COMBINATORIA. TEOREMA DEL BINOMIO.

(1) a) • $VR_6^3 = 6^3 = 216$ señales distintas.

• Empiezan por A: $VR_6^2 = 6^2 = 36$

• Acaban por B: $VR_6^2 = 6^2 = 36$

• Empiezan por A y acaban por B: $VR_6^1 = 6$.

b) • $V_6^3 = 6 \cdot 5 \cdot 4 = 120$ señales distintas.

• Empiezan por A: $V_5^2 = 5 \cdot 4 = 20$.

• Acaban por B: $V_5^2 = 5 \cdot 4 = 20$

• Empiezan por A y acaban por B: $V_4^1 = 4$.

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(2) $C_{602}^{600} = \binom{602}{600} = \binom{602}{2} = \frac{602 \cdot 601}{2} = 180901$ resultados distintos.

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(3) • $V_4^3 = 4 \cdot 3 \cdot 2 = 24$ números.

• Terminan en 64: $V_2^1 = 2$ números.

• Mayores de 500: empiezan por 6: $V_3^2 = 3 \cdot 2 = 6$

" " 8: $V_3^2 = 3 \cdot 2 = 6$

12 números en total.

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(4) a) $P_6 = 6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 = 720$

b) $P_5 = 5! = 5 \cdot 4 \cdot 3 \cdot 2 = 120$

c) $P_4 = 4! = 4 \cdot 3 \cdot 2 = 24$.

d) $P_5 = 5! = 120$.

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(5) a) $VR_{10}^8 = 10^8$ números $\neq$.

b) $V_9^8 = 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 = 362880$ números $\neq$.

c) $P_8^{2,3,3} = \dfrac{8!}{2!\,3!\,3!} = \dfrac{8\cdot 7\cdot \cancel{6}\cdot 5\cdot 4\cdot \cancel{3!}}{2!\,\cancel{3!}\,\cancel{3!}} = 560$ números.

— o —

6) $P_9^{3,3,2,1} = \dfrac{9!}{3!\,3!\,2!} = 7! = 5040$ formas.

— o —

7) a) $C_8^6 = \binom{8}{6} = \dfrac{8!}{6!\,2!} = 28$

b) $C_{16}^3 \cdot C_8^3 = \dfrac{16!}{13!\,3!} \cdot \dfrac{8!}{5!\,3!} = 31360$.

c) $C_{16}^6 = \dfrac{16!}{10!\,6!} = 8008$.

— o —

8) a) $\left(x^2 - \dfrac{2}{x}\right)^4 = \binom{4}{0}(x^2)^4 + \binom{4}{1}(x^2)^3\cdot\left(\dfrac{-2}{x}\right) + \binom{4}{2}(x^2)^2\left(-\dfrac{2}{x}\right)^2 + \binom{4}{3}(x^2)\cdot\left(\dfrac{-2}{x}\right)^3 +$

$+ \binom{4}{4}\left(-\dfrac{2}{x}\right)^4 = x^8 + 4x^6\cdot\left(-\dfrac{2}{x}\right) + 6x^4\cdot\dfrac{4}{x^2} + 4x^2\cdot\left(-\dfrac{8}{x^3}\right) + \dfrac{16}{x^4} =$

$= x^8 - 8x^5 + 24x^2 - \dfrac{32}{x} + \dfrac{16}{x^4}$.

b) $\left(2p + \dfrac{5}{p}\right)^3 = \binom{3}{0}(2p)^3 + \binom{3}{1}(2p)^2\left(\dfrac{5}{p}\right) + \binom{3}{2}(2p)\left(\dfrac{5}{p}\right)^2 + \binom{3}{3}\left(\dfrac{5}{p}\right)^3 =$

$= 8p^3 + 60p + \dfrac{150}{p} + \dfrac{125}{p^3}$

c) $\left(q + \dfrac{2}{p^3}\right)^5 = \binom{5}{0}q^5 + \binom{5}{1}q^4\left(\dfrac{2}{p^3}\right) + \binom{5}{2}q^3\left(\dfrac{2}{p^3}\right)^2 + \binom{5}{3}q^2\left(\dfrac{2}{p^3}\right)^3 + \binom{5}{4}q\left(\dfrac{2}{p^3}\right)^4 +$

$+ \binom{5}{5}\left(\dfrac{2}{p^3}\right)^5 = q^5 + \dfrac{10q^4}{p^3} + \dfrac{40q^3}{p^6} + \dfrac{80q^2}{p^9} + \dfrac{80q}{p^{12}} + \dfrac{32}{p^{15}}$

— o —

9) $\binom{7}{4}(3x)^3(-2y^2)^4 = 35\cdot 27x^3\cdot 16y^8 = 15120x^3y^8 \Rightarrow$ luego el coeficiente es 15120.

— o —

10 $(3x+2y)^4 = \binom{4}{0}(3x)^4 + \binom{4}{1}(3x)^3 2y + \binom{4}{2}(3x)^2(2y)^2 + \binom{4}{3} 3x(2y)^3 + \binom{4}{4}(2y)^4 =$

$= 81x^4 + 216x^3y + 216x^2y^2 + 96xy^3 + 16y^4$.

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11 a) 10 términos.

b) $\binom{9}{r}(3x^2)^{9-r}\cdot\left(-\frac{1}{x}\right)^r = \binom{9}{r}3^{9-r}\cdot(-1)^r\cdot x^{18-2r}\cdot\frac{1}{x^r} =$

$= \binom{9}{r}3^{9-r}\cdot(-1)^r\cdot x^{18-3r} \Rightarrow 18-3r=0 \Rightarrow \boxed{r=6}$

Luego: $\binom{9}{6}\cdot 3^{9-6}\cdot(-1)^6 = 84\cdot 27 = 2268$.

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12 El término que contiene a x^{10} contendrá: $(2x^2)^5$; luego:

$\binom{7}{5}\cdot 5^2\cdot(2x^2)^5 = 21\cdot 25\cdot 32\cdot x^{10} = 16800\,x^{10}$.

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13 a) 1, 6, 15, 20, 15, 6, 1 (fila de $\binom{6}{r}$)

b) $\binom{6}{4}\cdot 1^2\cdot(x^2)^4 = 15x^8 \Rightarrow$ el coeficiente es 15.

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14 a) 6 términos

b) $\binom{5}{3}(x^2)^2(-2)^3 = \binom{5}{3}x^4\cdot(-8) = 10x^4\cdot(-8) = -80x^4 \Rightarrow A=-80$.

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15 $\binom{10}{2}x^8(2y)^2 = 45\cdot x^8\cdot 4y^2 = 180\,x^8y^2 \Rightarrow a=180$

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16 $\binom{8}{5}\left(\frac{2}{3}x\right)^3(-3)^5 = 56\cdot\frac{8}{27}x^3\cdot(-243) = \frac{-108864}{27}x^3 = -4032\,x^3$

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(17) a) $n = 10$

b) $a = p$; $b = 2q$.

c) $\binom{10}{5} \cdot p^5 (2q)^5 = 252 \cdot p^5 \cdot 32 q^5 = 8064 p^5 q^5$

— o —

(18) $\binom{5}{r} \cdot (3x^2)^{5-r} \cdot \left(-\frac{2}{x}\right)^r = \binom{5}{r} \cdot 3^{5-r} \cdot x^{10-2r} \cdot \frac{(-2)^r}{x^r} = \binom{5}{r} 3^{5-r} \cdot (-2)^r \cdot x^{10-3r}$

$\Rightarrow 10 - 3r = 4 \Rightarrow 3r = 6 \Rightarrow \boxed{r = 2}$; luego:

$\binom{5}{2} \cdot (3x^2)^3 \cdot \left(\frac{-2}{x}\right)^2 = 10 \cdot 27x^6 \cdot \frac{4}{x^2} = 1080x^4$.

— o —

(19) $\left(1 + \frac{2}{3}x\right)^n = \binom{n}{0} \cdot 1^n + \binom{n}{1} \cdot 1^{n-1} \cdot \left(\frac{2}{3}x\right) + \binom{n}{2} \cdot 1^{n-2} \cdot \left(\frac{2}{3}x\right)^2 + \dots =$

$= 1 + n \cdot \frac{2}{3}x + \binom{n}{2} \cdot \frac{4}{9}x^2 + \dots$

$(3 + nx)^2 = 9 + 6nx + n^2x^2$.

Tèrmino en x serà: $1 \cdot 6nx + n \cdot \frac{2}{3}x \cdot 9 = 6nx + 6nx = 12nx$

$\Rightarrow 12nx = 84x \Rightarrow 12n = 84 \Rightarrow \boxed{n = 7}$.

— o —

(20) Tèrmino constante: $\binom{6}{r} \left(\frac{x}{a}\right)^{6-r} \cdot \left(\frac{a^2}{x}\right)^r = \binom{6}{r} \cdot \frac{x^{6-r}}{a^{6-r}} \cdot \frac{a^{2r}}{x^r} =$

$= \binom{6}{r} x^{6-2r} \cdot a^{3r-6} = 1280 \Rightarrow 6 - 2r = 0 \rightarrow \boxed{r = 3}$

$\binom{6}{3} \cdot a^3 = 1280 \rightarrow 20a^3 = 1280 \Rightarrow a^3 = 64 \rightarrow \boxed{a = 4}$

— o —

(21) Tèrmino constante: $x^2 \cdot \binom{8}{r} (3x^2)^{8-r} \cdot \left(\frac{K}{x}\right)^r = x^2 \binom{8}{r} \cdot 3^{8-r} \cdot x^{16-2r} \cdot \frac{K^r}{x^r} =$

$= \binom{8}{r} 3^{8-r} \cdot K^r \cdot x^{18-3r} \Rightarrow 18-3r=0 \rightarrow \boxed{r=6}$

$\binom{8}{6} \cdot 3^{8-6} \cdot K^6 = 16128 \Rightarrow 28 \cdot 9 \cdot K^6 = 16128 \Rightarrow K^6 = 64 \rightarrow K = \pm 2.$

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22 Tercer término: $\binom{8}{2} x^6 \cdot K^2 = 28 \cancel{x^6} K^2 = 63 \cancel{x^6} \Rightarrow 28K^2 = 63$

$\Rightarrow K^2 = \frac{63}{28} = \frac{9}{4} \Rightarrow K = \pm\sqrt{\frac{9}{4}} = \pm \frac{3}{2}.$

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