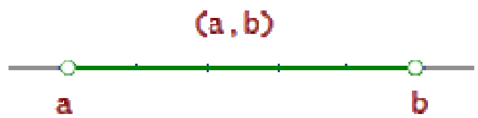
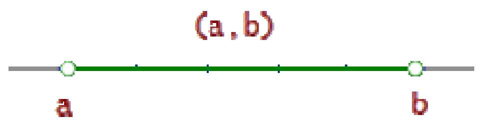
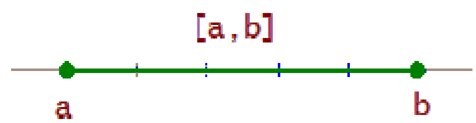
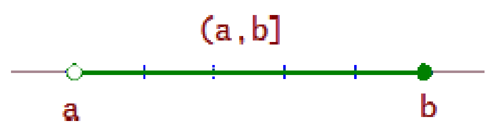
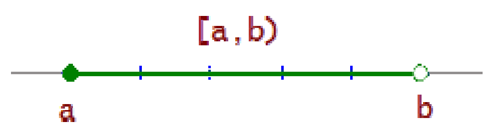
SET NOTATION

$\{x : \; -3 < x < 2\}$ reads "the set of all values that x can be such that x lies between -3 and 2".

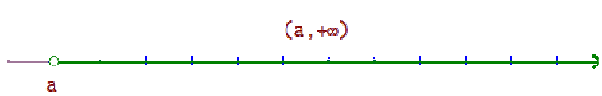
the set of all *such that*

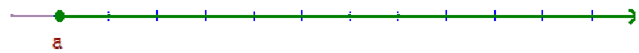
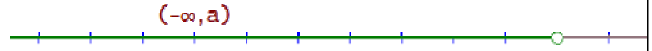
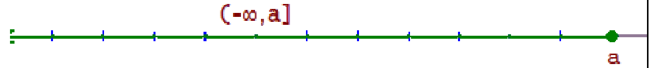
Intervals

Definition of an Interval	An **interval** is a to set of real numbers between two given points: **a and b**, which are called **ends of the interval**.	
Open Interval	**An open interval**, **(a, b)**, is the set **of all real numbers greater than a and smaller than b.**	**Set notation {x : a < x < b},** **from now we'll write:** **a < x < b** 
Closed Interval	**A closed interval**, **[a, b]**, is the set **of all real numbers greater than or equal to a and less than or equal to b**.	**a ≤ x ≤ b** 
Half-Closed Intervals	Half-closed intervals are also called half-open intervals.	**a < x ≤ b**  **a ≤ x < b** 

When there are a set of points formed by two or more of these intervals, the sign ∪(**Union**) is used between them.

Half-line

The **half-line** is determined by a number.	**x > a**	**a < x < +∞** 

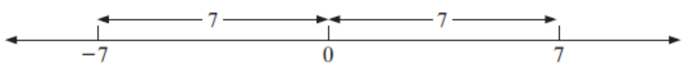
	$x \geq a$	$a \leq x < +\infty$ $[a, +\infty)$ 
	$x < a$	$-\infty < x < a$ $(-\infty, a)$ 
	$x \leq a$	$-\infty < x \leq a$ $(-\infty, a]$ 

ABSOLUTE VALUE (MODULUS)

The **modulus (absolute value)** of a real number is its size, ignoring its sign.

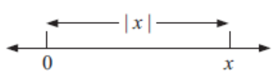
For example: the modulus (or absolute value) of 7 is 7, and
the modulus (or absolute value) of -7 is also 7.

Geometrically, the modulus of a real number can be interpreted as its *distance* from zero (0) on the number line. Because the modulus is distance, it cannot be negative.

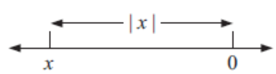


Thus, $|x|$ is the distance of x from 0 on the number line.

If $x > 0$



If $x < 0$



ALGEBRAIC DEFINITION

$$|x| = \begin{cases} x & \text{if } x \geqslant 0 \\ -x & \text{if } x < 0 \end{cases} \quad \text{or} \quad |x| = \sqrt{x^2}$$

EXERCISE F

1 Find the value of:

- **a** $5 - (-11)$
- **b** $|5| - |-11|$
- **c** $|5 - (-11)|$
- **d** $|(-2)^2 + 11(-2)|$
- **e** $|-6| - |-8|$
- **f** $|-6 - (-8)|$

2 If $a = -2$, $b = 3$, $c = -4$ find the value of:

- **a** $|a|$
- **b** $|b|$
- **c** $|a|\,|b|$
- **d** $|ab|$
- **e** $|a - b|$
- **f** $|a| - |b|$
- **g** $|a + b|$
- **h** $|a| + |b|$
- **i** $|a|^2$
- **j** a^2
- **k** $\left|\frac{c}{a}\right|$
- **l** $\frac{|c|}{|a|}$

Distance	The **distance** between **two real numbers** a and b, which writes **d(a, b)**, is defined as the absolute value **of the difference in both numbers**:	**d(a, b) = |b − a|**
	The **distance** between −5 and 4 is:	**d(−5, 4) = |4 − (−5)| = |4 + 5| = |9|**
"Entorno" = Neighbourhood	It is an interval with the form (a-r, a+r) where the point "a" is the centre and "r" is the radius. It could be an open or a closed neighbourhood. Using the modulus function, it is written as:	***Open neighbourhood*** $E(a,r) \Leftrightarrow \lvert x-a \rvert < r$ ***Closed neighbourhood*** $E[a,r] \Leftrightarrow \lvert x-a \rvert \le r$

Exercises:

1) Find the centre and the radius in the following "entornos". Draw the intervals. Express the intervals in other two ways.

   a) $[-3,5]$
   b) $|x-4| < 2$
   c) $[3,9]$
   d) $-2 < x < 9$
   e) $[-7,-4]$
   f) $-6 < x < -2$
   g) $|x-1| < 6$
   h) $|x+1| \le 3$
   i) $|x| \le 3$