PROBABILITY

Probability

Español	Inglés
Experimento aleatorio	Random experiment
Suceso	Event
Espacio muestral (E)	Sample space = the set of all possible outcomes of an experiment
Suceso elemental	Elementary event
Suceso compuesto	Compound event
Suceso cierto	Certain event
Suceso imposible	Impossible event
Suceso contrario	Complementary event the **complement** of any event A is the event [not A]
Espacio de Sucesos	Event space
Unión de sucesos	Union of events $\qquad A \bigcup B$
Intersección de sucesos	Intersection of events $\qquad A \bigcap B$
Sucesos incompatibles	Mutually exclusive events A and B are mutually exclusive events if $$A \bigcap B = 0$$
Sucesos compatibles	Not mutually exclusive events
Sucesos independientes	Independent events A and B are independent events if the occurrence (or non-occurrence) of one event does not affect the occurrence of the other
Sucesos dependientes	Dependent events
Diagrama en árbol	Tree diagram
Regla de Laplace: $$P(A) = \frac{\text{número de casos favorables al suceso } A}{\text{número de casos posibles}}$$	Theoretical probability = based on what we theoretically expect to occur. $$P(A) = \frac{\text{the number of members of the event } A}{\text{the total number of possible outcomes}}$$
Ley de los grandes números	Experimental probability = relative frecuency

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Probability. Vocabulary and some properties

<table>

| Summary of probabilities	
Event	**Probability**
A	$P(A) \in [0, 1]$
not A (complementary event of A)	$P(\overline{A}) = 1 - P(A)$
E = (Sample space)	$P(E) = 1$
Ø = (Impossible event)	$P(\varnothing) = 0$
A or B	P(either A or B) • $P(A \bigcup B) = P(A) + P(B)$ if A and B are mutually exclusive • $P(A \bigcup B) = P(A) + P(B) - P(A \bigcap B)$ if A and B are not mutually exclusive. Read as: P(either A <u>or</u> B) = P(A) + P(B) − P(both A <u>and</u> B)

</table>

Maths Dep.   *IES Sovelanos*

Mutually exclusive events (Sucesos incompatibles)

E

Times when I am truly happy (A)

Times when I am wearing pants (B)
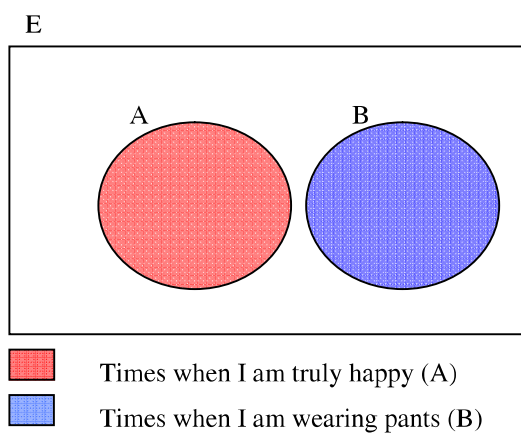
$$A \bigcap B = \varnothing$$

$$P(A \bigcap B) = 0$$

$$P(A \bigcup B) = P(A) + P(B)$$

Not mutually exclusive events (Sucesos compatibles)

E

Times when I am truly happy (A)

Times when I am wearing pants (B)
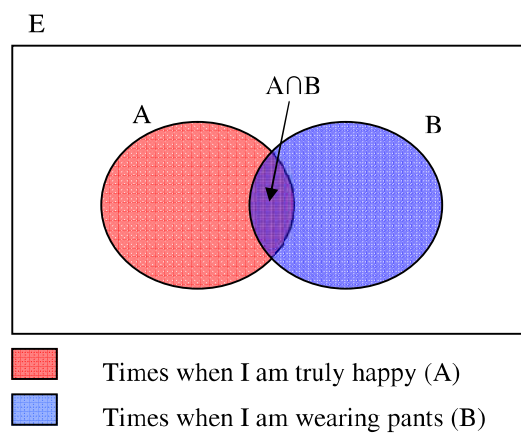
$$A \bigcap B \neq \varnothing$$

$$P(A \bigcap B) \neq 0$$

$$P(A \bigcup B) = P(A) + P(B) - P(A \bigcap B)$$

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Consider these statements:

> "The Wildcats will probably beat the Tigers on Saturday."
> "It is unlikely that it will rain today."
> "I will probably make the team."

Each of these statements indicates a **likelihood** or **chance** of a particular event happening.

We can indicate the likelihood of an event happening in the future by using a percentage.

> 0% indicates we believe the event **will not occur**.
> 100% indicates we believe the event **is certain to occur**.

All events can therefore be assigned a percentage between 0% and 100% (inclusive).

A number close to 0% indicates the event is **unlikely** to occur, whereas a number close to 100% means that it is **highly likely** to occur.

In mathematics, we usually write probabilities as either decimals or fractions rather than percentages. However, as $100\% = 1$, comparisons or conversions from percentages to fractions or decimals are very simple.

> An **impossible** event which has 0% chance of happening is assigned a probability of 0.
>
> A **certain** event which has 100% chance of happening is assigned a probability of 1.
>
> All other events can be assigned a probability between 0 and 1.

The assigning of probabilities is usually based on either:

- observing the results of an experiment (experimental probability),   or
- using arguments of symmetry (theoretical probability).

HISTORICAL NOTE

- **Girolamo Cardan** (1501 to 1576) admitted in his autobiography that he gambled "not only every year, but .... every day, and with the loss at once of thought, of substance, and of time". He wrote a handbook on gambling with tips on cheating and how to detect it. His book included discussion on equally likely events, frequency tables for dice probabilities, and expectations.

Girolamo Cardan

- **Pierre-Simon Laplace** (1749 - 1827) once described the theory of probability as "nothing but common sense reduced to calculation".

- **Blaise Pascal** (1623 - 1662) invented the first mechanical digital calculator. Pascal and his friend **Pierre de Fermat** (1601 - 1665) were first to develop probability theory as we know it today. Pascal also developed the syringe and the hydraulic press. He wrote a large number of articles on Christian beliefs and ethics.
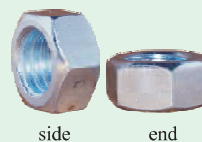
OPENING PROBLEM

When Karla dropped some metal nuts she noticed that they finished either on their ends or on their sides. She was interested to know what the probability was that a nut would finish on its end. So, she tossed a nut 200 times, and found that it finished on its end 137 times.

side   end

Later Sam repeated the experiment and the nut finished on its end 145 times.

For you to consider:

- What would Karla's best estimate be of the chance that the nut will finish on its end?
- What would Sam's estimate be?
- How can we obtain a better estimate of the chance of an end occurring?
- Hilda said that the best estimate would be obtained when the nut is tossed thousands of times. Is she correct?

A   EXPERIMENTAL PROBABILITY

We should use suitable language to help us describe what we are doing and the results we expect and get.

- The **number of trials** is the total number of times the experiment is repeated.
- The **outcomes** are the different results possible for one trial of the experiment.
- The **frequency** of a particular outcome is the number of times that this outcome is observed.
- The **relative frequency** of an outcome is the frequency of that outcome divided by the total number of trials.

For example, when tossing a tin can in the air 250 times, it comes to rest on an end 37 times. We say:

- the number of trials is 250
- the outcomes are *ends* and *sides*
- the frequency of *ends* is 37 and *sides* is 213
- the relative frequency of *ends* $= \frac{37}{250} \approx 0.148$
- the relative frequency of *sides* $= \frac{213}{250} \approx 0.852$.

EXPERIMENTAL PROBABILITY

Sometimes the only way of finding the probability of a particular event occurring is by experimentation.

Tossing a tin can is one such example. The probability of a can of this shape finishing on its end is the relative frequency found by experimentation.

Probability

We say that:

> the **estimated experimental probability** is the relative frequency of the event.

We write: Experimental P(end) ≈ 0.148.

The larger the number of trials, the more confident we are that the experimental probability obtained is accurate.

Example 1

Find the experimental probability of:

a tossing a head with one toss of a coin if it falls heads 96 times in 200 tosses

b rolling a *six* with a die given that when it was rolled 300 times, a *six* occurred 54 times.

a Experimental P(getting a head)
   $=$ relative frequency of getting
     a head
   $= \frac{96}{200}$
   $= 0.48$

b Experimental P(rolling a *six*)
   $=$ relative frequency of rolling a *six*
   $= \frac{54}{300}$
   $= 0.18$

EXERCISE 11A

1 Find the experimental probability of rolling *an odd number* with a die if *an odd number* occurred 33 times when the die was rolled 60 times.

2 Clem fired 200 arrows at a target and hit the target 168 times. Find the experimental probability of Clem hitting the target.

3 Ivy has free-range hens. Out of the first 123 eggs that they laid she found that 11 had double-yolks. Calculate the experimental probability of getting a double-yolk egg from her hens.

4 Jackson leaves for work at the same time each day. Over a period of 227 working days, on his way to work he had to wait for a train at the railway crossing on 58 days. Calculate the experimental probability that Jackson has to wait for a train on his way to work.

5 Ravi has a circular spinner marked P, Q and R on 3 equal sectors. Find the experimental probability of getting a Q if the spinner was twirled 417 times and finished on Q on 138 occasions.

6 Each time Claude shuffled a pack of cards before a game, he recorded the suit of the top card of the pack.
His results for 140 games were 34 Hearts, 36 Diamonds, 38 Spades and 32 Clubs.
Find the experimental probability that the top card of a shuffled pack is:

    a a Heart   **b** a Club or Diamond.

B   PROBABILITIES FROM TABLED DATA

If we are given a table of frequencies then we use **relative frequencies** to estimate the probabilities of the events.

Remember that: $$\text{relative frequency} = \frac{\text{frequency}}{\text{number of trials}}$$

Example 2

A marketing company surveys 80 randomly selected people to discover what brand of shoe cleaner they use. The results are shown in the table alongside:

Brand	Frequency
Shine	27
Brite	22
Cleano	20
No scuff	11

a Based on these results, what is the experimental probability of a community member using:

    i Brite   **ii** Cleano?

b Would you classify the estimate of **a** to be very good, good, or poor? Why?

a We start by calculating the relative frequency for each brand.

    i Experimental P(Brite) = 0.275

    ii Experimental P(Cleano) = 0.250

b Poor, as the sample size is very small.

Brand	Frequency	Relative Frequency
Shine	27	0.3375
Brite	22	0.2750
Cleano	20	0.2500
No scuff	11	0.1375

EXERCISE 11B

1 A marketing company was commissioned to investigate brands of products usually found in the bathroom. The results of a soap survey are given below:

  a How many people were randomly selected in this survey?

  b Calculate the relative frequency of use of each brand of soap.

  c Using the results obtained by the marketing company, what is the experimental probability that the soap used by a randomly selected person is:

    i Just Soap   **ii** Indulgence   **iii** Silktouch?

Brand	Freq	Relative Frequency
Silktouch	125	
Super	107	
Just Soap	93	
Indulgence	82	
Total		

2 Two coins were tossed 489 times and the *number of heads* occurring at each toss was recorded. The results are shown opposite:

  a Copy and complete the table given.

  b Estimate the chance of the following events occurring: **i** 0 heads   **ii** 1 head   **iii** 2 heads.

Outcome	Freq	Rel Freq
0 heads	121	
1 head		
2 heads	109	
Total		

3 At the Annual Show the toffee apple vendor estimated that three times as many people preferred red toffee apples to green toffee apples.

  a If 361 people wanted green toffee apples, estimate how many wanted red.

  b Copy and complete the table given.

  c Estimate the probability that the next customer will ask for:

  i a green toffee apple   **ii** a red toffee apple.

Colour	Freq	Rel Freq
Green	361	
Red		
Total		

4 The tickets sold for a tennis match were recorded as people entered the stadium. The results are shown:

  a How many tickets were sold in total?

  b Copy and complete the table given.

  c If a person in the stadium is selected at random, what is the probability that the person bought a Concession ticket?

Ticket Type	Freq	Rel Freq
Adult	3762	
Concession	1084	
Child	389	
Total		

5 The results of a local Council election are shown in the table. It is known that 6000 people voted in the election.

  a Copy and complete the table given.

  b What is the chance that a randomly selected person from this electorate voted for a female councillor?

Councillor	Freq	Rel Freq
Mr Tony Trimboli	2167	
Mrs Andrea Sims	724	
Mrs Sara Chong	2389	
Mr John Henry		
Total		

C   REPRESENTING COMBINED EVENTS

The possible outcomes for tossing two coins are listed below:

two heads   head and tail   tail and head   two tails

These results are the combination of two events: tossing coin 1 and tossing coin 2. If H represents a 'head' and T a 'tail', the sample space of possible outcomes is   HH, HT, TH and TT.

> A **sample space** is the set of all possible outcomes of an experiment.

Possible ways of representing sample spaces are:
- listing them
- using a 2-dimensional grid
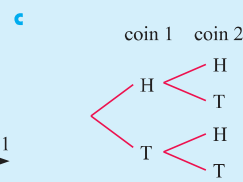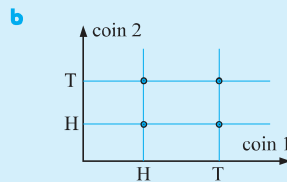- using a tree diagram
- using a Venn diagram.

Example 3

Represent the sample space for tossing two coins using:
  a a list
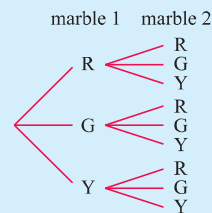 **b** a 2-D grid
  c a tree diagram

  a {HH, HT, TH, TT}
  b

  c

Example 4

Illustrate, using a tree diagram, the possible outcomes when drawing two marbles from a bag containing several marbles of each of the colours red, green and yellow.

Let
R be the event of getting a red
G be the event of getting a green
Y be the event of getting a yellow.

EXERCISE 11C

1 List the sample space for the following:
  a twirling a square spinner labelled A, B, C, D
  b the sexes of a 2-child family
  c the order in which 4 blocks A, B, C and D can be lined up
  d the 8 different 3-child families.
  e spinning a coin **i** twice **ii** three times **iii** four times.

2 Illustrate on a 2-dimensional grid the sample space for:
  a rolling a die and tossing a coin simultaneously
  b rolling two dice
  c rolling a die and spinning a spinner with sides A, B, C, D
  d twirling two square spinners: one labelled A, B, C, D and the other 1, 2, 3, 4.

3 Illustrate on a tree diagram the sample space for:
  a tossing a 5-cent and 10-cent coin simultaneously
  b tossing a coin and twirling an equilateral triangular spinner labelled A, B and C
  c twirling two equilateral triangular spinners labelled 1, 2 and 3 and X, Y and Z

     d drawing two tickets from a hat containing a number of pink, blue and white tickets.

     e drawing two beads from a bag containing 3 red and 4 blue beads.

4 Draw a Venn diagram to show a class of 20 students where 7 study History and Geography, 10 study History, 15 study Geography, and 2 study neither subject.

D   THEORETICAL PROBABILITY

From the methods of showing sample spaces in the previous section, we can find the probabilities of combined events.

These are theoretical probabilities which are calculated using

$$P(\text{event happens}) = \frac{\text{number of ways the event can happen}}{\text{total number of possible outcomes}}.$$

Example 5

Three coins are tossed. Write down a list of all possible outcomes.
Find the probability of getting:

  a 3 heads   **b** at least one head

  c 3 heads if it is known that there is at least one head.

Notice how we list the outcomes in a systematic way.

The sample space is: HHH   HHT   TTH   TTT
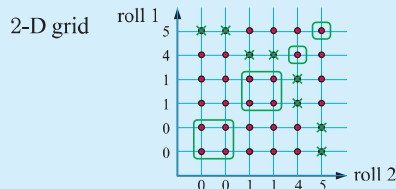   HTH   THT
                         THH   HTT

  a P(3 heads) $= \frac{1}{8}$

  b P(at least one H) $= \frac{7}{8}$ {all except TTT}

  c P(HHH knowing at least one H) $= \frac{1}{7}$
      {The sample space now excludes TTT}

Example 6

A die has the numbers 0, 0, 1, 1, 4 and 5. It is rolled *twice*. Illustrate the sample space using a 2-D grid. Hence find the probability of getting:

  a a total of 5   **b** two numbers which are the same.

There are $6 \times 6 = 36$ possible outcomes.

  a P(total of 5)
      $= \frac{8}{36}$ {those with a ×}

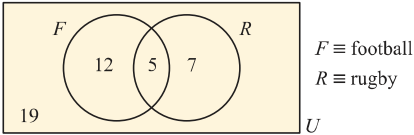 **b** P(same numbers)
      $= \frac{10}{36}$ {those circled}

EXERCISE 11D

1 **a** List all possible orderings of the letters O, D and G.

   b If these three letters are placed at random in a row, what is the probability of:

   i spelling DOG   **ii** O appearing first

   iii O not appearing first   **iv** spelling DOG or GOD?

2 The Venn diagram shows the sports played by boys at the local high school.
A student is chosen at random. Find the probability that he:

$F \equiv$ football
$R \equiv$ rugby

   a plays football   **b** plays both codes

   c plays football or rugby   **d** plays exactly one of these sports

   e plays neither of these sports

   f plays football, given that he is in at least one team

   g plays rugby, given that he plays football.

3 Draw the grid of the sample space when a 10-cent and a 50-cent coin are tossed simultaneously. Hence determine the probability of getting:

   a two heads   **b** two tails

   c exactly one head   **d** at least one head.

4 A coin and a pentagonal spinner with sectors 1, 2, 3, 4 and 5 are tossed and spun respectively.

   a Draw a grid to illustrate the sample space of possible outcomes.

   b How many outcomes are possible?

   c Use your grid to determine the chance of getting:

   i a head and a 4   **ii** a tail and an odd number

   iii an even number   **iv** a tail or a 3.

5 List the six different orders in which Alex, Bodi and Kek may sit in a row. If the three of them sit randomly in a row, determine the probability that:

   a Alex sits in the middle   **b** Alex sits at the left end

   c Alex sits at the right end   **d** Bodi and Kek are seated together.

6 **a** List the 8 possible 3-child families, according to the gender of the children.
For example, BGB means *"the first is a boy, the second is a girl, and the third is a boy"*.

   b Assuming that each of these is equally likely to occur, determine the probability that a randomly selected 3-child family consists of:

   i all boys   **ii** all girls

   iii boy, then girl, then girl   **iv** two girls and a boy

   v a girl for the eldest   **vi** at least one boy.

7 In a class of 24 students, 10 take Biology, 12 take Chemistry, and 5 take neither Biology nor Chemistry. Find the probability that a student picked at random from the class takes:

   a Chemistry but not Biology

   b Chemistry or Biology.
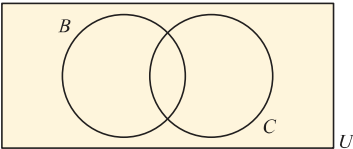
8 **a** List, in systematic order, the 24 different orders in which four people P, Q, R and S may sit in a row.

   b Hence, determine the probability that when the four people sit at random in a row:

      i P sits on one end

      ii Q sits on one of the two middle seats

      iii P and Q are seated together

      iv P, Q and R are seated together, not necessarily in that order.

9 A pair of dice is rolled.

   a Show that there are 36 members in the sample space of possible outcomes by displaying them on a grid.

   b Hence, determine the probability of a result with:

      i one die showing a 4 and the other a 5

      ii both dice showing the same result

      iii at least one of the dice showing a result of 3

      iv either a 4 or 6 being displayed

      v both dice showing even numbers

      vi the sum of the values being 7.

10 60 married men were asked whether they gave their wife flowers or chocolates for their last birthday. The results were: 26 gave chocolates, 21 gave flowers, and 5 gave both chocolates and flowers. If one of the married men was chosen at random, determine the probability that he gave his wife:

   a flowers but not chocolates   **b** neither chocolates nor flowers

   c chocolates or flowers.

11 List the possible outcomes when four coins are tossed simultaneously. Hence determine the probability of getting:

   a all heads   **b** two heads and two tails

   c more tails than heads   **d** at least one tail   **e** exactly one head.

12 **a** Copy and complete the grid alongside for the sample space of drawing one card from an ordinary pack.
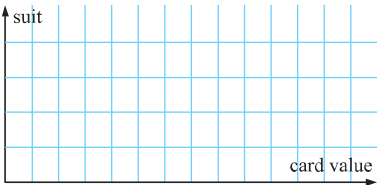
b Use your grid to determine the probability of getting:

i a Queen   **ii** the Jack of hearts   **iii** a spade

iv a picture card   **v** a red 7   **vi** a diamond or a club

vii a King or a heart   **viii** a Queen and a 3.

13 The medical records for a class of 28 children show whether they had previously had measles or mumps. The records show 22 have had measles, 13 have had measles and mumps, and 27 have had measles or mumps. If one child from the class is selected at random, determine the probability that he or she has had:

a measles   **b** mumps but not measles   **c** neither mumps nor measles.

E   COMPOUND EVENTS

We have previously used two-dimensional grids to represent sample spaces and hence find answers to certain probability problems.

Consider again a simple example of tossing a coin and rolling a die simultaneously.

To determine the probability of getting a head and a '5', we can illustrate the sample space on the two-dimensional grid shown. We can see that there are 12 possible outcomes but only one with the property that we want, so the answer is $\frac{1}{12}$.
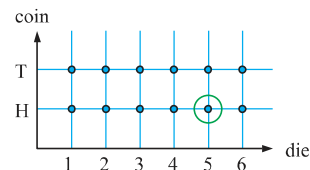
However, notice that P(a head) $= \frac{1}{2}$, P(a '5') $= \frac{1}{6}$ and $\frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$.

This suggests that P(a head **and** a '5') = P(a head) $\times$ P(a '5'), i.e., we multiply the separate probabilities.

It seems that if A and B are two events for which the occurrence of each one does not affect the occurence of the other, then P(A and B) = P(A) $\times$ P(B).

The two events 'getting a head' and 'rolling a 5' are events with this property, as the occurrence or non-occurrence of either one of them cannot affect the occurence of the other. We say they are **independent**.

> If two events A and B are **independent** then P(A **and** B) = P(A) $\times$ P(B).

Example 7

A coin is tossed and a die rolled simultaneously. Find the probability that a tail and a '2' result.

'Getting a tail' and 'rolling a 2' are independent events.

$\therefore$ P (a tail **and** a '2') = P(a tail) $\times$ P(a '2')

$= \frac{1}{2} \times \frac{1}{6}$

$= \frac{1}{12}$

Example 8

Sunil has probability $\frac{4}{5}$ of hitting a target and Monika has probability $\frac{5}{6}$.

If they both fire simultaneously at the target, determine the probability that:

a they both hit it

b they both miss it.

a P(both hit)
= P(Sunil hits and Monika hits)
= P(Sunil hits) $\times$ P(Monika hits)
= $\frac{4}{5} \times \frac{5}{6}$
= $\frac{2}{3}$

b P(both miss)
= P(Sunil misses and Monika misses)
= P(Sunil misses) $\times$ P(Monika misses)
= $\frac{1}{5} \times \frac{1}{6}$
= $\frac{1}{30}$

EXERCISE 11E.1

1 A coin and a pentagonal spinner with edges marked A, B, C, D and E are tossed and twirled simultaneously. Find the probabilities of getting:

    a a head and a D
    b a tail and either an A or a D.

2 A spinner with 6 equal sides has 3 red, 2 blue and 1 yellow edge. A second spinner with 7 equal sides has 4 purple and 3 green edges. Both spinners are twirled simultaneously. Find the probability of getting:

    a a red and a green
    b a blue and a purple.

3 Janice and Lee take set shots at a netball goal from 3 m. From past experience, Janice throws a goal on average 2 times in every 3 shots, whereas Lee throws a goal 4 times in every 7. If they both shoot for goals, determine the probability that:

    a both score a goal
    b both miss
    c Janice goals but Lee misses.

4 When a nut was tossed 400 times it finished on its edge 84 times and on its side for the rest. Use this information to estimate the probability that when two identical nuts are tossed:

    edge   side

    a they both fall on their edges
    b they both fall on their sides.

5 Tei has probability $\frac{1}{3}$ of hitting a target with an arrow, while See has probability $\frac{2}{5}$. If they both fire at the target, determine the probability that:

    a both hit the target
    b both miss the target
    c Tei hits the target and See misses
    d Tei misses the target and See hits.

6 A certain brand of drawing pin was tossed into the air 600 times. It landed on its back 243 times and on its side for the remainder. Use this information to estimate the probability that:

    a one drawing pin, when tossed, will fall on its **i** back   **ii** side
    b two drawing pins, when tossed, will both fall on their backs
    c two drawing pins, when tossed, will both fall on their sides.

DEPENDENT EVENTS

Suppose a cup contains 4 red and 2 green marbles. One marble is randomly chosen, its colour is noted, and it is then put aside. A second marble is then randomly selected. What is the chance that it is red?

If the first marble was red, $\quad$ P(second is red) $= \frac{3}{5}$ $\longleftarrow$ 3 reds remaining

$\longleftarrow$ 5 to choose from

If the first marble was green, $\quad$ P(second is red) $= \frac{4}{5}$ $\longleftarrow$ 4 reds remaining

$\longleftarrow$ 5 to choose from

So, the probability of the second marble being red **depends** on what colour the first marble was. We therefore have **dependent events**.

> Two or more events are **dependent** if they are **not independent**.
>
> **Dependent** events are events for which the occurrence of one of the events *does affect* the occurrence of the other event.

For compound events which are dependent, a similar product rule applies as to that for independent events:

> If A and B are dependent events then
>
> P(A **then** B) = P(A) × P(B given that A has occurred).

Example 9

A box contains 4 blue and 3 yellow buttons of the same size. Two buttons are randomly selected from the box without replacement. Find the probability that:

a both are yellow $\qquad$ **b** the first is yellow and the second is blue.

a $\quad$ P(both are yellow)

$= $ P(first is yellow *and* second is yellow)

$= $ P(first is yellow) × P(second is yellow given that the first is yellow)

$= \frac{3}{7} \times \frac{2}{6}$ $\longleftarrow$ 2 yellows remaining

$\longleftarrow$ 6 to choose from

$= \frac{1}{7}$

b $\quad$ P(first is Y and second is B)

$= $ P(first is Y) × P(second is B given that the first is Y)

$= \frac{3}{7} \times \frac{4}{6}$ $\longleftarrow$ 4 blues remaining

$\longleftarrow$ 6 to choose from

$= \frac{2}{7}$

EXERCISE 11E.2

1 A packet contains 8 identically shaped jelly beans. 5 are green and 3 are yellow. Two jelly beans are randomly selected without replacing the first before the second is drawn.

   a Determine the probability of getting:

     i two greens   **ii** a green then a yellow

     iii a yellow then a green   **iv** two yellows

   b Why do your answers in **a** add up to 1?

2 A pocket in a golf bag contains 6 white and 4 yellow golf balls. Two of them are selected at random without replacement.

   a Determine the probability that:

     i both are white   **ii** the first is white and the second is yellow

     iii one of each colour is selected.

   b Why do you answers in **a** not add up to 1?

3 A container has 4 purple, 3 blue and 1 gold ticket. Three tickets are selected without replacement. Find the probability that:

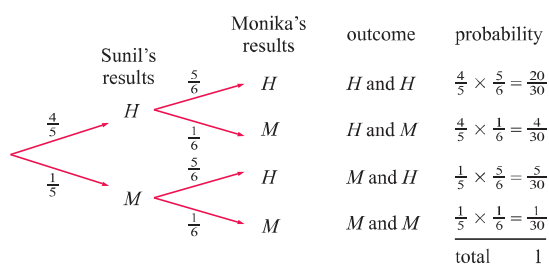   **a** all are purple   **b** all are blue   **c** the first two are purple and the third is gold.

F   USING TREE DIAGRAMS

Tree diagrams can be used to illustrate sample spaces, provided that the alternatives are not too numerous.

Once the sample space is illustrated, the tree diagram can be used for determining probabilities. Consider **Example 8** again. The tree diagram for this information is:

$H = $ hits
$M = $ misses

	Sunil's results	Monika's results	outcome	probability
	$\frac{4}{5}$ → H	$\frac{5}{6}$ → H	H and H	$\frac{4}{5} \times \frac{5}{6} = \frac{20}{30}$
		$\frac{1}{6}$ → M	H and M	$\frac{4}{5} \times \frac{1}{6} = \frac{4}{30}$
	$\frac{1}{5}$ → M	$\frac{5}{6}$ → H	M and H	$\frac{1}{5} \times \frac{5}{6} = \frac{5}{30}$
		$\frac{1}{6}$ → M	M and M	$\frac{1}{5} \times \frac{1}{6} = \frac{1}{30}$
			total	1

Notice that:

- The probabilities for hitting and missing are marked on the branches.
- There are *four* alternative paths and each path shows a particular outcome.
- All outcomes are represented and the probabilities of each outcome are obtained by **multiplying** the probabilities along that path.
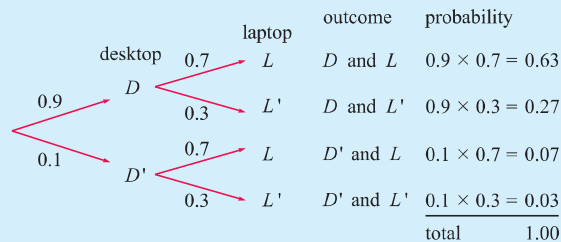
Example 10

Stephano is having problems. His desktop computer will only boot up 90% of the time and his laptop will only boot up 70% of the time.

  a Draw a tree diagram to illustrate this situation.

  b Use the tree diagram to determine the chance that:

     i both will boot up

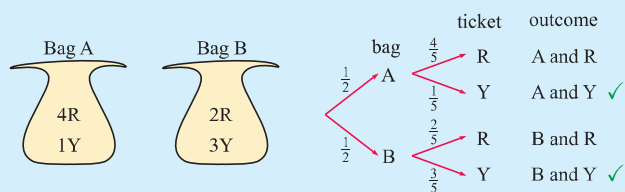   **ii** Stephano has no choice but to use his desktop computer.

a $D = $ desktop computer boots up $L = $ laptop boots up

	outcome	probability
L	D and L	$0.9 \times 0.7 = 0.63$
L'	D and L'	$0.9 \times 0.3 = 0.27$
L	D' and L	$0.1 \times 0.7 = 0.07$
L'	D' and L'	$0.1 \times 0.3 = 0.03$
	total	1.00

b **i**   P(both boot up)
      $= $ P(D and L)
      $= 0.9 \times 0.7$
      $= 0.63$

  ii P(desktop boots up but laptop does not)
      $= $ P(D and L')
      $= 0.9 \times 0.3$
      $= 0.27$

Example 11

Bag A contains 4 red jelly beans and 1 yellow jelly bean. Bag B contains 2 red and 3 yellow jelly beans. A bag is randomly selected by tossing a coin, and one jelly bean is removed from it. Determine the probability that it is yellow.
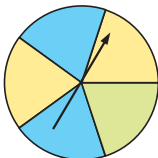

To get a yellow we take either the first branch ticked **or** the second one ticked. We **add** the probabilities for these outcomes.

$$P(\text{yellow}) = P(A \text{ and } Y) + P(B \text{ and } Y)$$
$$= \tfrac{1}{2} \times \tfrac{1}{5} + \tfrac{1}{2} \times \tfrac{3}{5} \quad \{\text{branches marked } \checkmark\}$$
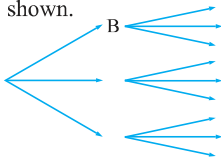$$= \tfrac{4}{10}$$
$$= \tfrac{2}{5}$$

EXERCISE 11F

1 Suppose this spinner is spun twice:

a Copy and complete the branches on the tree diagram shown.

b What is the probability that blue appears on both spins?

c What is the probability that green appears on both spins?

d What is the probability that different colours appear on both spins?

**e** What is the probability that blue appears on *either* spin?

2 In a particular board game there are nine tiles: five are green and the remainder are brown. The tiles start face down on the table so they all look the same.
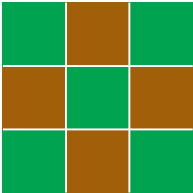
a If a player is required to pick a tile at random, determine the probability that it is:

  i green   **ii** brown.

b Suppose a player has to pick two tiles in a row, replacing the first and shuffling them before the second is selected. Copy and complete the tree diagram illustrating the possible outcomes.

tile 1   tile 2

c Using **b**, determine the probability that:

  i both tiles are green
  ii both tiles are brown
  iii tile 1 is brown and tile 2 is green
  iv one tile is brown and the other is green.
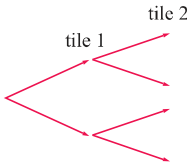
3 The probability of the race track being muddy next week is estimated to be $\frac{1}{4}$. If it is muddy, Rising Tide will start favourite with probability $\frac{2}{5}$ of winning. If it is dry he has a 1 in 20 chance of winning.
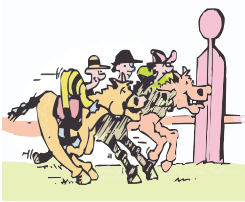
a Display the sample space of possible results on a tree diagram.

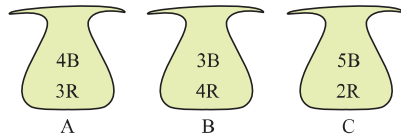**b** Determine the probability that Rising Tide will win next week.

4 Machine A cans 60% of the fruit at a factory. Machine B cans the rest. Machine A spoils 3% of its product, while Machine B spoils 4%. Determine the probability that the next can inspected at this factory will be spoiled.

5 Can A contains 2 blue and 3 red blocks and Can B contains 5 blue and 1 red block. A can is chosen at random (by the flip of a coin) and one block is taken at random from it. Determine the probability that the block is red.

6

4B
3R
A

3B
4R
B

5B
2R
C

Three bags contain different numbers of blue and red tickets. A bag is selected using a die which has three A faces, two B faces, and one C face.

One ticket is selected randomly from the chosen bag. Determine the probability that it is: **a** blue **b** red.

G SAMPLING WITH AND WITHOUT REPLACEMENT

Sampling is the process of selecting one object from a large group and inspecting it for some particular feature. The object is then either **put back** (sampling **with replacement**) or **put to one side** (sampling **without replacement**).

Sometimes the inspection process makes it impossible to return the object to the large group. Such processes include:

- Is the chocolate hard- or soft-centred? Bite it or squeeze it to see.
- Does the egg contain one or two yolks? Break it open and see.
- Is the object correctly made? Pull it apart to see.

The sampling process is used for quality control in industrial processes.

Example 12

A bin contains 4 blue and 5 green marbles. A marble is selected from this bin and its colour is noted. It is then *replaced*. A second marble is then drawn and its colour is noted. Determine the probability that:

a both are blue   **b** the first is blue and the second is green

c there is one of each colour.

Tree diagram:

B = blue
G = green

	outcome	probability
1st marble 2nd marble		
B $\frac{4}{9}$ → B	B and B	$\frac{4}{9} \times \frac{4}{9} = \frac{16}{81}$
$\frac{5}{9}$ → G	B and G	$\frac{4}{9} \times \frac{5}{9} = \frac{20}{81}$
G $\frac{4}{9}$ → B	G and B	$\frac{5}{9} \times \frac{4}{9} = \frac{20}{81}$
$\frac{5}{9}$ → G	G and G	$\frac{5}{9} \times \frac{5}{9} = \frac{25}{81}$
	total	1

a P(both blue)

$= \frac{4}{9} \times \frac{4}{9}$

$= \frac{16}{81}$

b P(first is B and second is G)

$= \frac{4}{9} \times \frac{5}{9}$

$= \frac{20}{81}$

c P(one of each colour)

$= $ P(B then G or G then B)

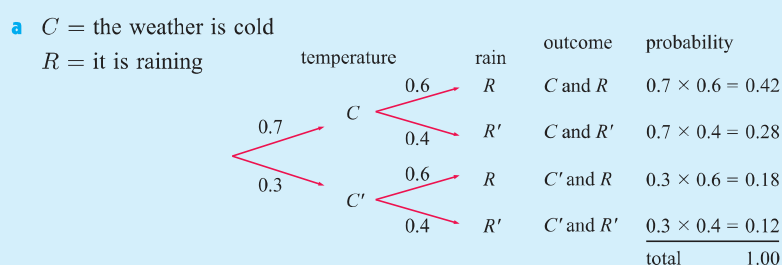$= $ P(B then G) + P(G then B)

$= \frac{4}{9} \times \frac{5}{9} + \frac{5}{9} \times \frac{4}{9}$

$= \frac{40}{81}$

Example 13

Sylke has bad luck with the weather when she takes her summer holidays. She estimates that it rains 60% of the time and it is cold 70% of the time.

a Draw a tree diagram to illustrate this situation.

b Use the tree diagram to determine the chance that for Sylke's holidays:

  i it is cold and raining   **ii** it is fine and cold.

a C = the weather is cold

    R = it is raining

	temperature	rain	outcome	probability
		0.6 → R	C and R	$0.7 \times 0.6 = 0.42$
	0.7 → C	0.4 → R'	C and R'	$0.7 \times 0.4 = 0.28$
	0.3 → C'	0.6 → R	C' and R	$0.3 \times 0.6 = 0.18$
		0.4 → R'	C' and R'	$0.3 \times 0.4 = 0.12$
			total	1.00

b **i**   P(it is cold and raining)

    = P(C and R)

    = 0.7×0.6

    = 0.42

ii   P(it is fine and cold)

    = P(R' and C)

    = 0.4×0.7

    = 0.28

EXERCISE 11G

1 A box contains 6 red and 3 yellow tickets. Two tickets are drawn at random (the first being *replaced* before the second is drawn). Draw a tree diagram to represent the sample space and use it to determine the probability that:

  a both are red   **b** both are yellow

  c the first is red and the second is yellow **d** one is red and the other is yellow.

2 7 tickets numbered 1, 2, 3, 4, 5, 6 and 7 are placed in a hat. Two of the tickets are taken from the hat at random *without replacement*. Determine the probability that:

  a both are odd   **b** both are even

  c the first is even and the second is odd **d** one is even and the other is odd.

3 Jessica has a bag of 9 acid drops which are all identical in shape. 5 are raspberry flavoured and 4 are orange flavoured. She selects one acid drop at random, eats it, and then takes another, also at random. Determine the probability that:

  a both acid drops were orange flavoured

  b both acid drops were raspberry flavoured

  c the first was raspberry and the second was orange

  d the first was orange and the second was raspberry.

Add your answers to **a**, **b**, **c** and **d**. Explain why the answer must be 1.

4 A cook selects an egg at random from a carton containing 7 ordinary eggs and 5 double-yolk eggs. She cracks the egg into a bowl and sees whether it has two yolks or not. She then selects another egg at random from the carton and checks it.

Let S represent "a single yolk egg" and D represent "a double yolk egg".

   a Draw a tree diagram to illustrate this sampling process.

   b What is the probability that both eggs had two yolks?

   c What is the probability that both eggs had only one yolk?

5 Freda selects a chocolate at random from a box containing 8 hard-centred and 11 soft-centred chocolates. She bites it to see whether it is hard-centred or not. She then selects another chocolate at random from the box and checks it.

Let H represent "a hard-centred chocolate" and S represent "a soft-centred chocolate".

   a Draw a tree diagram to illustrate this sampling process.

   b What is the probability that both chocolates have hard centres?

   c What is the probability that both chocolates have soft centres?

6 A sporting club runs a raffle in which 200 tickets are sold. There are two winning tickets which are drawn at random, in succession, without replacement. If Adam bought 8 tickets in the raffle, determine the probability that he:

   a wins first prize   **b** does not win first prize

   c wins second prize *given that* he did not win first prize.

H   MUTUALLY EXCLUSIVE AND NON-MUTUALLY EXCLUSIVE EVENTS

Suppose we select a card at random from a normal pack of 52 playing cards. Consider carefully these events:

Event X: the card is a heart   *Event Y:*   the card is an ace   *Event Z:*   the card is a 7
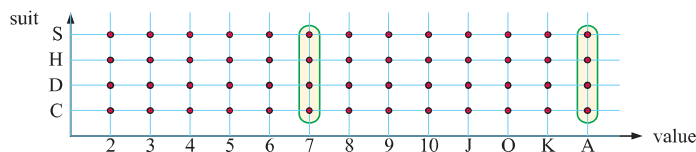
Notice that:

- X and Y have a common outcome: the Ace of hearts
- X and Z have a common outcome: the 7 of hearts
- Y and Z do not have a common outcome.
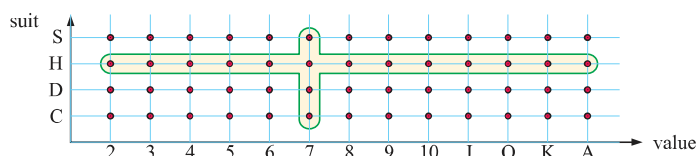
When considering a situation like this:

- if two events have no common outcomes we say they are **mutually exclusive** or **disjoint**
- if two events have common outcomes they are **not mutually exclusive**.

Notice that: $\quad$ P(ace **or** seven) $= \frac{8}{52}$ $\quad$ and $\quad$ P(ace) $+$ P(seven) $= \frac{4}{52} + \frac{4}{52} = \frac{8}{52}$

So, $\qquad\qquad$ if two events A and B are **mutually exclusive** then
$$\text{P}(A \textbf{ or } B) = \text{P}(A) + \text{P}(B)$$

Notice that: $\quad$ P(heart **or** seven) $= \frac{16}{52}$ $\quad$ and $\quad$ P(heart) $+$ P(seven) $= \frac{13}{52} + \frac{4}{52} = \frac{17}{52}$.

Actually, $\quad$ P(heart **or** seven) $=$ P(heart) $+$ P(seven) $-$ P(heart **and** seven).

So, $\qquad\qquad$ if two events A and B are **not mutually exclusive** then
$$\text{P}(A \textbf{ or } B) = \text{P}(A) + \text{P}(B) - \text{P}(A \textbf{ and } B).$$

EXERCISE 11H

1 An ordinary die with faces 1, 2, 3, 4, 5 and 6 is rolled once. Consider these events:

A: getting a 1 $\qquad\qquad$ B: getting a 3
C: getting an odd number $\qquad$ D: getting an even number
E: getting a prime number $\qquad$ F: getting a result greater than 3.

a List all possible pairs of events which are mutually exclusive.

b Find: $\quad$ **i** P(B or D) $\qquad$ **ii** P(D or E) $\qquad$ **iii** P(A or E)
$\qquad\qquad$ **iv** P(B or E) $\qquad$ **v** P(C or D) $\qquad$ **vi** P(A or B or F).

2 A coin and an ordinary die are tossed simultaneously.

a Draw a grid showing the 12 possible outcomes.

b Find the probabilites of getting: $\quad$ **i** a head and a 5 $\quad$ **ii** a head or a 5.

c Check that: $\quad$ P(H or 5) = P(H) + P(5) − P(H **and** 5).

3 Two ordinary dice are rolled.

a Draw a grid showing the 36 possible outcomes.

b Find the probability of getting: $\quad$ **i** a 3 and a 4 $\quad$ **ii** a 3 or a 4.

**c** Check that: $\quad$ P(3 or 4) = P(3) + P(4) − P(3 **and** 4).

1 VENN DIAGRAMS AND CONDITIONAL PROBABILITY

As we observed earlier in the chapter, Venn diagrams are useful for answering certain types of probability questions.

Example 14

In a class of 30 students, 19 play sport, 8 play the piano, and 3 both play sport and the piano. Display this information on a Venn diagram and hence determine the probability that a randomly selected class member plays:

a both sport and the piano
b at least one of sport and the piano
c sport, but not the piano
d exactly one of sport and the piano
e neither sport nor the piano
f the piano if it is known that the student plays sport.
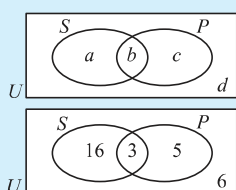
Let S represent the event of 'playing sport',
and P represent the event of 'playing the piano'.

Now $a + b = 19$ {as 19 play sport}
  $b + c = 8$ {as 8 play the piano}
  $b = 3$ {as 3 play both}
  $a + b + c + d = 30$ {as there are 30 in the class}

$\therefore \quad b = 3, \quad a = 16, \quad c = 5, \quad d = 6.$

a P(S and P)
$= \frac{3}{30}$ or $\frac{1}{10}$

b P(at least one of S and P)
$= \frac{16+3+5}{30}$
$= \frac{24}{30}$ (or $\frac{4}{5}$)

c P(S but not P)
$= \frac{16}{30}$
$= \frac{8}{15}$

d P(exactly one of S and P)
$= \frac{16+5}{30}$
$= \frac{7}{10}$

e P(neither S nor P)
$= \frac{6}{30}$
$= \frac{1}{5}$

f P(P given S)
$= \frac{3}{16+3}$
$= \frac{3}{19}$

EXERCISE 11I.1

1 In a class of 35 students, 18 swim, 14 play tennis, and 8 do neither of these sports. A student is randomly chosen from the class. Determine the probability that the student:

a plays tennis
b does not swim
c does at least one of the sports
d does exactly one of the sports
e swims, but does not play tennis.

2 On a hot day a group of 50 people at the beach were asked why they had come to the beach. 27 had come to swim, 19 had come to surf, and 3 had come to do both. If one person was chosen at random, determine the probability that he or she had come:

    a to surf but not swim   **b** neither to surf nor swim

    c to swim but not surf.

3 From the Venn diagram, $\quad P(A) = \dfrac{a+b}{a+b+c+d}$.
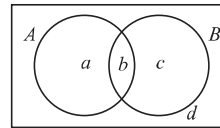
    a Use the Venn diagram to find:

        i $P(B)$ **ii** $P(A \text{ and } B)$ **iii** $P(A \text{ or } B)$

        iv $P(A) + P(B) - P(A \text{ and } B)$.

    b What is the connection between $\quad P(A \text{ or } B) \quad$ and $\quad P(A) + P(B) - P(A \text{ and } B)$?

UNION AND INTERSECTION

In **Chapter 1** we defined the **union** and **intersection** of sets.

> The **union** of two events A and B is denoted $\quad A \cup B$.   It includes all outcomes that are in A **or** B.
>
> The **intersection** of two events A and B is denoted $\quad A \cap B$.   It includes all outcomes that are in both A **and** B.

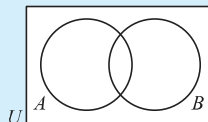In question **3** of the previous exercise we proved that:

> for two events A and B, $\quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

This is known as the **addition law of probability**, and can be written as

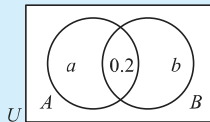> $P(\textbf{either } A \textbf{ or } B) = P(A) + P(B) - P(\textbf{both } A \textbf{ and } B)$.

Example 15

If $P(B) = 0.4$, $\quad P(A \cup B) = 0.7$
and $\quad P(A \cap B) = 0.2$, $\quad$ find $P(A)$.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$\therefore \quad 0.7 = P(A) + 0.4 - 0.2$$
$$\therefore \quad P(A) = 0.5$$

or   Using a Venn diagram with the probabilities on it,

$$b + 0.2 = 0.4 \qquad \therefore \quad b = 0.2$$
$$a + 0.2 + 0.2 = 0.7$$
$$\therefore \quad a = 0.3$$
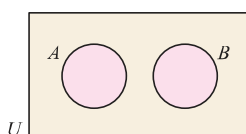$$\therefore \quad P(A) = 0.3 + 0.2 = 0.5$$

MUTUALLY EXCLUSIVE OR DISJOINT EVENTS

In **Chapter 1** we defined mutually exclusive sets.

If A and B are **mutually exclusive** events then $P(A \cap B) = 0$ and so the addition law becomes
$$P(A \cup B) = P(A) + P(B).$$

In a Venn diagram for mutually exclusive events, the circles for the events do not overlap.

Mutually exclusive events have no common outcomes.

Example 16

A piece of fruit is randomly selected from a basket containing 8 apples and 12 oranges. The event A represents selecting an apple, and O represents selecting an orange.

a Are A and O mutually exclusive?

b Find **i** $P(A)$ **ii** $P(O)$ **iii** $P(A \cap O)$ **iv** $P(A \cup O)$.

a A piece of fruit cannot be an apple *and* an orange.
∴ A and O are mutually exclusive.

b **i** $P(A) = \frac{8}{20} = \frac{2}{5}$ **ii** $P(O) = \frac{12}{20} = \frac{3}{5}$

    iii $P(A \cap O) = 0$ **iv** $P(A \cup O) = \frac{20}{20} = 1$

CONDITIONAL PROBABILITY

If we have two events A and B, then

$A \mid B$ is used to represent that 'A occurs knowing that B has occurred'.

$A \mid B$ is often read as 'A given B'.

Example 17

In a group of 25 students, 15 like chocolate milk (M) and 17 like iced coffee (C). Two students like neither and 9 students like both. One student is randomly selected from the class. What is the probability that the student:
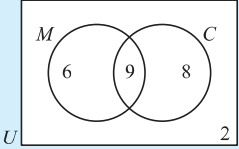
a likes chocolate milk

b likes chocolate milk given that he or she likes iced coffee?

The Venn diagram of the situation is shown.

a 15 of the 25 students like chocolate milk.
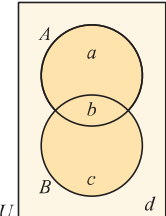
∴ P(chocolate milk) $= \frac{15}{25}$

b Of the 17 who like iced coffee, 9 like chocolate milk.

∴ P(chocolate milk | iced coffee) $= \frac{9}{17}$

If A and B are events then

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}.$$

Proof:

$P(A \mid B)$

$= \dfrac{b}{b+c}$ {Venn diagram}

$= \dfrac{b \div (a+b+c+d)}{(b+c) \div (a+b+c+d)}$
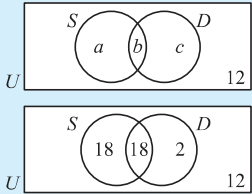
$= \dfrac{P(A \cap B)}{P(B)}$

It follows that $P(A \cap B) = P(A \mid B)\,P(B)$ or $P(A \cap B) = P(B \mid A)\,P(A)$.

Example 18

In a library group of 50 readers, 36 like science fiction, 20 like detective stories, and 12 dislike both.

If a reader is randomly selected, find the probability that he or she:

a likes science fiction and detective stories

b likes at least one of science fiction and detective stories

c likes science fiction given that he or she likes detective stories

d dislikes detective stories given that he or she likes science fiction.

S represents readers who like science fiction.
D represents readers who like detective stories.

We are given that $a + b = 36$
$b + c = 20$
$a + b + c = 38$

∴ $c = 38 - 36$ and so $b = 18$
$= 2$ and $a = 18$

a P(likes both) $= \frac{18}{50} = \frac{9}{25}$

b P(likes at least one) $= \frac{38}{50} = \frac{19}{25}$

c $P(S \mid D) = \frac{18}{20} = \frac{9}{10}$

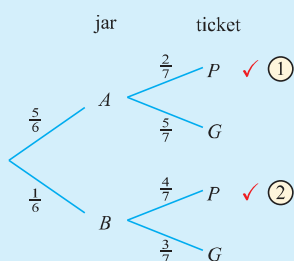**d** $P(D' \mid S) = \frac{18}{36} = \frac{1}{2}$

Example 19

Jar A contains 2 pink and 5 green tickets. Jar B contains 4 pink and 3 green tickets. A die with five faces marked A and one face marked B is rolled and used to select jar A or B. A ticket is then selected from this jar. Determine the probability that:

a the ticket is pink   **b** the ticket was chosen from B given it is pink.

a $\text{P}(P)$

$= \frac{5}{6} \times \frac{2}{7} + \frac{1}{6} \times \frac{4}{7}$ {the ✓ paths}

$= \frac{14}{42}$

$= \frac{1}{3}$

b $\text{P}(B \mid P)$

$= \dfrac{\text{P}(B \cap P)}{\text{P}(P)}$

$= \dfrac{\frac{1}{6} \times \frac{4}{7}}{\frac{1}{3}}$ ←— path ②
 ←— paths ① and ②

$= \frac{6}{21}$

$= \frac{2}{7}$

EXERCISE 11I.2

1 50 students went on a 'thrill seekers' holiday. 40 went white-water rafting, 21 went paragliding, and each student did at least one of these activities.

  a From a Venn diagram, find how many students did both activities.

  b If a student from this group is randomly selected, find the probability that he or she:

    i went white-water rafting but not paragliding

    ii went paragliding given that he or she went white-water rafting.

2 In a class of 25 students, 19 have fair hair, 15 have blue eyes, and 22 have fair hair, blue eyes or both. A child is selected at random. Determine the probability that the child has:

  a fair hair and blue eyes   **b** neither fair hair nor blue eyes

  c fair hair but not blue eyes   **d** blue eyes given that the child has fair hair.

3

28 students go tramping. 23 get sunburn, 8 get blisters, and 5 get both sunburn and blisters. Determine the probability that a randomly selected student:

  a did not get blisters

  b either got blisters or sunburn

  c neither got blisters nor sunburn

  d got blisters, given that the student was sunburnt

  e was sunburnt, given that the student did not get blisters.

4 An examination in French has two parts: aural and written. When 30 students sit for the examination, 25 pass aural, 26 pass written, and 3 fail both parts. Determine the probability that a student who:

   a passed aural also passed written   **b** passed aural, failed written.

5 In a small country there are 3 supermarkets: P, Q and R. 60% of the population shop at P, 36% shop at Q, 34% shop at R, 18% shop at P and Q, 15% shop at P and R, 4% shop at Q and R, and 2% shop at all 3 supermarkets. A person is selected at random. Determine the probability that the person shops at:

   a none of the supermarkets   **b** at least one of the supermarkets

   c exactly one of the supermarkets   **d** either P or Q

   e P, given that the person shops at at least one supermarket

   f R, given that the person shops at either P or Q or both.

6 Marius has 2 bags of peaches. Bag A has 4 ripe and 2 unripe peaches, and bag B has 5 ripe and 1 unripe peaches. Ingrid selects a bag by tossing a coin, and takes a peach from that bag.

   a Determine the probability that the peach is ripe.

   b Given that the peach is ripe, what is the probability it came from B?

7 When Sophia goes to the local shopping centre from Monday to Thursday, the probability that she finds a car park is 95%. When she goes on Friday or Saturday, the probability of finding a car park is 70%. Assuming that she is equally likely to shop on any day from Monday to Saturday, determine the probability that:

   a she finds a car park   **b** it is Saturday, given that she finds a car park.

8 On a given day, Claude's car has an 80% chance of starting first time and André's car has a 70% chance of the same. Given that at least one of the cars has started first time, what is the chance that André's car started first time?

REVIEW SET 11A

1 Donna kept records of the number of clients she interviewed over a period of consecutive days.

   a For how many days did the survey last?

   b Estimate Donna's chances of interviewing:

      i no clients on a day

      ii four or more clients on a day

      iii less than three clients on a day.

2 Illustrate on a 2-dimensional grid the possible outcomes when a coin and a pentagonal spinner with sides labelled A, B, C, D and E are tossed and spun simultaneously.

3 What is meant by saying that two events are "independent"?

4 Use a tree diagram to illustrate the sample space for the possible four-child families. Hence determine the probability that a randomly chosen four-child family:

    a is all boys   **b** has exactly two boys   **c** has more girls than boys.

5 In a shooting competition, Louise has 80% chance of hitting her target and Kayo has 90% chance of hitting her target. If they both have a single shot, determine the probability that:

    a both hit their targets   **b** neither hits her target

    c at least one hits her target   **d** only Kayo hits her target.

6 Two fair six-sided dice are rolled simultaneously. Determine the probability that the result is a 'double', i.e., both dice show the same number.

7 A bag contains 4 green and 3 red marbles. Two marbles are randomly selected from the bag, the first being replaced before the second is drawn. Determine the probability that:

    a both are green   **b** they are different in colour.

8 A circle is divided into 5 sectors with equal angles at the centre. It is made into a spinner, and the sectors are numbered 1, 2, 3, 4, and 5. A coin is tossed and the spinner is spun.

    a Use a 2-dimensional grid to show the sample space.

    b What is the chance of getting: **i** a head and a 5   **ii** a head or a 5?

9 Bag X contains three white and two red marbles. Bag Y contains one white and three red marbles. A bag is randomly chosen and two marbles are drawn from it. Illustrate the given information on a tree diagram and hence determine the probability of drawing two marbles of the same colour.

10 At a local girls school, 65% of the students play netball, 60% play tennis, and 20% play neither sport. Display this information on a Venn diagram, and hence determine the likelihood that a randomly chosen student plays:

    a netball   **b** netball but not tennis

    c at least one of these two sports   **d** exactly one of these two sports

    e tennis, given that she plays netball.

REVIEW SET 11B

1 Pierre conducted a survey to determine the ages of people walking through a shopping mall. The results are shown in the table alongside. Estimate, to 3 decimal places, the probability that the next person Pierre meets in the shopping mall is:

Age	Frequency
0 - 19	22
20 - 39	43
40 - 59	39
60+	14

    a between 20 and 39 years of age

    b less than 40 years of age   **c** at least 20 years of age.

2   **a** List the sample space of possible results when a tetrahedral die with four faces labelled A, B, C and D is rolled and a 20-cent coin is tossed simultaneously.

b Use a tree diagram to illustrate the sample spaces for the following:

   i Bags A, B and C contain green or yellow tickets. A bag is selected and then a ticket taken from it.

   ii Martina and Justine play tennis. The first to win three sets wins the match.

3 When a box of drawing pins was dropped onto the floor, it was observed that 49 pins landed on their backs and 32 landed on their sides. Estimate, to 2 decimal places, the probability of a drawing pin landing:

back   side

   a on its back   **b** on its side.

4 The letters A, B, C, D, ... N are put in a hat.

   a Determine the probability of drawing a vowel (A, E, I, O or U) if one of the letters is chosen at random.

   b If two letters are drawn without replacement, copy and complete the following tree diagram including all probabilities:

   c Use your tree diagram to determine the probability of drawing:

     i a vowel and a consonant

     ii at least one vowel.

1st draw   **2nd draw**

vowel — vowel / consonant

consonant — vowel / consonant

5 A farmer fences his rectangular property into 9 rectangular paddocks as shown alongside.

If a paddock is selected at random, what is the probability that it has:

   a no fences on the boundary of the property

   b one fence on the boundary of the property

   c two fences on the boundary of the property?

6 If $P(A) = \frac{7}{12}$, find $P(A')$.

7 Bag X contains 3 black and 2 red marbles. Bag Y contains 4 black and 1 red marble. A bag is selected at random and then two marbles are selected without replacement. Determine the probability that:

   a both marbles are red   **b** two black marbles are picked from Bag Y.

8 Two dice are rolled simultaneously. Illustrate this information on a 2-dimensional grid. Determine the probability of getting:

   a a double 5   **b** at least one 4

   c a sum greater than 9   **d** a sum of 7 or 11.

9 A class consists of 25 students. 15 have blue eyes, 9 have fair hair, and 3 have both blue eyes and fair hair. Represent this information on a Venn diagram.

Hence find the probability that a randomly selected student from the class:

   a has neither blue eyes nor fair hair   **b** has blue eyes, but not fair hair

   c has fair hair given that he or she has blue eyes

   d does not have fair hair given that he or she does not have blue eyes.