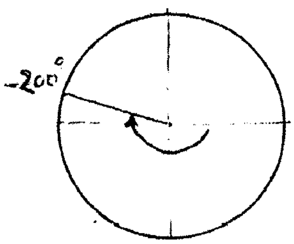
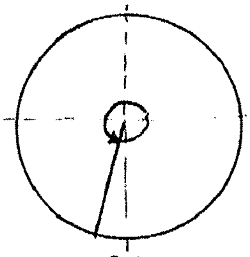
1]

ⓐ



SENO	+
COSENO	−
TANG	−

ⓑ

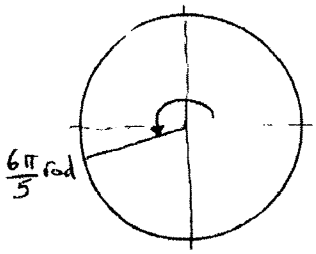


$3487 \,\lfloor\, 360$

$247 \quad 9$

SENO	−
COSENO	−
TANG	+

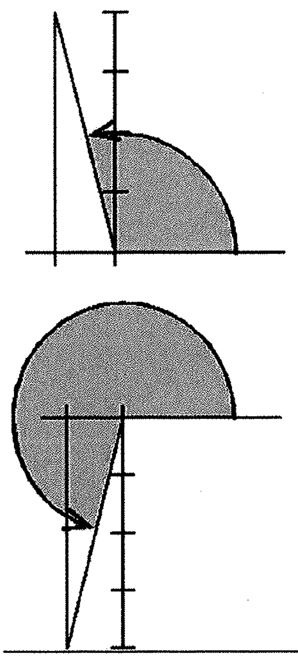
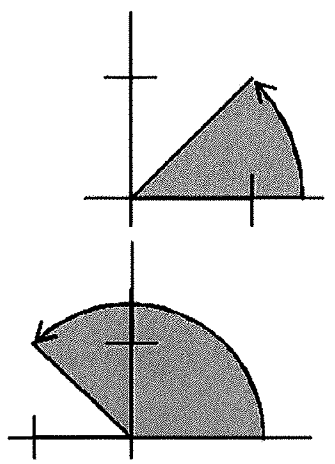
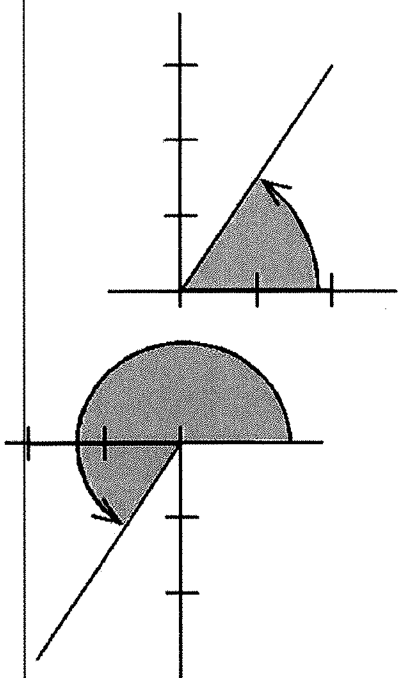
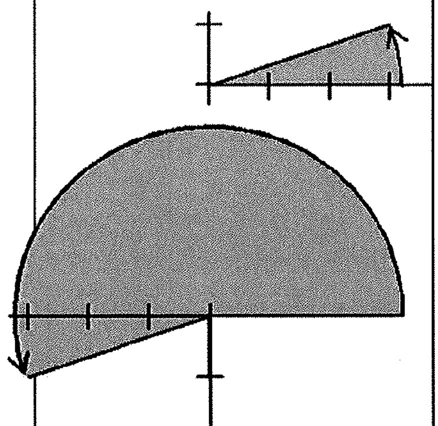
ⓒ



$\frac{6\pi}{5}\text{ rad} = \frac{6 \cdot 180°}{5} = 216°$

SENO	−
COSENO	−
TANG	+

2]

$\cos\alpha = -\frac{1}{4}$	$sen\alpha = \frac{\sqrt{2}}{2}$	$tg\alpha = 1'5$	$\cos\alpha = 3sen\alpha$
			

3]

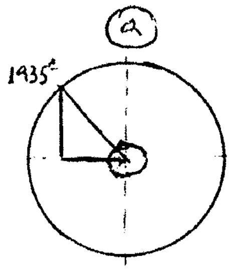
$$sen^2a + cos^2a = 1 \Rightarrow \frac{sen^2a}{sen^2a} + \frac{cos^2a}{sen^2a} = \frac{1}{sen^2a} \Rightarrow 1 + ctg^2a = cosec^2a \Rightarrow$$

$$\Rightarrow \boxed{cosec\,a = \pm\sqrt{1+ctg^2a}} \Rightarrow \boxed{sen\,a = \frac{\pm 1}{\sqrt{1+ctg^2a}}} \Rightarrow cos\,a = sen\,a \cdot ctg\,a = \boxed{\frac{\pm ctg\,a}{\sqrt{1+ctg^2a}}} \Rightarrow$$

$$\Rightarrow \boxed{sec\,a = \frac{\pm\sqrt{1+ctg^2a}}{ctg\,a}} \qquad \boxed{tg\,a = \frac{1}{ctg\,a}}$$

4

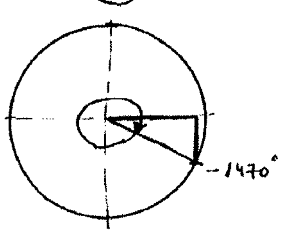
ⓐ



$$1935 \,\underline{|360}\quad \text{resto } 135,\ \text{cociente } 5$$

$$\boxed{\begin{aligned} \operatorname{Sen} 1935^\circ &= \operatorname{Sen} 135^\circ = \sqrt{2}/2 \\ \cos 1935^\circ &= \cos 135^\circ = -\sqrt{2}/2 \\ \operatorname{tg} 1935^\circ &= -1 \end{aligned}}$$

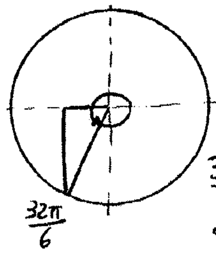
ⓑ



$$1470 \,\underline{|360}\quad \text{resto } 030,\ \text{cociente } 4$$

$$\boxed{\begin{aligned} \operatorname{sen}(-1470^\circ) &= \operatorname{sen}(-30) = -\operatorname{sen} 30 = -\tfrac{1}{2} \\ \cos(-1470^\circ) &= \cos(-30) = \cos 30 = \sqrt{3}/2 \\ \operatorname{tg}(-1470^\circ) &= -1/\sqrt{3} \end{aligned}}$$

ⓒ

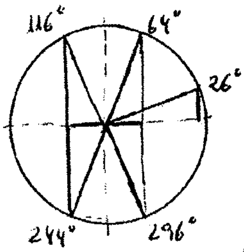


$$\frac{32\pi}{6} = \frac{32 \cdot 180}{6} = 960^\circ$$

$$960 \,\underline{|360}\quad \text{resto } 240,\ \text{cociente } 2$$

$$\boxed{\begin{aligned} \operatorname{Sen} \frac{32\pi}{6} &= \operatorname{sen} 240^\circ = -\sqrt{3}/2 \\ \cos \frac{32\pi}{6} &= \cos 240^\circ = -1/2 \\ \operatorname{tg} \frac{32\pi}{6} &= \sqrt{3} \end{aligned}}$$

5



$$\operatorname{cosec} 296^\circ = -1'1 \implies \operatorname{sen} 296^\circ = -\frac{1}{1'1} = -0'91$$

$$\cos 296^\circ = -\sqrt{1-(-0'91)^2} = +0'42$$

ⓐ	ⓑ	ⓒ	ⓓ
$\operatorname{sen} 64^\circ = 0'91$	$\operatorname{sen} 244^\circ = -0'91$	$\operatorname{sen} 26^\circ = 0'42$	$\operatorname{sen} 116^\circ = 0'91$
$\cos 64^\circ = 0'42$	$\cos 244^\circ = -0'42$	$\cos 26^\circ = 0'91$	$\cos 116^\circ = -0'42$

6

ⓐ $\left.\begin{array}{l}\operatorname{cosec} a = 2 \\ (\pi/2 < a < \pi)\end{array}\right| \implies \operatorname{sen} a = \boxed{\frac{1}{2}} \implies \cos a = \pm\sqrt{1-\frac{1}{4}} = \boxed{-\frac{\sqrt{3}}{2}} \implies \sec a = \boxed{-\frac{2}{\sqrt{3}}}$

$$\operatorname{tg} a = \frac{1/2}{-\sqrt{3}/2} = \boxed{-\frac{1}{\sqrt{3}}} \implies \boxed{\operatorname{ctg} a = -\sqrt{3}}$$

ⓑ $\left.\begin{array}{l}\operatorname{ctg} a = \frac{1}{3} \\ \pi < a < \frac{3\pi}{2}\end{array}\right| \implies \frac{\cos a}{\operatorname{sen} a} = \frac{1}{3} \implies \operatorname{sen} a = 3\cos a \implies (3\cos a)^2 + \cos^2 a = 1 \implies$

$$\implies 10\cos^2 a = 1 \implies \cos a = \boxed{-\frac{1}{\sqrt{10}}} \implies \operatorname{sen} a = \boxed{-\frac{3}{\sqrt{10}}} \implies \boxed{\operatorname{tg} a = 3}$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$\boxed{\sec a = -\sqrt{10}} \qquad \boxed{\operatorname{cosec} a = -\frac{\sqrt{10}}{3}}$$

7

a) $\cos^4\alpha - \operatorname{sen}^4\alpha = 2\cos^2\alpha - 1$

$$\cos^4\alpha - \operatorname{sen}^4\alpha = (\cos^2\alpha + \operatorname{sen}^2\alpha)(\cos^2\alpha - \operatorname{sen}^2\alpha) = 1\cdot[\cos^2\alpha - (1-\cos^2\alpha)] = 2\cos^2\alpha - 1 \ ✓$$

b) $(\operatorname{cosec}\alpha + \operatorname{ctg}\alpha)(\operatorname{cosec}\alpha - \operatorname{ctg}\alpha) = 1$

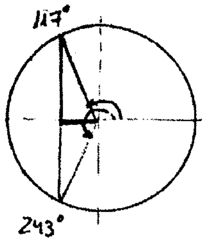
$$(\operatorname{cosec}\alpha + \operatorname{ctg}\alpha)(\operatorname{cosec}\alpha - \operatorname{ctg}\alpha) = \operatorname{cosec}^2\alpha - \operatorname{ctg}^2\alpha = \frac{1}{\operatorname{sen}^2\alpha} - \frac{\cos^2\alpha}{\operatorname{sen}^2\alpha} = \frac{1-\cos^2\alpha}{\operatorname{sen}^2\alpha} = \frac{\operatorname{sen}^2\alpha}{\operatorname{sen}^2\alpha} = 1 \ ✓$$

8

a) $$\frac{\sec^2\alpha - \cos^2\alpha}{\operatorname{tg}^2\alpha} = \frac{\frac{1}{\cos^2\alpha} - \cos^2\alpha}{\frac{\operatorname{sen}^2\alpha}{\cos^2\alpha}} = \frac{1-\cos^4\alpha}{\operatorname{sen}^2\alpha} = \frac{(1+\cos^2\alpha)(1-\cos^2\alpha)}{\operatorname{sen}^2\alpha} = \frac{(1+\cos^2\alpha)\cdot\cancel{\operatorname{sen}^2\alpha}}{\cancel{\operatorname{sen}^2\alpha}} = \boxed{1+\cos^2\alpha}$$

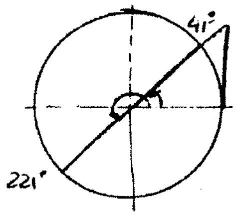
b) $$\frac{\operatorname{cosec}\alpha}{1+\operatorname{ctg}^2\alpha} = \frac{\frac{1}{\operatorname{sen}\alpha}}{1+\frac{\cos^2\alpha}{\operatorname{sen}^2\alpha}} = \frac{\operatorname{sen}\alpha}{\operatorname{sen}^2\alpha + \cos^2\alpha} = \frac{\operatorname{sen}\alpha}{1} = \boxed{\operatorname{sen}\alpha}$$

9] ⓐ



$$\boxed{\begin{array}{l}\alpha = 117^\circ + m\,360^\circ \\ \alpha = 243 + m\cdot 360^\circ \\ m \in \mathbb{Z}\end{array}}$$

ⓑ

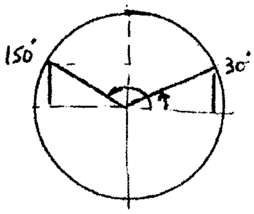


$$\boxed{\begin{array}{l}\alpha = 41^\circ + m\cdot 360^\circ \\ \alpha = 221^\circ + m\cdot 360^\circ \\ m \in \mathbb{Z}\end{array}}$$

O también:

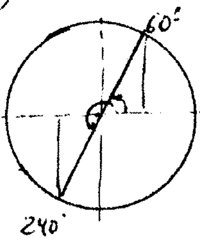
$$\boxed{\begin{array}{l}\alpha = 41^\circ + m\cdot 180^\circ \\ m \in \mathbb{Z}\end{array}}$$

ⓒ



$$\boxed{\begin{array}{l}\alpha = 30^\circ + m\cdot 360^\circ \\ \alpha = 150^\circ + m\cdot 360^\circ \\ m \in \mathbb{Z}\end{array}}$$

ⓓ



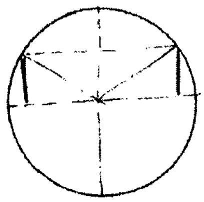
$$\boxed{\begin{array}{l}\alpha = 60^\circ + m\cdot 360^\circ \\ \alpha = 240^\circ + m\cdot 360^\circ \\ m \in \mathbb{Z}\end{array}}$$

O también:

$$\boxed{\begin{array}{l}\alpha = 60^\circ + m\cdot 180^\circ \\ m \in \mathbb{Z}\end{array}}$$

10] ⓐ $\operatorname{sen} x = 0'5983 \Rightarrow$

$$\boxed{\begin{array}{l}x = 37 + m\cdot 360^\circ \\ x = 143 + m\cdot 360^\circ \\ m \in \mathbb{Z}\end{array}}$$

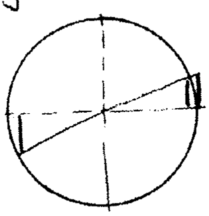


ⓑ $\operatorname{tg} x = 0'47960 \Rightarrow$

$$\boxed{\begin{array}{l}x = 26^\circ + m\cdot 360^\circ \\ x = 206^\circ + m\cdot 360^\circ \\ m \in \mathbb{Z}\end{array}}$$

O también:

$$\boxed{\begin{array}{l}x = 26^\circ + m\cdot 180^\circ \\ m \in \mathbb{Z}\end{array}}$$



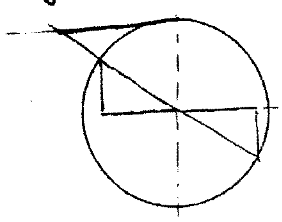
ⓒ $\operatorname{ctg} x = -1'19572 \Rightarrow x = -39'91$

$\Downarrow$

$$\boxed{\begin{array}{l}x = 320^\circ + m\cdot 360^\circ \\ x = 140^\circ + m\cdot 360^\circ \\ m \in \mathbb{Z}\end{array}}$$

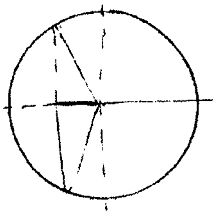
O también:

$$\boxed{\begin{array}{l}x = 140^\circ + m\cdot 180^\circ \\ m \in \mathbb{Z}\end{array}}$$



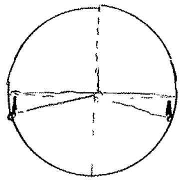
ⓓ $\cos x = -0'7583 \Rightarrow$

$$\boxed{\begin{array}{l}x = 139^\circ + m\cdot 360^\circ \\ x = 221^\circ + m\cdot 360^\circ \\ m \in \mathbb{Z}\end{array}}$$



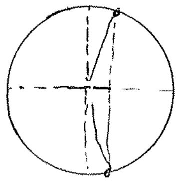
(e) $\operatorname{sen} x = -0'1045 \Rightarrow x = -6^\circ \equiv \boxed{354^\circ + m \cdot 360^\circ}$ $\quad m \in \mathbb{Z}$

$x = \boxed{186^\circ + m \cdot 360^\circ}$



(f) $\cos = 0'1908 \Rightarrow x = \boxed{79^\circ + m \cdot 360^\circ}$ $\quad m \in \mathbb{Z}$

$x = \boxed{281^\circ + m \cdot 360^\circ}$



11| Como el área de un sector $\left(\frac{R^2\theta}{2}\right)$ es proporcional al ángulo, que el mayor ángulo sea doble del menor significa que el mayor área será doble de la menor.

$A_1, A_2, \ldots, A_{20}$

$S_{20} = \pi R^2 \Rightarrow \frac{A_1 + A_{20}}{2} \cdot 20 = \pi R^2 \Rightarrow \frac{A_1 + 2A_1}{2} \cdot 20 = \pi R^2$; $30A_1 = \pi R^2$; $A_1 = \frac{\pi R^2}{30}$

$A_{20} = 2A_1$

$A_1 = \frac{R^2\theta_1}{2} \Rightarrow \frac{\cancel{R^2}\theta_1}{2} = \frac{\pi \cancel{R^2}}{30} \Rightarrow \theta_1 = \boxed{\frac{\pi}{15} \text{ rad}}$

<u>También</u>

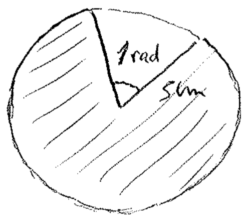
Al ser áreas proporcionales a ángulos, que las primeras estén en progresión aritmética implicaría que lo están los ángulos: $\theta_1, \theta_2, \ldots, \theta_{20}$

$S_{20} = 2\pi \Rightarrow \frac{\theta_1 + \theta_{20}}{2} \cdot 20 = 2\pi$; $\frac{\theta_1 + 2\theta_1}{2} \cdot 20 = 2\pi$; $30\theta_1 = 2\pi \Rightarrow \theta_1 = \frac{\pi}{15}$ ✓

12|

2π rad —— $2\pi \cdot 5$

$2\pi - 1$ rad —— Arco



$\text{Arco} = \frac{\cancel{2\pi} \cdot 5 \cdot (2\pi - 1)}{\cancel{2\pi}} = 10\pi - 5 \simeq 26'4$ cm

$\text{Perímetro} = \text{Arco} + 2 \cdot R = 10\pi - 5 + 2 \cdot 5 =$

$= \boxed{10\pi + 5} \simeq 36'4$ cm

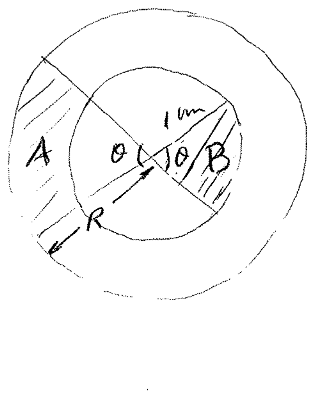
2π rad —— $\pi \cdot 5^2$

$2\pi - 1$ rad —— Sector

$\text{Sector} = \frac{\cancel{\pi} \cdot 25 \cdot (2\pi - 1)}{2\cancel{\pi}} = \boxed{\frac{50\pi - 25}{2}} \simeq 66'04 \text{ cm}^2$

13|

$\text{Area } B = \frac{\pi \cdot 1^2 \cdot \theta}{2} = \frac{\pi\theta}{2}$



$\text{Area } A = \frac{\pi \cdot R^2\theta}{2} - \frac{\pi \cdot 1^2\theta}{2} = \frac{\pi(R^2-1)\theta}{2}$

$\text{Area } A = 2 \cdot \text{Area } B \Rightarrow \frac{\cancel{\pi}(R^2-1)\cancel{\theta}}{\cancel{2}} = 2\frac{\cancel{\pi\theta}}{\cancel{2}}$

$R^2 - 1 = 2$

$R^2 = 3$; $R = \boxed{\sqrt{3} \text{ cm}}$

14

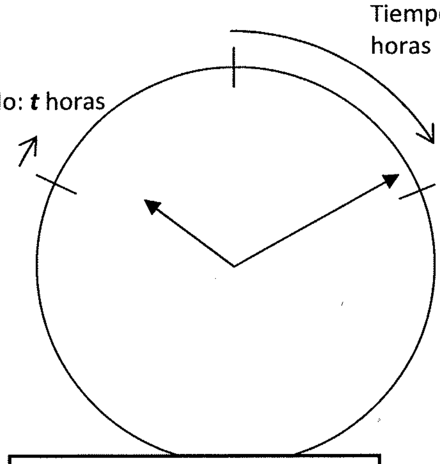
Las velocidades angulares de las agujas son:

Aguja Horaria = $30 \,{}^{\circ}\!/_{h}$

Aguja Minutera = $360 \,{}^{\circ}\!/_{h}$

Aguja Horaria
Tiempo transcurrido: ***t*** horas
Angulo girado: ***a***

Aguja Minutera
Tiempo transcurrido: ***t*** horas



A partir de las diez en punto, y en un mismo tiempo ***t***:

$$\left.\begin{array}{r} 30 = \dfrac{a}{t} \rightarrow a = 30t \\ 360 = \dfrac{60 - a}{t} \end{array}\right\} \Rightarrow 360t = 60 - 30t$$

$$390t = 60 \Rightarrow t = \frac{60}{390} = \frac{2}{13} h \approx 9\,\text{min}\,14 seg$$

$\boxed{\text{Son casi las 10h 9min 14seg}}$