LOGARITMOS. EJERCICIOS

1) a) $\log_2 128 = \log_2 2^7 = 7$; b) $\log_5 625 = \log_5 5^4 = 4$;

c) $\log \sqrt{10} = \log 10^{\frac{1}{2}} = \frac{1}{2}$; d) $\log 40 + \log 25 = \log(40 \cdot 25) = \log 1000 = 3$

e) $\log 80 - \log 8 = \log \frac{80}{8} = \log 10 = 1$; f) $\log_3 \sqrt[4]{3^5} = \log_3 3^{\frac{5}{4}} = \frac{5}{4}$

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2) $\log_2 64 + \log_2 \frac{1}{4} - \log_3 9 - \log_2 \sqrt{2} = \log_2 2^6 + \log_2 2^{-2} - \log_3 3^2 - \log_2 2^{\frac{1}{2}} =$

$= 6 - 2 - 2 - \frac{1}{2} = \frac{3}{2}$

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3) a) $\log \sqrt[3]{0'002} = \log(0'002)^{\frac{1}{3}} = \frac{1}{3} \log(2 \cdot 10^{-3}) = \frac{1}{3}\left(\log 2 + \log 10^{-3}\right) =$

$= \frac{1}{3}(0'301 - 3) = -0'89966\ldots \simeq -0'900$ (con 3 c.s.)

b) $\log \frac{1}{\sqrt[3]{16}} = \log \frac{1}{2^{4/3}} = \log 2^{-\frac{4}{3}} = -\frac{4}{3} \log 2 = -\frac{4}{3} \cdot 0'301 = 0'401.$

c) $\log 25 = \log \frac{100}{4} = \log 100 - \log 4 = 2 - \log 2^2 = 2 - 2\log 2 = 2 - 2 \cdot 0'301 =$
$= 1'398 \simeq$
$\simeq 1'40$

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4) $\log_5 30 = \frac{\log 30}{\log 5} = 2'11$; $\log_7 4 = \frac{\log 4}{\log 7} = 0'712$;

$\log_3 10 = \frac{\log 10}{\log 3} = \frac{1}{\log 3} = 2'10$

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5) a) $\log 5 + \log 20 - \log 25 = \log \frac{5 \cdot 20}{25} = \log \frac{100}{25} = \log 4$

b) $2 \log_4 7 + \log_4 2 - \log_4 5 = \log_4 49 + \log_4 2 - \log_4 5 = \log_4 \frac{49 \cdot 2}{5} = \log_4 \frac{98}{5}$

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6) a) $\ln p^2 q - \ln\left(\frac{1}{pq}\right) = \ln \frac{p^2 q}{\frac{1}{pq}} = \ln p^3 q^2$

b) $\log ab - \log \sqrt{ab} = \log ab - \log(ab)^{\frac{1}{2}} = \log ab - \frac{1}{2} \log ab = \frac{1}{2} \log ab =$

$= \log \sqrt{ab}$

c) $x \log y + x \log\left(\frac{1}{y}\right) = x \log y + x\left(\log 1 - \log y\right) = x \log y + x\left(0 - \log y\right) =$

$= x \log y + x\left(-\log y\right) = x \log y - x \log y = 0$

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(7) $\log\left(\frac{P}{QR^3}\right)^2 = 2\log\left(\frac{P}{QR^3}\right) = 2\left(\log P - \log Q - \log R^3\right) =$

$= 2\log P - 2\log Q - 6\log R = 2x - 2y - 6z.$

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(8) a) $\log_2 32 = \log_2 2^5 = 5$

b) $\log_2\left(\frac{32^x}{8^y}\right) = \log_2 32^x - \log_2 8^y = x \log_2 32 - y \log_2 8 =$

$= 5x - 3y. \Rightarrow p = 5 \ ; \ q = -3$

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(11) a) $\log_2 5 = \frac{\log_a 5}{\log_a 2} = \frac{y}{x}$

b) $\log_a 20 = \log_a\left(2^2 \cdot 5\right) = \log_a 2^2 + \log_a 5 = 2\log_a 2 + \log_a 5 = 2x + y$

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(12) a) $\log_5 x^2 = 2\log_5 x = 2y$

b) $\log_5\left(\frac{1}{x}\right) = \log_5 x^{-1} = -\log_5 x = -y$

c) $\log_{25} x = \frac{\log_5 x}{\log_5 25} = \frac{y}{2}$

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(13) a) $\log_8 x = 3^{-1} = \frac{1}{3} \Rightarrow 8^{\frac{1}{3}} = x \Rightarrow x = \sqrt[3]{8} = 2$

b) $8^{-x} = \left(\frac{1}{4}\right)^3 \Rightarrow \left(2^3\right)^{-x} = \left(2^{-2}\right)^3 \Rightarrow 2^{-3x} = 2^{-6} \Rightarrow -3x = -6 \Rightarrow x = 2$

c) $\log_x 49 = 2 \Rightarrow x^2 = 49 \Rightarrow x = 7$

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(14) $\log_{27} x = 1 - \log_{27}(x - 0'4) \Rightarrow \log_{27} x + \log_{27}(x - 0'4) = 1 \Rightarrow$

$\Rightarrow \log_{27}\left(x \cdot (x - 0'4)\right) = 1 \Rightarrow x^2 - 0'4x = 27 \Rightarrow x^2 - 0'4x - 27 = 0 \Rightarrow$

$\Rightarrow x = \begin{cases} \frac{27}{5} \\ -5 \end{cases}$ (No válida)

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15 $9^{2x}=27^{(1-x)} \to (3^2)^{2x}=(3^3)^{(1-x)} \to 3^{4x}=3^{3-3x} \to 4x=3-3x$

$\to 7x=3 \to x=\frac{3}{7}$

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17 a) $\log x = 1+\log(22-x) \to \log x = \log 10+\log(22-x) \to$

$\to \log x = \log(10\cdot(22-x)) \to x=220-10x \to 11x=220 \to x=20$ ✓

b) $2\log x - \log(x-16)=2 \to \log x^2-\log(x-16)=2 \to \log\frac{x^2}{x-16}=2$

$\to \frac{x^2}{x-16}=100 \to x^2=100x-1600 \to x^2-100x+1600=0 \to x= \begin{cases} 20 \checkmark \\ 80 \checkmark \end{cases}$

c) $\log(x^2+1)-\log(x^2-1)=\log\frac{13}{12} \to \log\frac{x^2+1}{x^2-1}=\log\frac{13}{12} \to$

$\to \frac{x^2+1}{x^2-1}=\frac{13}{12} \to 12x^2+12=13x^2-13 \to x^2=25 \to x=\pm 5$ ✓

d) $\log_2 x+\log_2(x-7)=3 \to \log_2(x\cdot(x-7))=3 \to x^2-7x=8 \to$

$\to x^2-7x-8=0 \to x= \begin{cases} 8 \checkmark \\ -1 \text{ No válida.} \end{cases}$

e) $\log_9 81+\log_9\left(\frac{1}{9}\right)+\log_9 3=\log_9 x \to \log_9 9^2+\log_9 9^{-1}+\log_9 9^{\frac{1}{2}}=\log_9 x$

$\to 2-1+\frac{1}{2}=\log_9 x \to \frac{3}{2}=\log_9 x \to x=9^{\frac{3}{2}}=\sqrt{9^3}=\sqrt{729}=27$

f) $\log_2(x+2)+\log_2(x-1)=\log_2 4 \to \log_2(x+2)(x-1)=\log_2 4 \to$

$\to x^2-x+2x-2=4 \to x^2+x-6=0 \to x= \begin{cases} 2 \checkmark \\ -3 \text{ No válida} \end{cases}$

g) $\log_7 x+\log_7 3=\log_7(x-1) \to$

$\to \log_7(3x)=\log_7(x-1) \to 3x=x-1 \to 2x=-1 \to x=-\frac{1}{2}$ No válida.

h) $1+2\ln(x-1)=2 \to 2\ln(x-1)=1 \to \ln(x-1)=\frac{1}{2} \to x-1=e^{\frac{1}{2}}$

$\to x=\sqrt{e}+1$

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18 a) $2^x=1024 \to 2^x=2^{10} \to x=10$

b) $3^{x+1}=729 \to 3^{x+1}=3^6 \to x+1=6 \to x=5$

c) $2^{x-1}+2^x+2^{x+1}=7 \rightarrow 2^x\cdot 2^{-1}+2^x+2^x\cdot 2=7 \rightarrow \boxed{2^x=t} \rightarrow$

$\rightarrow \frac{1}{2}t+t+2t=7 \rightarrow \frac{7}{2}t=7 \rightarrow t=2 \rightarrow 2^x=2 \rightarrow \boxed{x=1}$

d) $3^{2x+2}-28\cdot 3^x+3=0 \rightarrow 3^{2x}\cdot 3^2-28\cdot 3^x+3=0 \rightarrow \boxed{3^x=t} \rightarrow$

$\rightarrow 9t^2-28t+3=0 \rightarrow t=\begin{cases} 3 \rightarrow 3^x=3 \rightarrow \boxed{x=1} \\ \frac{1}{9} \rightarrow 3^x=\frac{1}{9} \rightarrow \boxed{x=-2} \end{cases}$

e) $2^{-2x+3}-\frac{1}{2\sqrt{2}}=0 \rightarrow 2^{-2x+3}-\frac{1}{2\cdot 2^{\frac{1}{2}}}=0 \rightarrow 2^{-2x+3}=\frac{1}{2^{3/2}}$

$\rightarrow 2^{-2x+3}=2^{-\frac{3}{2}} \rightarrow -2x+3=-\frac{3}{2} \rightarrow -2x=-\frac{3}{2}-3 \rightarrow -2x=-\frac{9}{2} \rightarrow$

$\rightarrow x=\frac{9}{4}$

f) $3^{x+2}+3^{x+1}+3^x=\frac{13}{9} \rightarrow 3^x\cdot 3^2+3^x\cdot 3+3^x=\frac{13}{9} \rightarrow \boxed{3^x=t}$

$\rightarrow 9t+3t+t=\frac{13}{9} \rightarrow 13t=\frac{13}{9} \rightarrow t=\frac{1}{9} \rightarrow 3^x=\frac{1}{9} \rightarrow \boxed{x=-2}$

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19 a) $\ln(x+2)=3 \rightarrow x+2=e^3 \rightarrow x=e^3-2$

b) $10^{2x}=500 \rightarrow \log 10^{2x}=\log 500 \rightarrow 2x=\log 500 \rightarrow$

$\rightarrow x=\frac{\log 500}{2}=1'35$

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20 a) $e^{\ln x}=x$;

b) $e^{\ln x+\ln y}=e^{\ln(x\cdot y)}=x\cdot y$

c) $\ln(e^{x+y})^2=2\ln e^{x+y}=2(x+y)$

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21 a) $\log_3 x-\log_3(x-5)=\log_3 A \rightarrow \log_3\frac{x}{x-5}=\log_3 A \rightarrow \frac{x}{x-5}=A$

b) $\log_3 x-\log_3(x-5)=1 \rightarrow \log_3\frac{x}{x-5}=1 \rightarrow \frac{x}{x-5}=3 \rightarrow$

$\rightarrow x=3x-15 \rightarrow -2x=-15 \rightarrow x=\frac{15}{2}$

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22 a) $e^{2x} - 3e^x + 2 = 0 \Rightarrow \boxed{e^x = t} \Rightarrow t^2 - 3t + 2 = 0 \Rightarrow$

$\Rightarrow t = \begin{cases} 1 \rightarrow e^x = 1 \Rightarrow \boxed{x = 0} \\ 2 \rightarrow e^x = 2 \Rightarrow \boxed{x = \ln 2} \end{cases}$

b) $\ln\sqrt{x-2} = 1 \Rightarrow \ln(x-2)^{\frac{1}{2}} = 1 \rightarrow \frac{1}{2}\ln(x-2) = 1 \Rightarrow$

$\rightarrow \ln(x-2) = 2 \Rightarrow x - 2 = e^2 \rightarrow x = 2 + e^2.$

c) $\frac{3}{1+e^t} = \frac{1}{1-e^t} \Rightarrow 3(1-e^t) = 1 + e^t \Rightarrow 3 - 3e^t = 1 + e^t \Rightarrow$

$\rightarrow 2 = 4e^t \Rightarrow e^t = \frac{1}{2} \Rightarrow t = \ln\frac{1}{2}$

d) $2^x = 5'6^{x-1} \Rightarrow \log 2^x = \log 5'6^{x-1} \rightarrow x\log 2 = (x-1)\log 5'6$

$\rightarrow x\log 2 = x\log 5'6 - \log 5'6 \Rightarrow x\log 2 - x\log 5'6 = -\log 5'6 \Rightarrow$

$\rightarrow x(\log 2 - \log 5'6) = -\log 5'6 \Rightarrow x = \frac{\log 5'6}{\log 5'6 - \log 2} = 1'67$

e) $e^{-\frac{1}{4}x} = 100 \Rightarrow -\frac{1}{4}x = \ln 100 \Rightarrow -x = 4\ln 100 \Rightarrow x = -4\ln 100$

$\rightarrow x = -18'4.$

f) $-3 + e^{-x} = 2 \Rightarrow e^{-x} = 5 \rightarrow -x = \ln 5 \Rightarrow x = -\ln 5 = -1'61$

g) $\frac{2}{1-e^{-2x}} = 4 \Rightarrow 2 = 4 - 4e^{-2x} \Rightarrow 4e^{-2x} = 2 \Rightarrow e^{-2x} = \frac{1}{2} \Rightarrow$

$\rightarrow -2x = \ln\frac{1}{2} \Rightarrow x = \frac{\ln 1/2}{-2} = 0'347.$

h) $2^{x-3} = 5^{1-x} \rightarrow \log 2^{x-3} = \log 5^{1-x} \rightarrow (x-3)\log 2 = (1-x)\log 5$

$\rightarrow x\log 2 - 3\log 2 = \log 5 - x\log 5 \rightarrow x\log 2 + x\log 5 = \log 5 + 3\log 2 \Rightarrow$

$\rightarrow x(\log 2 + \log 5) = \log 5 + 3\log 2 \rightarrow x = \frac{\log 5 + 3\log 2}{\log 2 + \log 5} = \frac{\log 5 + \log 8}{\log 10} =$

$= \log 40 = 1'60$

i) $\log_x 5 = 12 \Rightarrow x^{12} = 5 \Rightarrow x = \sqrt[12]{5} = 1'14$

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(23) a) $N = 4'7^{t+1}$

Si $t = 5$ h: $N = 4'7^{5+1} = 10779'21 \simeq 10779$ bacterias.

b) $5000 = 4'7^{t+1} \Rightarrow \log 5000 = (t+1) \cdot \log 4'7 \Rightarrow t+1 = \dfrac{\log 5000}{\log 4'7}$

$\Rightarrow t = \dfrac{\log 5000}{\log 4'7} - 1 = 4'50$ h Después de 4h y media aprox.

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(24) a) $2^6 = 64$ números.

b) $4096 = 2^n \Rightarrow 2^{12} = 2^n \Rightarrow n = 12$ dígitos.

c) $2^n = 10^6 \Rightarrow n \log 2 = 6 \Rightarrow n = \dfrac{6}{\log 2} = 19'93 \simeq 20$ dígitos.

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(25) $A = B \Rightarrow 0'9 \cdot 1'2^t = 1'6 \cdot 1'1^t \Rightarrow \dfrac{1'2^t}{1'1^t} = \dfrac{1'6}{0'9} \Rightarrow \left(\dfrac{1'2}{1'1}\right)^t = \dfrac{1'6}{0'9} \Rightarrow$

$\Rightarrow t \ln\left(\dfrac{1'2}{1'1}\right) = \ln \dfrac{1'6}{0'9} \Rightarrow t = \dfrac{\ln \frac{1'6}{0'9}}{\ln \frac{1'2}{1'1}} = 6'61$ minutos.

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(26) a) $n = 0 \Rightarrow R = 2'3 \ln(0+2) = 1'59$

b) $n = 5 \Rightarrow R = 2'3 \ln(5+2) = 4'48$

c) $\cancel{2'3} \ln(n+2) = 2 \cdot \cancel{2'3} \ln 2 \Rightarrow \ln(n+2) = \ln 2^2 \Rightarrow$

$\Rightarrow \ln(n+2) = \ln 4 \Rightarrow n+2 = 4 \Rightarrow \boxed{n = 2}$ semanas

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(9) a) $\log_3 p^2 = 2 \log_3 p = 2 \cdot 6 = 12$

b) $\log_3\left(\dfrac{p}{q}\right) = \log_3 p - \log_3 q = 6 - 7 = -1$

c) $\log_3(9p) = \log_3 9 + \log_3 p = 2 + 6 = 8$

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10 a) $\log_6 36 = \log_6 6^2 = 2\log_6 6 = 2$

b) $\log_6 4 + \log_6 9 = \log_6 (4\cdot 9) = \log_6 36 = 2$

c) $\log_6 2 - \log_6 12 = \log_6 \frac{2}{12} = \log_6 \frac{1}{6} = \log_6 6^{-1} = -1.$

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16 $\log_2 x + \log_2 (x-2) = 3\ ,\quad x > 2$

$\log_2 (x\cdot(x-2)) = 3 \Rightarrow \log_2 (x^2 - 2x) = 3 \Rightarrow x^2 - 2x = 2^3 \Rightarrow$

$\Rightarrow x^2 - 2x - 8 = 0 \Rightarrow x = \begin{cases} 4 \\ -2 \quad \text{(No válida)} \end{cases}$
