(1)

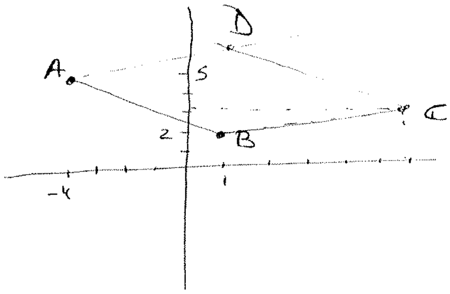
$\vec{BC} = \begin{pmatrix} 6-1 \\ 3-2 \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$

$\vec{OD} = \vec{OA} + \begin{pmatrix} 5 \\ 1 \end{pmatrix} = \begin{pmatrix} -4 \\ 5 \end{pmatrix} + \begin{pmatrix} 5 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 6 \end{pmatrix}$

$\boxed{D = (1,6)}$

$\vec{CB} = \begin{pmatrix} 1-6 \\ 2-3 \end{pmatrix} = \begin{pmatrix} -5 \\ -1 \end{pmatrix}$

$\vec{OD} = \vec{OA} + \begin{pmatrix} -5 \\ -1 \end{pmatrix} = \begin{pmatrix} -4 \\ 5 \end{pmatrix} + \begin{pmatrix} -5 \\ -1 \end{pmatrix} = \begin{pmatrix} -9 \\ 4 \end{pmatrix}$ $\boxed{D = (-9,4)}$



(2)

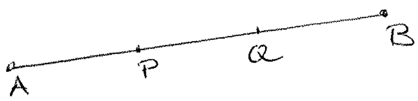
$y = \dfrac{10-5x}{8} \longrightarrow m = -\dfrac{5}{8} \Rightarrow tg\,\alpha = -\dfrac{5}{8} \Rightarrow \alpha \simeq -32° \equiv \boxed{148°}$

(3)

$\vec{AB} = \begin{pmatrix} 6-3 \\ 15-5 \end{pmatrix} = \begin{pmatrix} 3 \\ 10 \end{pmatrix}$

$\vec{OP} = \vec{OA} + \frac{1}{3}\vec{AB} = \begin{pmatrix} 3 \\ 5 \end{pmatrix} + \begin{pmatrix} 1 \\ 10/3 \end{pmatrix} = \begin{pmatrix} 4 \\ 25/3 \end{pmatrix}$

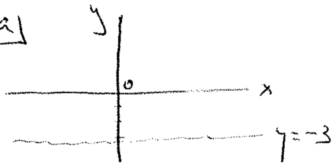
$\vec{OQ} = \vec{OA} + \frac{2}{3}\vec{AB} = \begin{pmatrix} 3 \\ 5 \end{pmatrix} + \begin{pmatrix} 2 \\ 20/3 \end{pmatrix} = \begin{pmatrix} 5 \\ 35/3 \end{pmatrix}$



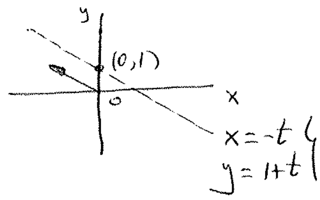
$\boxed{P(4, \frac{25}{3})}$ $\boxed{Q(5, \frac{35}{3})}$

(4)

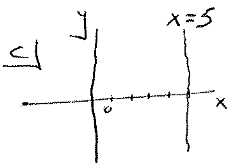
a)



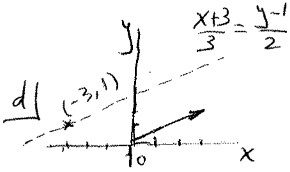
b)



c)



d)

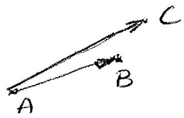


(5)

$\vec{AB} = \begin{pmatrix} 5+1/2 \\ -1-2 \end{pmatrix} = \begin{pmatrix} 11/2 \\ -3 \end{pmatrix} \Rightarrow$ pendiente $= \dfrac{-3}{11/2} = -\dfrac{6}{11}$

$\vec{AC} = \begin{pmatrix} 3+1/2 \\ -1/3-2 \end{pmatrix} = \begin{pmatrix} 7/2 \\ -7/3 \end{pmatrix} \Rightarrow$ pendiente $= \dfrac{-7/3}{7/2} = -\dfrac{2}{3}$

Al ser distintas no están alineados



(6)

$-x+y+3=0 \longrightarrow y = x-3 \longrightarrow$ pendiente $= 1 \longrightarrow y+8 = 1\cdot(x-1)$; $\boxed{-x+y+9=0}$

También: $-x+y+C=0$; $-1-8+C=0 \Rightarrow C=9 \longrightarrow -x+y+9=0$ ✓

(7)

$y = \dfrac{3-5x}{6} \Rightarrow m_1 = -\dfrac{5}{6} \longrightarrow \vec{v}_1 = \begin{pmatrix} 6 \\ -5 \end{pmatrix}$

$\left.\begin{array}{l} x = -1+2r \\ y = 3-r \end{array}\right\} \Rightarrow \vec{v}_2 = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$

$\cos\theta = \dfrac{\begin{pmatrix} 6 \\ -5 \end{pmatrix}\cdot\begin{pmatrix} 2 \\ -1 \end{pmatrix}}{\left|\begin{pmatrix} 6 \\ -5 \end{pmatrix}\right| \cdot \left|\begin{pmatrix} 2 \\ -1 \end{pmatrix}\right|} = \dfrac{12+5}{\sqrt{36+25}\sqrt{4+1}} = \dfrac{17}{\sqrt{305}} \Rightarrow \boxed{\theta = 13°}$

También: $\left.\begin{array}{l} m_1 = -5/6 \\ m_2 = -1/2 \end{array}\right\} \Rightarrow tg\,\theta = \dfrac{-1/2 + 5/6}{1 + \frac{-5}{6}\cdot\frac{-1}{2}} = \dfrac{2/6}{1+5/12} = \dfrac{4}{17} \Rightarrow \theta = 13°$ ✓

(8)

a) $\left.\begin{array}{l} 6x+2y-5=0 \\ 3x+y+7=0 \end{array}\right\}$ $\dfrac{6}{3} = \dfrac{2}{1} \neq \dfrac{-5}{7} \Rightarrow$ PARALELAS

b) $\left.\begin{array}{l} x+3y-4=0 \\ x-3y-5=0 \end{array}\right\}$ $\dfrac{1}{1} \neq \dfrac{1}{-3} \Rightarrow$ SECANTES

c) $\left.\begin{array}{l} -x-y+3=0 \\ 2x+2y-6=0 \end{array}\right\}$ $-\dfrac{1}{2} = -\dfrac{1}{2} = \dfrac{3}{-6} \Rightarrow$ COINCIDENTES

(9) $\left.\begin{array}{l}4x+y-14=0\\7x+2y-28=0\end{array}\right\} \rightarrow \begin{array}{l}8x+2y-28=0\\7x+2y-28=0\\\hline \boxed{x\quad=0}\end{array} \rightarrow \boxed{y=14}$ $\quad P(0,14)$

$(0,14)\in 3x+my-7=0 \Rightarrow 0+14m-7=0 \Rightarrow \boxed{m=\frac{1}{2}}$

(10) $r_1 \equiv x=3$

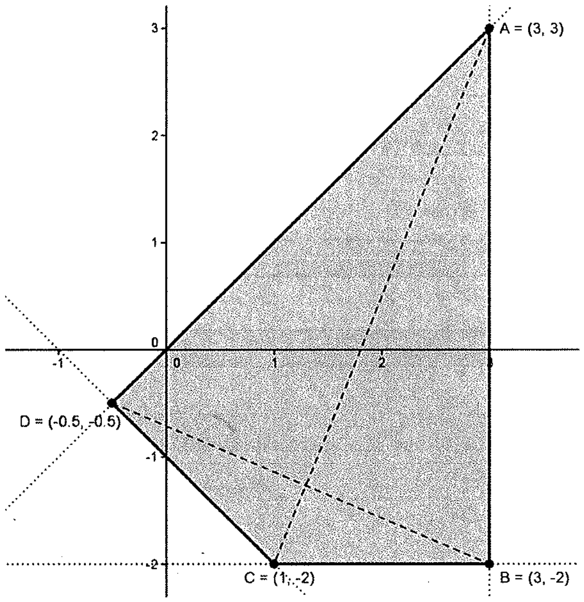
$r_2 \equiv y-x=0$

$r_3 \equiv y+x+1=0$

$r_4 \equiv y+2=0$

$\left.\begin{array}{l}r_1\\r_2\end{array}\right\}\Rightarrow (3,3)$ $\quad \left.\begin{array}{l}r_1\\r_3\end{array}\right\}\Rightarrow \cancel{(3,-4)}$ $\quad \left.\begin{array}{l}r_1\\r_4\end{array}\right\}\Rightarrow (3,-2)$

$\left.\begin{array}{l}r_2\\r_3\end{array}\right\}\Rightarrow \left(-\frac{1}{2},-\frac{1}{2}\right)$ $\quad \left.\begin{array}{l}r_2\\r_4\end{array}\right\}\Rightarrow \cancel{(-2,-2)}$ $\quad \left.\begin{array}{l}r_3\\r_4\end{array}\right\}\Rightarrow (1,-2)$



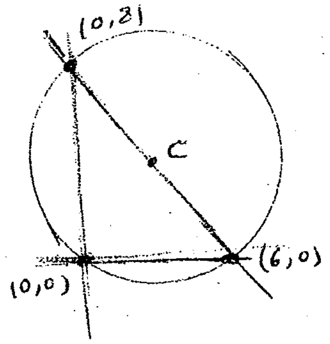
$\vec{AC} = (-2,-5)$ $\quad \frac{x-3}{2}=\frac{y-3}{5} \rightarrow \boxed{y=\frac{5x-9}{2}}$

$\vec{DB} = \left(3+\frac{1}{2}, -2+\frac{1}{2}\right) = \left(7/2, -3/2\right)$ $\quad \frac{x-3}{7}=\frac{y+2}{-3}$

$\boxed{y=\frac{-3x-5}{7}}$

$\boxed{\text{Diagonales: } \begin{array}{l}-5x+2y+9=0\\3x+7y+5=0\end{array}}$

(11)



$4x+3y-24=0$ $\quad \begin{array}{l}x=0 \Rightarrow y=8 \quad (0,8)\\ y=0 \Rightarrow x=6 \quad (6,0)\end{array}$

Como es un Triángulo Rectángulo, el punto medio es el centro de la circunferencia: $C(3,4)$

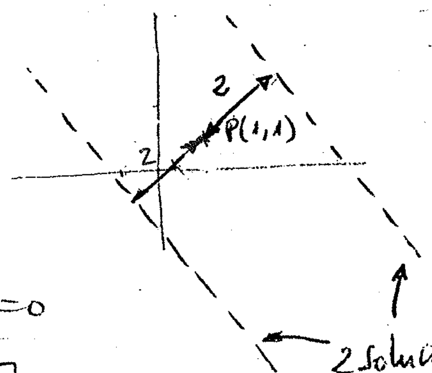
$R=\sqrt{3^2+4^2}=5$ $\quad \boxed{\text{Area} = 25\pi \text{ u.s.}}$

(12) $y=\frac{2-4x}{3} \rightarrow 3y=2-4x \rightarrow 4x+3y-2=0$

$4x+3y+C=0$

$2=\frac{|4+3+C|}{\sqrt{16+9}} \Rightarrow |7+C|=10$

$7+C=10$	$7+C=-10$
$\boxed{C=3}$	$\boxed{C=-17}$
$4x+3y+3=0$	$4x+3y-17=0$
$\boxed{y=\frac{-4x-3}{3}}$	$\boxed{y=\frac{17-4x}{3}}$



2 soluciones

(13) $\dfrac{|3x-y-1|}{\sqrt{9+1}} = \dfrac{|-6x+2y-8|}{\sqrt{36+4}}$; $\dfrac{3x-y-1}{\sqrt{10}} = \pm\dfrac{-6x+2y-8}{\sqrt{40}}$; $\dfrac{3x-y-1}{\cancel{\sqrt{10}}} = \pm\dfrac{-6x+2y-8}{2\cancel{\sqrt{10}}}$;

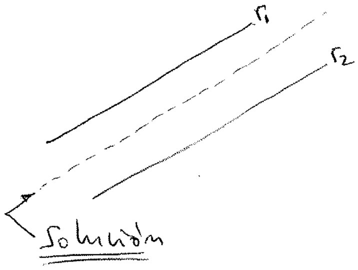
$3x-y-1 = \pm(-3x+y-4)$

$3x-y-1=-3x+y-4$

$\boxed{6x-2y+3=0}$

$\cancel{3x}-\cancel{y}-1=\cancel{3x}-\cancel{y}+4$

Absurdo



Al ser paralelas las rectas dadas, sólo habrá una recta solución que será también paralela.

(14) $\dfrac{|2x+y-3|}{\sqrt{4+1}} = 3\,\dfrac{|2x+4y-6|}{\sqrt{4+16}}$; $\dfrac{2x+y-3}{\sqrt{5}} = \pm 3\,\dfrac{2x+4y-6}{\sqrt{20}}$; $\dfrac{2x+y-3}{\cancel{\sqrt{5}}} = \pm 3\,\dfrac{2x+4y-6}{2\cancel{\sqrt{5}}}$;

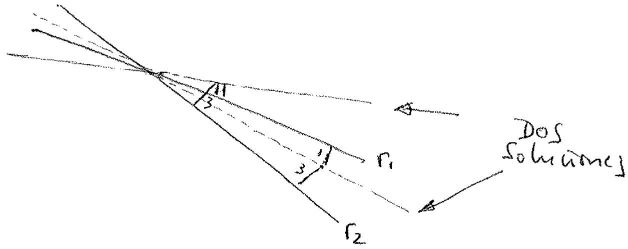
$2x+y-3 = \pm(3x+6y-9)$

$2x+y-3=3x+6y-9$

$\boxed{0=x+5y-6}$

$2x+y-3=-3x-6y+9$

$\boxed{5x+7y-12=0}$



(15) Base

$\vec{AC} = \begin{pmatrix} 4+3 \\ -1+4 \end{pmatrix} = \begin{pmatrix} 7 \\ 3 \end{pmatrix}$

$|\vec{AC}| = \sqrt{49+9} = \sqrt{58}$

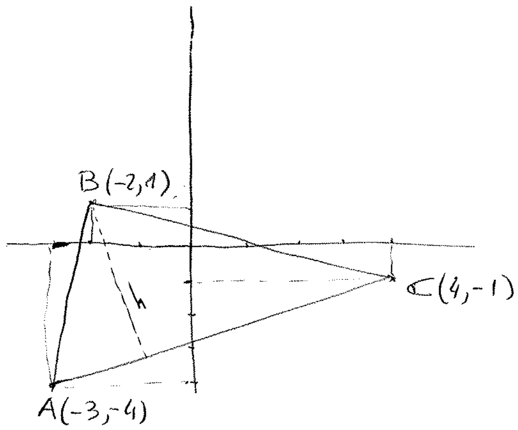
Ecuación Base

$\vec{AC} = \begin{pmatrix} 7 \\ 3 \end{pmatrix} \longrightarrow m = \frac{3}{7}$

$y+1 = \frac{3}{7}(x-4)$

$7y+7 = 3x-12$

$-3x+7y+19=0$



Altura

$h = d(B; AC) = \dfrac{|-3\cdot(-2)+7\cdot 1+19|}{\sqrt{9+49}} = \dfrac{32}{\sqrt{58}}$

$\text{Área} = \dfrac{\cancel{\sqrt{58}}\cdot\dfrac{32}{\cancel{\sqrt{58}}}}{2} = \boxed{16 \text{ u.s.}}$

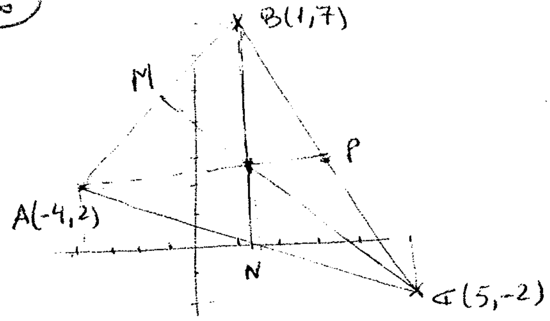
También

$\cos\hat{A} = \dfrac{\vec{AB}\cdot\vec{AC}}{|\vec{AB}|\,|\vec{AC}|} = \dfrac{\begin{pmatrix}1\\5\end{pmatrix}\cdot\begin{pmatrix}7\\3\end{pmatrix}}{\left|\begin{matrix}1\\5\end{matrix}\right|\,\left|\begin{matrix}7\\3\end{matrix}\right|} = \dfrac{7+15}{\sqrt{1+25}\sqrt{49+9}} = \dfrac{22}{\sqrt{1508}}$

$\sin\hat{A} = \sqrt{1-\left(\dfrac{22}{\sqrt{1508}}\right)^2} = \sqrt{\dfrac{1024}{1508}} = \dfrac{32}{\sqrt{1508}}$

$\text{Área} = \dfrac{\overline{AC}\cdot\overline{AB}\cdot\sin\hat{A}}{2} = \dfrac{\cancel{\sqrt{58}}\cdot\cancel{\sqrt{26}}\cdot 32/\cancel{\sqrt{1508}}}{2} = 16$ ✓

16



$M = \left(-\frac{3}{2}, \frac{9}{2}\right)$

$\vec{MC} = \left(5+\frac{3}{2}, -2-\frac{9}{2}\right) = \left(\frac{13}{2}, -\frac{13}{2}\right) \rightarrow$ Pend $= -1$

Mediana vértice C: $y+2 = -1(x-5)$ $\boxed{y = 3-x}$

$N = \left(1/2, 0\right)$

$\vec{BN} = \left(\frac{1}{2}-1, 0-7\right) = \left(-\frac{1}{2}, -7\right) \rightarrow$ Pend $= 14$

Mediana vértice B: $y-7 = 14(x-1)$ $\boxed{y = 14x-7}$

$P = \left(3, 5/2\right)$

$\vec{AP} = \left(3+4, \frac{5}{2}-2\right) = \left(7, \frac{1}{2}\right) \rightarrow$ Pend $= \frac{1}{14}$

Mediana vértice C: $y-2 = \frac{1}{14}(x+4)$ $\boxed{y = \frac{x+32}{14}}$

$$\left.\begin{array}{l} y = 3-x \\ y = 14x-7 \end{array}\right\} \rightarrow 3-x = 14x-7$$

$-15x = -10$

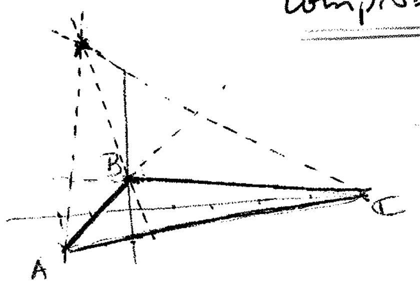
$x = \frac{10}{15} = \frac{2}{3}$

$y = 3-\frac{2}{3} = \frac{7}{3}$

$\boxed{\text{Baricentro} = \left(\frac{2}{3}, \frac{7}{3}\right)}$

Comprobación: $x = \frac{2}{3} \rightarrow y = \frac{2/3+32}{14} = \frac{7}{3}$ ✓

17



A(-2,-1)
B(0,1)
C(5,0)

$\vec{AB} = (2, 2) \rightarrow$ Pend $= 1$

Altura vértice C: $y = -(x-5)$ $\boxed{y = -x+5}$

$\vec{AC} = (7, 1) \rightarrow$ Pend $= \frac{1}{7}$

Altura vértice B: $y-1 = -7x$ $\boxed{y = 1-7x}$

$\vec{BC} = (5, -1) \rightarrow$ Pend $= -\frac{1}{5}$

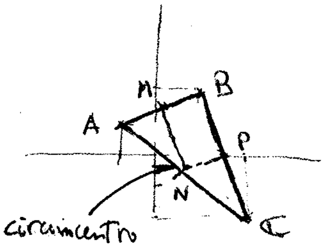
Altura vértice A: $y+1 = 5(x+2)$ $\boxed{y = 5x+9}$

$$\left.\begin{array}{l} y = -x+5 \\ y = 1-7x \end{array}\right\} \quad -x+5 = 1-7x \quad ; \quad 6x = -4 \quad x = -\frac{2}{3} \Rightarrow y = \frac{2}{3}+5 = \frac{17}{3}$$

$\boxed{\text{Ortocentro} \left(-\frac{2}{3}, \frac{17}{3}\right)}$

Comprobación: $x = -\frac{2}{3} \rightarrow y = 5\cdot\frac{-2}{3}+9 = \frac{17}{3}$ ✓

18



A(-1,1)
B(1,2)
C(2,-2)

Comprobación: $x = \frac{5}{6} \rightarrow y = \frac{2\cdot\frac{5}{6}-3}{8} = -\frac{1}{6}$ ✓

$M = \left(0, \frac{3}{2}\right)$

$\vec{AB} = (2,1) \rightarrow$ Pend $= \frac{1}{2}$

Mediatriz lado $\overline{AB}$: $y-\frac{3}{2} = -2x$ $\boxed{y = \frac{3}{2}-2x}$

$N = \left(\frac{1}{2}, -\frac{1}{2}\right)$

$\vec{AC} = (3,-3) \rightarrow$ Pend $= -1$

Mediatriz lado $\overline{AC}$: $y+\frac{1}{2} = 1\cdot\left(x-\frac{1}{2}\right)$ $\boxed{y = x-1}$

$P = \left(\frac{3}{2}, 0\right)$

$\vec{BC} = (1,-4) \rightarrow$ Pend $= -4$

Mediatriz lado $\overline{BC}$: $y = \frac{1}{4}\left(x-\frac{3}{2}\right)$ $\boxed{y = \frac{2x-3}{8}}$

$$\left.\begin{array}{l} y = \frac{3}{2}-2x \\ y = x-1 \end{array}\right\} \quad x-1 = \frac{3}{2}-2x \quad ; \quad 3x = \frac{5}{2} \quad x = \frac{5}{6} \Rightarrow y = \frac{5}{6}-1 = -\frac{1}{6}$$

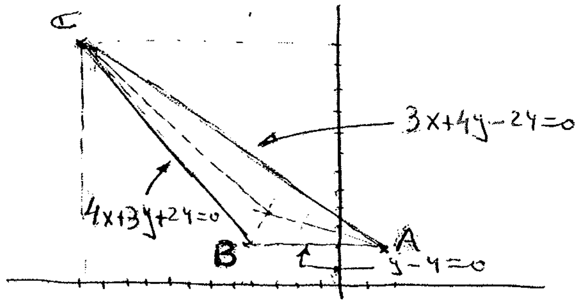
$\boxed{\text{Circuncentro} \left(\frac{5}{6}, -\frac{1}{6}\right)}$

(19)

$$\left.\begin{array}{l} y-4=0 \\ 3x+4y-24=0 \end{array}\right\} \rightarrow \boxed{\begin{array}{l} x=\frac{8}{3} \\ y=4 \end{array}} \quad A\left(\tfrac{8}{3},4\right)$$

$$\left.\begin{array}{l} y-4=0 \\ 4x+3y+24=0 \end{array}\right\} \rightarrow \boxed{\begin{array}{l} x=-9 \\ y=4 \end{array}} \quad B(-9,4)$$

$$\left.\begin{array}{l} 3x+4y-24=0 \\ 4x+3y+24=0 \end{array}\right\} \rightarrow \boxed{\begin{array}{l} x=-24 \\ y=24 \end{array}} \quad C(-24,24)$$



$$\frac{|y-4|}{1}=\frac{|3x+4y-24|}{5} \begin{cases} y-4=\dfrac{3x+4y-24}{5} \rightarrow 5y-20=3x+4y-24 \rightarrow \cancel{y=3x-4} \\ y-4=-\dfrac{3x+4y-24}{5} \rightarrow 5y-20=-3x-4y+24 \rightarrow \boxed{y=\dfrac{-3x+44}{9}} \end{cases}$$

$$\frac{|y-4|}{1}=\frac{|4x+3y+24|}{5} \begin{cases} y-4=\dfrac{4x+3y+24}{5} \rightarrow 5y-20=4x+3y+24 \rightarrow \boxed{y=2x+22} \\ y-4=-\dfrac{4x+3y+24}{5} \rightarrow 5y-20=-4x-3y-24 \rightarrow \cancel{y=\dfrac{-x-1}{2}} \end{cases}$$

$$\frac{|3x+4y-24|}{5}=\frac{|4x+3y+24|}{5} \begin{cases} 3x+4y-24=4x+3y+24 \rightarrow \cancel{y=x+48} \\ 3x+4y-24=-4x-3y-24 \rightarrow \boxed{y=-x} \end{cases}$$

$$\left.\begin{array}{l} y=2x+22 \\ y=-x \end{array}\right\} \rightarrow 2x+22=-x \rightarrow \boxed{\begin{array}{l} x=-\frac{22}{3} \\ y=\frac{22}{3} \end{array}} \qquad \boxed{\text{Incentro } \left(-\tfrac{22}{3}, \tfrac{22}{3}\right)}$$

Comprobación: $x=-22/3 \rightarrow y=\dfrac{-3\cdot\frac{22}{3}+44}{9}=\dfrac{22}{3}$ ✓

(20)

$\boxed{\begin{array}{l} \vec{a}(3,-4) \\ \vec{b}(5,x) \end{array}}$

a) $\vec{a}\parallel\vec{b} \Longrightarrow \dfrac{5}{3}=\dfrac{x}{-4} \Longrightarrow \boxed{x=-\dfrac{20}{3}}$

b) $\vec{a}\perp\vec{b} \Longrightarrow (3,-4)\cdot(5,x)=0 \Longrightarrow 15-4x=0 \Longrightarrow \boxed{x=\dfrac{15}{4}}$

c) $\alpha=60^\circ \Longrightarrow$

$$\vec{a}\cdot\vec{b}=|\vec{a}|\,|\vec{b}|\cos 60^\circ$$

$$15-4x=\sqrt{9+16}\,\sqrt{25+x^2}\cdot\frac{1}{2}$$

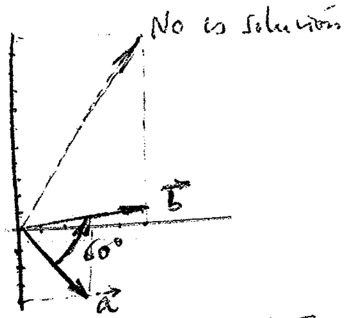
$$30-8x=5\sqrt{25+x^2}$$

$$(30-8x)^2=25(25+x^2)$$

$$900-480x+64x^2=625+25x^2$$

$$39x^2-480x+275=0$$

$$x=\frac{480\pm\sqrt{480^2-4\times 39\times 275}}{78}=\frac{480\pm 250\sqrt{3}}{78}=\frac{240\pm 125\sqrt{3}}{39} \begin{array}{l} \nearrow \cancel{11'4} \\ \searrow \boxed{0'602} \end{array}$$



Otro método: $(3-4i)\cdot 1_{60^\circ}=(3-4i)\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)=\left(\frac{3}{2}+2\sqrt{3}\right)+i\left(\frac{3\sqrt{3}}{2}-2\right)$

El vector $(5,x)$ debe ser paralelo a $\left(\frac{3}{2}+2\sqrt{3},\ \frac{3\sqrt{3}}{2}-2\right)$ luego:

$$\frac{5}{\frac{3}{2}+2\sqrt{3}}=\frac{x}{\frac{3\sqrt{3}}{2}-2} \Longrightarrow$$

$$x=\frac{\frac{15\sqrt{3}}{2}-10}{\frac{3}{2}+2\sqrt{3}}=\frac{15\sqrt{3}-20}{3+4\sqrt{3}}=\frac{(15\sqrt{3}-20)(3-4\sqrt{3})}{(3+4\sqrt{3})(3-4\sqrt{3})}=\frac{45\sqrt{3}-180-60+80\sqrt{3}}{9-48}=\frac{125\sqrt{3}-240}{-39}=\boxed{\frac{240-125\sqrt{3}}{39}}$$ ✓

21) $\vec{a}\cdot\vec{b}=16 \quad |\vec{a}|=2 \quad |\vec{b}|=12$

$$(3\vec{a}-\vec{b})^2 = 9\,\vec{a}\cdot\vec{a} - 6(\vec{a}\cdot\vec{b}) + \vec{b}\cdot\vec{b} = 9|\vec{a}|^2 - 6(\vec{a}\cdot\vec{b}) + |\vec{b}|^2 = 36 - 96 + 144 = \boxed{84}$$

$$(4\vec{a}+5\vec{b})(4\vec{a}-5\vec{b}) = 16\,\vec{a}\cdot\vec{a} - \cancel{20\,\vec{a}\cdot\vec{b}} + \cancel{20\,\vec{b}\cdot\vec{a}} - 25\,\vec{b}\cdot\vec{b} =$$

$$= 16|\vec{a}|^2 - 25|\vec{b}|^2 = 16\cdot 4 - 25\cdot 144 = \boxed{-3536}$$

22)

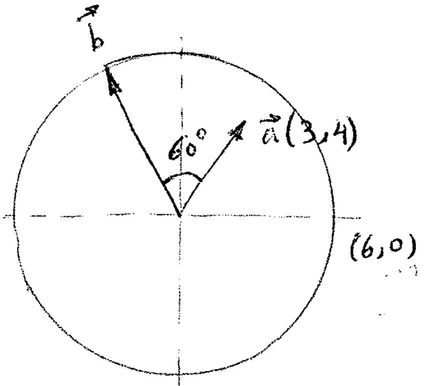
$$|\vec{a}+\vec{b}|=8 \Rightarrow (\vec{a}+\vec{b})(\vec{a}+\vec{b}) = 64 \Rightarrow \vec{a}\cdot\vec{a} + 2\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{b} = 64$$

$$|\vec{a}-\vec{b}|=6 \Rightarrow (\vec{a}-\vec{b})(\vec{a}-\vec{b}) = 36 \Rightarrow \vec{a}\cdot\vec{a} - 2\vec{a}\vec{b} + \vec{b}\vec{b} = 36$$

$$4\,\vec{a}\cdot\vec{b} = 28$$

$$\boxed{\vec{a}\cdot\vec{b} = 7}$$

23)



$\vec{b} = (b_1, b_2)$

$\vec{a}\cdot\vec{b} = |\vec{a}|\,|\vec{b}|\cos 60°$

$3b_1 + 4b_2 = 5\cdot 6\cdot \frac{1}{2}$

$3b_1 + 4b_2 = 15 \rightarrow b_2 = \frac{15-3b_1}{4}$

$b_1^2 + b_2^2 = 36$

$$b_1^2 + \frac{(15-3b_1)^2}{16} = 36 \quad ; \quad 16b_1^2 + 225 - 90b_1 + 9b_1^2 = 576$$

$$25b_1^2 - 90b_1 - 351 = 0$$

$$b_1 = \frac{90 \pm \sqrt{8100+35100}}{50} = \frac{90 \pm 10\sqrt{432}}{50} = \frac{90 \pm 120\sqrt{3}}{50} = \frac{9 \pm 12\sqrt{3}}{5}$$

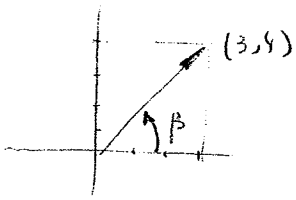
Como $\vec{b}$ pertenece al 2º cuadrante, escojo la solución negativa:

$$\boxed{b_1 = \frac{9-12\sqrt{3}}{5}} \Rightarrow b_2 = \frac{15 - 3\,\frac{9-12\sqrt{3}}{5}}{4} = \frac{75-27+36\sqrt{3}}{20} =$$

$$= \frac{48+36\sqrt{3}}{20} = \boxed{\frac{12+9\sqrt{3}}{5}}$$

$$\boxed{\vec{b} = \left(\frac{9-12\sqrt{3}}{5}, \frac{12+9\sqrt{3}}{5}\right)} = \boxed{(-2'357,\ 5'518)}$$

Otra forma:



$$\operatorname{tg}\beta = \frac{4}{3} \Rightarrow \beta = 53,13°$$

Hay que girar $6+0i$ un ángulo de $60 + 53,13° = 113,13°$

$$b_1 + i b_2 = (6+0i)\cdot 1_{113,13°} = 6(\cos 113,13° + i \operatorname{sen} 113,13°) =$$

$$= -2'357 + i\,5'518$$

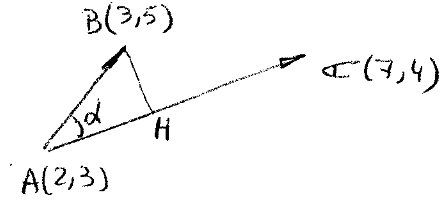
$$\boxed{\vec{b} = (-2'357,\ 5'518)}$$

Otra forma:

$$\vec{b} = (3+4i)\cdot\left(\frac{6}{5}\right)_{60°} = (3+4i)\cdot\frac{6}{5}(\cos 60° + i\operatorname{sen}60°) = \frac{6}{5}(3+4i)\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right) =$$

$$= \frac{6}{5}\left(\frac{3}{2} + \frac{3\sqrt{3}}{2}i + \frac{4}{2}i - \frac{4\sqrt{3}}{2}\right) = \frac{3}{5}\left(3-4\sqrt{3} + i(3\sqrt{3}+4)\right) \Rightarrow \boxed{\vec{b} = \left(\frac{9-12\sqrt{3}}{5}, \frac{9\sqrt{3}+12}{5}\right)}$$

(24)



$\vec{AB} = (1,2)$
$\vec{AC} = (5,1)$

$\vec{AB}\cdot\vec{AC} = |\vec{AB}|\,|\vec{AC}|\cos\alpha$

$\cos\alpha = \dfrac{(1,2)(5,1)}{\sqrt{1+4}\sqrt{25+1}} = \dfrac{7}{\sqrt{130}} \Rightarrow \boxed{\alpha = 52^{\circ}}$

$\overline{AH} = \overline{AB}\cdot\cos\alpha = \sqrt{1+4}\cdot\dfrac{7}{\sqrt{130}} = \boxed{\dfrac{7}{\sqrt{26}}} = 1'373$

$H = A + \dfrac{\overline{AH}}{\overline{AC}}\vec{AC} = (2,3) + \dfrac{7/\sqrt{26}}{\sqrt{26}}(5,1) = (2,3) + \dfrac{7}{26}(5,1) =$

$= \left(2+\dfrac{35}{26},\ 3+\dfrac{7}{26}\right) = \boxed{\left(\dfrac{87}{26},\ \dfrac{85}{26}\right)} = (3'346,\ 3'269)$

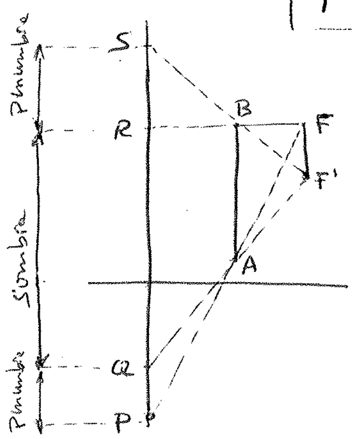
otro método:

$\left.\begin{array}{l} \dfrac{x-2}{5} = \dfrac{y-3}{1} \\ \dfrac{x-3}{1} = \dfrac{y-5}{-5} \end{array}\right\} \longrightarrow \left.\begin{array}{l} x-2 = 5y-15 \\ -5x+15 = y-5 \end{array}\right\} \longrightarrow \left.\begin{array}{l} x-5y+13=0 \\ -5x-y+20=0 \end{array}\right\} \longrightarrow \left.\begin{array}{l} 5x-25y+65=0 \\ -5x-y+20=0 \end{array}\right\}$

$-26y+85=0$

$\boxed{y = \dfrac{85}{26}} \Rightarrow \boxed{x = \dfrac{87}{26}}$

(25)



Recta $\overline{BF'}$

$\vec{BF'} = (3,-2) \rightarrow m = -\frac{2}{3}$

$y-6 = -\frac{2}{3}(x-4)$

$3y-18 = -2x+8$

$y = \dfrac{26-2x}{3}$

$x=0 \Rightarrow y = \frac{26}{3}$ $\boxed{S\left(0,\frac{26}{3}\right)}$

$\boxed{R(0,6)}$

Recta $\overline{AF}$

$\vec{AF} = (3,5) \rightarrow m = \frac{5}{3}$

$y-1 = \frac{5}{3}(x-4)$

$3y-3 = 5x-20$

$y = \dfrac{5x-17}{3}$

$x=0 \rightarrow y = \frac{-17}{3}$ $\boxed{P\left(0,-\frac{17}{3}\right)}$

Recta $\overline{AF'}$

$\vec{AF'} = (3,3) \rightarrow m = 1$

$y-1 = 1\cdot(x-4)$

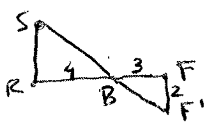
$y = x-3$

$x=0 \rightarrow y=-3$ $\boxed{Q(0,-3)}$

Sombra $= 6+3 = \boxed{9 \text{ unidades}}$

$\left.\begin{array}{l} \text{Penumbra}_1 = \frac{26}{3} - 6 = \frac{8}{3} \\ \text{Penumbra}_2 = \frac{17}{3} - 3 = \frac{8}{3} \end{array}\right\}$ Penumbra $= \frac{8}{3} + \frac{8}{3} = \boxed{\frac{16}{3} \text{ unidades}}$

otro método (Talis):



$\dfrac{RS}{4} = \dfrac{2}{3} \rightarrow \overline{RS} = \dfrac{8}{3}$ ✓

26 a) $\vec{CB} = \boxed{\vec{b} - \vec{c}}$

$\vec{AC} = \vec{OC} - \vec{OA} = \vec{OC} + \vec{OB} = \boxed{\vec{b} + \vec{c}}$

porque AB es un diámetro

b) $\vec{AC} \cdot \vec{CB} = (\vec{b} + \vec{c})(\vec{b} - \vec{c}) = \vec{b} \cdot \vec{b} - \cancel{\vec{b} \cdot \vec{c}} + \cancel{\vec{c} \cdot \vec{b}} - \vec{c} \cdot \vec{c} =$

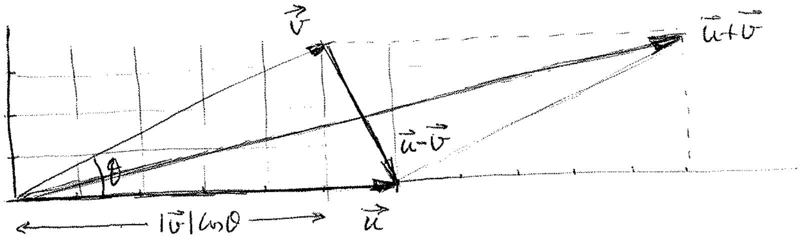
$= |\vec{b}|^2 - |\vec{c}|^2 = R^2 - R^2 = 0 \Rightarrow \vec{AC} \perp \vec{CB} \Rightarrow \boxed{\angle = 90^\circ}$

27 $\vec{v} = 6\vec{i} - 8\vec{j} = \begin{pmatrix} 6 \\ -8 \end{pmatrix} \longrightarrow |\vec{v}| = \sqrt{36+64} = 10$

Vectores Paralelos: $\pm \frac{15}{10} \vec{v} = \pm \frac{3}{2} \vec{v} = \pm \frac{3}{2} \begin{pmatrix} 6 \\ -8 \end{pmatrix} = \pm \begin{pmatrix} 9 \\ -12 \end{pmatrix} = \boxed{9\vec{i} - 12\vec{j}}$, $\boxed{-9\vec{i} + 12\vec{j}}$

Vectores Perpendiculares $\pm \frac{15}{10} \begin{pmatrix} 8 \\ 6 \end{pmatrix} = \pm \frac{3}{2} \begin{pmatrix} 8 \\ 6 \end{pmatrix} = \pm \begin{pmatrix} 12 \\ 9 \end{pmatrix} = \boxed{12\vec{i} + 9\vec{j}}$, $\boxed{-12\vec{i} - 9\vec{j}}$

28



b) $\vec{u} \cdot \vec{v} = |\vec{u}| \cdot |\vec{v}| \cos\theta = 6 \cdot 5 = \boxed{30}$

29 $\vec{OA} \perp \vec{BC} \Rightarrow \vec{a} \cdot (\vec{c} - \vec{b}) = 0 \rightarrow \vec{a} \cdot \vec{c} - \vec{a} \cdot \vec{b} = 0$

$\vec{OB} \perp \vec{CA} \Rightarrow \vec{b} \cdot (\vec{a} - \vec{c}) = 0 \rightarrow \vec{b} \cdot \vec{a} - \vec{b} \cdot \vec{c} = 0$ +

$\vec{a} \cdot \vec{c} - \vec{b} \cdot \vec{c} = 0 \Rightarrow (\vec{a} - \vec{b}) \cdot \vec{c} = 0 \Rightarrow \vec{BA} \cdot \vec{OC} = 0$ ✓