+D|}{\sqrt{1+9+1}}=\sqrt{44}$$

$$|D-8|=22 \to \begin{cases} D-8=22; & D=30 \\ D-8=-22; & D=-14 \end{cases}$$

$$\boxed{\begin{array}{l} \pi_1 \equiv x-3y+z+30=0 \\ \pi_2 \equiv x-3y+z-14=0 \end{array}}$$

(16)

$$2(4+3r)-3(1-r)+(-3r)+1=0$$

$$8+6r-3+\cancel{3r}-\cancel{3r}+1=0\ ;\ 6r=-6\ ;\ r=-1 \longrightarrow \boxed{P(1,2,3)}$$

La recta corta al plano en el punto $P(1,2,3)$

(17) a) $\vec{AB}=(4,2,-2)$

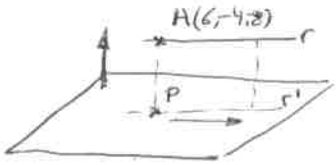
$$r\equiv \frac{x-6}{4}=\frac{y+4}{2}=\frac{z-8}{-2} \longrightarrow \left.\begin{array}{l} x=6+4r \\ y=-4+2r \\ z=8-2r \end{array}\right\}$$

$$2(6+4r)-3(-4+2r)+(8-2r)-4=0$$

$$12+8r+12-6r+8-2r-4=0$$

$$0\cdot r=-28$$

Absurdo. La recta y el plano son paralelos.



b)

$$\left.\begin{array}{l} x=6+2r \\ y=-4-3r \\ z=8+r \\ 2x-3y+z-4=0 \end{array}\right\} \quad \begin{array}{l} 12+4r+12+9r+8+r-4=0 \\ 14r=-28 \\ r=-2 \Rightarrow P(2,2,6) \Rightarrow \end{array} \quad \boxed{r'\equiv \begin{cases} x=2+2r \\ y=2-3r \\ z=6+r \end{cases}}$$

18 a) $2x-3z-2+k\cdot(-2x-2y+z)=0 \quad (k\in\mathbb{R})$

$\boxed{(2-2k)x-2ky+(k-3)z-2=0}$ Son todos los planos del haz salvo $-2x-2y+z=0$

b) $(2-2k)\cdot 1-2k\cdot 1+(k-3)\cdot 1-2=0$

$2-2k-2k+k-3-\cancel{2}=0 \;;\; -3k=3 \;;\; k=-1 \longrightarrow \boxed{4x+2y-4z-2=0}$

19 $\boxed{\frac{x-4}{3}=\frac{y-1}{-1}=\frac{z}{-3}}$

$-x+4=3y+3 \qquad\qquad -3y+3=-z$

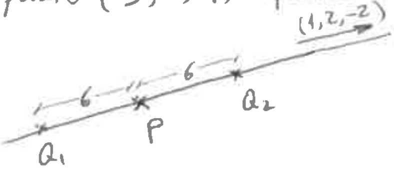
$-x-3y+1=0 \qquad\qquad -3y+z+3=0$

$-x-3y+1+k\cdot(-3y+z+3)=0 \quad (k\in\mathbb{R})$

$\boxed{-x-(3k+3)y+kz+(3k+1)=0}$ Son todos los planos del haz salvo $-3y+z+3=0$

20 $-3+2=\frac{-1-1}{2}=\frac{1+1}{-2}$

$-1=-1=-1$ ✓ Es decir, que el punto $(-3,-1,1)$ pertenece a la recta.



$Q=(-2+r,1+2r,-1-2r)$

$\vec{PQ}=(-2+r+3,1+2r+1,-1-2r-1)=$

$=(1+r,2+2r,-2-2r)$

$|\vec{PQ}|=6 \Rightarrow \sqrt{(1+r)^2+(2+2r)^2+(-2-2r)^2}=6$

$1+2r+r^2+4+8r+4r^2+4+8r+4r^2=36$

$9r^2+18r+9=36$

$9r^2+18r-27=0$

$r^2+2r-3=0 \;;\; r=\begin{cases}1 \to \\ -3 \to\end{cases}$ $\boxed{\begin{matrix}Q_1(-1,3,-3)\\ Q_2(-5,-5,5)\end{matrix}}$

También

$|1,2,-2|=\sqrt{1+4+4}=3$

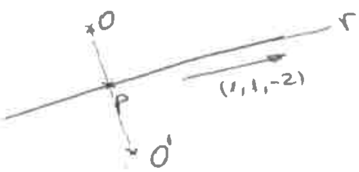
$Q=P\pm 2\cdot(1,2,-2)=$

$=(-3,-1,1)\pm(2,4,-4)=$

$=\begin{cases}(-1,3,-3)\\(-5,-5,5)\end{cases}$ ✓

21 a)

$P(6+r,7+r,4-2r)$



$\vec{OP}=(6+r,7+r,4-2r)$

$\vec{OP}\cdot(1,1,-2)=0$

$6+r+7+r-8+4r=0$

$6r=2$

$r=1/3 \Rightarrow \boxed{P\left(\frac{19}{3},\frac{22}{3},\frac{10}{3}\right)}$

b) $d(O,r)=|\vec{OP}|=\sqrt{\left(\frac{19}{3}\right)^2+\left(\frac{22}{3}\right)^2+\left(\frac{10}{3}\right)^2}=\boxed{\frac{\sqrt{945}}{3}}$

c) $O'=O+2\cdot\vec{OP}=\boxed{\left(\frac{38}{3},\frac{44}{3},\frac{20}{3}\right)}$

22 **a)**

$$\left.\begin{array}{l} \left\{\begin{array}{l} x=r\\ y=r\\ z=r \end{array}\right. \\ \left\{\begin{array}{l} x=1+t\\ y=2+2t\\ z=2t \end{array}\right. \end{array}\right\} \quad \left.\begin{array}{l} r=1+t\\ r=2+2t\\ r=2t \end{array}\right\} \quad \left.\begin{array}{l} r-t=1\\ r-2t=2\\ r-2t=0 \end{array}\right\}$$

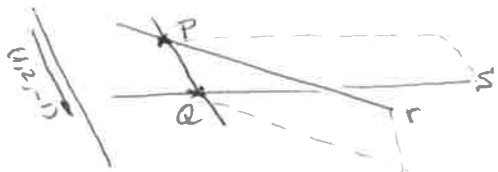
$$M=\begin{pmatrix} 1 & -1\\ 1 & -2\\ 1 & -2 \end{pmatrix} \qquad \begin{vmatrix} 1 & -1\\ 1 & -2 \end{vmatrix} = -1 \neq 0 \Rightarrow \boxed{r(M)=2}$$

$$M^{*}=\begin{pmatrix} 1 & -1 & 1\\ 1 & -2 & 2\\ 1 & -2 & 0 \end{pmatrix}$$

$$\begin{vmatrix} 1 & -1 & 1\\ 1 & -2 & 2\\ 1 & -2 & 0 \end{vmatrix} = -2-2+2+4=6\neq 0 \Rightarrow \boxed{r(M^{*})=3}$$

El sistema es incompatible. Las rectas no tienen puntos comunes. Como no son paralelas, las rectas $\boxed{\text{se cruzan}}$

b) Plano paralelo a la recta y contiene a r



$$\begin{vmatrix} x & y & z\\ 1 & 1 & 1\\ 1 & 2 & -1 \end{vmatrix}=0$$

$$\boxed{-3x+2y+z=0}$$

Plano paralelo a la recta y contiene a s

$$\begin{vmatrix} x-1 & y-2 & z\\ 1 & 2 & 2\\ 1 & 2 & -1 \end{vmatrix}=0$$

$$-6(x-1)+3(y-2)=0$$

$$-6x+\cancel{6}+3y-\cancel{6}=0 \ ; \quad \boxed{-2x+y=0}$$

Recta intersección

$$\left.\begin{array}{l} -3x+2y+z=0\\ -2x+y=0 \end{array}\right\} \quad \left.\begin{array}{l} -3x+2y=-z\\ -2x+y=0 \end{array}\right\} \quad z\in\mathbb{R}$$

$$x=\frac{\begin{vmatrix} -z & 2\\ 0 & 1 \end{vmatrix}}{\begin{vmatrix} -3 & 2\\ -2 & 1 \end{vmatrix}}=\frac{-z}{1}=-z$$

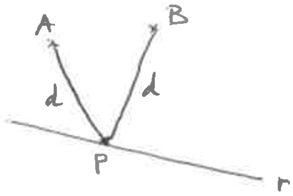
$$y=\frac{\begin{vmatrix} -3 & -z\\ -2 & 0 \end{vmatrix}}{1}=\frac{-2z}{1}=-2z$$

$$\boxed{\begin{array}{l} x=-r\\ y=-2r\\ z=r \end{array}}$$

También se puede hacer parametrizando P y Q y hallando los parámetros de manera que $\vec{PQ}$ sea paralelo a $(1,2,-1)$.

23

$$\left.\begin{array}{l} y=2+x\\ z=3+2x \end{array}\right\} \rightarrow \left.\begin{array}{l} x=r\\ y=2+r\\ z=3+2r \end{array}\right\} \rightarrow P(r,2+r,3+2r)$$



$$\vec{AP}=(r-1,2+r,2+2r)$$

$$\vec{BP}=(r,-2+r,1+2r)$$

$$|\vec{AP}|=|\vec{BP}| \Rightarrow \sqrt{(r-1)^2+(2+r)^2+(2+2r)^2}=\sqrt{r^2+(-2+r)^2+(1+2r)^2} \Rightarrow$$

$$\Rightarrow \cancel{r^2}-2r+\cancel{1}+\cancel{4}+\cancel{4r}+\cancel{r^2}+4+8r+\cancel{4r^2}=\cancel{r^2}+\cancel{4}-4r+\cancel{r^2}+\cancel{1}+\cancel{4r}+\cancel{4r^2}$$

$$10r=-4 \Rightarrow r=-4/10=-2/5 \Rightarrow \boxed{P(-2/5,\ 8/5,\ 11/5)}$$

(24)

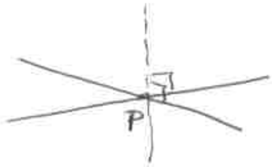
$$\left.\begin{array}{l} x=-2+r \\ y=-3+2r \\ z=1-2r \end{array}\right\} \quad \left.\begin{array}{l} x=1-t \\ y=-1+2t \\ z=-1-2t \end{array}\right\}$$

$$\left.\begin{array}{l} -2+r=1-t \\ -3+2r=-1+2t \\ 1-2r=-1-2t \end{array}\right\} ; \quad \left.\begin{array}{l} r+t=3 \\ 2r-2t=2 \\ -2r+2t=-2 \end{array}\right\} ; \quad \left.\begin{array}{l} r+t=3 \\ r-t=1 \\ \cancel{r-t=1} \end{array}\right\} \longrightarrow \boxed{\begin{array}{l} r=2 \\ t=1 \end{array}}$$

$r=2 \longrightarrow \boxed{P(0,1,-3)}$ Es el punto de corte de las dos rectas.

$t=1$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & -2 \\ -1 & 2 & -2 \end{vmatrix} = (0,4,4)$$



$$\boxed{\left.\begin{array}{l} x=0 \\ y=1+4r \\ z=-3+4r \end{array}\right\}}$$

(25)

$$M=\begin{pmatrix} 2 & -3 & 1 \\ 1 & -1 & 1 \\ 2 & 1 & 1 \end{pmatrix} \quad \begin{vmatrix} 2 & -3 \\ 1 & -1 \end{vmatrix} = 1 \Rightarrow r(M)\geq 2$$

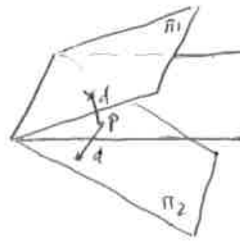
$$\begin{vmatrix} 2 & -3 & 1 \\ 1 & -1 & 1 \\ 2 & 1 & 1 \end{vmatrix} = -2+1-6+\cancel{2}+3-\cancel{2} = 2 \Rightarrow \left.\begin{array}{l} r(M)=3 \\ r(M^*)=3 \\ n=3 \end{array}\right\} \Rightarrow S.C.D.$$

Los tres planos se cortan en un punto.

Resuelto con calculadora gráfica: $\boxed{P(1,0,1)}$

(26)

$$d(P(x,y,z),\ 2x-3y+6z-3=0)=d(P(x,y,z),\ x-2y+2z-2=0)$$



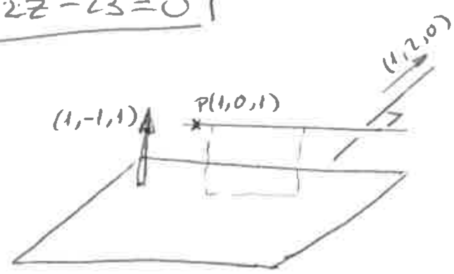
$$\frac{|2x-3y+6z-3|}{\sqrt{4+9+36}} = \frac{|x-2y+2z-2|}{\sqrt{1+4+4}}$$

$$3(2x-3y+6z-3) = \pm 7(x-2y+2z-2)$$

$$6x-9y+18z-9=7x-14y+14z-14 \qquad 6x-9y+18z-9=-7x+14y-14z+14$$

$$\boxed{-x+5y+4z+5=0} \qquad \boxed{13x-23y+32z-23=0}$$

(27)



$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & 1 \\ 1 & 2 & 0 \end{vmatrix} = (-2,1,3)$$

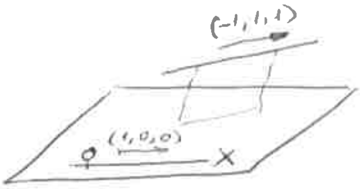
$$\left.\begin{array}{l} x=1-2r \\ y=r \\ z=3+r \end{array}\right\}$$

$$O(0,0,0) \longrightarrow \begin{array}{l} 0=1-2r \\ 0=r \\ 0=3+r \end{array} \begin{array}{l} \to r=1/2 \\ \to r=0 \\ \to r=-3 \end{array}$$ No pertenece a la recta.

(28)

$$\begin{vmatrix} x & y & z \\ 1 & -2 & 3 \\ -3 & 2 & -1 \end{vmatrix} = 0 \ ; \ \boxed{-4x-8y-4z=0}$$

(29) $\left.\begin{array}{l} y=2+x\\ z=3+y\end{array}\right\} \to \left.\begin{array}{l}x=2-y\\ z=3+y\end{array}\right\} \to \left.\begin{array}{l}x=2-r\\ y=r\\ z=3+r\end{array}\right\} \to \vec{v}_1(-1,1,1)$



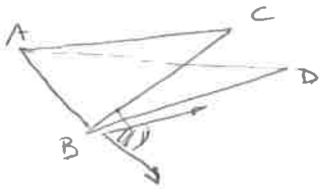
Eje X $\to$ $\vec{v}_2(1,0,0)$

$$\begin{vmatrix} x & y & z\\ -1 & 1 & 1\\ 1 & 0 & 0\end{vmatrix}=0 \ ; \ \boxed{y-z=0}$$

(30) a) $\left.\begin{array}{l}\vec{BD}=(3,1,-2)\\ \vec{BC}=(3,0,1)\end{array}\right| \quad \cos\alpha=\dfrac{(3,1,-2)(3,0,1)}{\sqrt{9+1+4}\cdot\sqrt{9+0+1}}=\dfrac{7}{\sqrt{140}} \Rightarrow \boxed{\alpha=\quad}$

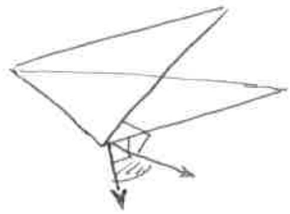
b) $\left.\begin{array}{l}\vec{AB}=(-2,0,-1)\\ \vec{AC}=(1,0,0)\end{array}\right| \quad \begin{vmatrix} \vec{i} & \vec{j} & \vec{k}\\ -2 & 0 & -1\\ 1 & 0 & 0\end{vmatrix}=(0,1,0)$

$$\cos\alpha=\frac{(3,1,-2)\cdot(0,1,0)}{\sqrt{13}\ \sqrt{1}}=\frac{1}{\sqrt{13}} \Rightarrow \boxed{\alpha=\quad}$$



c) $\left.\begin{array}{l}\vec{AB}=(3,1,-2)\\ \vec{AD}=(1,1,-3)\end{array}\right| \quad \begin{vmatrix} \vec{i} & \vec{j} & \vec{k}\\ 3 & 1 & -2\\ 1 & 1 & -3\end{vmatrix}=(-1,7,2)$

$$\cos\alpha=\frac{(-1,7,2)\cdot(0,1,0)}{\sqrt{1+49+4}\cdot\sqrt{1}}=\frac{7}{\sqrt{54}} \Rightarrow \boxed{\alpha=\quad}$$



(31) $\vec{u}+\vec{v}+\vec{w}=\vec{0}$

- $(\vec{u}+\vec{v}+\vec{w})\times\vec{v}=\vec{0}\times\vec{v} \Rightarrow \vec{u}\times\vec{v}+\underbrace{\vec{v}\times\vec{v}}_{=\vec{0}}+\vec{w}\times\vec{v}=\vec{0}\ ;\ \vec{u}\times\vec{v}+\vec{w}\times\vec{v}=\vec{0} \Rightarrow$

  $\Rightarrow \vec{u}\times\vec{v}=-\vec{w}\times\vec{v}\ ;\ \boxed{\vec{u}\times\vec{v}=\vec{v}\times\vec{w}}$ ✓

- $\vec{u}\times(\vec{u}+\vec{v}+\vec{w})=\vec{u}\times\vec{0} \Rightarrow \underbrace{\vec{u}\times\vec{u}}_{=\vec{0}}+\vec{u}\times\vec{v}+\vec{u}\times\vec{w}=\vec{0}\ ;\ \vec{u}\times\vec{v}=-\vec{u}\times\vec{w} \Rightarrow$

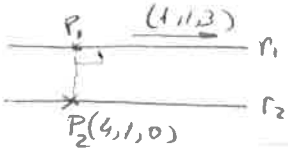
  $\Rightarrow \boxed{\vec{u}\times\vec{v}=\vec{w}\times\vec{u}}$ ✓

(32) a) $\left.\begin{array}{l} r_1 \to \vec{v}_1(1,1,3)\\ r_2 \to \vec{v}_2(-1,-1,-3)\end{array}\right| \quad \dfrac{1}{-1}=\dfrac{1}{-1}=\dfrac{3}{-3} \Rightarrow$ vectores paralelos

$P_2(4,1,0)\in r_2 \longrightarrow 4-2 \overset{¿?}{=} \dfrac{1-1}{1} \overset{¿?}{=} \dfrac{0+3}{3}\ ;\ 2\neq 0\neq 1 \Rightarrow P_2\notin r_1$

Por lo tanto r_1 y r_2 son paralelas pero no coincidentes

b) $P_1(2+r, 1+r, -3+3r)$



$\vec{P_2P_1}=(2+r-4, 1+r-1, -3+3r-0)=$
$=(-2+r, r, -3+3r)$

$\vec{P_2P_1}\cdot(1,1,3) \Rightarrow -2+r+r-9+9r=0\ ;\ r=1 \Rightarrow P_1(3,2,0)$

$\vec{P_2P_1}=(-1,1,0)$

$d(r_1,r_2)=|\vec{P_2P_1}|=\sqrt{1+1+0}=\boxed{\sqrt{2}}$

33) **a)**

$$\left.\begin{array}{l} \left.\begin{array}{l} x=-2+r \\ y=5-2r \\ z=r \end{array}\right\} \\ \left.\begin{array}{l} x=2+t \\ y=1+t \\ z=-3+4t \end{array}\right\} \end{array}\right\} \quad \left.\begin{array}{l} -2+r=2+t \\ 5-2r=1+t \\ r=-3+4t \end{array}\right\} \quad \left.\begin{array}{l} r-t=4 \\ -2r-t=-4 \\ r-4t=-3 \end{array}\right\}$$

$$M^{*}=\begin{pmatrix} 1 & -1 \\ -2 & -1 \\ 1 & -4 \end{pmatrix} \quad \begin{vmatrix} 1 & -1 \\ -2 & -1 \end{vmatrix} = -3 \neq 0 \Rightarrow r(M)=2$$

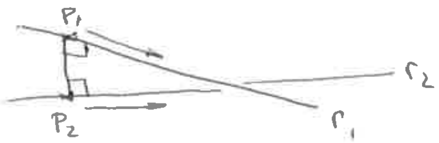
$$M^{*}=\begin{pmatrix} 1 & -1 & 4 \\ -2 & -1 & -4 \\ 1 & -4 & -3 \end{pmatrix}$$

$$|M^{*}| = 3+32+4+4-6-16 = 12 \neq 0 \Rightarrow r(M^{*})=3$$

El sistema es incompatible. Las rectas no tienen puntos comunes.
Como no son paralelas, $\boxed{\text{se cruzan.}}$

b)

$P_1(-2+r, 5-2r, r)$
$P_2(2+t, 1+t, -3+4t)$



$$\overrightarrow{P_1P_2} = (4+t-r, -4+t+2r, -3+4t-r)$$

$$\left.\begin{array}{l} \overrightarrow{P_1P_2}\cdot(1,-2,1)=0 \Rightarrow 4+t-r+8-2t-4r-3+4t-r=0 \\ \overrightarrow{P_1P_2}\cdot(1,1,4)=0 \Rightarrow \cancel{4}+t-r-\cancel{4}+t+2r-12+16t-4r=0 \end{array}\right\}$$

$$\left.\begin{array}{l} 3t-6r=-9 \\ 18t-3r=12 \end{array}\right\} \quad \begin{array}{l} t=1 \\ r=2 \end{array} \rightarrow \begin{array}{l} \overrightarrow{P_1P_2} = (3,1,-1) \\ P_1 = (0,1,2) \\ P_2 = (3,2,1) \end{array}$$

$$\boxed{\left.\begin{array}{l} x=3r \\ y=1+r \\ z=2-r \end{array}\right\}}$$

c) $d = |\overrightarrow{P_1P_2}| = \sqrt{9+1+1} = \boxed{\sqrt{11}}$

El apartado (b) también se puede hacer por intersección de dos planos, pero no daría datos para el apartado (c).

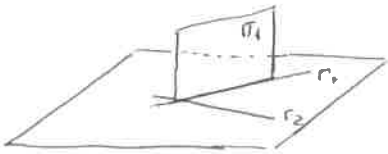
34) **a)**

$$\left.\begin{array}{l} P(2,1,-3)\in r_1 \\ P(2,1,-3)\in r_2 \end{array}\right\} \Rightarrow r_1 \text{ y } r_2, \text{ ó son coincidentes, ó se cortan en } P(2,1,-3)$$

$$\left.\begin{array}{l} \vec{u}_1(-1,2,2) \\ \vec{u}_2(3,-2,6) \end{array}\right\} \Rightarrow \vec{u}_1 \text{ y } \vec{u}_2 \text{ no son paralelos.}$$

Por lo tanto, r_1 y r_2 se cortan en el punto $P(2,1,-3)$

b)



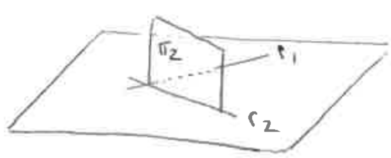
$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 2 & 2 \\ 3 & -2 & 6 \end{vmatrix} = (16, 12, -4)$$

$$\begin{vmatrix} x-2 & y-1 & z+3 \\ 16 & 12 & -4 \\ -1 & 2 & 2 \end{vmatrix} = 0$$

$$32(x-2)-28(y-1)+44(z+3)=0$$

$$8(x-2)-7(y-1)+11(z+3)=0 \Rightarrow \boxed{\pi_1 \equiv 8x-7y+11z+24=0}$$

c)



$$\begin{vmatrix} x-2 & y-1 & z+3 \\ 16 & 12 & -4 \\ 3 & -2 & 6 \end{vmatrix} = 0$$

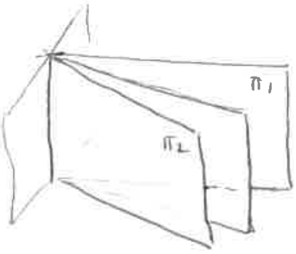
$$64(x-2)-108(y-1)-68(z+3)=0$$

$$16(x-2)-27(y-1)-17(z+3)=0$$

$$\boxed{\pi_2 \equiv 16x-27y-17z-56=0}$$

d) $d(P(x,y,z), \pi_1) = d(P(x,y,z), \pi_2)$

$$\frac{|8x-7y+11z+24|}{\sqrt{8^2+7^2+11^2}} = \frac{|16x-27y-17z-56|}{\sqrt{16^2+27^2+17^2}}$$



$$\frac{8x-7y+11z+24}{3\cancel{\sqrt{26}}} = \pm\frac{16x-27y-17z-56}{7\cancel{\sqrt{26}}}$$

$$56x-49y+77z+168 = \pm(48x-81y-51z-168)$$

$8x+32y+128z+336=0$; $104x-130y+26z=0$

$\boxed{x+4y+16z+42=0}$; $\boxed{4x-5y+z=0}$

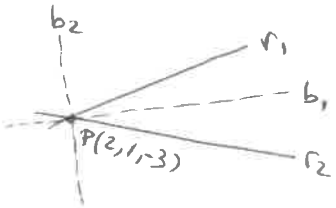
e) Plano que contiene a r_1 y r_2:
$$\begin{vmatrix} x+2 & y-1 & z+3 \\ -1 & 2 & 2 \\ 3 & -2 & 6 \end{vmatrix} = 0$$

$$16(x+2)+12(y-1)-4(z+3)=0$$
$$4(x+2)+3(y-1)-(z+3)=0$$
$$\boxed{4x+3y-z+2=0}$$

Bisectrices:

$\left.\begin{array}{l} x+4y+16z+42=0 \\ 4x+3y-z+2=0 \end{array}\right\} \longrightarrow \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 4 & 16 \\ 4 & 3 & -1 \end{vmatrix} = (-52,65,-13)$

Paralelo: $(-4,5,-1)$.



$$\boxed{b_1 \equiv \begin{cases} x=2-4r \\ y=1+5r \\ z=-3-r \end{cases}}$$

$\left.\begin{array}{l} 4x-5y+z=0 \\ 4x+3y-z+2=0 \end{array}\right\} \longrightarrow \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & -5 & 1 \\ 4 & 3 & -1 \end{vmatrix} = (2,8,32)$

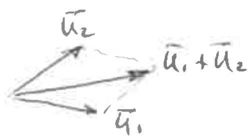
Paralelo: $(1,4,16)$

$$\boxed{b_2 \equiv \begin{cases} x=2+r \\ y=1+4r \\ z=-3+16r \end{cases}}$$

35 a) $|-1,2,2| = \sqrt{9} = 3 \Rightarrow \bar{u}_1 = \frac{1}{3}(-1,2,2) = \boxed{(-1/3, 2/3, 2/3)}$

$|3,-2,6| = \sqrt{49} = 7 \Rightarrow \bar{u}_2 = \frac{1}{7}(3,-2,6) = \boxed{(3/7, -2/7, 6/7)}$

b) Al tener $\bar{u}_1$ y $\bar{u}_2$ el mismo módulo, $\bar{u}_1+\bar{u}_2$ formará idéntico ángulo con $\bar{u}_1$ que con $\bar{u}_2$



c) $\bar{u}_1+\bar{u}_2 = \left(-\frac{1}{3}+\frac{3}{7}, \frac{2}{3}-\frac{2}{7}, \frac{2}{3}+\frac{6}{7}\right) = \left(\frac{2}{21}, \frac{8}{21}, \frac{32}{21}\right)$

Paralelo sería: $(1,4,16) \Longrightarrow \boxed{b_2 \equiv \begin{cases} x=2+r \\ y=1+4r \\ z=-3+16r \end{cases}}$ ✓

d) $\bar{u}_1-\bar{u}_2$ (ó $\bar{u}_2-\bar{u}_1$) nos serviría para la otra bisectriz:

$\bar{u}_1-\bar{u}_2 = \left(-\frac{1}{3}-\frac{3}{7}, \frac{2}{3}+\frac{2}{7}, \frac{2}{3}-\frac{6}{7}\right) = \left(-\frac{16}{21}, \frac{20}{21}, \frac{-4}{21}\right)$

Paralelo sería: $(-4,5,-1) \Longrightarrow \boxed{b_1 \equiv \begin{cases} x=2-4r \\ y=1+5r \\ z=-3-r \end{cases}}$ ✓

(36) $5x+1 = 9-5y = -5z = r \Rightarrow \begin{cases} x = \frac{r-1}{5} \\ y = \frac{9-r}{5} \\ z = -r/5 \end{cases}$; $\boxed{\begin{aligned} x &= -\frac{1}{5} + \frac{1}{5}r \\ y &= \frac{9}{5} - \frac{1}{5}r \\ z &= -\frac{1}{5}r \end{aligned}}$

La recta es paralela al vector $(1,-1,-1)$

$\pi_1 \equiv 2x+3y-z=5 \longrightarrow (2,3,-1)$; $(2,3,-1)\cdot(1,-1,-1) = 2-3+1=0$ ✓

$\pi_2 \equiv x-y+2z=k \longrightarrow (1,-1,2)$; $(1,-1,2)\cdot(1,-1,-1) = 1+1-2=0$ ✓

Por lo tanto la recta, ó es paralela a π_1 y a π_2, ó está contenida en ambos.

$P(-1/5, 9/5, 0) \in$ Recta.

$2\cdot\frac{-1}{5} + 3\cdot\frac{9}{5} - 0 = -\frac{2}{5} + \frac{27}{5} = \frac{25}{5} = 5$ ✓ $\Rightarrow P \in \pi_1$

$-\frac{1}{5} - \frac{9}{5} + 2\cdot 0 = -\frac{10}{5} = -2 \Rightarrow \boxed{K=-2}$ para que $P \in \pi_2$

(37) $|\vec{a}| = 3 \Rightarrow \vec{a}\cdot\vec{a} = 3^2 = 9$

$|\vec{b}| = 1 \Rightarrow \vec{b}\cdot\vec{b} = 1^2 = 1$

$|\vec{c}| = 4 \Rightarrow \vec{c}\cdot\vec{c} = 4^2 = 6$

$\vec{a}+\vec{b}+\vec{c} = 0 \Rightarrow (\vec{a}+\vec{b}+\vec{c})\cdot(\vec{a}+\vec{b}+\vec{c}) = 0$

$\vec{a}\cdot\vec{a} + 2(\vec{a}\cdot\vec{b}) + 2(\vec{a}\cdot\vec{c}) + \vec{b}\cdot\vec{b} + 2(\vec{b}\cdot\vec{c}) + \vec{c}\cdot\vec{c} = 0$

$9 + 2\{\vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{c} + \vec{b}\cdot\vec{c}\} + 1 + 6 = 0$

$\boxed{\vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{c} + \vec{b}\cdot\vec{c} = -8}$