

POLINOMIOS Y FRACCIONES ALGEBRAICAS

- ① a) Grado : 3; coeficiente principal : -3; término independiente: -4
- b) " : 4; " " : -2; " " : 0
- c) " : 4; " " : -5; " " : 6

② $R(-\frac{1}{2}) = 6 \cdot (-\frac{1}{2})^3 - 4 \cdot (-\frac{1}{2})^2 - 2 \cdot (-\frac{1}{2}) + 3 = 6 \cdot (-\frac{1}{8}) - 4 \cdot \frac{1}{4} - 2 \cdot (-\frac{1}{2}) + 3 =$
 $= -\frac{6}{8} - \frac{4}{4} + \frac{2}{2} + 3 = -\frac{3}{4} - 1 + 1 + 3 = \frac{9}{4}$

③ a) $3P(x) + 2Q(x) = 3(-2x^3 + 3x^2 + 1) + 2(3x^2 - 2x - 2) = -6x^3 + 9x^2 + 3 + 6x^2 - 4x - 4 =$
 $= -6x^3 + 15x^2 - 4x - 1.$

b) $3Q(x) - 5P(x) = 3(3x^2 - 2x - 2) - 5(-2x^3 + 3x^2 + 1) = 9x^2 - 6x - 6 + 10x^3 - 15x^2 - 5 =$
 $= 10x^3 - 6x^2 - 6x - 11.$

④ a) $B(x) = I(x) - C(x) = (-\frac{1}{4}x^2 + 6x + 50) - (-\frac{1}{10}x^2 + 2x + 20) =$
 $= -\frac{1}{4}x^2 + 6x + 50 + \frac{1}{10}x^2 - 2x - 20 = -\frac{3}{20}x^2 + 4x + 30.$

b) $B(10) = -\frac{3}{20} \cdot 10^2 + 4 \cdot 10 + 30 = -\frac{300}{20} + 40 + 30 = -15 + 70 = 55.$

c) $B(x) = I(x) - \frac{1}{2}C(x) = (-\frac{1}{4}x^2 + 6x + 50) - \frac{1}{2}(-\frac{1}{10}x^2 + 2x + 20) =$
 $= -\frac{1}{4}x^2 + 6x + 50 + \frac{1}{20}x^2 - x - 10 = -\frac{1}{5}x^2 + 5x + 40.$

⑤ a) $(2x^3 - 3x^2 - 2x + 1)(4x^2 + 6x - 3) =$
 $= 8x^5 + 12x^4 - 6x^3 - 12x^4 - 18x^3 + 9x^2 - 8x^3 - 12x^2 + 6x + 4x^2 + 6x - 3 =$
 $= 8x^5 - 32x^3 + x^2 + 12x - 3$

b) $(7x^3 - 6x^2 + 8x - 4) \cdot (3x^2 + 7x + 4) =$
 $= 21x^5 + 49x^4 + 28x^3 - 18x^4 - 42x^3 - 24x^2 + 24x^3 + 56x^2 + 32x - 12x^2 - 28x - 16 =$

$$= 21x^5 + 31x^4 + 10x^3 + 20x^2 + 4x - 16$$

$$c) (2x^2 - 3x + 1)^2 = (2x^2 - 3x + 1) \cdot (2x^2 - 3x + 1) =$$

$$= 4x^4 - 6x^3 + 2x^2 - 6x^3 + 9x^2 - 3x + 2x^2 - 3x + 1 = 4x^4 - 12x^3 + 13x^2 - 6x + 1$$

$$d) (x^2 - x - 2)^3 = (x^2 - x - 2)(x^2 - x - 2)(x^2 - x - 2) =$$

$$= (x^4 - x^3 - 2x^2 - x^3 + x^2 + 2x - 2x^2 + 2x + 4)(x^2 - x - 2) =$$

$$= (x^4 - 2x^3 - 3x^2 + 4x + 4)(x^2 - x - 2) =$$

$$= x^6 - x^5 - \cancel{2x^4} - 2x^5 + \cancel{2x^4} + 4x^3 - 3x^4 + 3x^3 + 6x^2 + 4x^3 - \cancel{4x^2} - 8x + \cancel{4x^2} - 4x - 8 =$$

$$= x^6 - 3x^5 - 3x^4 + 11x^3 + 6x^2 - 12x - 8.$$

$$\textcircled{6} \text{ a) } \begin{array}{r} x^5 + 7x^3 - 5x + 1 \\ -x^5 - 2x^3 \\ \hline 5x^3 - 5x + 1 \\ -5x^3 - 10x \\ \hline -15x + 1 \end{array} \quad \begin{array}{l} \underline{x^3 + 2x} \\ x^2 + 5 \end{array}$$

Cociente: $x^2 + 5$

Resto: $-15x + 1$.

$$\text{b) } \begin{array}{r} -2x^4 + 5x^3 - x + 4 \\ 2x^4 - 6x^2 \\ \hline 5x^3 - 6x^2 - x + 4 \\ -5x^3 + 15x \\ \hline -6x^2 + 14x + 4 \\ 6x^2 - 18 \\ \hline 14x - 14 \end{array} \quad \begin{array}{l} \underline{x^2 - 3} \\ -2x^2 + 5x - 6 \end{array}$$

Cociente: $-2x^2 + 5x - 6$

Resto: $14x - 14$.

$$\text{c) } \begin{array}{r} 3x^4 + 4x^3 + 6x^2 - 2x + 6 \\ -3x^4 - 6x^3 - 9x^2 \\ \hline -2x^3 - 3x^2 - 2x + 6 \\ 2x^3 + 4x^2 + 6x \\ \hline x^2 + 4x + 6 \\ -x^2 - 2x - 3 \\ \hline 2x + 3 \end{array} \quad \begin{array}{l} \underline{x^2 + 2x + 3} \\ 3x^2 - 2x + 1 \end{array}$$

Cociente: $3x^2 - 2x + 1$

Resto: $2x + 3$

$$d) \begin{array}{r|rrrr} 3 & 3 & 1 & -24 & -1 \\ & & 9 & 30 & 18 \\ \hline & 3 & 10 & 6 & 17 \end{array}$$

Cociente: $3x^2 + 10x + 6$

Resto: 17.

$$e) \begin{array}{r|rrrrr} -1 & -1 & 2 & 0 & -3 & 1 \\ & & 1 & -3 & 3 & 0 \\ \hline & -1 & 3 & -3 & 0 & 1 \end{array}$$

Cociente: $-x^3 + 3x^2 - 3x$

Resto: 1.

$$f) \begin{array}{r|rrrrr} -2 & 1 & 1 & 0 & 4 & -6 \\ & & -2 & 2 & -4 & 0 \\ \hline & 1 & -1 & 2 & 0 & -6 \end{array}$$

Cociente: $x^3 - x^2 + 2x$

Resto: -6.

7) $(x^3 + 2x) \cdot (x^2 + 5) + (-15x + 1) = x^5 + 5x^3 + 2x^3 + 10x - 15x + 1 = x^5 + 7x^3 - 5x + 1$ ✓

8) a) $1^3 + 2 \cdot 1^2 - 13 \cdot 1 + 10 = 1 + 2 - 13 + 10 = 0 \Rightarrow$ Si.

b) $(-2)^3 - (-2)^2 - 4 \cdot (-2) + 6 = -8 - 4 + 8 + 6 = 2 \Rightarrow$ No.

c) $2^4 - 2^3 - 4 \cdot 2^2 + 4 \cdot 2 = 16 - 8 - 16 + 8 = 0 \Rightarrow$ Si.

9) a) $1^3 - 3 \cdot 1^2 + m \cdot 1 = 0 \Rightarrow 1 - 3 + m = 0 \Rightarrow \boxed{m = 2}$

b) $5 \cdot 2^3 - m \cdot 2^2 - 5 \cdot 2 + 10 = 0 \Rightarrow 40 - 4m - 10 + 10 = 0 \Rightarrow 4m = 40 \Rightarrow \boxed{m = 10}$

c) $2 \cdot (-1)^4 + m(-1)^3 - 30 \cdot (-1)^2 - 10 \cdot (-1) + 28 = 2 - m - 30 + 10 + 28 \Rightarrow \boxed{m = 10}$

10) $-(-2)^3 + 3 \cdot (-2)^2 + k \cdot (-2) + 7 = -7 \Rightarrow 8 + 12 - 2k + 7 = -7 \Rightarrow 2k = 34 \Rightarrow \boxed{k = 17}$

- 11) a) $x^2 - 4 = (x - 2)(x + 2)$
 b) $9x^2 - 4 = (3x - 2)(3x + 2)$
 c) $4x^4 - x^2 = (2x^2 - x)(2x^2 + x)$
 d) $12x - 3x^3 = 3x(4 - x^2) = 3x(2 - x)(2 + x)$
 e) $x^2 + 2x + 1 = (x + 1)^2$
 f) $x^2 - 4x + 4 = (x - 2)^2$
 g) $x^4 - 18x^2 + 81 = (x^2 - 9)^2$
 h) $12x^3 + 12x^2 + 3x = 3x(4x^2 + 4x + 1) = 3x(2x + 1)^2$

- 12) a) $5(x + 1)(x - 1)(x - 2)$; b) $(x + 2)(x + 1)^2(x - 2)$
 c) $3(x + 2)(x + 1)(x - 2)(x - 3)$; d) $3x(x - 1)(x + 1)(x + 2)$
 e) $(x - 1)^3(x + 2)$; f) $x(x - 2)^3$
 g) $2(x - 1)(x + 1)(x - 2)(x - \frac{1}{2})$; h) $(x - 1)^2(x + 2)^2$

13) $2(x - 1)(x + 1)(x - 2) = 2(x^2 - 1)(x - 2) = 2x^3 - 4x^2 - 2x + 4$

14) a) $\frac{(3x - 2)^2}{9x^2 - 4} = \frac{(3x - 2)^{\cancel{2}}}{(\cancel{3x - 2})(3x + 2)} = \frac{3x - 2}{3x + 2}$
 b) $\frac{x(2x^2 - 16x + 32)}{x^2 - 16} = \frac{x \cdot 2(x^2 - 8x + 16)}{(x - 4)(x + 4)} = \frac{2x(x - 4)^2}{(\cancel{x - 4})(x + 4)} = \frac{2x(x - 4)}{x + 4}$
 c) $\frac{(3x + 12)(x - 4)}{2x^2 - 32} = \frac{3(x + 4)(x - 4)}{2(x^2 - 16)} = \frac{3(\cancel{x + 4})(x - 4)}{2(\cancel{x - 4})(x + 4)} = \frac{3}{2}$

15) a) $\frac{2x^4 + x^3 - 11x^2 + 11x - 3}{2x^3 + 3x^2 - 8x + 3} = \frac{(x-1)^2(x+3)(2x-1)}{(x-1)(x+3)(2x-1)} = x-1$

Para $x=2 \Rightarrow 2-1=1$

b) $\frac{x^3 - 2x^2 - 9x + 18}{x^3 - 7x^2 + 16x - 12} = \frac{(x-2)(x-3)(x+3)}{(x-2)^2(x-3)} = \frac{x+3}{x-2}$

Para $x=2 \Rightarrow \frac{2+3}{2-2} = \frac{5}{0}$ No existe la fracción para $x=2$.

16) a)
$$\begin{array}{r} 6x \quad | \quad 2x-5 \\ -6x+15 \\ \hline 15 \end{array} \quad \frac{6x}{2x-5} = 3 + \frac{15}{2x-5}$$

b)
$$\begin{array}{r} x^3 - 4x^2 + 5x + 2 \quad | \quad x^2 - 2 \\ -x^3 \quad \quad + 2x \\ \hline -4x^2 + 7x + 2 \\ 4x^2 \quad \quad - 8 \\ \hline 7x - 6 \end{array} \quad \frac{x^3 - 4x^2 + 5x + 2}{x^2 - 2} = x - 4 + \frac{7x - 6}{x^2 - 2}$$

c)
$$\begin{array}{r|rrr} & 1 & 0 & -1 \\ -4 & & -4 & 16 \\ \hline & 1 & -4 & 15 \end{array} \quad \frac{x^2 - 1}{x + 4} = x - 4 + \frac{15}{x + 4}$$

17) a)
$$\frac{x+2}{x^2-9} \cdot \frac{2x+6}{3x+6} - \frac{1}{2} \cdot \frac{1-x}{x-3} = \frac{(x+2) \cdot 2(x+3)}{(x+3)(x-3) \cdot 3(x+2)} - \frac{1-x}{2(x-3)}$$

$$= \frac{2}{3(x-3)} - \frac{1-x}{2(x-3)} = \frac{4 - 3(1-x)}{6(x-3)} = \frac{4 - 3 + 3x}{6(x-3)} = \frac{1+3x}{6(x-3)}$$

b)
$$6 \cdot \frac{1}{1-x} - x \cdot \frac{1}{x^2-x} = \frac{6}{1-x} - \frac{x}{x(x-1)} = \frac{6}{1-x} + \frac{1}{1-x} =$$

$$= \frac{7}{1-x}$$

$$c) \frac{4}{3x^2-3x-18} - \frac{3}{2x+4} = \frac{4}{3(x-3)(x+2)} - \frac{3}{2(x+2)} =$$

$$= \frac{8 - 3 \cdot 3(x-3)}{6(x-3)(x+2)} = \frac{8 - 9x + 27}{6(x-3)(x+2)} = \frac{-9x + 35}{6(x-3)(x+2)}$$

$$d) \frac{3}{3x^2+6x} + \frac{1}{x} - \frac{2-x}{6x+12} = \frac{3}{3x(x+2)} + \frac{1}{x} - \frac{2-x}{6(x+2)} =$$

$$= \frac{6 + 6(x+2) - (2-x) \cdot x}{6x(x+2)} = \frac{6 + 6x + 12 - 2x + x^2}{6x(x+2)} = \frac{x^2 + 4x + 18}{6x(x+2)}$$

_____ 0 _____

$$\frac{5-x}{x-x} = \frac{5+x^2+x^2-x}{x-x}$$

$$\frac{2+2x}{3-x} = \frac{2+2x}{3-x}$$

$$\frac{2}{1+x} + 1-x = \frac{1-x}{1+x}$$

$$\frac{1-x}{1-x} = \frac{1-x}{1-x}$$

$$\frac{x-1}{(x-1)(x-1)} = \frac{(x-1)(x-1)}{(x-1)(x-1)} = \frac{x-1}{x-1} = \frac{x-1}{x-1}$$

$$\frac{x-1}{(x-1)(x-1)} = \frac{x-1}{(x-1)(x-1)} = \frac{x-1}{(x-1)(x-1)}$$

$$\frac{1}{x-1} + \frac{x}{x-1} = \frac{1+x}{(x-1)(x-1)} = \frac{1+x}{(x-1)(x-1)}$$