JUN 94

$x-2y+8=0 \longrightarrow y=\frac{x+8}{2} \Rightarrow$ pendiente $=\frac{1}{2}$

$y^2=4x \longrightarrow y=\pm\sqrt{4x}=\pm 2\sqrt{x}$

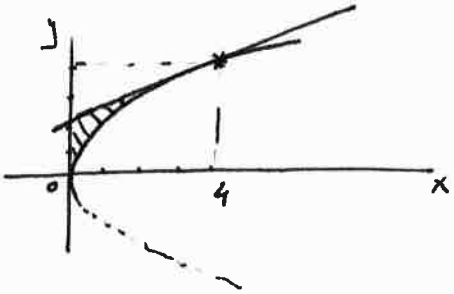
$y'=\pm 2\frac{1}{2\sqrt{x}}=\frac{\pm 1}{\sqrt{x}}$

$y'=\frac{1}{2} \Rightarrow \frac{1}{2}=\frac{\pm 1}{\sqrt{x}} \Rightarrow \boxed{x=4} \longrightarrow y=+2\sqrt{4}=4 \quad \boxed{P(4,4)}$

$y-4=\frac{1}{2}(x-4)$; $y-4=\frac{x-4}{2}$; $2y-8=x-4$; $\boxed{x-2y+4=0}$

$y^2=4x$
$x-2y+4=0 \longrightarrow y=\frac{x+4}{2}$

$\left(\frac{x+4}{2}\right)^2=4x$; $\frac{x^2+8x+16}{4}=4x$; $x^2+8x+16=16x$;
$x^2-8x+16=0$
$\boxed{x=4}$



$$\text{Area}=\int_0^4\left(\frac{x+4}{2}-2\sqrt{x}\right)dx=\int_0^4\left(\frac{1}{2}x+2-2x^{1/2}\right)dx=$$
$$=\frac{1}{2}\frac{x^2}{2}+2x-2\frac{x^{3/2}}{3/2}\Big|_0^4=\frac{x^2}{4}+2x-\frac{4\sqrt{x^3}}{3}\Big|_0^4=$$
$$=\frac{16}{4}+8-\frac{4\sqrt{64}}{3}=4+8-\frac{32}{3}=12-\frac{32}{3}=\boxed{\frac{4}{3}}$$

SEPT 94

$y=ax^3+bx^2+cx+d$; $y'=3ax^2+2bx+c$; $y''=6ax+2b$

$P(2,1) \Rightarrow 1=8a+4b+2c+d$
$P(0,0) \Rightarrow \boxed{0=d}$
inflexión en $x=2$ $\Rightarrow 0=12a+2b$
Tangente horizontal en $x=2$ $\Rightarrow 0=12a+4b+c$

$$\left.\begin{aligned}8a+4b+2c&=1\\12a+2b&=0\\12a+4b+c&=0\end{aligned}\right\}$$

$$a=\frac{\begin{vmatrix}1&4&2\\0&2&0\\0&4&1\end{vmatrix}}{\begin{vmatrix}8&4&2\\12&2&0\\12&4&1\end{vmatrix}}=\frac{2}{16}=\frac{1}{8}$$

$$b=\frac{\begin{vmatrix}8&1&2\\12&0&0\\12&0&1\end{vmatrix}}{16}=\frac{-12}{16}=\frac{-6}{8}$$

$$c=\frac{\begin{vmatrix}8&4&1\\12&2&0\\12&4&0\end{vmatrix}}{6}=\frac{24}{16}=\frac{12}{8}$$

$$y=\frac{1}{8}x^3-\frac{6}{8}x^2+\frac{12}{8}x=\boxed{\frac{x^3-6x^2+12x}{8}}$$

$O(0,0)$
$P(2,1)$ $\boxed{y=\frac{1}{2}x}$

$$\left.\begin{aligned}y&=\frac{x^3-6x^2+12x}{8}\\y&=\frac{1}{2}x\end{aligned}\right\}\quad \frac{1}{2}x=\frac{x^3-6x^2+12x}{8} ;\quad 4x=x^3-6x^2+12x ;\quad x^3-6x^2+8x=0$$

$x(x^2-6x+8)=0$

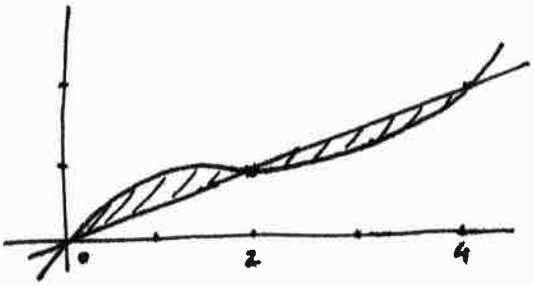
$x=\frac{6\pm\sqrt{36-32}}{2}=\frac{6\pm 2}{2}\begin{cases}4\\2\end{cases}$

$x=0$

$y = \dfrac{x^3-6x^2+12x}{8}$ $\qquad$ $y = \dfrac{x}{2}$

x	y
2	1
4	2
3	9/8
1	3/4
0	0

x	y
2	1
4	2
3	3/2
1	1/2
0	0



$$\text{Area} = \int_0^2 \left(\frac{x^3-6x^2+12x}{8} - \frac{x}{2}\right)dx + \int_2^4 \left(\frac{x}{2} - \frac{x^3-6x^2+12x}{8}\right)dx =$$

$$= \left(\frac{x^4}{32} - \frac{6x^3}{24} + \frac{12x^2}{16} - \frac{x^2}{4}\right)\Big|_0^2 + \left(\frac{x^2}{4} - \frac{x^4}{32} + \frac{6x^3}{24} - \frac{12x^2}{16}\right)\Big|_2^4 =$$

$$= \left(\frac{16}{32} - \frac{48}{24} + \frac{48}{16} - \frac{4}{4}\right) + \left(\frac{16}{4} - \frac{256}{32} - \frac{384}{24} - \frac{192}{16}\right) - \left(\frac{4}{4} - \frac{16}{32} + \frac{48}{24} - \frac{48}{16}\right) =$$

$$= \frac{1}{2} + \frac{1}{2} = \boxed{1}$$

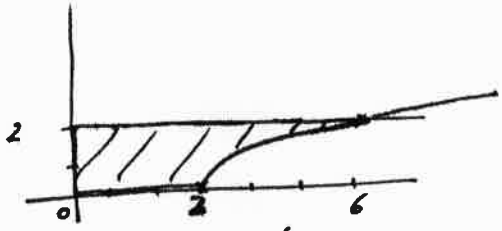
JUN 95

$\left.\begin{array}{l} y = \sqrt{x-2} \\ x = 0 \end{array}\right\}$ Sin intersección

$\left.\begin{array}{l} y = \sqrt{x-2} \\ y = 2 \end{array}\right\} \Rightarrow 2 = \sqrt{x-2}\ ;\ 4 = x-2\ ;\ x = 6$

$y = \sqrt{x-2}$ $\quad$ Dominio $= [2, +\infty)$

x	y
2	0
6	2



$$\text{Area} = 2\cdot 2 + \int_2^6 \left(2 - \sqrt{x-2}\right)dx = 4 + \int_2^6 \left(2 - (x-2)^{1/2}\right)dx = 4 + \left(2x - \frac{(x-2)^{3/2}}{3/2}\right)\Big|_2^6 =$$

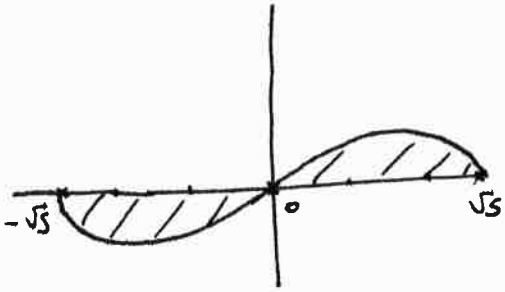
$$= 4 + \left(12 - \frac{2}{3}4^{3/2}\right) - (4 - 0) = 12 - \frac{16}{3} = \boxed{\frac{20}{3}}$$

SEPT 95

$\left.\begin{array}{l} f(x) = x\sqrt{5-x^2} \\ y = 0 \end{array}\right\}$ $\quad 0 = x\sqrt{5-x^2} \Rightarrow x = \begin{cases} 0 \\ \pm\sqrt{5} \end{cases}$

$y = x\sqrt{5-x^2}$ $\quad$ Dominio $= [-5, 5]$

x	y
0	0
$\sqrt{5}$	0
$-\sqrt{5}$	0
1	2
-1	-2



$$\text{Area} = 2\int_0^{\sqrt{5}} x\sqrt{5-x^2}\,dx = -\int_0^{\sqrt{5}} (-2x)(5-x^2)^{1/2}\,dx = -\frac{(5-x^2)^{3/2}}{3/2}\Bigg|_0^{\sqrt{5}} =$$

$$= -\frac{2}{3}\cdot 0 + \frac{2}{3}\,5^{3/2} = \boxed{\frac{2}{3}\sqrt{125}}$$

JUN 96

$$\int\left(tg^2x+1-\frac{1}{x}+tg\,x\right)dx = \int(tg^2x+1)\,dx - \int\frac{1}{x}\,dx + \int\frac{\sin x}{\cos x}\,dx =$$

$$= \boxed{tg\,x - \ln x - \ln|\cos x| + K}$$

$$P(\pi,0) \Rightarrow \quad 0 = tg\,\pi - \ln\pi - \ln\cos\pi + K\ ; \quad 0 = 0 - \ln\pi - \ln 1 + K \Rightarrow \boxed{K = \ln\pi}$$

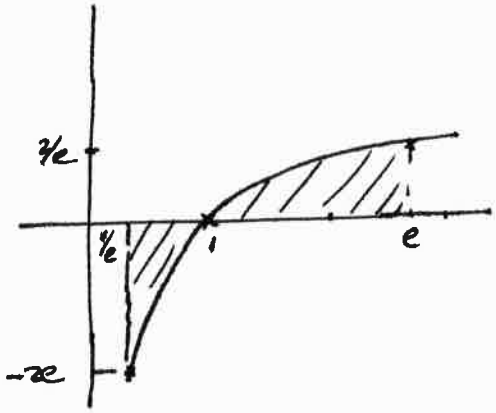
$$\boxed{tg\,x - \ln x - \ln|\cos x| + \ln\pi}$$

SEPT 96

$$\left.\begin{array}{l} y = 2\dfrac{\ln x}{x} \\ y = 0 \end{array}\right\} \quad 0 = 2\frac{\ln x}{x} \Rightarrow \ln x = 0 \Rightarrow x = 1$$

$y = \dfrac{2\ln x}{x}$ Dominio $= (0,+\infty)$

x	y
1	0
e	$2/e$
$1/e$	$-2e$



$$\text{Area} = -\int_{1/e}^{1} 2\frac{\ln x}{x}\,dx + \int_{1}^{e} 2\frac{\ln x}{x}\,dx = -\ln^2 x\Big|_{1/e}^{1} + \ln^2 x\Big|_{1}^{e} = -\ln^2 1 + \ln^2 1/e + \ln^2 e - \ln^2 1 =$$

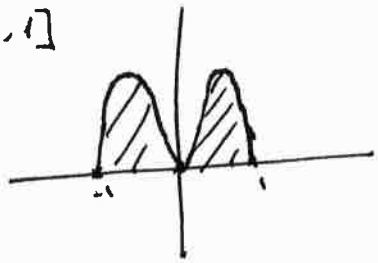
$$= -0 + (-1)^2 + 1^2 - 0 = \boxed{2}$$

JUN 97

$$\left.\begin{array}{l} y = \sqrt{x^2-x^4} \\ y = 0 \end{array}\right\} \quad \sqrt{x^2-x^4} = 0\ ; \quad x^2 - x^4 = 0\ ; \quad x^2(1-x^2) = 0 \Rightarrow x = \begin{cases} 0 \\ 1 \\ -1 \end{cases}$$

x		-1		0		1	
Signo x^2-4x	$\ominus$	0	$\oplus$	0	$\oplus$	0	$\ominus$

Dominio $= [-1,1]$



$$\text{Area} = 2\int_0^1 \sqrt{x^2-x^4}\,dx = 2\int_0^1 x\sqrt{1-x^2}\,dx =$$

$$= -\int_0^1 (-2x)\,(1-x^2)^{1/2}\,dx = -\frac{(1-x^2)^{3/2}}{3/2}\Bigg|_0^1 = -\frac{2}{3}0^{3/2} + \frac{2}{3}1^{3/2} = \boxed{\frac{2}{3}}$$

SEPT 97

$$y=\sqrt{x-2}$$
$$y'=\frac{1}{2\sqrt{x-2}}$$

$x=6 \to y=\sqrt{4}=2$

$\to y'=\frac{1}{2\sqrt{4}}=\frac{1}{4}$

$y-2=\frac{1}{4}(x-6)$; $4y-8=x-6$; $4y=x+2$; $\boxed{y=\frac{x+2}{4}}$

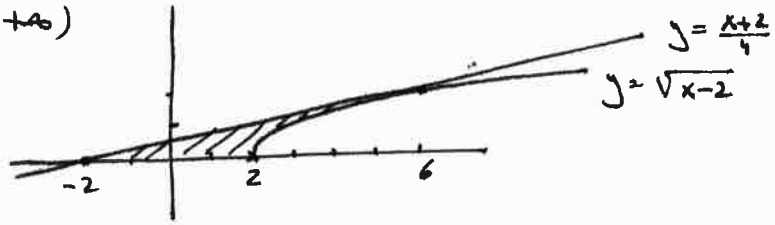
$\begin{cases} y=\sqrt{x-2} \\ y=0 \end{cases}$ $\sqrt{x-2}=0$; $x-2=0$; $x=2$

$\begin{cases} y=\sqrt{x-2} \\ y=\frac{x+2}{4} \end{cases}$ $\sqrt{x-2}=\frac{x+2}{4}$; $x-2=\left(\frac{x+2}{4}\right)^2$; $x-2=\frac{x^2+4x+4}{16}$;

$16x-32=x^2+4x+4$; $0=x^2-12x+36$; $x=6$

$y=\sqrt{x-2}$ Dominio $=[2,+\infty)$

x	y
2	0
6	2



$y=\frac{x+2}{4}$

x	y
-2	0
2	1

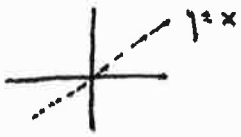
$$\text{Area}=\int_{-2}^{2}\frac{x+2}{4}\,dx+\int_{2}^{6}\left(\frac{x+2}{4}-\sqrt{x-2}\right)dx=$$

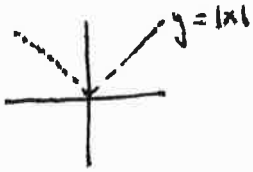
$$=\left(\frac{x^2}{8}+\frac{x}{2}\right)\Big|_{-2}^{2}+\left(\frac{x^2}{8}+\frac{x}{2}-\frac{(x-2)^{3/2}}{3/2}\right)\Big|_{2}^{6}=$$

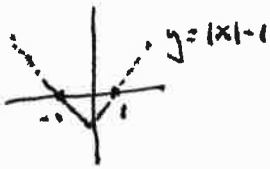
$$=\left(\cancel{\frac{4}{8}}+\cancel{\frac{2}{2}}\right)-\left(\frac{4}{8}-\frac{2}{2}\right)+\left(\frac{36}{8}+\frac{6}{2}-\frac{2}{3}4^{3/2}\right)-\left(\cancel{\frac{4}{8}}+\cancel{\frac{2}{2}}-0\right)=$$

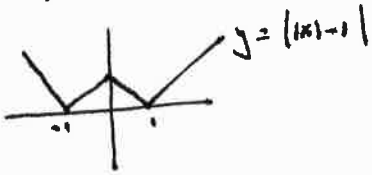
$$=-\frac{1}{2}+1+\frac{9}{2}+3-\frac{2}{3}\cdot 8=4+4-\frac{16}{3}=\boxed{\frac{8}{3}}$$

JUN 98









$$|x|-1=\begin{cases} x-1 & \text{si } x\geq 0 \\ -x-1 & \text{si } x<0 \end{cases}$$

$$||x|-1|=\begin{cases} x-1 & \text{si } 1\leq x \\ -x+1 & \text{si } 0\leq x<1 \\ x+1 & \text{si } -1\leq x<0 \\ -x-1 & \text{si } x<-1 \end{cases}$$

$$\int_{-a}^{a}||x|-1|\,dx=2\int_{0}^{a}||x|-1|\,dx=2\left[\int_{0}^{1}(-x+1)\,dx+\int_{1}^{a}(x-1)\,dx\right]=2\left[\left(-\frac{x^2}{2}+x\right)\Big|_{0}^{1}+\left(\frac{x^2}{2}-x\right)\Big|_{1}^{a}\right]=$$

$$= 2\left[\left(-\frac{1}{2}+1\right)-0+\left(\frac{a^2}{2}-a\right)-\left(\frac{1}{2}-1\right)\right] = 2\left(-\cancel{\frac{1}{2}}+\cancel{1}+\frac{a^2}{2}-a-\cancel{\frac{1}{2}}+1\right) = a^2-2a+2$$

$$a^2-2a+2=4 \Rightarrow a^2-2a-2=0$$

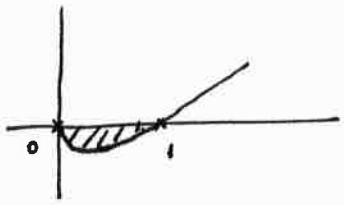
$$a=\frac{2\pm\sqrt{4+8}}{2} = \frac{2\pm\sqrt{12}}{2} = 1\pm\sqrt{3} \qquad \boxed{a=1+\sqrt{3}}$$

SEPT 98

$y=(x-1)\sqrt{x}$ Dominio $=[0,+\infty)$

$$\left.\begin{array}{l} y=(x-1)\sqrt{x} \\ \text{Eje } X \end{array}\right\} \rightarrow \left.\begin{array}{l} y=(x-1)\sqrt{x} \\ y=0 \end{array}\right\} \rightarrow (x-1)\sqrt{x}=0 \begin{array}{l} \nearrow x=0 \\ \searrow x=1 \end{array}$$

x	0		1		$+\infty$
Signo $(x-1)\sqrt{x}$	0	$-$	0	$+$	



$$\text{Area} = -\int_0^1 (x-1)\sqrt{x}\,dx = -\int_0^1 (x\sqrt{x}-\sqrt{x})\,dx = -\int_0^1 (x^{3/2}-x^{1/2})\,dx =$$

$$= -\left(\frac{x^{5/2}}{5/2}-\frac{x^{3/2}}{3/2}\right)\Bigg|_0^1 = -\left(\frac{1}{5/2}-\frac{1}{3/2}\right)+0 = -\left(\frac{2}{5}-\frac{2}{3}\right) = \boxed{\frac{4}{15}}$$

JUN 99

$y=x^2+\alpha$

$y'=2x$

$|x|=1 \rightarrow$

$x=1 \rightarrow y=1+\alpha,\ y'=2$ $\quad \boxed{y-(1+\alpha)=2(x-1)} \rightarrow y=2x-2+1+\alpha$; $\boxed{y=2x+(\alpha-1)}$

$x=-1 \rightarrow y=1+\alpha,\ y'=-2$ $\quad \boxed{y-(1+\alpha)=-2(x+1)} \rightarrow y=-2x-2+1+\alpha$; $\boxed{y=-2x+(\alpha-1)}$

$(0,0) \Rightarrow 0=2\cdot 0+(\alpha-1) \Rightarrow \boxed{\alpha=1}$

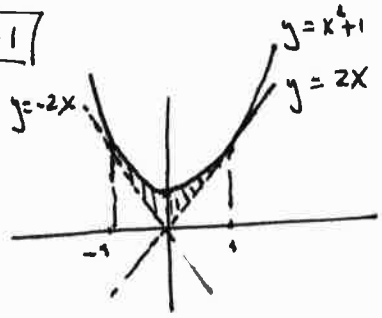
$(0,0) \Rightarrow 0=-2\cdot 0+(\alpha-1) \Rightarrow \boxed{\alpha=1}$

Para $\alpha=1$:

$y=x^2+1$

$y=2x$

$y=-2x$



$$\text{Area} = 2\int_0^1 (x^2+1)-2x\,dx = 2\left(\frac{x^3}{3}+x-x^2\right)\Bigg|_0^1 = 2\left[\left(\frac{1}{3}+\cancel{1}-\cancel{1}\right)-0\right] = \boxed{\frac{2}{3}}$$

SEPT 99

$y = x^2+2x+2$

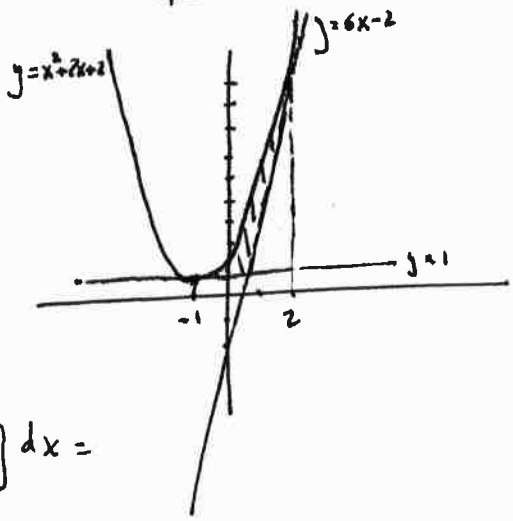
$y' = 2x+2$

$y'=0 \Rightarrow 2x+2=0 \; ; \; \boxed{x=-1}$

$x=-1 \to y = 1-2+2 = 1$, $y' = 0$ $\qquad y-1 = 0\cdot(x+1) \qquad \boxed{y=1}$

$y'=6 \Rightarrow 2x+2=6 \; ; \; 2x=4 \; ; \; \boxed{x=2}$

$x=2 \to y = 4+4+2 = 10$, $y' = 2\cdot 2+2 = 6$ $\qquad y-10 = 6(x-2) \qquad \boxed{y = 6x-2}$



$\left.\begin{array}{l} y=1 \\ y=6x-2 \end{array}\right\} \; 6x-2=1 \; ; \; 6x=3 \; ; \; x=1/2$

$$\text{Area} = \int_{-1}^{1/2} \left[(x^2+2x+2)-1\right] dx + \int_{1/2}^{2} \left[(x^2+2x+2)-(6x-2)\right] dx =$$

$$= \int_{-1}^{1/2} (x^2+2x+1)\,dx + \int_{1/2}^{2} (x^2-4x+4)\,dx = \left(\frac{x^3}{3}+x^2+x\right)\Big|_{-1}^{1/2} + \left(\frac{x^3}{3}-2x^2+4x\right)\Big|_{1/2}^{2} =$$

$$= \left(\frac{1/8}{3}+\frac{1}{4}+\frac{1}{2}\right) - \left(-\frac{1}{3}\cancel{+1}\cancel{-1}\right) + \left(\frac{8}{3}\cancel{-8}\cancel{+8}\right) - \left(\frac{1/8}{3}-\frac{2}{4}+\frac{4}{2}\right) =$$

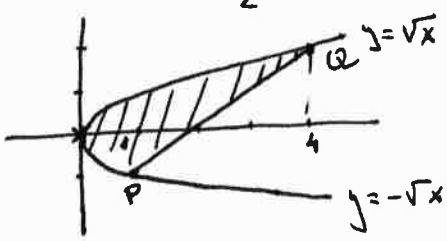
$$= \cancel{\frac{1}{24}}+\frac{1}{4}+\frac{1}{2}+\frac{1}{3}+\frac{8}{3}-\cancel{\frac{1}{24}}+\frac{1}{2}-2 = \boxed{\frac{9}{4}}$$

JUN 00

$P(1,-1)$, $Q(4,2)$ $\quad \vec{PQ} = (3,3) \to$ pendiente $= \frac{3}{3} = 1 \qquad y+1 = 1\cdot(x-1) \qquad \boxed{y = x-2}$

$\left.\begin{array}{l} y^2 = x \\ y = x-2 \end{array}\right\} \; (x-2)^2 = x \; ; \; x^2-4x+4 = x \; ; \; x^2-5x+4=0 \; ; \; x = \frac{5\pm\sqrt{25-16}}{2} = \frac{5\pm 3}{2} \begin{cases} 4 \\ 1 \end{cases}$

$$\text{Area} = 2\int_0^1 \sqrt{x}\,dx + \int_1^4 \sqrt{x}-(x-2)\,dx =$$



$y = \operatorname{sen}^2 3x$ es siempre positiva.

$$\text{Area} = \int_0^{\pi/6} \operatorname{sen}^2 3x \, dx = \frac{x}{2} - \frac{\operatorname{sen} 3x \cos 3x}{6} \Bigg|_0^{\pi/6} = \left(\frac{\pi}{12} - 0\right) - \left(\frac{\operatorname{sen}\pi/2 \cos \pi/2}{6} - \frac{\operatorname{sen} 0 \cos 0}{6}\right) =$$

$$= \frac{\pi}{12} - \left(\frac{1\cdot 0}{6} - \frac{0\cdot 1}{6}\right) = \boxed{\frac{\pi}{12}}$$

SEPT 01

$$\int \frac{x^2}{x^2+1} dx = \int \left(1 + \frac{-1}{x^2+1}\right) dx =$$

$$\begin{array}{r|l} x^2 & x^2+1 \\ \hline -x^2-1 & 1 \\ \hline -1 & \end{array}$$

$$= \boxed{x - \operatorname{arctg} x + K}$$

$$\int_0^1 \left(3\frac{x^2}{x^2+1} + 2\right) dx = 3\int_0^1 \frac{x^2}{x^2+1} dx + \int_0^1 2\, dx = 3\,(x - \operatorname{arctg} x)\Big|_0^1 + 2x\Big|_0^1 =$$

$$= 3(1 - \operatorname{arctg} 1) - 3(0 - \operatorname{arctg} 0) + 2 - 0 =$$

$$= 3\left(1 - \frac{\pi}{4}\right) - 3(0-0) + 2 = \boxed{5 - \frac{3\pi}{4}}$$

JUN 02

$$\int \frac{6x}{x^2+1} dx = 3\int \frac{2x}{x^2+1} dx = \boxed{3 \ln(x^2+1) + K}$$

$$\left.\begin{array}{l} y = \dfrac{6x}{x^2+1} \\ y = 0 \end{array}\right\} \Rightarrow \frac{6x}{x^2+1} = 0 \; ; \; 6x = 0, \; x = 0$$

x	$-\infty$		0		$+\infty$
Signo $\frac{6x}{x^2+1}$		$-$	0	$+$	

$$\text{Area} = \int_2^5 \frac{6x}{x^2+1} dx = 3\ln(x^2+1)\Big|_2^5 = 3\ln 26 - 3\ln 5 = \boxed{3\ln\left(\frac{26}{5}\right)}$$

SEPT 02

$$\int x^2 e^x dx = x^2 e^x - \int 2x e^x dx = x^2 e^x - 2x e^x + \int 2 e^x dx = \boxed{x^2 e^x - 2x e^x + 2e^x + K}$$

$$\left|\begin{array}{l} u = x^2 \longrightarrow u' = 2x \\ v' = e^x \longrightarrow v = e^x \end{array}\right| \qquad \begin{array}{l} u = 2x \longrightarrow u' = 2 \\ v' = e^x \longrightarrow v = e^x \end{array}$$

$$\int_1^2 x^2 e^x dx = (x^2 - 2x + 2)e^x \Big|_1^2 = (4-4+2)e^2 - (1-2+2)e^1 = 2e^2 - e = \boxed{e(2e-1)}$$

$$= 2\int_0^1 x^{1/2}\,dx + \int_1^4 (x^{1/2} - x + 2)\,dx = 2\,\frac{x^{3/2}}{3/2}\Big|_0^1 + \left(\frac{x^{3/2}}{3/2} - \frac{x^2}{2} + 2x\right)\Big|_1^4 =$$

$$= \left(\frac{4}{3}\cdot 1\right) - 0 + \left(\frac{2}{3}4^{3/2} - \cancel{\frac{16}{2}} + \cancel{8}\right) - \left(\frac{2}{3} - \frac{1}{2} + 2\right) =$$

$$= \frac{4}{3} + \frac{16}{3} - \frac{2}{3} + \frac{1}{2} - 2 = \boxed{\frac{9}{2}}$$

SEPT 00

$$f(x) = (x-1)^2$$

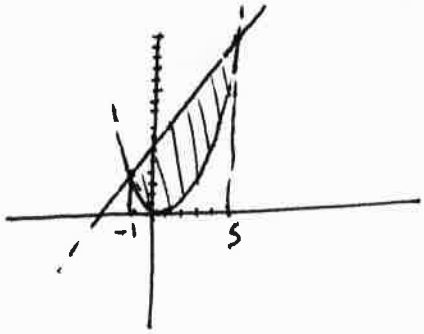
$$y' = 2(x-1) = 2x-2$$

$$x = 2 \longrightarrow y' = 4-2 = 2$$

$$\left.\begin{array}{l} P(0,6) \\ \text{pend} = 2 \end{array}\right\} \longrightarrow y - 6 = 2(x-0) \qquad \boxed{y = 2x+6}$$

$$\left.\begin{array}{l} y = (x-1)^2 \\ y = 2x+6 \end{array}\right\} \quad (x-1)^2 = 2x+6 \;;\quad x^2 - 2x + 1 = 2x+6 \;;\quad x^2 - 4x - 5 = 0$$

$$x = \frac{4 \pm \sqrt{16+20}}{2} = \frac{4 \pm 6}{2} \begin{cases} 5 \\ -1 \end{cases}$$



$$\text{Area} = \int_{-1}^{5} \left[(2x+6) - (x-1)^2\right] dx =$$

$$= \int_{-1}^{5} (2x+6 - x^2 + 2x - 1)\,dx = \int_{-1}^{5} (-x^2 + 4x + 5)\,dx =$$

$$= \left(-\frac{x^3}{3} + 2x^2 + 5x\right)\Big|_{-1}^{5} = \left(-\frac{125}{3} + 50 + 25\right) - \left(+\frac{1}{3} + 2 - 5\right) = -\frac{125}{3} + 50 + 25 - \frac{1}{3} - 2 + 5 = \boxed{36}$$

JUN 01

$$\int \sin^2(3x)\,dx = -\frac{1}{3}\sin 3x \cos 3x + \int \cancel{3}\cos 3x\, \frac{1}{\cancel{3}}\cos 3x\,dx = -\frac{1}{3}\sin 3x \cos 3x + \int \cos^2 3x\,dx =$$

$$\boxed{\begin{array}{l} u = \sin 3x \longrightarrow u' = 3\cos 3x \\ v' = \sin 3x \longrightarrow v = -\frac{1}{3}\cos 3x \end{array}}$$

$$= -\frac{1}{3}\sin 3x \cos 3x + \int (1 - \sin^2 3x)\,dx = -\frac{1}{3}\sin 3x \cos 3x + x - \int \sin^2 3x\,dx$$

$$2\int \sin^2 3x\,dx = -\frac{1}{3}\sin 3x \cos 3x + x \quad \Longrightarrow \quad \boxed{\int \sin^2 3x\,dx = \frac{x}{2} - \frac{1}{6}\sin 3x \cos 3x + K}$$

JUN 03

$y = x$
$y = x^2$
$y = x^2/4$

$\left.\begin{array}{l} y = x \\ y = x^2 \end{array}\right\} \rightarrow x^2 = x \;;\; x^2 - x = 0 \;,\; x(x-1) = 0 \rightarrow \boxed{\begin{array}{l} x = 0 \\ x = 1 \end{array}}$

$\left.\begin{array}{l} y = x \\ y = \frac{x^2}{4} \end{array}\right\} \rightarrow \frac{x^2}{4} = x \;;\; x^2 = 4x \;;\; x^2 - 4x = 0 \;;\; x(x-4) = 0 \rightarrow \boxed{\begin{array}{l} x = 0 \\ x = 4 \end{array}}$

$\left.\begin{array}{l} y = x^2 \\ y = \frac{x^2}{4} \end{array}\right\} \rightarrow x^2 = \frac{x^2}{4} \;;\; 4x^2 = x^2 \;;\; 3x^2 = 0 \;;\; \boxed{x = 0}$

$y = x$

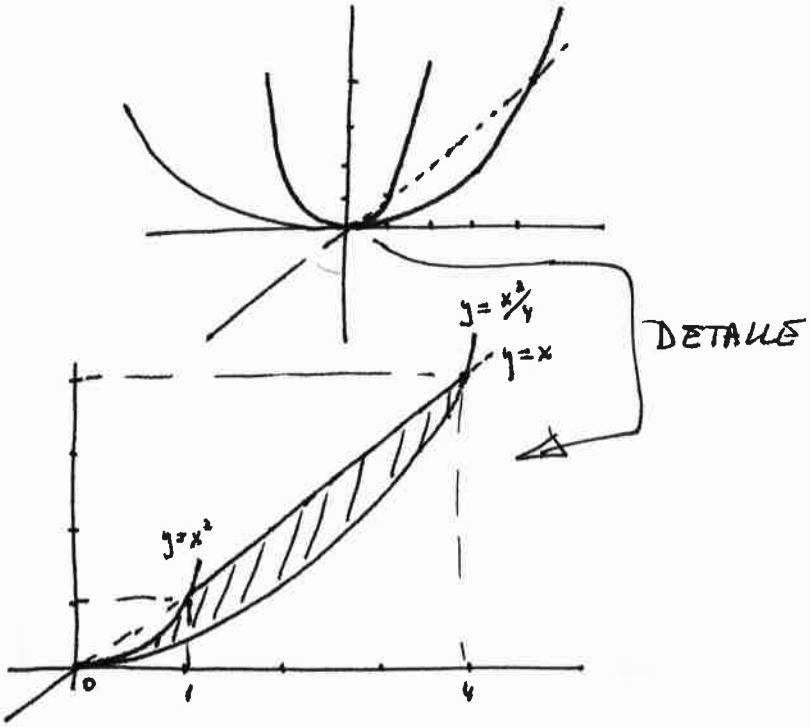
x	y
0	0
1	1
4	4

$y = x^2$

x	y
0	0
1	1

$y = \frac{x^2}{4}$

x	y
0	0
4	4



$$\text{Área} = \int_0^1 \left(x^2 - \frac{x^2}{4}\right) dx + \int_1^4 \left(x - \frac{x^2}{4}\right) dx =$$

$$= \int_0^1 \frac{3x^2}{4}\, dx + \int_1^4 \left(x - \frac{x^2}{4}\right) dx =$$

$$= \frac{\cancel{3}}{4} \frac{x^3}{\cancel{3}} \Big|_0^1 + \left(\frac{x^2}{2} - \frac{1}{4}\frac{x^3}{3}\right)\Big|_1^4 = \left(\frac{1}{4} - 0\right) + \left(\frac{16}{2} - \frac{64}{12}\right) - \left(\frac{1}{2} - \frac{1}{12}\right) = \boxed{\frac{5}{2}}$$

SEPT 03

$$f(x) = \begin{cases} x \ln x & \text{si } x > 0 \\ 0 & \text{si } x = 0 \end{cases}$$

$$f'(x) = x \cdot \frac{1}{x} + 1 \cdot \ln x = 1 + \ln x \quad \text{si } x > 0$$

$f'(x) = 0 \rightarrow 1 + \ln x = 0 \;;\; \ln x = -1 \;;\; x = e^{-1} = \frac{1}{e}$

x	0		$1/e$		$+\infty$
$f(x)$		↘		↗	
$f'(x)$		−	0	+	

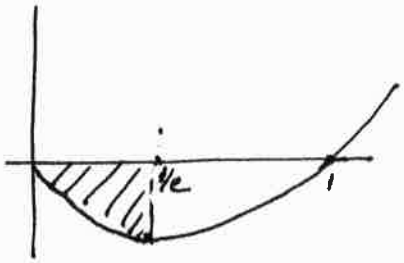
Mínimo en $x = 1/e$

$\left.\begin{array}{l} y = x \ln x \\ y = 0 \end{array}\right\} \quad x \ln x = 0 \rightarrow \boxed{x = 0} \;;\; \ln x = 0 \;;\; \boxed{x = 1}$

x	0		1		$+\infty$
Signo $x \ln x$	0	−	0	+	

$$\text{Area} = -\int_0^{1/e} x\ln x\,dx = -\ln x\cdot\frac{x^2}{2}\Big|_0^{1/e} + \int_0^{1/e}\frac{1}{x}\frac{x^2}{2}\,dx =$$

$$\boxed{\begin{array}{l} u=\ln x \to u'=\frac{1}{x} \\ v'=x \to v=x^2/2 \end{array}}$$



$$= \frac{-x^2\ln x}{2}\Big|_0^{1/e} + \int_0^{1/e}\frac{x}{2}\,dx = \left(\frac{-x^2\ln x}{2} + \frac{x^2}{4}\right)\Big|_0^{1/e} = \left(\frac{-(1/e)^2\ln(1/e)}{2} + \frac{(1/e)^2}{4}\right) - \left(\frac{0}{2}+0\right) =$$

$$= \frac{1}{2e^2} + \frac{1}{4e^2} = \boxed{\frac{3}{4e^2}}$$

JUN 04

$$\left.\begin{array}{l} y = 4-(x-2)^2 \\ y=0 \end{array}\right\} \quad 0 = 4-(x-2)^2 \;;\quad (x-2)^2 = 4 \;;\quad x-2=\pm 2 \;;\quad x = \begin{cases} 2+2=4 \\ 2-2=0 \end{cases}$$

$$\boxed{C(4,0)}$$

Recta r_4: $\left.\begin{array}{l}(0,0)\\(-4,4)\end{array}\right| \to \vec{v}=(-4,4) \to m=-1 \qquad \boxed{y=-x}$

Recta r_2: $\left.\begin{array}{l}(-4,0)\\m=-1\end{array}\right| \to y=-(x+4) \qquad \boxed{y=4-x} \qquad x=0 \to y=-4 \qquad \boxed{D=(0,-4)}$

Recta r_1: $\left.\begin{array}{l}D(0,-4)\\C(4,0)\end{array}\right| \to \vec{v}=(4,4) \to m=1 \qquad y+4=x \;;\; \boxed{y=x-4}$

$$\text{Area} = \int_{-4}^{0}\left(-x-(4-x)\right)dx + \int_0^4\left(4-(x-2)^2-(x-4)\right)dx = \int_{-4}^{0}4\,dx + \int_0^4\left(4-x^2+4x-\cancel{4}-x+\cancel{4}\right)dx =$$

$$= \int_{-4}^{0}4\,dx + \int_0^4\left(-x^2+3x+4\right)dx = 4x\Big|_{-4}^{0} + \left(-\frac{x^3}{3}+\frac{3x^2}{2}+4x\right)\Big|_0^4 = 0+16+\left(\frac{-64}{3}+24+16\right)-0 =$$

$$= 16+24+16-\frac{64}{3} = 56-\frac{64}{3} = \boxed{\frac{104}{3}}$$

SEPT 04

$$f(x) = \frac{2x+1}{x-2} \qquad f'(x) = \frac{2(x-2)-(2x+1)}{(x-2)^2} = \frac{-5}{(x-2)^2}$$

a) $x=3 \to f(3)=\frac{7}{1}=7$, $\to f'(3)=-5$ $\qquad y-7=-5(x-3) \qquad \boxed{y=22-5x}$

b) $\lim_{x\to\infty}\frac{2x+1}{x-2} = \frac{\infty}{\infty} = \lim_{x\to\infty}\frac{2}{1} = 2 \qquad \boxed{y=2 \text{ A. Horizontal}}$

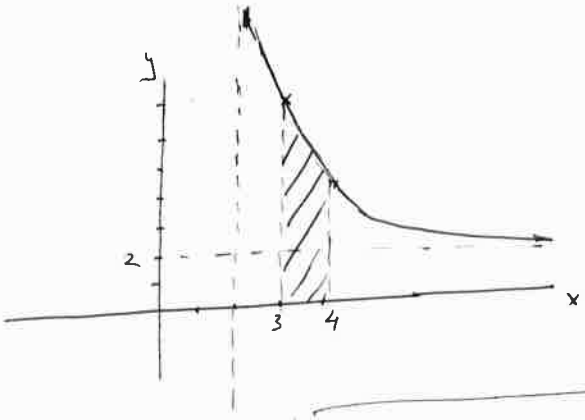
$$\left.\begin{array}{l} y=22-5x \\ y=2 \end{array}\right\} \quad 2=22-5x \;;\quad 5x=20 \;;\quad x=4 \qquad \boxed{P(4,2)}$$

$f(x) = \dfrac{2x+1}{x-2}$ $\Big\}$ $\dfrac{2x+1}{x-2} = 0$; $2x+1=0$; $x = -1/2$

$y=0$

$y = \dfrac{2x+1}{x-2}$

x	y
$-1/2$	0
2	$\nexists$
3	7
4	$9/2$



$$\text{Area} = \int_3^4 \frac{2x+1}{x-2}\,dx = \int_3^4 \left(2 + \frac{5}{x-2}\right) dx =$$

$$\begin{array}{r|l} 2x+1 & x-2 \\ \hline -2x+4 & 2 \\ \hline 5 & \end{array}$$

$$= \Big(2x + 5\ln(x-2)\Big)\Big|_3^4 = (8 + 5\ln 2) - (6 + 5\ln 1) = \boxed{2 + 5\ln 2}$$

JUN 05 $\quad f(x) = \dfrac{\operatorname{sen} x}{2-\cos x}$

a) $2-\cos x = 0$, $\cos x = 2$ $\nexists$

$\boxed{\text{Dominio} = \mathbb{R}}$ $\quad$ Periodo $= 2\pi$ $\quad$ $\boxed{\text{Dominio Restringido} = [0, 2\pi)}$

$$f'(x) = \frac{\cos x\,(2-\cos x) - \operatorname{sen} x \cdot \operatorname{sen} x}{(2-\cos x)^2} = \frac{2\cos x - \cos^2 x - \operatorname{sen}^2 x}{(2-\cos x)^2} = \frac{2\cos x - 1}{(2-\cos x)^2}$$

$f'(x) = 0 \Rightarrow 2\cos x - 1 = 0$; $\cos x = \frac{1}{2} \Rightarrow \boxed{\begin{array}{l} x = \pi/3 \\ x = \frac{5\pi}{3} \end{array}}$

x	0		$\pi/3$		$5\pi/3$		2π
$f(x)$		$\nearrow$		$\searrow$		$\nearrow$	
$f'(x)$		$+$	0	$-$	0	$+$	

$\boxed{\begin{array}{l} f(x) \text{ tiene un máximo relativo en } x = \pi/3 \\ \text{" " " mínimo " " } x = 5\pi/3 \end{array}}$

b) $$\int_0^{\pi/3} \frac{\operatorname{sen} x}{2-\cos x}\,dx = \ln(2-\cos x)\Big|_0^{\pi/3} = \ln\left(2-\tfrac{1}{2}\right) - \ln(2-1) = \ln\tfrac{3}{2} - \ln 1 = \boxed{\ln\tfrac{3}{2}}$$

SEPT 05 $\quad f(x) = x\sqrt{4-x^2}$

$\boxed{\text{Dominio} = [-2, 2]}$

a) $$f'(x) = 1\cdot\sqrt{4-x^2} + x\cdot\frac{-2x}{2\sqrt{4-x^2}} = \sqrt{4-x^2} - \frac{x^2}{\sqrt{4-x^2}} = \frac{4-x^2-x^2}{\sqrt{4-x^2}} = \frac{4-2x^2}{\sqrt{4-x^2}}$$

$x=0 \to y=0$

$\to y' = \dfrac{4}{\sqrt{4}} = \dfrac{4}{2} = 2$ $\qquad y - 0 = 2(x-0)$; $\boxed{y = 2x}$

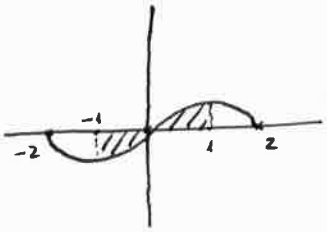
b) $\displaystyle\int_{-1}^{1} x\sqrt{4-x^2}\,dx = \frac{1}{-2}\int_{-1}^{1}(-2x)(4-x^2)^{1/2}\,dx = -\frac{1}{2}\,\frac{(4-x^2)^{\frac{1}{2}+1}}{\frac{1}{2}+1}\Bigg|_{-1}^{1} =$

$\displaystyle = -\frac{1}{2}\,\frac{(4-x^2)^{3/2}}{3/2}\Bigg|_{-1}^{1} = -\frac{\sqrt{(4-x^2)^3}}{3}\Bigg|_{-1}^{1} = -\frac{\sqrt{27}}{3}+\frac{\sqrt{27}}{3} = \boxed{0}$

c) $y = x\sqrt{4-x^2}$

$y=0 \longrightarrow x\sqrt{4-x^2}=0 \longrightarrow x=0,\ x=\pm 2$

x	-2		0		2
$f(x)$	0	$-$	0	$+$	0



$\displaystyle \text{Area} = 2\int_0^1 x\sqrt{4-x^2}\,dx = -\int_0^1(-2x)(4-x^2)^{1/2}\,dx = -\frac{(4-x^2)^{3/2}}{3/2}\Bigg|_0^1 =$

$\displaystyle = -\frac{2}{3}\sqrt{(4-x^2)^3}\Bigg|_0^1 = -\frac{2}{3}\sqrt{27}+\frac{2}{3}\sqrt{64} = \boxed{\frac{2}{3}(8-3\sqrt{3})}$

JUN 06 $\quad f(x) = \dfrac{3x^3}{x^2-4} \qquad D = \mathbb{R} - \{\pm 2\}$

a) $\displaystyle\lim_{x\to 2^-} f(x) = \frac{24}{-0} = -\infty$

$\displaystyle\lim_{x\to 2^+} f(x) = \frac{24}{+0} = +\infty$ $\Longrightarrow$ $\boxed{\text{A. Vertical } x=2}$

$\displaystyle\lim_{x\to -2^-} f(x) = \frac{24}{+0} = +\infty$

$\displaystyle\lim_{x\to -2^+} f(x) = \frac{24}{-0} = -\infty$ $\Longrightarrow$ $\boxed{\text{A. Vertical } x=-2}$

$\displaystyle\lim_{x\to\infty} f(x) = \frac{\infty}{\infty} = \lim_{x\to\infty}\frac{9x^2}{2x} = \frac{\infty}{\infty} = \lim_{x\to\infty}\frac{18x}{2} = \infty \longrightarrow$ No tiene A. Horizontal

$\displaystyle\lim_{x\to\infty}\frac{f(x)}{x} = \lim_{x\to\infty}\frac{3x^3}{x^3-4x} = \frac{\infty}{\infty} = \lim_{x\to\infty}\frac{9x^2}{3x^2-4} = \frac{\infty}{\infty} = \lim_{x\to\infty}\frac{18\cancel{x}}{6\cancel{x}} = 3 = m$

$\displaystyle\lim_{x\to\infty}(f(x)-mx) = \lim_{x\to\infty}\left(\frac{3x^3}{x^2-4}-3x\right) = \lim_{x\to\infty}\frac{\cancel{3x^3}-\cancel{3x^3}+12x}{x^2-4} = \lim_{x\to\infty}\frac{12x}{x^2-4} = \frac{\infty}{\infty} =$

$\displaystyle = \lim_{x\to\infty}\frac{12}{2x} = 0 = n$ $\qquad$ $\boxed{\text{A. Oblicua } y=3x}$

<u>Otro</u> procedimiento para hallar la Asíntota oblicua

$$\begin{array}{l|l} 3x^3 & x^2-4 \\ \hline -3x^3+12x & 3x \\ \hline 12x & \end{array}$$

$f(x) = \underset{\substack{\uparrow \\ \text{A. Oblicua}}}{3x} + \dfrac{12x}{x^2-4}$

b) $\displaystyle\int_{-1}^{1} \frac{3x^3}{x^2-4}\,dx = \boxed{0}$ Por ser f(x) una función impar.

Otro procedimiento:

$$\int_{-1}^{1} \frac{3x^3}{x^2-4}\,dx = \int_{-1}^{1}\left(3x + \frac{12x}{x^2-4}\right)dx = \frac{3x^2}{2} + 6\ln|x^2-4|\Big|_{-1}^{1} = \left(\frac{3}{2}+6\ln 3\right)-\left(\frac{3}{2}+6\ln 3\right)=0$$

SEPT 06

$y = x^2-2x+1$

$x=0 \rightarrow y=1 \qquad P(0,1)$

$y=0 \rightarrow x^2-2x+1=0$

$x=1 \qquad Q(1,0)$

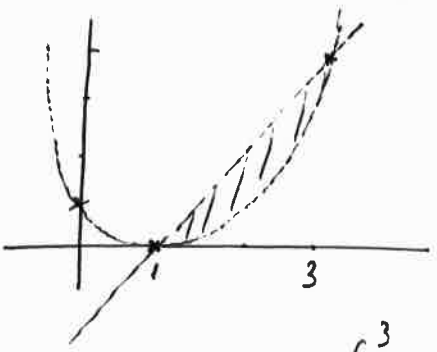
$y'=0 \rightarrow 2x-2=0$

$x=1 \qquad V(1,0)$

$A(1,0)$, $B(3,4)$ $\quad \vec{AB} = (2,4) \rightarrow m = \frac{4}{2} = 2$

$y-0 = 2\cdot(x-1)$

$y = 2x-2$



$\left.\begin{array}{l} y = x^2-2x+1 \\ y = 2x-2 \end{array}\right\}$ $\quad x^2-2x+1 = 2x-2$

$x^2-4x+3=0$

$x = \begin{cases} 1 \\ 3 \end{cases}$

$$\text{Area} = \int_1^3 (2x-2)-(x^2-2x+1)\,dx = \int_1^3 (-x^2+4x-3)\,dx = -\frac{x^3}{3}+2x^2-3x\Big|_1^3 =$$

$$= \left(-\frac{27}{3}+18-9\right)-\left(-\frac{1}{3}+2-3\right) = 0 = \frac{1}{3}-2+3 = \frac{1-6+9}{3} = \boxed{\frac{4}{3}\ \text{u.a.}}$$

JUN 07

$f(x) = ax^2+bx\cos x+c \quad ; \quad f'(x) = 2ax+b\cos x - bx\sin x$

$P(0,1) \rightarrow \boxed{1=c}$

Recta tangente paralela a $y=x$ en $P(0,1)$ $\rightarrow \boxed{1=b}$

$$\int_0^{\pi}(ax^2+bx\cos x+c)\,dx = \int_0^{\pi}(ax^2+c)\,dx + b\int_0^{\pi} x\cos x\,dx = \int_0^{\pi}(ax^2+c)\,dx + bx\sin x\Big|_0^{\pi} - b\int_0^{\pi}\sin x\,dx =$$

$$\boxed{\begin{array}{l|l} u = x & du = dx \\ dv = \cos x\,dx & v = \sin x \end{array}}$$

$$= \frac{ax^3}{3}+cx+bx\sin x+b\cos x\Big|_0^{\pi} = \left(\frac{a\pi^3}{3}+\pi c-b\right)-(b) =$$

$$= \frac{a\pi^3}{3}+\pi c-2b = \boxed{\pi\left(\frac{a}{3}\pi^2+c\right)-2b} \rightarrow \boxed{a=2}$$

SEPT 07

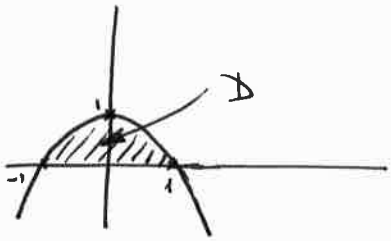
$y = 1 - x^2$

$x = 0 \longrightarrow y = 1 \qquad P(0,1)$

$y = 0 \longrightarrow x = \pm 1 \qquad Q_1(1,0)$

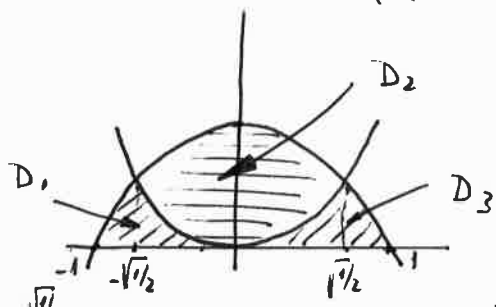
$\qquad\qquad\qquad\qquad Q_2(-1,0)$

$y' = 0 \longrightarrow x = 0 \qquad V(0,1)$



a) $\text{Area } D = \int_{-1}^{1} (1-x^2)\,dx = 2\int_0^1 (1-x^2)\,dx = 2\left(x - \frac{x^3}{3}\right)\Big|_0^1 = 2\left(1 - \frac{1}{3}\right) = \boxed{\frac{4}{3} \text{ u.s.}}$

b) $\left.\begin{array}{l} y = 1 - x^2 \\ y = x^2 \end{array}\right\} \quad x^2 = 1 - x^2 \;;\quad 2x^2 = 1 \;;\quad x = \pm\sqrt{1/2} \longrightarrow \begin{array}{l} R_1\left(\sqrt{1/2},\ 1/2\right) \\ R_2\left(-\sqrt{1/2},\ 1/2\right) \end{array}$



$$\text{Area } D_2 = \int_{-\sqrt{1/2}}^{\sqrt{1/2}} \left[(1-x^2) - x^2\right] dx = 2\int_0^{\sqrt{1/2}} (1-2x^2)\,dx = 2\left(x - \frac{2x^3}{3}\right)\Big|_0^{\sqrt{1/2}} =$$

$$= 2x\left(1 - \frac{2x^2}{3}\right)\Big|_0^{\sqrt{1/2}} = 2\sqrt{1/2}\left(1 - \frac{2\,1/2}{3}\right) = 2\sqrt{\frac{1}{2}}\,\frac{2}{3} = \boxed{\frac{4}{3\sqrt{2}} \text{ u.s}} = \boxed{\frac{2\sqrt{2}}{3} \text{ u.s}}$$

$\simeq 0'943$ u.s.

SEPT 08

$y = -x^2 + 4 \longrightarrow y' = -2x$

$y' = 0 \longrightarrow x = 0$

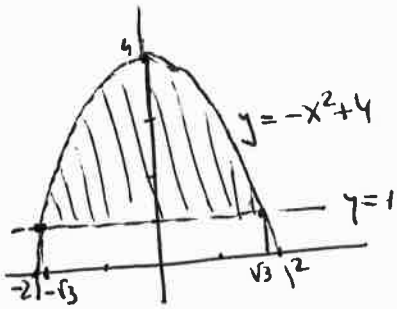
	x	y
	-2	0
	-1	3
V	0	4
	1	3
	2	0

$\left.\begin{array}{l} y = -x^2 + 4 \\ y = 1 \end{array}\right\} \longrightarrow 1 = -x^2 + 4 \;;\quad x^2 = 3 \;;\quad x = \pm\sqrt{3}$

$$\text{Area} = \int_{-\sqrt{3}}^{\sqrt{3}} (-x^2+4) - 1\,dx =$$

$$= 2\int_0^{\sqrt{3}} (-x^2+3)\,dx =$$

$$= 2\left[-\frac{x^3}{3} + 3x\right]_0^{\sqrt{3}} = 2\left(-\sqrt{3} + 3\sqrt{3}\right) = \boxed{4\sqrt{3}\ u^2} \simeq 6'93\ u^2$$



JUN 08

$f(x) = \dfrac{x}{x^2+1}$ $\quad$ dom f = $\mathbb{R}$

a) $\lim\limits_{x\to-\infty} f(x) = \dfrac{\infty}{\infty} = \lim\limits_{x\to-\infty} \dfrac{1}{2x} = 0$

$\lim\limits_{x\to+\infty} f(x) = \dfrac{\infty}{\infty} = \lim\limits_{x\to+\infty} \dfrac{1}{2x} = 0$ $\Big\}$ Asíntota Horizontal $y=0$

$f'(x) = \dfrac{1\cdot(x^2+1) - x\cdot 2x}{(x^2+1)^2} = \dfrac{1-x^2}{(x^2+1)^2}$

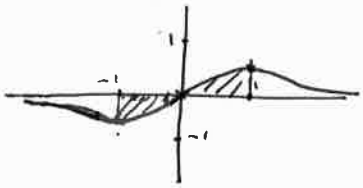
$f'(x)=0 \Rightarrow x=\pm 1$ Posibles Max/Min

		-1		1	
$f'(x)$	$-$		$+$		$-$
$f(x)$	$\searrow$		$\nearrow$		$\searrow$

Mínimo Relativo en $x=-1$ $\boxed{(-1,-1/2)}$

Máximo Relativo en $x=1$ $\boxed{(1,1/2)}$

b)



$f(-x) = \dfrac{-x}{(-x)^2+1} = \dfrac{-x}{x^2+1} = -f(x)$

f es simétrica respecto del origen $\Rightarrow$ Función IMPAR

c) $\text{Area} = 2\displaystyle\int_0^1 \frac{x}{x^2+1}\,dx = \int_0^1 \frac{2x}{x^2+1}\,dx = \Big[\ln|x^2+1|\Big]_0^1 = \ln 2 - \ln 1 = \boxed{\ln 2}$

SEPT 08

$f(x) = 2 - \dfrac{x}{x^2+1}$ $\quad$ dom f = $\mathbb{R}$

a) $f'(x) = -\dfrac{1\cdot(x^2+1) - x\cdot 2x}{(x^2+1)^2} = \dfrac{x^2-1}{(x^2+1)^2}$

$f'(x)=0 \Rightarrow x=\pm 1$ Posibles Max/Min

		-1		1	
f'	$+$		$-$		$+$
f	$\nearrow$		$\searrow$		$\nearrow$

Máximo Relativo en $x=-1$ $\boxed{(-1,5/2)}$

Mínimo Relativo en $x=1$ $\boxed{(1,3/2)}$

$f''(x) = \dfrac{2x(x^2+1)^{\cancel{2}} - (x^2-1)\cdot 2\cancel{(x^2+1)}\cdot 2x}{(x^2+1)^{\cancel{4}\,3}} = \dfrac{2x(x^2+1-2x^2+2)}{(x^2+1)^3} = \dfrac{2x(3-x^2)}{(x^2+1)^3}$

$f''(x)=0 \Rightarrow x = \begin{cases} 0 \\ \pm\sqrt{3} \end{cases}$ Posibles Puntos Inflexión

		$-\sqrt{3}$		0		$\sqrt{3}$	
f''	$+$		$-$		$+$		$-$
f	cóncava		conv.		cóncava		conv.

Punto Inflexión en $x=0$ $\boxed{(0,2)}$

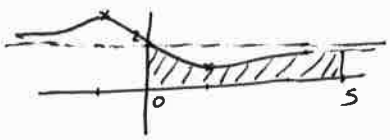
" " " $x=-\sqrt{3}$ $\boxed{\left(-\sqrt{3}, \dfrac{8+\sqrt{3}}{4}\right)}$

" " " $x=\sqrt{3}$ $\boxed{\left(\sqrt{3}, \dfrac{8-\sqrt{3}}{4}\right)}$

b) $y=0 \longrightarrow 0 = 2 - \dfrac{x}{x^2+1}$; $\dfrac{x}{x^2+1} = 2$; $x = 2x^2+2$; $2x^2 - x + 2 = 0$

$x = \dfrac{+1 \pm \sqrt{1-16}}{4} = \cancel{\dfrac{1\pm\sqrt{-15}}{4}}$

$f(x) > 0$ para todo $x \in$ dom f.



$\text{Area} = \displaystyle\int_0^5 \left(2 - \frac{x}{x^2+1}\right)dx = \left[2x - \frac{1}{2}\ln|x^2+1|\right]_0^5 =$

$= \left(10 - \frac{1}{2}\ln 26\right) - \left(0 - \frac{1}{2}\ln 1\right) = \boxed{10 - \dfrac{\ln 26}{2}}$

JUN 09

$$y = -x^2 + x + \frac{7}{4}$$

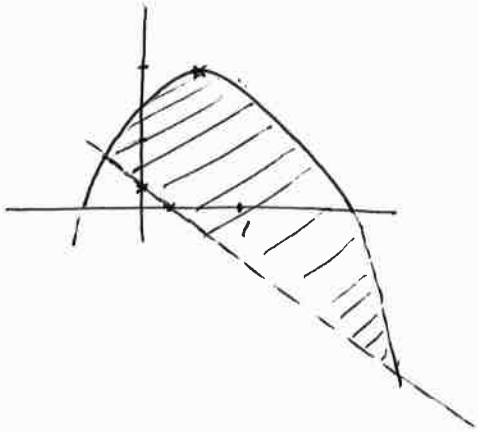
$$y' = -2x + 1$$

$y' = 0 \rightarrow x = 1/2 \rightarrow$ Mínimo en $x = \frac{1}{2}$ $\boxed{(1/2, 2)}$

$x = 0 \rightarrow y = 7/4$

$y = 0 \rightarrow 0 = -x^2 + x + 7/4$; $x = \frac{-1 \pm \sqrt{1+7}}{-2} = \frac{-1 \pm \sqrt{8}}{-2} = \frac{1}{2} \pm \sqrt{2}$

$A(0, 1/4)$
$B(1/6, 0)$ | $\vec{AB} = (1/6, -1/4) \rightarrow m = \frac{-1/4}{1/6} = -\frac{6}{4} = -\frac{3}{2}$



$$y - 0 = -\frac{3}{2}\left(x - \frac{1}{6}\right)$$

$$y = \frac{3}{12} - \frac{3x}{2}$$

$$\left.\begin{array}{l} y = -x^2 + x + \frac{7}{4} \\ y = \frac{3}{12} - \frac{3x}{2} \end{array}\right\} \quad \frac{3}{12} - \frac{3x}{2} = -x^2 + x + \frac{7}{4}$$

$$3 - 18x = -12x^2 + 12x + 21$$

$$12x^2 - 30x - 18 = 0$$

$$2x^2 - 5x - 3 = 0$$

$$x = \frac{5 \pm \sqrt{25+24}}{4} = \frac{5 \pm 7}{4} \begin{cases} 3 \\ -\frac{2}{4} = -\frac{1}{2} \end{cases}$$

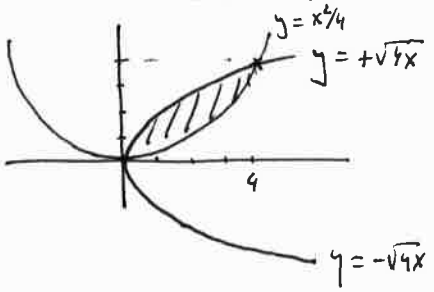
$$\text{Area} = \int_{-1/2}^{3} \left[\left(-x^2 + x + \frac{7}{4}\right) - \left(\frac{3}{12} - \frac{3x}{2}\right)\right] dx = \int_{-1/2}^{3} \left(-x^2 + \frac{5x}{2} + \frac{18}{12}\right) dx =$$

$$= \left[-\frac{x^3}{3} + \frac{5x^2}{4} + \frac{18x}{12}\right]_{-1/2}^{3} = \frac{1}{12}\left[-4x^3 + 15x^2 + 18x\right]_{-1/2}^{3} =$$

$$= \frac{1}{12}(-108 + 135 + 54) - \frac{1}{12}\left(\frac{1}{2} + \frac{15}{4} - 9\right) = \frac{-108 + 135 + 54 - \frac{1}{2} - \frac{15}{4} + 9}{12} = \boxed{\frac{343}{48}} = 7'14583\overline{3}$$

SEPT 09

$y^2 = 4x$
$x^2 = 4y \rightarrow y = x^2/4$ $\rightarrow \left(\frac{x^2}{4}\right)^2 = 4x$; $x^4 = 64x \rightarrow x(x^3 - 64) = 0 \rightarrow \boxed{\begin{array}{l} x = 0 \rightarrow y = 0 \\ x = 4 \rightarrow y = 4 \end{array}}$



$$\text{Area} = \int_0^4 \left(\sqrt{4x} - \frac{x^2}{4}\right) dx = \int_0^4 \left(2 \cdot x^{1/2} - \frac{1}{4}x^2\right) dx = \left[\frac{2x^{3/2}}{3/2} - \frac{1}{4}\frac{x^3}{3}\right]_0^4 = \left[\frac{4\sqrt{x^3}}{3} - \frac{x^3}{12}\right]_0^4 =$$

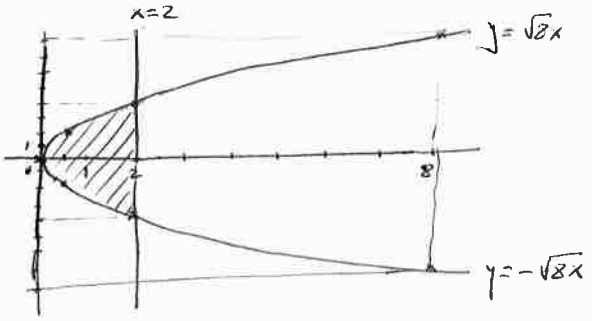
$$= \frac{32}{3} - \frac{16}{3} = \boxed{\frac{16}{3}}$$

JUN 10 fase E

a) $y^2 = 8x$; $y = \pm\sqrt{8x}$

dominio $= [0, +\infty)$

x	y
0	0
1/2	±2
2	±4
8	±8



b) $\text{Area} = 2\int_0^2 \sqrt{8x}\, dx =$

$= 2\sqrt{8}\int_0^2 \sqrt{x}\, dx = 4\sqrt{2}\int_0^2 x^{1/2}\, dx = 4\sqrt{2}\cdot\left(\frac{x^{3/2}}{3/2}\right)_0^2 = \frac{8\sqrt{2}}{3}\left(\sqrt{x^3}\right)_0^2 = \frac{8\sqrt{2}}{3}\left(\sqrt{8} - 0\right) =$

$= \frac{8\sqrt{16}}{3} = \boxed{\frac{32}{3} \text{ u.s.}}$

JUN 10 fase E

$\left.\begin{array}{l} y = \frac{4}{2-x} \\ y = 4 \end{array}\right\}$ $4 = \frac{4}{2-x}$; $2 - x = 1$; $x = 1$ $\quad P(1,4)$

$\left.\begin{array}{l} y = \frac{4}{2-x} \\ x = 0 \end{array}\right\}$ $y = \frac{4}{2} = 2$ $\quad P(0,2)$

$f(x) = \frac{4}{2-x}$

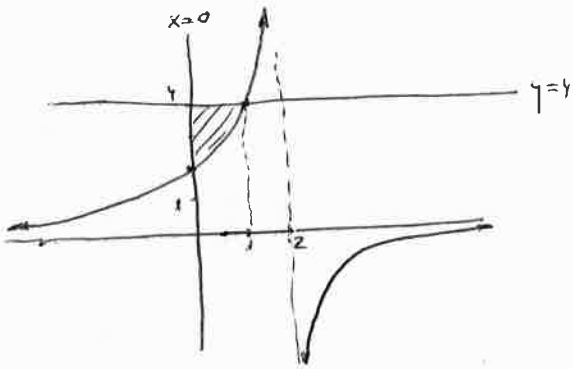
dominio $= \mathbb{R} - \{2\}$

$\lim_{x\to 2^-} f(x) = \frac{4}{0^+} = +\infty$

$\lim_{x\to 2^+} f(x) = \frac{4}{0^-} = -\infty$

$\lim_{x\to -\infty} f(x) = \frac{4}{+\infty} = 0$

$\lim_{x\to +\infty} f(x) = \frac{4}{-\infty} = 0$



$\text{Area} = \int_0^1 \left(4 - \frac{4}{2-x}\right) dx = \int_0^1 4\,dx + 4\int_0^1 \frac{-1}{2-x}\, dx = \left[4x + 4\ln|2-x|\right]_0^1 =$

$= (4 + 4\overset{0}{\ln 1}) - (0 + 4\ln 2) = \boxed{4 - 4\ln 2} = 1'2274$ u.s.

JUN 10 fase G

$F(x) = \int x(1-\ln x)\, dx = (1-\ln x)\cdot\frac{x^2}{2} - \int \frac{-1}{x}\cdot\frac{x^2}{2}\, dx = (1-\ln x)\frac{x^2}{2} + \frac{1}{2}\int x\, dx = (1-\ln x)\frac{x^2}{2} + \frac{1}{2}\frac{x^2}{2} =$

$\boxed{\begin{array}{l|l} u = 1-\ln x & du = -\frac{1}{x}dx \\ dv = x\,dx & v = \frac{x^2}{2} \end{array}}$

$= \frac{x^2}{2}\left[1 - \ln x + \frac{1}{2}\right] = \frac{x^2}{2}\cdot\frac{3-2\ln x}{2} = \boxed{\frac{(3-2\ln x)x^2}{4} + K}$

$P(1,3) \Rightarrow 3 = \frac{(3-2\overset{0}{\ln 1})\cdot 1^2}{4} + K$; $3 = \frac{3}{4} + K$; $K = 9/4$

$F(x) = \frac{(3-2\ln x)x^2}{4} + \frac{9}{4} = \boxed{\frac{(3-2\ln x)x^2 + 9}{4}}$

JUN 10 fase G

$$\int \frac{e^x - 4e^{2x}}{1+e^x}\,dx = \int \frac{t-4t^2}{1+t}\cdot\frac{1}{t}\,dt = \int \frac{\cancel{t}(1-4t)}{(1+t)\cdot\cancel{t}}\,dt = \int \frac{1-4t}{1+t}\,dt =$$

$$\boxed{\begin{array}{l} t = e^x \\ x = \ln t \\ dx = \frac{1}{t}\,dt \end{array}}$$

$$\boxed{\begin{array}{ll} -4t + 1 & \underline{|\,t+1} \\ \underline{+4t + 4} & -4 \\ \quad 5 & \end{array}}$$

$$= \int\left(-4 + \frac{5}{t+1}\right)dt = -4t + 5\ln(t+1) = \boxed{-4e^x + 5\ln(1+e^x) + K}$$

SEPT 10 fase E

$$\left.\begin{array}{l} y = 6x - x^2 \\ y = 0 \end{array}\right\} \quad 0 = 6x - x^2 \;;\quad 0 = x(6-x) \;:\; x = \begin{cases} 0 \\ 6 \end{cases} \quad \begin{array}{l}(0,0)\\(6,0)\end{array}$$

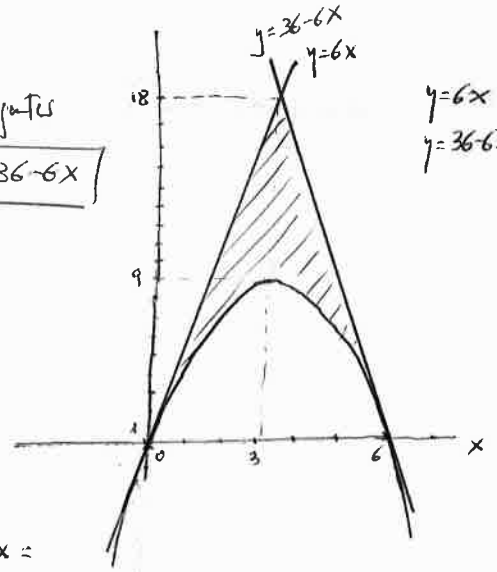
$y' = 6 - 2x$

$x = 0 \to y = 0$, $\to y' = 6$ $\qquad y - 0 = 6(x-0)$; $\boxed{y = 6x}$

$x = 6 \to y = 0$, $\to y' = 6 - 12 = -6$ $\qquad y - 0 = -6(x-6)$; $\boxed{y = 36 - 6x}$

Rectas Tangentes

$$\left.\begin{array}{l} y = 6x \\ y = 36 - 6x \end{array}\right\} \to \begin{array}{l} x = 3 \\ y = 18 \end{array}$$



$y = 6x - x^2$

$y' = 0 \to 6 - 2x = 0$; $x = 3 \Rightarrow$ Vértice en $x = 3$

x	y
0	0
3	9
6	0

a) $$\text{Area} = \int_0^3 (6x - (6x - x^2))\,dx + \int_3^6 ((36-6x) - (6x - x^2))\,dx =$$

$$= \int_0^3 x^2\,dx + \int_3^6 (36 - 12x + x^2)\,dx = \left(\frac{x^3}{3}\right)_0^3 + \left(36x - 6x^2 + \frac{x^3}{3}\right)_3^6 =$$

$$= \frac{27}{3} - 0 + (\cancel{216} - \cancel{216} + 72) - (108 - 54 + 9) = \cancel{9} + 72 - 108 + 54 - \cancel{9} = \boxed{18 \text{ us}}$$

<u>También</u>. Por la simetría del recinto, $\text{Area} = 2\int_0^3 (6x - (6x - x^2))\,dx = 2\int_0^3 x^2\,dx =$

$$= 2\left(\frac{x^3}{3}\right)_0^3 = 2\cdot\frac{27}{3} = 18 \;\checkmark$$

SEPT 10 fase E

$$f(x) = \begin{cases} 2x - 2 & \text{si } x < 2 \\ e^{x-2} + K^2 & \text{si } x \geq 2 \end{cases}$$

- $2x - 2$ está definida y es continua en $\mathbb{R}$ por ser polinómica (lineal)
- $e^{x-2} + K^2$ " " " " " " $\mathbb{R}$ " " exponencial.

Por lo tanto está asegurada la continuidad en $(-\infty, 2) \cup (2, +\infty)$. Veamos la continuidad en $x = 2$:

$$\left.\begin{array}{l} \lim_{x\to 2^-} f(x) = 2\cdot 2 - 2 = 2 \\ f(2) = e^{2-2} + K^2 = e^0 + K^2 = K^2 + 1 \\ \lim_{x\to 2^+} f(x) = e^{2-2} + K^2 = e^0 + K^2 = K^2 + 1 \end{array}\right\} \quad K^2 + 1 = 2 \;;\; K^2 = 1 \;;\; \boxed{K = \pm 1}$$

Para cualquiera de estos valores, $f(x)$ será continua en $\mathbb{R}$, y por lo tanto en $[0,4]$.

b) $K=1 \longrightarrow f(x)=\begin{cases} 2x-2 & \text{si } x<2 \\ e^{x-2}+1 & \text{si } x\geq 2 \end{cases} \longrightarrow f'(x)=\begin{cases} 2 & \text{si } x<2 \\ e^{x-2} & \text{si } x>2 \end{cases}$

$x=3 \to f(3)=e^{3-2}+1=e+1$

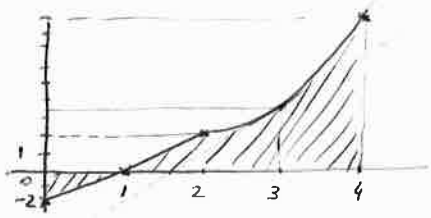
$\to f'(3)=e^{3-2}=e$

$y-(e+1)=e(x-3)$; $y=ex-3e+e+1$

$\boxed{y=ex+(1-2e)}$ Recta Tangente en $x=3$

c)

x	$f(x)$
0	-2
1	0
2^-	2
2	2
3	$e+1$
4	e^2+1



$$\text{Area} = -\int_0^1 (2x-2)\,dx + \int_1^2 (2x-2)\,dx + \int_2^4 (e^{x-2}+1)\,dx = -\left(x^2-2x\right)_0^1 + \left(x^2-2x\right)_1^2 + \left(e^{x-2}+x\right)_2^4 =$$

$$= -(1-2)+0+(4-4)-(1-2)+(e^2+4)-(e^0+2) = \cancel{2}+e^2+4-1-\cancel{2} = \boxed{e^2+3} = 10'3891 \text{ us}$$

SEPT 10 fase G

$$I=\int e^x \cos 3x\,dx = e^x\cos 3x + 3\int e^x \sin 3x\,dx = e^x\cos 3x + 3\left(e^x \sin 3x - 3\int e^x \cos 3x\,dx\right) =$$

$u=\cos 3x$	$du=-3\sin 3x\,dx$
$dv=e^x dx$	$v=e^x$

$u=\sin 3x$	$du=3\cos 3x\,dx$
$dv=e^x dx$	$v=e^x$

$$= e^x\cos 3x + 3e^x \sin 3x - 9\int e^x \cos 3x\,dx$$

$I = e^x(\cos 3x + 3\sin 3x) - 9I \Rightarrow 10I = e^x(\cos 3x + 3\sin 3x)$; $I=\frac{1}{10}e^x(\cos 3x+3\sin 3x)$

$$\int e^x \cos 3x\,dx = \boxed{\frac{e^x(\cos 3x+3\sin 3x)}{10}+K}$$

SEPT 10 fase G

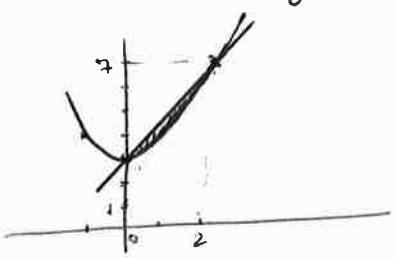
$\left.\begin{array}{l} y=x^2+3 \\ y=2x+3 \end{array}\right\}$ $x^2+\cancel{3}=2x+\cancel{3}$; $x^2-2x=0$; $x=\begin{cases}2\\0\end{cases}$

$y=x^2+3$

x	y
-1	4
0	3
1	4
2	7

$y=2x+3$

x	y
0	3
2	7



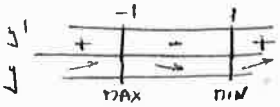
$$\text{Area} = \int_0^2 \left((2x+3)-(x^2+3)\right)dx = \int_0^2 (2x-x^2)\,dx = \left(x^2-\frac{x^3}{3}\right)_0^2 = \left(4-\frac{8}{3}\right)-0 = \boxed{\frac{4}{3} \text{ us}}$$

JUN 11 fase E

$$\begin{cases} y = x^3 - 3x \\ y = x \end{cases} \quad x^3 - 3x = x \;;\quad x^3 - 4x = 0 \;;\quad x(x^2-4) = 0 \;;\quad x = \begin{cases} 0 \\ \pm 2 \end{cases}$$

$y = x^3 - 3x$

$y' = 3x^2 - 3 \;;\quad y' = 0 \Rightarrow x = \pm 1$

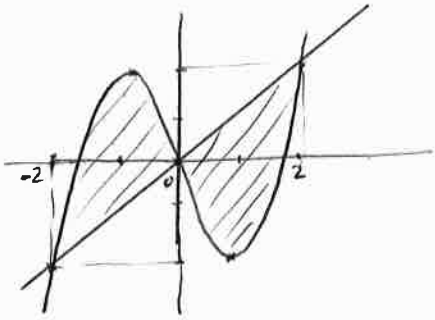


$y = x^3 - 3x$

	x	y
	-2	-2
MAX	-1	2
	0	0
MIN	1	-2
	2	2

$y = x$

x	y
-2	-2
0	0
2	2



Por simetría:

$$\text{Area} = 2 \cdot \int_0^2 x - (x^3 - 3x)\, dx = 2 \int_0^2 (4x - x^3)\, dx = 2\left(2x^2 - \frac{x^4}{4}\right)_0^2 = 2(8-4) = \boxed{8 \text{ u.s}}$$

JUN 11 fase E

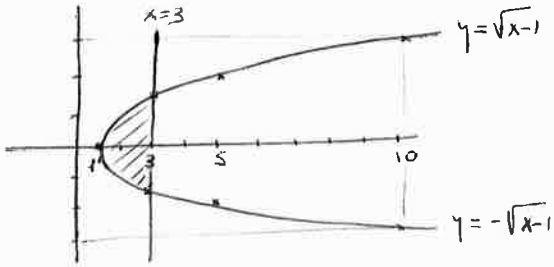
$$\begin{cases} x = y^2 + 1 \\ x = 3 \end{cases} \quad y^2 + 1 = 3 \;;\quad y^2 = 2 \;;\quad y = \pm\sqrt{2}$$

$x = y^2 + 1 \rightarrow y^2 = x - 1 \;;\quad y = \pm\sqrt{x-1}$

dominio $= [1, +\infty)$

$y = \pm\sqrt{x-1}$

x	y
1	0
3	$\pm\sqrt{2}$
5	± 2
10	± 3



b) $$\text{Area} = 2\int_1^3 \sqrt{x-1}\, dx = 2\int_1^3 (x-1)^{1/2}\, dx = 2\left[\frac{(x-1)^{3/2}}{3/2}\right]_1^3 = \left(\frac{4}{3}\sqrt{(x-1)^3}\right)_1^3 =$$

$$= \frac{4}{3}\sqrt{8} - 0 = \boxed{\frac{8\sqrt{2}}{3}} = 3'7712 \text{ u.s.}$$

JUN 11 fase G

$$f(x) = \begin{cases} x^2 & \text{si } x < 0 \\ mx + n & \text{si } 0 \le x < 1 \\ 2 & \text{si } x \ge 1 \end{cases}$$

x^2, $mx+n$, 2 son funciones continuas en $\mathbb{R}$, por lo que está asegurada la continuidad en $(-\infty, 0) \cup (0,1) \cup (1, +\infty)$. Veamos ahora en $x=0$, y en $x=1$:

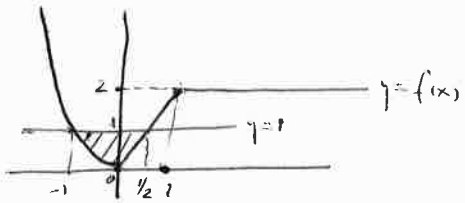
$$\left.\begin{array}{l} \lim_{x \to 0^-} f(x) = 0^2 = 0 \\ f(0) = m \cdot 0 + n = n \\ \lim_{x \to 0^+} f(x) = m \cdot 0 + n = n \end{array}\right\} \Rightarrow \boxed{n = 0}$$

$$\left.\begin{array}{l} \lim_{x \to 1^-} f(x) = m \cdot 1 + n = m + n \\ f(1) = 2 \\ \lim_{x \to 1^+} f(x) = 2 \end{array}\right\} \Rightarrow m + n = 2 \;;\quad \boxed{m = 2}$$

Para que $f(x)$ sea continua en $\mathbb{R}$

b) $f(x) = \begin{cases} x^2 & \text{si } x<0 \\ 2x & \text{si } 0\le x<1 \\ 2 & \text{si } x\ge 1 \end{cases}$; $y=1$ $\begin{cases} x^2=1 \ ; \ x=\begin{cases}-1\\ \cancel{1}\end{cases} \text{ porque } 1\not<0 \\ 2x=1 \ ; \ x=1/2 \\ 2=1 \ \text{Absurdo}\end{cases}$

x	y
$-\infty$	$+\infty$
-1	1
0^-	0
0	0
$1/2$	1
1^-	2
1	2
$+\infty$	2



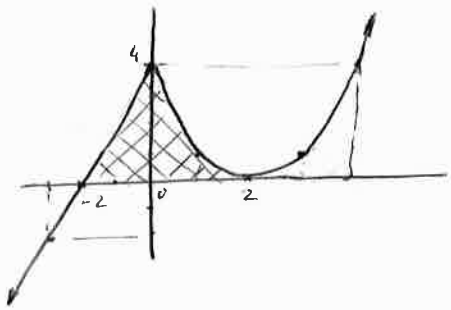
$$\text{Area} = \int_{-1}^{0}(1-x^2)\,dx + \int_{0}^{1/2}(1-2x)\,dx = \left(x-\frac{x^3}{3}\right)_{-1}^{0} + (x-x^2)_{0}^{1/2} =$$

$$= 0-\left(-1+\frac{1}{3}\right)+\left(\frac{1}{2}-\frac{1}{4}\right)-0 = \frac{2}{3}+\frac{1}{4} = \boxed{\frac{11}{12}\ u.s}$$

JUN 11 fase G

$f(x) = \begin{cases} 2x+4 & \text{si } x\le 0 \\ (x-2)^2 & \text{si } x>0 \end{cases}$ $\quad y=0 \begin{cases} 2x+4=0 \ ; \ x=-2 \\ (x-2)^2=0 \ ; \ x=2 \end{cases}$

x	y
-3	-2
-2	0
0	4
0^+	4
1	1
2	0
3	1
4	4



$$\text{Area} = \int_{-2}^{0}(2x+4)\,dx + \int_{0}^{2}(x-2)^2\,dx =$$

$$= (x^2+4x)_{-2}^{0} + \left[\frac{(x-2)^3}{3}\right]_{0}^{2} =$$

$$= 0-(4-8)+0-\left(\frac{-8}{3}\right) = \boxed{\frac{20}{3}\ u\,s}$$

JUL 11 fase E

$$\int \operatorname{arctg}(3x)\,dx = x\operatorname{arctg}3x - \int\frac{3x}{1+9x^2}\,dx = x\operatorname{arctg}x - \frac{1}{6}\int\frac{18x}{1+9x^2}\,dx = \boxed{x\operatorname{arctg}x - \frac{1}{6}\ln(1+9x^2)+K}$$

$$\boxed{\begin{array}{l|l} u=\operatorname{arctg}(3x) & du=\dfrac{3}{1+(3x)^2}\,dx \\ dv=dx & v=x \end{array}}$$

JUL 11 fase G

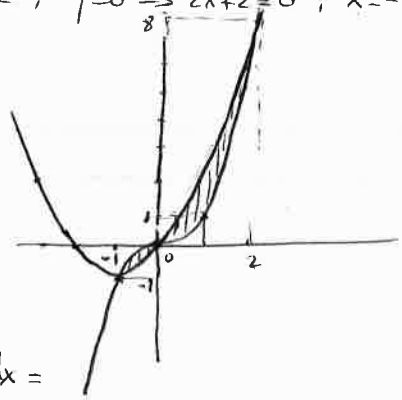
$\left.\begin{array}{l} y=x^3 \\ y=x^2+2x \end{array}\right\} \longrightarrow x^3=x^2+2x \ ; \ x^3-x^2-2x=0 \ ; \ x(x^2-x-2)=0 \begin{cases} x=0 \\ x^2-x-2=0 \ ; \ x=\begin{cases}-1\\ 2\end{cases} \end{cases}$

$y=x^3$

x	y
$-\infty$	$-\infty$
-1	-1
0	0
1	1
2	8
$+\infty$	$+\infty$

$y=x^2+2x \longrightarrow y'=2x+2 \ ; \ y'=0 \Rightarrow 2x+2=0 \ ; \ x=-1$ (vértice)

x	y
-3	2
-2	0
-1	-1
0	0
1	3
2	8



$$\text{Area} = \int_{-1}^{0}\left[x^3-(x^2+2x)\right]dx + \int_{0}^{2}\left[(x^2+2x)-x^3\right]dx =$$

$$= \int_{-1}^{0}(x^3-x^2-2x)\,dx + \int_{0}^{2}(x^2+2x-x^3)\,dx = \left(\frac{x^4}{4}-\frac{x^3}{3}-x^2\right)_{-1}^{0} + \left(\frac{x^3}{3}+x^2-\frac{x^4}{4}\right)_{0}^{2} =$$

$$= 0-\left(\frac{1}{4}+\frac{1}{3}-1\right)+\left(\frac{8}{3}+\cancel{4}-\cancel{4}\right)-0 = \boxed{\frac{37}{12}\ u\,s}$$

JUN 12 fase E

$$\int \frac{e^x}{e^{2x}-1}\,dx = \int \frac{\cancel{t}}{t^2-1}\cdot\frac{1}{\cancel{t}}\,dt = \int \frac{1}{t^2-1}\,dt =$$

$$\boxed{\begin{array}{l} t=e^x \\ x=\ln t \\ dx=\frac{1}{t}dt \end{array}}$$

$$\frac{1}{t^2-1} = \frac{1}{(t+1)(t-1)} = \frac{A}{t+1} + \frac{B}{t-1} = \frac{A(t-1)+B(t+1)}{(t+1)(t-1)}$$

$$1 = A(t-1)+B(t+1)$$

$t=1 \Rightarrow 1 = 2B$; $B = 1/2$

$t=-1 \Rightarrow 1 = -2A$; $A = -1/2$

$$= \int \frac{-1/2}{t+1}\,dt + \int \frac{1/2}{t-1}\,dt = -\frac{1}{2}\ln|t+1| + \frac{1}{2}\ln|t-1| = \frac{1}{2}\left[\ln|t-1| - \ln|t+1|\right] =$$

$$= \frac{1}{2}\ln\left|\frac{t-1}{t+1}\right| = \boxed{\frac{1}{2}\ln\left|\frac{e^x-1}{e^x+1}\right| + K}$$

JUN 12 fase E

$$\int_0^{\pi/2} (e^{2x} + x\cos x)\,dx = (*)$$

- $\int e^{2x} = \frac{1}{2}\int 2e^{2x}\,dx = \frac{1}{2}e^{2x} + K$
- $\int x\cos x\,dx = x\,\mathrm{sen}\,x - \int \mathrm{sen}\,x\,dx = x\,\mathrm{sen}\,x - (-\cos x) = x\,\mathrm{sen}\,x + \cos x + K$

$$\boxed{\begin{array}{c|c} u=x & du=dx \\ dv=\cos x\,dx & v=\mathrm{sen}\,x \end{array}}$$

$$(*) = \left[\frac{1}{2}e^{2x} + x\,\mathrm{sen}\,x + \cos x\right]_0^{\pi/2} = \left(\frac{1}{2}e^{\pi} + \frac{\pi}{2}\overset{1}{\mathrm{sen}\,\pi/2} + \overset{0}{\cos \pi/2}\right) - \left(\frac{1}{2}\overset{1}{e^0} + 0\,\overset{0}{\mathrm{sen}\,0} + \overset{1}{\cos 0}\right) =$$

$$= \boxed{\frac{1}{2}e^{\pi} + \frac{\pi}{2} - \frac{3}{2}} = \boxed{\frac{e^{\pi}+\pi-3}{2}}$$

JUN 12 fase G

$$\int_1^2 \frac{dx}{x^2+3x} = \int_1^2 \left(\frac{1/3}{x} - \frac{1/3}{x+3}\right)dx = \left[\frac{1}{3}\ln|x| - \frac{1}{3}\ln|x+3|\right]_1^2 = \left(\frac{1}{3}\ln 2 - \frac{1}{3}\ln 5\right) - \left(\frac{1}{3}\overset{0}{\ln 1} - \frac{1}{3}\ln 4\right)$$

$$\frac{1}{x^2+3x} = \frac{1}{x(x+3)} = \frac{A}{x} + \frac{B}{x+3} = \frac{A(x+3)+Bx}{x(x+3)}$$

$$1 = A(x+3) + Bx$$

$x=-3 \Rightarrow 1 = -3B$; $B = -1/3$

$x=0 \Rightarrow 1 = 3A$; $A = 1/3$

$$= \frac{1}{3}\left[\ln 2 - \ln 5 + \ln 4\right] = \frac{1}{3}\ln\frac{2\cdot 4}{5} = \frac{1}{3}\ln\frac{8}{5} = \boxed{\ln\sqrt[3]{8/5}} = 0'1567$$

JUN 12 fase G

$\left.\begin{array}{l} y=\ln^2 x \\ y=0 \end{array}\right\} \rightarrow \ln^2 x = 0$; $\ln x = 0$; $x=1$; $P(1,0)$

$\left.\begin{array}{l} y=\ln^2 x \\ x=1 \end{array}\right\} \rightarrow y = \ln^2 1 = 0$ $P(1,0)$

$\left.\begin{array}{l} y=\ln^2 x \\ x=e \end{array}\right\} \rightarrow y = \ln^2 e = 1$ $P(e,1)$

$y = \ln^2 x$ $\boxed{\text{dominio} = (0,+\infty)}$

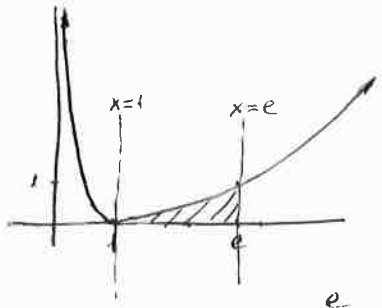
$y' = 2\ln x\cdot\frac{1}{x} = \frac{2\ln x}{x}$

$y'=0 \Rightarrow \frac{2\ln x}{x} = 0$; $2\ln x = 0$; $x=1$

	0		1	
y'		$-$		$+$
y		$\searrow$	MIN	$\nearrow$

$y = \ln^2 x$

	x	y
	0^+	$+\infty$
MIN	1	0
	e	1
	$+\infty$	$+\infty$



$$\text{Area} = \int_1^e \ln^2 x\,dx = \left(x\ln^2 x\right)_1^e - \int_1^e 2\ln x \cdot \frac{1}{\cancel{x}} \cdot \cancel{x}\,dx = \left(x\ln^2 x\right)_1^e - \int_1^e 2\ln x\,dx =$$

$$\boxed{\begin{array}{l|l} u = \ln^2 x & du = 2\ln x \cdot \frac{1}{x}\,dx \\ dv = dx & v = x \end{array}} \qquad \boxed{\begin{array}{l|l} u = 2\ln x & du = 2\cdot\frac{1}{x}\,dx \\ dv = dx & v = x \end{array}}$$

$$= \left(x\ln^2 x\right)_1^e - \left(2\ln x \cdot x\right)_1^e + \int_1^e 2\cdot\frac{1}{\cancel{x}}\cdot\cancel{x}\,dx = \left(x\ln^2 x - 2x\ln x\right)_1^e + \int_1^e 2\,dx =$$

$$= \left(x\cdot\ln^2 x - 2x\ln x + 2x\right)_1^e = \left(e\,\overset{1}{\ln^2 e} - 2e\,\overset{1}{\ln e} + 2e\right) - \left(\overset{0}{\ln^2 1} - 2\,\overset{0}{\ln 1} + 2\right) =$$

$$= e - \cancel{2e} + \cancel{2e} - 2 = \boxed{e-2} \text{ u.s.}$$

JUL 12 fase E

$f'(x) = (x+2)(x^2-9)$

a) $f'(x) = 0 \Rightarrow (x+2)(x^2-9) = 0$ → $x+2=0$; $x=-2$; → $x^2-9=0$; $x=\pm 3$

	$-\infty$		-3		-2		3		$+\infty$
$f'(x)$		−		+		−		+	
$f(x)$		↘	MIN	↗	MAX	↘	MIN	↗	

$f(x)$ es creciente en $(-3,-2) \cup (3,+\infty)$
$f(x)$ es decreciente en $(-\infty,-3) \cup (-2,3)$
$f(x)$ tiene un máximo relativo en $x=-2$
$f(x)$ tiene mínimos relativos en $x=-3$, $x=3$

b) $f(x) = \int f'(x)\,dx = \int (x+2)(x^2-9)\,dx = \int (x^3 - 9x + 2x^2 - 18)\,dx = \frac{x^4}{4} - \frac{9x^2}{2} + \frac{2x^3}{3} - 18x + K$

$f(0) = \frac{1}{5} \Rightarrow \frac{1}{5} = 0 - 0 + 0 - 0 + K$; $K = \frac{1}{5}$ $\boxed{f(x) = \frac{x^4}{4} - \frac{9x^2}{2} + \frac{2x^3}{3} + 18x + \frac{1}{5}}$

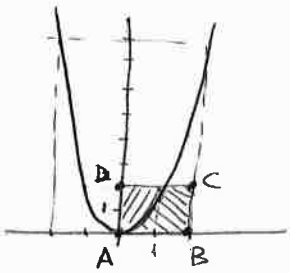
JUL 12 fase E

$y = 2x^2$

$y=0 \rightarrow 0 = 2x^2$; $x=0$

$y=2 \rightarrow 2 = 2x^2$; $x^2 = 1$; $x = \pm 1$

x	y
−2	8
−1	2
0	0
1	2
2	8



$$\text{Area}_1 = \int_0^1 (2 - 2x^2)\,dx = \left(2x - \frac{2x^3}{3}\right)_0^1 = \left(2 - \frac{2}{3}\right) - 0 = \boxed{\frac{4}{3}} \text{ u.s.}$$

$$\text{Area}_2 = 2^2 - \frac{4}{3} = \boxed{\frac{8}{3}} \text{ u.s.}$$

JUL 12 fase 6

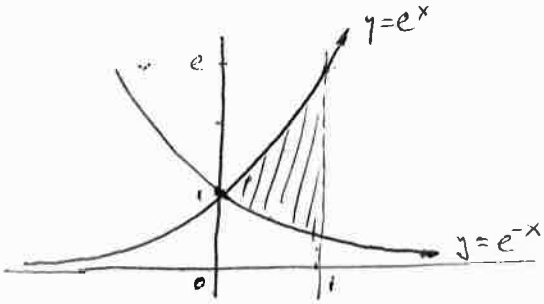
$$\left.\begin{array}{l} y=e^x \\ y=e^{-x} \end{array}\right\} \quad e^x=e^{-x} \;;\; e^x=\frac{1}{e^x} \;;\; e^{2x}=1 \;;\; 2x=0 \;;\; x=0 \qquad P(0,1)$$

$y=e^x$

x	y
$-\infty$	0
0	1
1	e
$+\infty$	$+\infty$

$y=e^{-x}$

x	y
$-\infty$	$+\infty$
0	1
1	$e^{-1}=1/e$
$+\infty$	0



$$\text{Area} = \int_0^1 (e^x - e^{-x})\,dx = \left(e^x + e^{-x}\right)\Big|_0^1 = (e+e^{-1}) - (1+1) = \boxed{e+\frac{1}{e}-2} \text{ u.s.}$$

JUL 12 fase 6

$y = x^3 - 2x^2 + x$

$y' = 3x^2 - 4x + 1$

$x=0 \begin{cases} \to y=0 \\ \to y'=1 \end{cases} \qquad y-0 = 1\cdot(x-0) \;;\; \boxed{y=x}$ Recta Tangente en $x=0$

$$\left.\begin{array}{l} y=x^3-2x^2+x \\ y=x \end{array}\right\} \to x^3-2x^2+x=x \;;\; x^3-2x^2=0 \;;\; x^2(x-2)=0 \;;\; x=\begin{cases} 0 \\ 2 \end{cases} \quad \begin{array}{l} P(0,0) \\ P(2,2) \end{array}$$

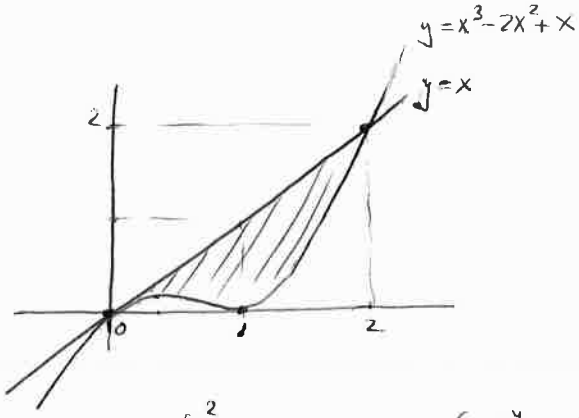
$y = 3x^2 - 4x + 1$

$y'=0 \to 3x^2-4x+1=0 \;;\; x = \frac{4\pm\sqrt{16-12}}{6} = \frac{4\pm 2}{6} = \begin{cases} 1 \\ 1/3 \end{cases}$

		$1/3$		1	
y'	$+$		$-$		$+$
y	$\nearrow$	MAX	$\searrow$	MIN	$\nearrow$

$y = x^3 - 2x^2 + x$

	x	y
	$-\infty$	$-\infty$
	0	0
MAX	$1/3$	$4/27$
MIN	1	0
	2	2
	$+\infty$	$+\infty$



$$\text{Area} = \int_0^2 \left[x - (x^3-2x^2+x)\right] dx = \int_0^2 (-x^3+2x^2)\,dx = \left(-\frac{x^4}{4} + \frac{2x^3}{3}\right)\Big|_0^2 =$$

$$= \left(-\frac{16}{4} + \frac{16}{3}\right) - 0 = \boxed{\frac{4}{3} \text{ u.s}}$$

JUN 13
fase General

$$f(x)=\begin{cases} 4x+12 & \text{si } x\le -1 \\ x^2-4x+3 & \text{si } x>-1 \end{cases}$$

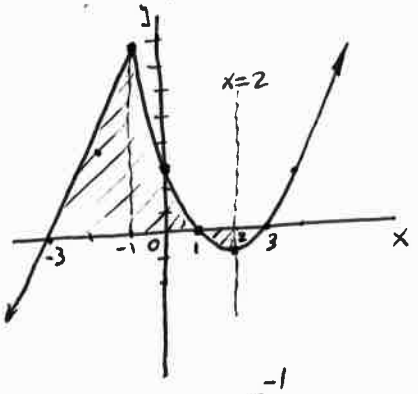
a) $y=4x+12$

x	y
-2	4
-1	8
-3	0

$4x+12=0$
$x=-3$

$y=x^2-4x+3$
$y'=2x-4$
$y'=0 \rightarrow x=2$
$x^2-4x+3=0$
$x=\frac{4\pm\sqrt{16-12}}{2}=\frac{4\pm 2}{2}\begin{matrix}3\\1\end{matrix}$

x	y
-1	8
0	3
1	0
V 2	-1
3	0
4	3



b) $$\text{Area}=\int_{-3}^{-1}(4x+12)\,dx+\int_{-1}^{1}(x^2-4x+3)\,dx-\int_{1}^{2}(x^2-4x+3)\,dx=$$

$$=\left(2x^2+12x\right)_{-3}^{-1}+\left(\frac{x^3}{3}-2x^2+3x\right)_{-1}^{1}-\left(\frac{x^3}{3}-2x^2+3x\right)_{1}^{2}=$$

$$=(2-12)-(18-36)+\left(\frac{1}{3}-2+3\right)-\left(-\frac{1}{3}-2-3\right)-\left(\frac{8}{3}-8+6\right)+\left(\frac{1}{3}-2+3\right)=$$

$$=-10+18+\frac{1}{3}+1+\frac{1}{3}+5-\frac{8}{3}+2+\frac{1}{3}+1=\boxed{14\ u^2}$$

JUN 13
fase General

$y=x^2-3x+6$
$y'=2x-3$
$y'=0 \rightarrow x=3/2$
$x^2-3x+6=0$; $x=\frac{3\pm\sqrt{9-24}}{2}$ Absurdo

$y=x^2-3x+6$

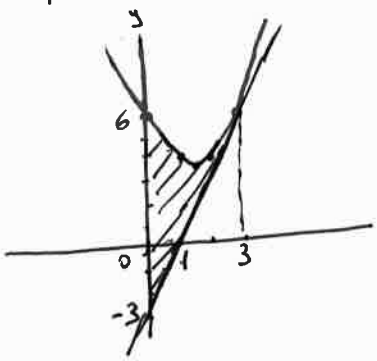
x	y
0	6
1	4
V 3/2	15/4
2	4
3	6

a) $x=3 \rightarrow y=3^2-3\cdot 3+6=6$
$\rightarrow y'=2\cdot 3-3=3$

$y-6=3(x-3)$; $\boxed{y=3x-3}$

x	y
0	-3
1	0
3	6

b)



$$\text{Area}=\int_0^3 (x^2-3x+6)-(2x-3)\,dx-\int_0^1(2x-3)\,dx=$$

$$=\int_0^3(x^2-5x+9)\,dx-\int_0^1(2x-3)\,dx=$$

$$=\left[\frac{x^3}{3}-\frac{5x^2}{2}+9x\right]_0^3-\left[x^2-3x\right]_0^1=$$

$$=\left(\frac{27}{3}-\frac{45}{2}+27\right)-(1-2)=\boxed{14'5\ u^2}$$

JUN 13
fase Específica

$$\int_0^{\pi/2} (x-a)\cos x\,dx = \left[(x-a)\sin x\right]_0^{\pi/2} - \int_0^{\pi/2} \sin x\,dx = \left[(x-a)\sin x + \cos x\right]_0^{\pi/2} =$$

$u = x-a$	$du = dx$
$dv = \cos x\,dx$	$v = \sin x$

$$= \left[\left(\frac{\pi}{2}-a\right)\cdot 1 + 0\right] - \left[-a\cdot 0 + 1\right] = \frac{\pi}{2} - a - 1$$

$$\frac{\pi}{2} - a - 1 = \frac{\pi}{2} - 2 \Rightarrow \boxed{a = 1}$$

JUN 13
fase Específica

a) $\left.\begin{array}{l} y = x^3-3x-2 \\ y = x^2-x-2 \end{array}\right\}$ $x^3-3x-2 = x^2-x-2$; $x^3-x^2-2x=0$;

$x(x^2-x-2)=0 \to x=0$

$\to x^2-x-2=0$

$x = \frac{1\pm\sqrt{1+8}}{2} = \frac{1\pm 3}{2} \begin{cases} 2 \\ -1 \end{cases}$

b) $f(x) = x^3-3x-2$

$f'(x) = 3x^2-3$

$f'(x)=0 \to 3x^2=3$; $x=\pm 1$

		-1		1	
f'	$+$		$-$		$+$
f	$\nearrow$	MAX	$\searrow$	MIN	$\nearrow$

$y = x^3-3x-2$

	x	y
	$-\infty$	$-\infty$
MAX	-1	0
	0	-2
MIN	1	-4
	2	0
	$+\infty$	$+\infty$

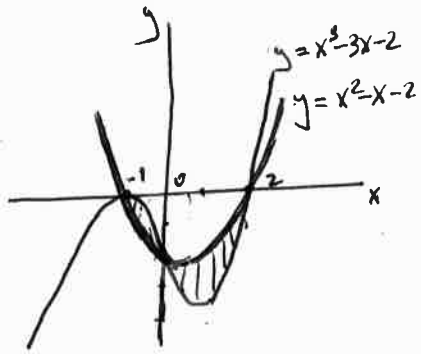
$g(x) = x^2-x-2$

$g'(x) = 2x-1$

$g'(x)=0 \to 2x=1$; $x=1/2$

		$1/2$	
g'	$-$		$+$
g	$\searrow$	MIN	$\nearrow$

	x	y
	-1	0
	0	-2
MIN	$1/2$	$-9/4$
	2	0



$$\text{Area} = \int_{-1}^{0} (x^3-3x-2)-(x^2-x-2)\,dx + \int_0^2 (x^2-x-2)-(x^3-3x-2)\,dx =$$

$$= \int_{-1}^{0} (x^3-x^2-2x)\,dx + \int_0^2 (-x^3+x^2+2x)\,dx = \left[\frac{x^4}{4} - \frac{x^3}{3} - x^2\right]_{-1}^{0} + \left[-\frac{x^4}{4} + \frac{x^3}{3} + x^2\right]_0^2 =$$

$$= -\left(\frac{1}{4}+\frac{1}{3}-1\right) + \left(-\frac{16}{4}+\frac{8}{3}+4\right) = \boxed{\frac{37}{12}\ u^2} = \boxed{3'08\overline{3}\ u^2}$$

JUL 13
fase General

$$\int_0^{\pi/2}(\operatorname{sen}(2x)+x\operatorname{sen}x)\,dx = \left[-\frac{1}{2}\cos 2x\right]_0^{\pi/2} + \int_0^{\pi/2} x\operatorname{sen}x\,dx =$$

$$\left|\begin{array}{c|c} u=x & du=dx \\ dv=\operatorname{sen}x\,dx & v=-\cos x \end{array}\right|$$

$$= \left[-\frac{1}{2}\cos 2x\right]_0^{\pi/2} + \left[-x\cos x\right]_0^{\pi/2} + \int_0^{\pi/2}\cos x\,dx =$$

$$= \left[-\frac{1}{2}\cos 2x - x\cos x + \operatorname{sen}x\right]_0^{\pi/2} = \left[+\frac{1}{2} - \frac{\pi}{2}\cdot 0 + 1\right] - \left[-\frac{1}{2} - 0 + 0\right] =$$

$$= \frac{1}{2} + 1 + \frac{1}{2} = \boxed{2}$$

JUL 13
fase General

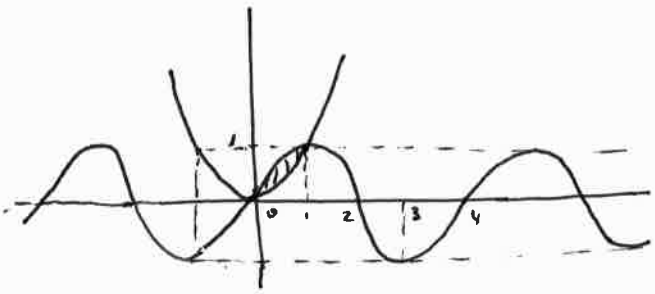
$y = \operatorname{sen}\left(\frac{\pi}{2}x\right)$

a) Periodo = 4

x	y
0	0
1	1
2	0
3	-1
4	0

$y = x^2$

x	y
-2	4
-1	1
0	0
1	1
2	4



b) $$\text{Area} = \int_0^1 \left(x^2 - \operatorname{sen}(2x)\right)dx = \left[\frac{x^3}{3} + \frac{1}{2}\cos(2x)\right]_0^1 = \left(\frac{1}{3}+\frac{1}{2}\right) - \left(0+\frac{1}{2}\right) = \boxed{\frac{1}{3}\,u^2}$$

JUL 13
fase Específica

$$f(x) = \begin{cases} (x-1)^2 & \text{si } x \le 1 \\ \ln x & \text{si } x > 1 \end{cases}$$

$y = (x-1)^2 = x^2 - 2x + 1$

$y' = 2x - 2$

$y' = 0 \rightarrow x = 1$

$(x-1)^2 = 0 \rightarrow x = 1$

x	y
-1	4
0	1
V 1	0
~~2~~	~~1~~
~~3~~	~~4~~

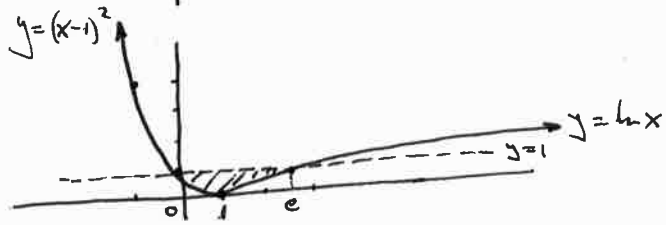
$y = \ln x$

$y' = \frac{1}{x}$

$y' = 0 \rightarrow \frac{1}{x} = 0$ Absurdo

$\ln x = 0 \rightarrow \cancel{x = 1}$ No pertenece al segundo Trozo

x	y
~~0^+~~	~~$-\infty$~~
~~1~~	~~0~~
e	1
e^2	2



$$\text{Area} = \int_0^1 \left[1-(x-1)^2\right]dx + \int_1^e (1-\ln x)\,dx = (*)$$

Hagamos primero la integral no inmediata:

$\int \ln x$

$$\int \ln x\,dx = x\ln x - \int x\cdot\frac{1}{x}\,dx = x\ln x - \int 1\,dx = x\ln x - x$$

$$\boxed{\begin{array}{c|c} u=\ln x & du=\frac{1}{x}dx \\ dv=dx & v=x \end{array}}$$

$$(*) = \left[x - \frac{(x-1)^3}{3}\right]_0^1 + \left[x - x\ln x + x\right]_1^e = (1-0)-\left(0+\frac{1}{3}\right)+(2e-e)-(1-0+1) =$$

$$= 1-\frac{1}{3}+e-2 = \boxed{e-\frac{4}{3}\ u^2}$$

JUN 14 fase General

$$\int \frac{2x^3-3x^2-2x-1}{x^2-x-2}\,dx = \int\left(2x-1+\frac{x-3}{x^2-x-2}\right)dx =$$

$$= \int\left(2x-1+\frac{x-3}{(x-2)(x+1)}\right)dx = (*)$$

$$\begin{array}{l|l} 2x^3-3x^2-2x-1 & x^2-x-2 \\ \hline -2x^3+2x^2+4x & 2x-1 \\ \quad -x^2+2x-1 & \\ \quad\ x^2-x-2 & \\ \qquad\quad x-3 & \end{array}$$

$$x^2-x-2=0$$
$$x=\frac{1\pm\sqrt{1+8}}{2} = \frac{1\pm 3}{2} \begin{cases} 2 \\ -1 \end{cases}$$

$$\frac{x-3}{(x-2)(x+1)} = \frac{A}{x-2}+\frac{B}{x+1} = \frac{A(x+1)+B(x-2)}{(x-2)(x+1)}$$

$$x-3 = A(x+1)+B(x-2)$$

$x=-1 \rightarrow -4=-3B$; $B=4/3$

$x=2 \rightarrow -1=3A$; $A=-1/3$

$$(*) = \int\left(2x-1-\frac{1/3}{x-2}+\frac{4/3}{x+1}\right)dx = x^2-x-\frac{1}{3}\ln(x-2)+\frac{4}{3}\ln(x+1) =$$

$$= \boxed{x^2-x+\frac{1}{3}\ln\frac{(x+1)^4}{x-2}+C}$$

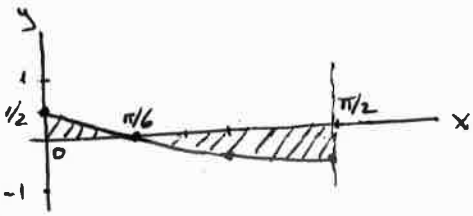
JUN 14 fase General

$$f(x) = \frac{1}{2}-\operatorname{sen}x$$

$y=0 \rightarrow \frac{1}{2}-\operatorname{sen}x=0$;

$\operatorname{sen}x = 1/2 \Rightarrow x=\pi/6$

	x	y
	0	1/2
	$\pi/6$	0
MIN	$\pi/2$	$-1/2$



$$\text{Area} = \int_0^{\pi/6}\left(\frac{1}{2}-\operatorname{sen}x\right)dx - \int_{\pi/6}^{\pi/2}\left(\frac{1}{2}-\operatorname{sen}x\right)dx = \left[\frac{x}{2}+\cos x\right]_0^{\pi/6} - \left[\frac{x}{2}+\cos x\right]_{\pi/6}^{\pi/2} =$$

$$= \left(\frac{\pi}{12}+\frac{\sqrt{3}}{2}\right)-(0+1)-\left(\frac{\pi}{4}+0\right)+\left(\frac{\pi}{12}+\frac{\sqrt{3}}{2}\right) =$$

$$= \boxed{\sqrt{3}-1-\frac{\pi}{12}\ u^2} \simeq 0'470\ u^2$$

JUN 14 fase Específica

$$\int\sqrt{x}\ln^2 x\,dx = \int x^{1/2}\ln^2 x\,dx = \frac{2}{3}x^{3/2}\ln^2 x - \frac{4}{3}\int \ln x\cdot\frac{x^{3/2}}{x}\,dx =$$

$$\boxed{\begin{array}{c|c} u=\ln^2 x & du=2\ln x\cdot\frac{1}{x}dx \\ dv=x^{1/2}dx & v=\frac{x^{3/2}}{3/2}=\frac{2}{3}x^{3/2} \end{array}}$$

$$= \frac{2}{3}\sqrt{x^3}\ln^2 x - \frac{4}{3}\int \ln x\cdot x^{1/2}\,dx = \frac{2}{3}\sqrt{x^3}\ln^2 x - \frac{4}{3}\left(\frac{2}{3}x^{3/2}\ln x - \frac{2}{3}\int\frac{x^{3/2}}{x}\,dx\right)$$

$$\boxed{\begin{array}{c|c} u=\ln x & du=\frac{1}{x}dx \\ dv=x^{1/2}dx & v=\frac{2}{3}x^{3/2} \end{array}}$$

JUN 14
fase Específica

$$\int \sqrt{x}\,\ln^2 x\,dx = \int x^{1/2}\cdot \ln^2 x\,dx = \frac{2}{3}x^{3/2}\ln^2 x - \frac{4}{3}\int \ln x\cdot\frac{x^{3/2}}{x}\,dx =$$

$$\boxed{\begin{array}{l|l} u=\ln^2 x & du = 2\ln x\cdot\frac{1}{x}\,dx \\ dv = x^{1/2}dx & v = \frac{x^{3/2}}{3/2} = \frac{2}{3}x^{3/2}\end{array}}$$

$$= \frac{2}{3}\sqrt{x^3}\ln^2 x - \frac{4}{3}\int \ln x\cdot x^{1/2}\,dx =$$

$$\boxed{\begin{array}{l|l} u=\ln x & du = \frac{1}{x}\,dx \\ dv = x^{1/2}dx & v = \frac{2}{3}x^{3/2}\end{array}}$$

$$= \frac{2}{3}\sqrt{x^3}\ln^2 x - \frac{4}{3}\left(\frac{2}{3}x^{3/2}\ln x - \frac{2}{3}\int\frac{x^{3/2}}{x}\,dx\right) =$$

$$= \frac{2}{3}\sqrt{x^3}\ln^2 x - \frac{8}{9}\sqrt{x^3}\ln x + \frac{8}{9}\int x^{1/2}\,dx =$$

$$= \frac{2}{3}\sqrt{x^3}\ln^2 x - \frac{8}{9}\sqrt{x^3}\ln x + \frac{8}{9}\cdot\frac{x^{3/2}}{3/2} =$$

$$= \frac{2}{3}\sqrt{x^3}\ln^2 x - \frac{8}{9}\sqrt{x^3}\ln x + \frac{16}{27}\sqrt{x^3} =$$

$$= \boxed{\frac{2}{27}\sqrt{x^3}\cdot\left(9\ln^2 x - 12\ln x + 8\right) + C}$$

JUL 14
fase General

$$\int \frac{x^3-3x+5}{\sqrt[3]{x}}\,dx = \int\left(x^{3-1/3} - 3x^{1-1/3} + 5x^{-1/3}\right)dx = \int\left(x^{8/3} - 3x^{2/3} + 5x^{-1/3}\right)dx =$$

$$= \frac{x^{11/3}}{11/3} - 3\,\frac{x^{5/3}}{5/3} + 5\,\frac{x^{2/3}}{2/3} =$$

$$= \frac{3}{11}\sqrt[3]{x^{11}} - \frac{9}{5}\sqrt[3]{x^5} + \frac{15}{2}\sqrt[3]{x^2} =$$

$$= \frac{3x^3}{11}\sqrt[3]{x^2} - \frac{9x}{5}\sqrt[3]{x^2} + \frac{15}{2}\sqrt[3]{x^2} = \boxed{\left(\frac{3x^3}{11} - \frac{9x}{5} + \frac{15}{2}\right)\sqrt[3]{x^2} + C}$$

JUL 14
fase General

$$f''(x) = xe^x \longrightarrow f'(x) = \int xe^x\,dx = xe^x - \int e^x\,dx = xe^x - e^x + C_1 \longrightarrow$$

$$\boxed{\begin{array}{l|l} u=x & du=dx \\ dv=e^x dx & v=e^x\end{array}}$$

integral hecha para f'

$$\longrightarrow f(x) = \int\left(xe^x - e^x + C_1\right)dx = xe^x - e^x - e^x + C_1 x + C_2 =$$

$$= \boxed{(x-2)e^x + C_1 x + C_2}$$

$A(0,2) \longrightarrow 2 = -2 + C_2 \longrightarrow C_2 = 4$

$B(2,0) \longrightarrow 0 = 2C_1 + C_2 \longrightarrow C_1 = -2$

$$\boxed{f(x) = (x-2)e^x - 2x + 4}$$

JUN 13
fase General

$y = x^3 - 3x^2 + 1 \longrightarrow y' = 3x^2 - 6x$

$y' = 0 \Rightarrow 3x^2 - 6x = 0$; $3x(x-2) = 0$; $x = 0,\ 2$

		0		2	
y'	+		−		+
y	↗	MAX	↘	MIN	↗

$x = 0 \longrightarrow y = 1$, $y' = 0$

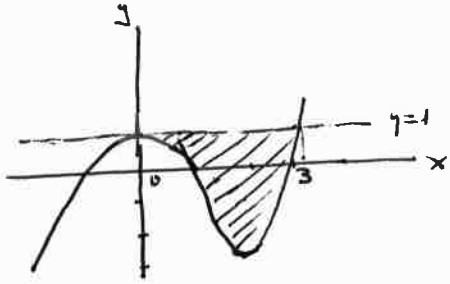
$y - 1 = 0\cdot(x - 0)$; $\boxed{y = 1}$

Recta Tangente

$$\left.\begin{array}{l} y = x^3 - 3x^2 + 1 \\ y = 1 \end{array}\right\} \quad x^3 - 3x^2 + 1 = 1 \; ; \quad x^3 - 3x^2 = 0 \; ; \quad x^2 \cdot (x-3) = 0 \; ; \quad x = \begin{cases} 0 \\ 3 \end{cases}$$

$y = x^3 - 3x^2 + 1$

	x	y
	$-\infty$	$-\infty$
MAX	0	1
MIN	2	-3
	3	1
	$+\infty$	$+\infty$



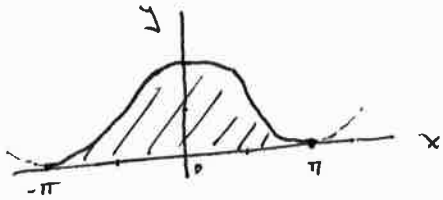
$$\text{Area} = \int_0^3 1 - (x^3 - 3x^2 + 1)\, dx = \int_0^3 (-x^3 + 3x^2)\, dx = \left[-\frac{x^4}{4} + x^3\right]_0^3 = -\frac{81}{4} + 27 = \boxed{\frac{27}{4}\, u^2}$$

JUL 14
fase Específica

$$\left.\begin{array}{l} y = 1 + \ln x \\ y = 0 \end{array}\right\} \quad 1 + \ln x = 0 \; ; \quad \ln x = -1 \; ; \quad x = \pi$$

Como $\ln x \geq -1 \quad \forall x \in \mathbb{R} \longrightarrow 1 + \ln x \geq 0$

$$\text{Area} = \int_{-\pi}^{\pi} (1 + \ln x)\, dx =$$



$$= 2\int_0^{\pi} (1 + \ln x)\, dx =$$

$$= \left[2(x + \operatorname{sen} x)\right]_0^{\pi} = 2(\pi + 0) - 2(0 + 0) = \boxed{2\pi\, u^2}$$

JUN 15
fase General

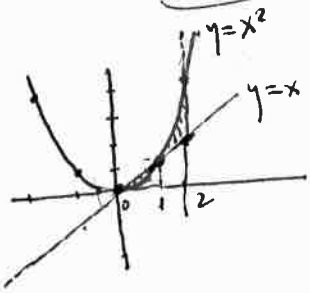
$$\left.\begin{array}{l} y = x^2 \\ y = x \end{array}\right\} \quad x^2 = x \; ; \quad x^2 - x = 0 \; ; \quad x(x-1) = 0 \; ; \quad x = \begin{cases} 0 \\ 1 \end{cases}$$

$y = x$

x	y
-1	-1
0	0
1	1
2	2

$y = x^2$

x	y
-1	1
0	0
1	1
2	4



$$\text{Area} = \int_0^1 (x - x^2)\, dx + \int_1^2 (x^2 - x)\, dx = \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 + \left[\frac{x^3}{3} - \frac{x^2}{2}\right]_1^2 =$$

$$= \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{8}{3} - 2\right) - \left(\frac{1}{3} - \frac{1}{2}\right) = \boxed{1\, u^2}$$

JUN 15
fase General

$$I=\int e^{2x+1}\cdot\cos x\,dx = e^{2x+1}\cdot\sin x-\int 2e^{2x+1}\cdot\sin x\,dx = e^{2x+1}\sin x+2e^{2x+1}\cos x-4\int e^{2x+1}\cos x\,dx$$

$$\boxed{\begin{array}{l|l} u=e^{2x+1} & du=2e^{2x+1}dx \\ dv=\cos x\,dx & v=\sin x \end{array}} \qquad \boxed{\begin{array}{l|l} u=2e^{2x+1} & du=4e^{2x+1}dx \\ dv=\sin x\,dx & v=-\cos x \end{array}}$$

$$I=e^{2x+1}\cdot\sin x+2e^{2x+1}\cos x-4I$$

$$5I=e^{2x+1}(\sin x+2\cos x) \Rightarrow I=\boxed{\frac{1}{5}e^{2x+1}(\sin x+2\cos x)}$$

JUN 15
fase General

$$\int\frac{\ln x}{\sqrt{x}}dx=\int\ln x\cdot x^{-1/2}dx=2\ln x\cdot x^{1/2}-2\int\frac{x^{1/2}}{x}dx=2\ln x\sqrt{x}-2\int x^{-1/2}dx=$$

$$\boxed{\begin{array}{l|l} u=\ln x & du=\frac{1}{x}dx \\ dv=x^{-1/2}dx & v=\dfrac{x^{-1/2+1}}{-\frac{1}{2}+1}=2x^{1/2} \end{array}}$$

$$=2\ln x\sqrt{x}-2\frac{x^{-1/2+1}}{-1/2+1}=2\ln x\sqrt{x}-4x^{1/2}=\boxed{2\sqrt{x}(\ln x-2)+C}$$

JUL 15
fase General

$$\left.\begin{array}{l} y=4x-x^2 \\ y=x^2-6 \end{array}\right\} \quad x^2-6=4x-x^2\ ;\quad 2x^2-4x-6=0\ ;\quad x^2-2x-3=0$$

$$x=\frac{2\pm\sqrt{4+12}}{2}=\frac{2\pm4}{2}=\left\{\begin{array}{l}3\\-1\end{array}\right.$$

$y=4x-x^2 \longrightarrow y'=4-2x$

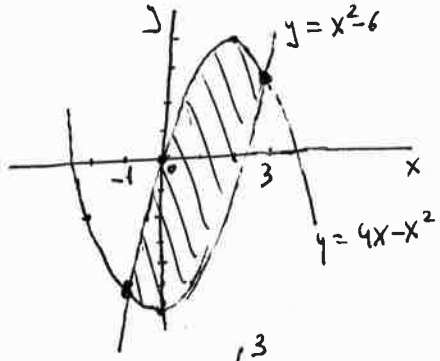
$y'=0 \longrightarrow x=2$

x	y
-1	-5
0	0
2	4
3	3
4	0

$y=x^2-6 \longrightarrow y'=2x$

$y'=0 \longrightarrow x=0$

x	y
-2	-2
-1	-5
0	-6
1	-5
3	3



$$\text{Area}=\int_{-1}^{3}(4x-x^2)-(x^2-6)\,dx=\int_{-1}^{3}(-2x^2+4x+6)\,dx=\left[-\frac{2x^3}{3}+2x^2+6x\right]_{-1}^{3}=$$

$$=(-18+18+18)-\left(\frac{2}{3}+2-6\right)=\boxed{\frac{64}{3}u^2}\simeq 21'3\ u^2$$

JUL 15
fase General

$f'(x) = (x+1)(x^2-4) = x^3+x^2-4x-4$

a) $f(x) = \int (x^3+x^2-4x-4)\,dx = \frac{x^4}{4} + \frac{x^3}{3} - 2x^2 - 4x + C$

$f(0) = \frac{1}{7} \Rightarrow C = 1/7 \longrightarrow \boxed{f(x) = \frac{x^4}{4} + \frac{x^3}{3} - 2x^2 - 4x + \frac{1}{7}}$ $\quad \text{dom} f = \mathbb{R}$

b) $f'(x) = 0 \Rightarrow (x+1)(x^2-4) = 0 \begin{cases} x+1=0 \;;\; x=-1 \\ x^2-4=0 \;;\; x=\pm 2 \end{cases}$

		-2		-1		2	
f'	$-$		$+$		$-$		$+$
f	$\searrow$	MIN	$\nearrow$	MAX	$\searrow$	MIN	$\nearrow$

f es creciente en $(-2,-1) \cup (2,+\infty)$

f es decreciente en $(-\infty,-2) \cup (-1,2)$

f tiene mínimos locales en $x=-2$, $x=2$

f tiene máximo local en $x=-1$

NOTA: Como $\lim_{x\to-\infty} f(x) = +\infty$, $\lim_{x\to+\infty} f(x) = +\infty$ $\Big| \rightarrow$ los extremos locales no son absolutos.

JUL 15
fase Específica

$\left.\begin{array}{l} y = x^2-4x+13 \\ y = 2x^2-8x+16 \end{array}\right\} \rightarrow 2x^2-8x+16 = x^2-4x+13 \;;\; x^2-4x+3=0$

$x = \frac{4 \pm \sqrt{16-12}}{2} = \frac{4\pm 2}{2} = \begin{cases} 3 \\ 1 \end{cases}$

$y = x^2-4x+13 \rightarrow y' = 2x-4$, $y'=0 \rightarrow x=2$

x	y
0	13
1	10
2	9
3	10
4	13

V

$y = 2x^2-8x+16 \rightarrow y' = 4x-8$, $y'=0 \rightarrow x=2$

x	y
0	16
1	10
2	8
3	10
4	16

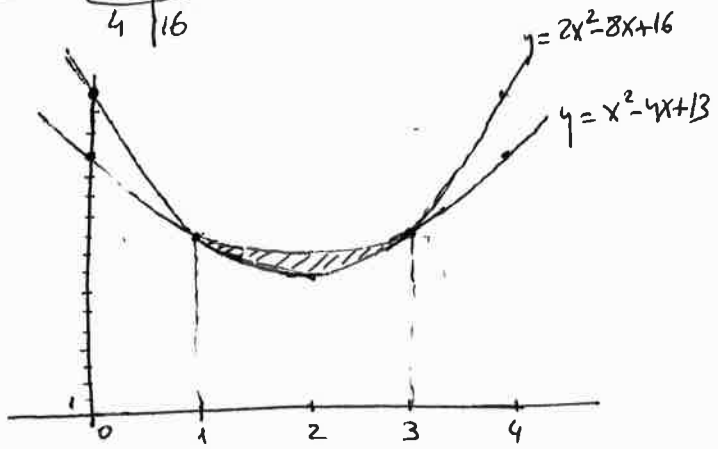
V

$\text{Area} = \int_1^3 (x^2-4x+13) - (2x^2-8x+16)\,dx =$

$= \int_1^3 (-x^2+4x-3)\,dx =$

$= \left[-\frac{x^3}{3} + 2x^2 - 3x\right]_1^3 =$

$= (-9+18-9) - \left(-\frac{1}{3}+2-3\right) = \boxed{\frac{4}{3}u^2}$



JUN16 fase General

$$\left.\begin{array}{l} f(x)=e^x \\ g(x)=e^{-x} \end{array}\right\} \rightarrow e^x=e^{-x}\ ;\ e^x=\frac{1}{e^x}\ ;\ (e^x)^2=1\ ;\ e^{2x}=1\ ;\ 2x=0\ ;\ x=0$$

$$\left.\begin{array}{l} f(x)=e^x \\ h(x)=e^2 \end{array}\right\} \rightarrow e^x=e^2\ ;\ x=2$$

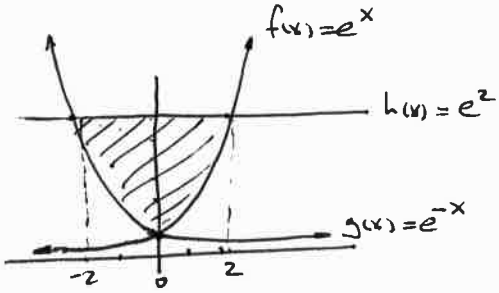
$$\left.\begin{array}{l} g(x)=e^{-x} \\ h(x)=e^2 \end{array}\right\} \rightarrow e^{-x}=e^2\ ;\ x=-2$$

$f(x)=e^x$

x	y
$-\infty$	0
0	1
2	e^2
$+\infty$	$+\infty$

$g(x)=e^{-x}$

x	y
$-\infty$	$+\infty$
0	1
-2	e^2
$+\infty$	0



$$\text{Area} = \int_{-2}^{0}(e^2-e^{-x})\,dx + \int_{0}^{2}(e^2-e^x)\,dx \underset{\text{por simetría}}{=} 2\int_{0}^{2}(e^2-e^x)\,dx =$$

$$= 2\left[e^2x-e^x\right]_0^2 = 2(2e^2-e^2)-2(0-1) = \boxed{2e^2+2}$$

JUN16 fase General

$$f''(x)=e^x-\frac{1}{x^2}=e^x-x^{-2}$$

$$f'(x)=\int(e^x-x^{-2})\,dx = e^x-\frac{x^{-1}}{-1} = e^x+\frac{1}{x}+c$$

$$f'(1)=e+2 \Rightarrow \cancel{e}+2=\cancel{e}+\frac{1}{1}+c\ ;\ c=1 \Rightarrow f'(x)=e^x+\frac{1}{x}+1$$

$$f(x)=\int\left(e^x+\frac{1}{x}+1\right)dx = e^x+\ln x+x+c$$

$$f(1)=e+2 \Rightarrow \cancel{e}+2=\cancel{e}+\ln 1+1+c\ ;\ c=1 \Rightarrow \boxed{f(x)=e^x+\ln x+x+1}$$

JUN16 fase Específica

$$\int_0^2 \frac{x^2+15x-16}{1-x^2}\,dx =$$

$$\begin{array}{r|l} x^2+15x-16 & -x^2+1 \\ \hline -x^2\quad\ +1 & -1 \\ \hline 15x-15 & \end{array}$$

$$= \int_0^2\left[-1+\frac{15x-15}{1-x^2}\right]dx = \int_0^2\left[-1+\frac{15\cancel{(x-1)}}{-\cancel{(x-1)}(x+1)}\right]dx = \int_0^2\left(-1-\frac{15}{x+1}\right)dx =$$

$$= \left[-x-15\ln(x+1)\right]_0^2 = (-2-15\ln 3)-(-0-15\ln 1) = \boxed{-2-15\ln 3}$$

JUN16 fase Específica

a) $f'(x)=2x \rightarrow f(x)=\int 2x\,dx = x^2+c$

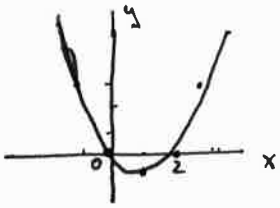
$$f(-3)=7 \Rightarrow 7=(-3)^2+c\ ;\ c=-2 \Rightarrow \boxed{f(x)=x^2-2x}$$

b) $y=x^2-2x$

$y'=2x-2$

$y'=0 \rightarrow x=1$

		1	
y'	$-$		$+$
y	$\searrow$	MIN	$\nearrow$



x	y
-1	3
0	0
1	-1
2	0
3	3

$y''=2$

$y''>0 \Rightarrow$ Cóncava

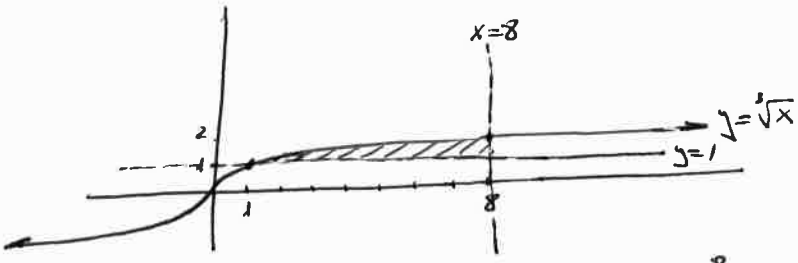
JUL 16
fase Específica

$$\begin{cases} y=\sqrt[3]{x} \\ x=8 \end{cases} \quad y=\sqrt[3]{8}=2 \qquad P(8,2)$$

$$\begin{cases} y=\sqrt[3]{x} \\ y=1 \end{cases} \quad 1=\sqrt[3]{x}\ ; \quad x=1^3=1 \qquad Q(1,1)$$

$y=\sqrt[3]{x}$

x	y
$-\infty$	$-\infty$
0	0
1	1
8	2
$+\infty$	$+\infty$



$$\text{Area} = \int_1^8 (\sqrt[3]{x}-1)\,dx = \int_1^8 (x^{1/3}-1)\,dx = \left[\frac{x^{4/3}}{4/3}-x\right]_1^8 = \left[\frac{3\sqrt[3]{x^4}}{4}-x\right]_1^8 =$$

$$= \left(\frac{3\sqrt[3]{8^4}}{4}-8\right)-\left(\frac{3\sqrt[3]{1^4}}{4}-1\right) = (12-8)-\left(\frac{3}{4}-1\right) = 5-\frac{3}{4} = \boxed{\frac{17}{4}\,u^2}$$

JUL 16
fase General

$$\int_1^2 \frac{4x^2+8x+1}{x^2+2x}\,dx = \int_1^2 \left(4+\frac{1}{x^2+2x}\right)dx = (*)$$

$$\begin{array}{l|l} 4x^2+8x+1 & x^2+2x \\ \hline -4x^2-8x & 4 \\ \hline \quad 1 & \end{array}$$

$$\frac{1}{x^2+2x} = \frac{1}{x(x+2)} = \frac{A}{x}+\frac{B}{x+2}$$

$$1 = A(x+2)+Bx$$

$$x=0:\ 1 = 2A+0\ ;\ A=1/2$$

$$x=-2:\ 1 = 0-2B\ ;\ B=-1/2$$

$$(*) = \int_1^2 \left(4+\frac{1/2}{x}-\frac{1/2}{x+2}\right)dx = \left[4x+\frac{1}{2}\ln x-\frac{1}{2}\ln(x+2)\right]_1^2 =$$

$$= \left(8+\frac{1}{2}\ln 2-\frac{1}{2}\ln 4\right)-\left(4+\frac{1}{2}\ln 1-\frac{1}{2}\ln 3\right) = 4+\frac{1}{2}(\ln 2-\ln 4+\ln 3) =$$

$$= 4+\frac{1}{2}\ln\frac{2\cdot 3}{4} = \boxed{4+\frac{1}{2}\ln\left(\frac{3}{2}\right)} = \boxed{4+\ln\sqrt{\frac{3}{2}}}$$

JUL 16
fase Específica

$$\int (x+1)^2\ln(3x)\,dx = \frac{(x+1)^3\ln(3x)}{3} - \int \frac{1}{x}\,\frac{(x+1)^3}{3}\,dx = \frac{1}{3}(x+1)^3\ln(3x) - \frac{1}{3}\int \frac{x^3+3x^2+3x+1}{x}\,dx$$

$$\begin{array}{l|l} u=\ln(3x) & du = \cancel{3}\cdot\frac{1}{\cancel{3}x}\,dx \\ dv=(x+1)^2dx & v=\frac{(x+1)^3}{3} \end{array}$$

$$= \frac{1}{3}(x+1)^3\ln(3x) - \frac{1}{3}\int\left(x^2+3x+3+\frac{1}{x}\right)dx =$$

$$= \frac{1}{3}(x+1)^3\ln(3x) - \frac{1}{3}\left(\frac{x^3}{3}+\frac{3x^2}{2}+3x+\ln x\right) =$$

$$= \boxed{\frac{1}{18}\left[6(x+1)^3\ln(3x)-2x^3-27x^2-54x+18\ln x\right]+C}$$

JUL 16
fase Específica

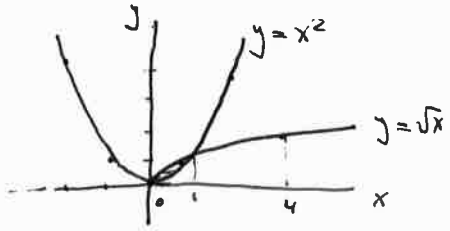
$$\begin{cases} y=\sqrt{x} \\ y=x^2 \end{cases} \quad \sqrt{x}=x^2 \;;\; x=x^4 \;;\; 0=x^4-x \;;\; 0=x(x^3-1) \begin{cases} \to x=0 \\ \to x^3-1=0;\; x=1 \end{cases}$$

$y=\sqrt{x}$

x	y
0	0
1	1
4	2

$y=x^2$

x	y
-2	4
-1	1
0	0
1	1
2	4



$$\text{Area} = \int_0^1 (\sqrt{x}-x^2)\,dx = \int_0^1 (x^{1/2}-x^2)\,dx = \left[\frac{x^{3/2}}{3/2} - \frac{x^3}{3}\right]_0^1 =$$

$$= \left[\frac{2\sqrt{x^3}}{3} - \frac{x^3}{3}\right]_0^1 = \left(\frac{2}{3}-\frac{1}{3}\right)-(0-0) = \boxed{\frac{1}{3}\ u^2}$$

Modelo 17

a) $y=x^3-3x$ dominio $=\mathbb{R}$

$y'=3x^2-3$

$y'=0 \to 3x^2-3=0 \;;\; x^2=1 \;;\; x=\pm 1$

		-1		1	
y'	+		−		+
y	↗	MAX	↘	MIN	↗

$y(x)$ es creciente en $(-\infty,-1)\cup(1,+\infty)$

$y(x)$ " decreciente en $(-1,1)$

$y(x)$ tiene un máximo local en $x=-1$

$y(x)$ " " mínimo " " $x=1$

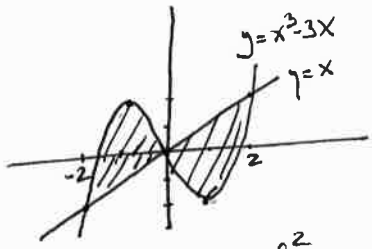
b) $\begin{cases} y=x^3-3x \\ y=x \end{cases} \quad x^3-3x=x \;;\; x^3-4x=0 \;;\; x(x^2-4)=0 \;;\; x=\begin{cases} 0 \\ \pm 2 \end{cases}$

$y=x^3-3x$

x	y
-2	-2
-1	2
0	0
1	-2
2	2

$y=x$

x	y
-2	-2
0	0
2	2



$$\text{Area} = \int_{-2}^{0} (x^3-3x)-x\,dx + \int_0^2 x-(x^3-3x)\,dx \underset{\text{por simetría}}{=} 2\int_0^2 x-(x^3-3x)\,dx =$$

$$= 2\int_0^2 (x-x^3+3x)\,dx = 2\int_0^2 (4x-x^3)\,dx = 2\left[2x^2-\frac{x^4}{4}\right]_0^2 =$$

$$= 2\left(8-\frac{16}{4}\right) - 2(0-0) = \boxed{8\ u^2}$$