JUN 94

i) rango = nº de filas (o columnas) de una matriz que resulten ser linealmente independientes

ii) rango $\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = 3 \Rightarrow \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} \neq 0$

Si eliminamos por ejemplo la 1ª columna, quedaría $\begin{pmatrix} b & c \\ e & f \\ h & i \end{pmatrix}$.

Si rango $\begin{pmatrix} b & c \\ e & f \\ h & i \end{pmatrix} \leq 1 \Rightarrow \begin{matrix} \begin{vmatrix} b & c \\ e & f \end{vmatrix} = 0 \\ \begin{vmatrix} b & c \\ h & i \end{vmatrix} = 0 \\ \begin{vmatrix} e & f \\ h & i \end{vmatrix} = 0 \end{matrix} \Rightarrow \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a \cdot \begin{vmatrix} e & f \\ h & i \end{vmatrix} - d \begin{vmatrix} b & c \\ h & i \end{vmatrix} + g \cdot \begin{vmatrix} b & c \\ e & f \end{vmatrix} = 0$ Absurdo.

Luego: $\boxed{\text{rango} \begin{pmatrix} b & c \\ e & f \\ h & i \end{pmatrix} = 2}$

Si eliminamos por ejemplo la 1ª fila y la 1ª columna: $\begin{pmatrix} e & f \\ h & i \end{pmatrix}$

Podría ser de rango 1: Por ejemplo, $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 1 & 1 \\ 5 & 1 & 1 \end{pmatrix}$ tiene rango 3

mientras que $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ tiene rango 1.

SEPT 94

$\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 1-x & 1 & 1 \\ 1 & 1 & 2-x & 1 \\ 1 & 1 & 1 & 3-x \end{vmatrix}$ contiene sólo tres veces la variable x y se multiplican entre sí sólo una vez al no repetir ni fila ni columna, por lo que $1 \cdot (1-x)(2-x)(3-x)$ daría un término en x^3 que no podría simplificarse con ningún otro, luego su grado es **3**. La ecuación $f(x)=0$ tendrá entonces 3 soluciones

Vemos que para **x=0** se igualan las dos primeras filas, dando el determinante nulo, por lo que cumpliría $f(x)=0$

Lo mismo sucede con $x=1$ (1ª fila = 3ª fila)

y con $x=2$ (1ª fila = 4ª fila)

Por lo tanto las 3 soluciones son $\boxed{\begin{matrix} x=0 \\ x=1 \\ x=2 \end{matrix}}$

JUN 95

a) $A \cdot C \cdot B$

$(m \times n) \cdot (q \times r) \cdot (n \times p) = (m \times p)$ es el orden del resultado (de poder hacerse

Debe cumplirse $\left.\begin{matrix} n=q \\ r=n \end{matrix}\right\} \rightarrow \boxed{n=q=r}$

b) $A \cdot (B + C)$

$(m \times n) \cdot [(n \times p) + (q \times r)] = \begin{matrix} (m \times p) \\ (m \times r) \end{matrix}$ es el orden del resultado (de poder hacerse

Debe cumplirse: $\boxed{\begin{matrix} n=q \\ p=r \end{matrix}}$

c] $\begin{pmatrix}3&4\\1&2\end{pmatrix}A=\begin{pmatrix}3&4\\1&2\end{pmatrix}B \Rightarrow A=\begin{pmatrix}3&4\\1&2\end{pmatrix}^{-1}\cdot\begin{pmatrix}3&4\\1&2\end{pmatrix}B = IB = B \Rightarrow \boxed{A=B}$

$\begin{vmatrix}3&4\\1&2\end{vmatrix}=6-4=2\neq 0$, por lo que existe su inversa.

JUN 95] i] rango = nº filas (o columnas) que forman un conjunto linealmente independ

conjunto linealmente independiente (de filas) = ninguna fila se puede poner como una combinación lineal de las restantes filas del conjunto.

Combinación lineal (de filas) = suma de filas multiplicadas por constantes

ii] $\begin{vmatrix}a&b&c\\d&e&f\\g&h&i\end{vmatrix}=0$ y existe algún menor de orden 2 que es $\neq 0$.
es decir que las 3 columnas no son proporcionales.

$$\begin{vmatrix}a&b&c+a\\d&e&f+d\\g&h&i+g\end{vmatrix}=\begin{vmatrix}a&b&c\\d&e&f\\g&h&i\end{vmatrix}+\begin{vmatrix}a&b&a\\d&e&d\\g&h&g\end{vmatrix}=0+0=0$$

columnas iguales

Luego el rango de $\begin{pmatrix}a&b&c+a\\d&e&f+d\\g&h&i+g\end{pmatrix}$ no será 3

Veamos que no puede ser menor de 2.

Si no alcanza el rango es que sus menores de orden 2 son todos nulos: $\begin{vmatrix}a&b\\d&e\end{vmatrix}=0$, $\begin{vmatrix}a&b\\g&h\end{vmatrix}=0$, $\begin{vmatrix}d&e\\g&h\end{vmatrix}=0$, $\begin{vmatrix}a&c+a\\d&f+d\end{vmatrix}=0$, $\begin{vmatrix}a&c+a\\g&i+g\end{vmatrix}=0$,

$\begin{vmatrix}d&f+d\\g&i+g\end{vmatrix}=0$, $\begin{vmatrix}b&c+a\\e&f+d\end{vmatrix}=0$, $\begin{vmatrix}b&c+a\\h&i+g\end{vmatrix}=0$, $\begin{vmatrix}e&f+d\\h&i+g\end{vmatrix}=0$

Veremos como esto implica que todos los menores de la matriz primitiva deben ser nulos.

- Los 3 de las dos primeras columnas son comunes.
- Los de la 1ª y 3ª columnas:

$$\begin{vmatrix}a&c+a\\d&f+d\end{vmatrix}=0 \Rightarrow \begin{vmatrix}a&c\\d&f\end{vmatrix}+\begin{vmatrix}a&a\\d&d\end{vmatrix}=0 \Rightarrow \begin{vmatrix}a&c\\d&f\end{vmatrix}+0=0 \Rightarrow \begin{vmatrix}a&c\\d&f\end{vmatrix}=0$$

Igual se haría con los otros dos.

- Los de la 2ª y 3ª columnas:

$$\left.\begin{matrix}\begin{vmatrix}b&c+a\\e&f+d\end{vmatrix}=0\\ \begin{vmatrix}a&b\\d&e\end{vmatrix}=0\end{matrix}\right\} \Rightarrow 0=\begin{vmatrix}b&c\\e&f\end{vmatrix}+\begin{vmatrix}b&a\\e&d\end{vmatrix}=\begin{vmatrix}b&c\\e&f\end{vmatrix}-0=\begin{vmatrix}b&c\\e&f\end{vmatrix}$$

Igual se haría con los otros dos.

En definitiva, que la matriz primitiva no tendría rango 2,

por lo tanto $\Rightarrow$ $\boxed{\text{rango}\begin{pmatrix}a&b&c+a\\d&e&f+d\\g&h&i+g\end{pmatrix}=\text{rango}\begin{pmatrix}a&b&c\\d&e&f\\g&h&i\end{pmatrix}=2}$

JUN 96

i)

$$A=\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$B=(2^{i-j}\cdot a_{ij})=\begin{pmatrix} a_{11} & \frac{1}{2}a_{12} & \frac{1}{4}a_{13} \\ 2a_{21} & a_{22} & \frac{1}{2}a_{23} \\ 4a_{31} & 2a_{32} & a_{33} \end{pmatrix}$$

$$|B|=\begin{vmatrix} a_{11} & \frac{1}{2}a_{12} & \frac{1}{4}a_{13} \\ 2a_{21} & a_{22} & \frac{1}{2}a_{23} \\ 4a_{31} & 2a_{32} & a_{33} \end{vmatrix} = 2\begin{vmatrix} \frac{1}{2}a_{11} & \frac{1}{2}a_{12} & \frac{1}{4}a_{13} \\ a_{21} & a_{22} & \frac{1}{2}a_{23} \\ 2a_{31} & 2a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} \frac{1}{2}a_{11} & \frac{1}{2}a_{12} & \frac{1}{2}a_{13} \\ a_{21} & a_{22} & a_{23} \\ 2a_{31} & 2a_{32} & 2a_{33} \end{vmatrix} =$$

$1/2$ $\quad$ 2

$$= 2\cdot\frac{1}{2}\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = |A|$$

ii)

$$A=(a_{ij})=(i+j)=\begin{pmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{pmatrix}$$

$$|A|=\begin{vmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{vmatrix}=0 \Rightarrow \boxed{\nexists\ A^{-1}}$$

SEPT 96

$$\begin{vmatrix} x & 1 & 1 & 1 \\ 1 & x & 1 & 1 \\ 1 & 1 & x & 1 \\ 1 & 1 & 1 & x \end{vmatrix} = \begin{vmatrix} x & 1 & 1 & 1 \\ 1-x & x-1 & 0 & 0 \\ 1-x & 0 & x-1 & 0 \\ 1-x^2 & 1-x & 1-x & 0 \end{vmatrix} = -1\cdot\begin{vmatrix} 1-x & x-1 & 0 \\ 1-x & 0 & x-1 \\ 1-x^2 & 1-x & 1-x \end{vmatrix} = -\begin{vmatrix} -(x-1) & x-1 & 0 \\ -(x-1) & 0 & x-1 \\ -(x+1)(x-1) & -(x-1) & -(x-1) \end{vmatrix}$$

$$\boxed{\begin{array}{l} F2 \to F2-F1 \\ F3 \to F3-F1 \\ F4 \to F4-xF1 \end{array}}$$

$$= -(x-1)^3\begin{vmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ -(x+1) & -1 & -1 \end{vmatrix} = (x-1)^3\begin{vmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ x+1 & -1 & -1 \end{vmatrix} = (x-1)^3\begin{vmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ x+2 & -1 & 0 \end{vmatrix} = -(x-1)^3\begin{vmatrix} 1 & 1 \\ x+2 & -1 \end{vmatrix} =$$

$$\boxed{F3 \to F3+F2}$$

$$= -(x-1)^3\begin{vmatrix} 1 & 1 \\ x+3 & 0 \end{vmatrix} = +(x-1)^3(x+3) = (x-1)^3(x+3)$$

$$\boxed{F2 \to F2+F1}$$

$$(x-1)^3(x+3)=0 \Rightarrow \boxed{\begin{array}{l} x=+1 \text{ (raíz triple)} \\ x=-3 \end{array}}$$

SEPT 96

$$M=\begin{pmatrix} a & 1 & 1 & 1 \\ 1 & a & 1 & a \\ 1 & 1 & a & a^2 \end{pmatrix}$$

$$\begin{vmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & a \end{vmatrix} = a^3+1+1-a-a-a = a^3-3a+2$$

	1	0	-3	2
1		1	1	-2
	1	1	-2	0
1		1	2	
	1	2	0	
-2		-2		
	1	0		

- Si $\left.\begin{array}{l} a\neq 1 \\ a\neq -2 \end{array}\right| \Rightarrow \boxed{\text{rango } M=3}$

• Si $a=1 \Rightarrow M=\begin{pmatrix}1&1&1&1\\1&1&1&1\\1&1&1&1\end{pmatrix} \longrightarrow \boxed{\text{rango } M=1}$

• Si $a=-2 \Rightarrow M=\begin{pmatrix}-2&1&1&1\\1&-2&1&-2\\1&1&-2&4\end{pmatrix}$

$\begin{vmatrix}-2&1\\1&-2\end{vmatrix}=4-1=3 \Rightarrow$ rango $M\geq 2$

$\begin{vmatrix}-2&1&1\\1&-2&1\\1&1&-2\end{vmatrix}=0$

$\begin{vmatrix}-2&1&1\\1&-2&-2\\1&1&4\end{vmatrix}=16+1-\cancel{2}+\cancel{2}-4-4=9 \Rightarrow \boxed{\text{rango } M=3}$

Resumen: El rango es 3 para cualquier valor de a salvo para a=1 en el que el rango es 1.

JUN 97 i)

$$\begin{pmatrix}1&7&8\\3&1&K\end{pmatrix}=P\cdot\begin{pmatrix}1&2&3\\2&-1&1\end{pmatrix}$$

$$(2\times3)=(2\times2)\cdot(2\times3)$$

P tendría que ser una matriz (2×2)

$$\begin{pmatrix}1&7&8\\3&1&K\end{pmatrix}=\begin{pmatrix}a&b\\c&d\end{pmatrix}\begin{pmatrix}1&2&3\\2&-1&1\end{pmatrix}=\begin{pmatrix}a+2b&2a-b&3a+b\\c+2d&2c-d&3c+d\end{pmatrix}$$

$\left.\begin{matrix}a+2b=1\\2a-b=7\\3a+b=8\end{matrix}\right\}$ $\begin{vmatrix}1&2&1\\2&-1&7\\3&1&8\end{vmatrix}=0 \Rightarrow$ rango $A^{+}=2$

$$a=\frac{\begin{vmatrix}1&2\\7&-1\end{vmatrix}}{\begin{vmatrix}1&2\\2&-1\end{vmatrix}}=\frac{-1-14}{-1-4}=\frac{-15}{-5}=3$$

$$b=\frac{\begin{vmatrix}1&1\\2&7\end{vmatrix}}{\begin{vmatrix}1&2\\2&-1\end{vmatrix}}=\frac{5}{-5}=-1$$

$\left.\begin{matrix}c+2d=3\\2c-d=1\\3c+d=K\end{matrix}\right\} \longrightarrow$ $c=\frac{\begin{vmatrix}3&2\\1&-1\end{vmatrix}}{\begin{vmatrix}1&2\\2&-1\end{vmatrix}}=\frac{-5}{-5}=1$, $d=\frac{\begin{vmatrix}1&3\\2&1\end{vmatrix}}{-5}=\frac{-5}{-5}=1$ $\Rightarrow K=3c+d=3\cdot1+1=4$

$\boxed{\text{K debe ser 4, y la matriz } P=\begin{pmatrix}3&-1\\1&1\end{pmatrix}}$

ii)

$$M\cdot N^t=\begin{pmatrix}1&2&3\\2&-1&1\end{pmatrix}\begin{pmatrix}1&3\\7&1\\8&K\end{pmatrix}=\begin{pmatrix}39&3K+5\\3&K+5\end{pmatrix}$$

Tiene sentido hablar de la inversa de $M\cdot N^t$ puesto que es cuadrada, pero podría no existir si su determinante fuese nulo:

$$\begin{vmatrix}39&3K+5\\3&K+5\end{vmatrix}=39K+195-9K-15=30K+180$$

$30K+180=0 \Rightarrow \boxed{K=-6}$

Para $K\neq-6$, existe $(M\cdot N^t)^{-1}$.

Para $K=0 \Rightarrow M\cdot N^t=\begin{pmatrix}39&5\\3&5\end{pmatrix} \longrightarrow \boxed{(MN^t)^{-1}=\begin{pmatrix}\frac{5}{180}&-\frac{5}{180}\\-\frac{3}{180}&\frac{39}{180}\end{pmatrix}}$

SEPT 97

i) $A^2 = I$, la única solución no tiene porqué ser I, por ejemplo: $A = -I$, $(-I)\cdot(-I) = +I$ ✓

NOTA: para matrices 2x2 se puede estudiar todas sus soluciones:

$$A^2 = I \Rightarrow |A^2| = |I| \Rightarrow |A||A| = 1 \Rightarrow |A|^2 = 1 \Rightarrow |A| = \pm 1$$

$$\Rightarrow A = A^{-1}$$

Caso $|A| = 1$:

$$\left.\begin{array}{l} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} \\ |A| = 1 \end{array}\right\} \Rightarrow \left.\begin{array}{l} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \\ |A| = 1 \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} a = d \\ b = -b \\ c = -c \\ d = a \\ ad - bc = 1 \end{array}\right\} \Rightarrow \left.\begin{array}{l} \boxed{b = 0} \\ \boxed{c = 0} \\ a = d \\ ad = 1 \end{array}\right\} \rightarrow a^2 = 1 \Rightarrow \boxed{a = \begin{cases} 1 \Rightarrow d = 1 \\ -1 \Rightarrow d = -1 \end{cases}}$$

Hay dos soluciones: $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

Caso $|A| = -1$:

$$\left.\begin{array}{l} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} \\ |A| = -1 \end{array}\right\} \Rightarrow \left.\begin{array}{l} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} -d & b \\ c & -a \end{pmatrix} \\ |A| = 1 \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} a = -d \\ b = b \\ c = c \\ d = -a \\ ad - bc = -1 \end{array}\right\} \Rightarrow \left.\begin{array}{l} a = -d \\ ad - bc = -1 \\ \boxed{\begin{array}{l} b \in \mathbb{R} \\ c \in \mathbb{R} \end{array}} \end{array}\right\} \rightarrow \begin{array}{l} -d^2 - bc = -1 \\ d^2 + bc = 1 \\ d = \boxed{\pm\sqrt{1-bc}} \rightarrow a = \boxed{\mp\sqrt{1-bc}} \end{array}$$

Hay infinitas soluciones del tipo: $\begin{pmatrix} \sqrt{1-bc} & b \\ c & -\sqrt{1-bc} \end{pmatrix}$, $\begin{pmatrix} -\sqrt{1-bc} & b \\ c & \sqrt{1-bc} \end{pmatrix}$

ii) $$A^3 = I \Rightarrow \left.\begin{array}{l} A \cdot A^2 = I \\ A^2 \cdot A = I \end{array}\right\} \Rightarrow \boxed{A^{-1} = A^2}$$

iii) $$\left.\begin{array}{l} (1)\ AB = A \\ (2)\ BA = B \end{array}\right\} \Rightarrow A^2 = (AB)\cdot(AB) = A\underbrace{(BA)}_{(2)}B = \overbrace{(AB)}^{(1)}B = \underbrace{AB}_{(1)} = A$$

JUN 98

$$X \cdot \begin{pmatrix} 2 & 1 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}$$

$(3\times2)\cdot(2\times2) = (3\times2)$ X es (3×2)

$$X = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix} \cdot \begin{pmatrix} 2 & 1 \\ 3 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix} \cdot \begin{pmatrix} -1 & 1 \\ 3 & -2 \end{pmatrix} = \boxed{\begin{pmatrix} 5 & -3 \\ 9 & -5 \\ 13 & -7 \end{pmatrix}}$$

La solución es única porque $\begin{pmatrix} 2 & 1 \\ 3 & 1 \end{pmatrix}$ tiene inversa.

SEPT 98 | i) M triangular superior $\iff a_{ij}=0 \quad \forall i>j$

$$\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ 0 & a_{22} & \cdots & a_{2m} \\ 0 & 0 & \ddots & \vdots \\ \vdots & \vdots & & \\ 0 & 0 & \cdots & a_{mm} \end{vmatrix} = a_{11}\, a_{22} \cdots a_{mm}$$

ii) $M=\begin{pmatrix} a_{11} & a_{12} \\ 0 & a_{22}\end{pmatrix}$

$$M^2=I \Rightarrow \begin{pmatrix} a_{11} & a_{12} \\ 0 & a_{22}\end{pmatrix}\begin{pmatrix} a_{11} & a_{12} \\ 0 & a_{22}\end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1\end{pmatrix} \Rightarrow \left.\begin{array}{l} a_{11}^2=1 \\ a_{11}a_{12}+a_{12}a_{22}=0 \\ 0=0 \\ a_{22}^2=1 \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} a_{11}=\pm 1 \\ a_{22}=\pm 1 \\ a_{12}\cdot(a_{11}+a_{22})=0 \end{array}\right\}$$

- $\boxed{\begin{array}{l} a_{11}=1 \\ a_{22}=1 \end{array}}$ $\quad 2a_{12}=0 \rightarrow \boxed{a_{12}=0}$ $\qquad M=\begin{pmatrix} 1 & 0 \\ 0 & 1\end{pmatrix}$
- $\boxed{\begin{array}{l} a_{11}=1 \\ a_{22}=-1 \end{array}}$ $\quad a_{12}\cdot 0=0 \rightarrow \boxed{a_{12}\in\mathbb{R}}$ $\qquad M=\begin{pmatrix} 1 & a_{12} \\ 0 & -1\end{pmatrix}$
- $\boxed{\begin{array}{l} a_{11}=-1 \\ a_{22}=1 \end{array}}$ $\quad a_{12}\cdot 0=0 \rightarrow \boxed{a_{12}\in\mathbb{R}}$ $\qquad M=\begin{pmatrix} -1 & a_{12} \\ 0 & 1\end{pmatrix}$
- $\boxed{\begin{array}{l} a_{11}=-1 \\ a_{22}=-1 \end{array}}$ $\quad -2a_{12}=0 \rightarrow \boxed{a_{12}=0}$ $\qquad M=\begin{pmatrix} -1 & 0 \\ 0 & -1\end{pmatrix}$

JUN 99 |

$$(x \;\; y \;\; z)\begin{pmatrix} 1 & -2 \\ 2 & 1 \\ 1 & 2\end{pmatrix} = (0 \;\; 0) \rightarrow (x+2y+z \quad -2x+y+2z) = (0 \quad 0) \rightarrow$$

$$\rightarrow \left.\begin{array}{l} x+2y+z=0 \\ -2x+y+2z=0 \end{array}\right\} \rightarrow \begin{pmatrix} 1 & 2 & 1 \\ -2 & 1 & 2\end{pmatrix}\begin{pmatrix} x \\ y \\ z\end{pmatrix} = \begin{pmatrix} 0 \\ 0\end{pmatrix} \qquad \boxed{A=\begin{pmatrix} 1 & 2 & 1 \\ -2 & 1 & 2\end{pmatrix}}$$

$$\left.\begin{array}{l} x+2y=-z \\ -2x+y=-2z \end{array}\right\} \qquad x=\frac{\begin{vmatrix} -z & 2 \\ -2z & 1\end{vmatrix}}{\begin{vmatrix} 1 & 2 \\ -2 & 1\end{vmatrix}} = \frac{-z+4z}{5} = \frac{3}{5}z$$

$$y=\frac{\begin{vmatrix} 1 & -z \\ -2 & -2z\end{vmatrix}}{5} = \frac{-2z-2z}{5} = -\frac{4}{5}z$$

$z\in\mathbb{R}$

$$\sqrt{x^2+y^2+z^2}=1 \Rightarrow \left(\frac{3}{5}z\right)^2+\left(-\frac{4}{5}z\right)^2+z^2=1 \Rightarrow \left(\frac{9}{25}+\frac{16}{25}+1\right)z^2=1 \Rightarrow$$

$$\Rightarrow z^2=\frac{1}{2} \Rightarrow z=\pm\frac{\sqrt{2}}{2}$$

Soluciones con módulo 1 : $\boxed{\left(\frac{3\sqrt{2}}{10}, -\frac{4\sqrt{2}}{10}, \frac{\sqrt{2}}{2}\right) \qquad \left(-\frac{3\sqrt{2}}{10}, \frac{4\sqrt{2}}{10}, -\frac{\sqrt{2}}{2}\right)}$

JUN 99

$|A| = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & 0 & b \end{vmatrix} = b$

i) $|A| = \sin\alpha \Rightarrow b = \sin\alpha \Rightarrow \boxed{b \in [-1,1]}$

ii) Existe si $b \neq 0$:

$A_{11} = \begin{vmatrix} 1 & 0 \\ 0 & b \end{vmatrix} = b$ | $A_{21} = -\begin{vmatrix} 0 & 0 \\ 0 & b \end{vmatrix} = 0$ | $A_{31} = \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} = 0$

$A_{12} = -\begin{vmatrix} 0 & 0 \\ a & b \end{vmatrix} = 0$ | $A_{22} = \begin{vmatrix} 1 & 0 \\ a & b \end{vmatrix} = b$ | $A_{32} = -\begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0$

$A_{13} = \begin{vmatrix} 0 & 1 \\ a & 0 \end{vmatrix} = -a$ | $A_{23} = -\begin{vmatrix} 1 & 0 \\ a & 0 \end{vmatrix} = 0$ | $A_{33} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$

$\boxed{A^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -a/b & 0 & 1/b \end{pmatrix}}$ siendo $b \neq 0$

iii) $A = A^{-1} \Rightarrow \left.\begin{array}{l} a = -\frac{a}{b} \\ b = \frac{1}{b} \end{array}\right\} \rightarrow \left.\begin{array}{l} ab + a = 0 \\ b^2 = 1 \end{array}\right\} \rightarrow \left.\begin{array}{l} a(b+1) = 0 \\ b^2 = 1 \end{array}\right\} \rightarrow b = \pm 1$

- $\boxed{b=1} \rightarrow 2a = 0 \Rightarrow \boxed{a = 0}$
- $\boxed{b=-1} \rightarrow a \cdot 0 = 0 \Rightarrow \boxed{a \in \mathbb{R}}$

SEPT 99

i) $|A| = \begin{vmatrix} \lambda & 1 & 1 \\ -1 & 2 & \lambda \\ 1 & 1 & 3 \end{vmatrix} = 6\lambda - 1 + \lambda - 2 + 3 - \lambda^2 = -\lambda^2 + 7\lambda$

$|A| = 0 \Rightarrow -\lambda^2 + 7\lambda = 0 \ ; \ \lambda = \begin{cases} 0 \\ 7 \end{cases}$

Para $\lambda = 0$ o $\lambda = 7$ A no tiene inversa.

Para $\left.\begin{array}{l}\lambda \neq 0 \\ \lambda \neq 7\end{array}\right\}$ A tiene inversa.

ii) $|bA| = b^3 |A| = b^3(7\lambda - \lambda^2) = 1 \Rightarrow \boxed{b = \sqrt[3]{\frac{1}{7\lambda - \lambda^2}}}$

JUN 00

i) $A = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$ $\quad |A| = ab$

$A = A^{-1} \Rightarrow \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} = \begin{pmatrix} \frac{1}{a} & 0 \\ 0 & \frac{1}{b} \end{pmatrix} \Rightarrow \left.\begin{array}{l} a = 1/a \\ b = 1/b \end{array}\right\} \Rightarrow \left.\begin{array}{l} a^2 = 1 \\ b^2 = 1 \end{array}\right\} \Rightarrow \boxed{\begin{array}{l} a = \pm 1 \\ b = \pm 1 \end{array}}$

$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

ii) $A^2 = A \cdot A = A \cdot A^{-1} = \boxed{I}$

SEPT 00

i) $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$|A| = |A + I| \Rightarrow \begin{vmatrix} a & b \\ c & d \end{vmatrix} = \begin{vmatrix} a+1 & b \\ c & d+1 \end{vmatrix} \Rightarrow ad - \cancel{bc} = (a+1)(d+1) - \cancel{bc} \Rightarrow$

$\Rightarrow ad = ad + a + d + 1 \Rightarrow \boxed{a + d + 1 = 0} \rightarrow \boxed{\begin{array}{l} a \in \mathbb{R} \\ d = -a-1 \\ b \in \mathbb{R} \\ c \in \mathbb{R} \end{array}}$

$\boxed{A = \begin{pmatrix} a & b \\ c & -a-1 \end{pmatrix}}$

ii) $|A| = |A+I|$, $b = c = 0$, $|A| = 0$ $\Rightarrow \begin{vmatrix} a & 0 \\ 0 & -a-1 \end{vmatrix} = 0 \Rightarrow a(-a-1) = 0 \Rightarrow a = 0$ ó $a = -1$

$$\boxed{A = \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix}} \qquad \boxed{A = \begin{pmatrix} -1 & 0 \\ 0 & 0 \end{pmatrix}}$$

iii) Casi nunca se cumple. Por ejemplo:

$$|I + I| = \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} = 4$$

$$|I| + |I| = 1 + 1 = 2$$

JUN 01) a) $B \cdot A$ = matriz fila

$(1 \times m)(m \times m)$ = matriz fila $\qquad$ $\boxed{B \text{ debe ser } 1 \times m}$

b) $A \cdot B$ = matriz fila

$(m \times m) \cdot (m \times 1)$ = matriz fila $\qquad$ Tendría que ser $m = 1$, de ser así, B debería tener m filas

c) $B \cdot \begin{pmatrix} 1 & 2 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \end{pmatrix}$

$(1 \times 3) \cdot (3 \times 2) = (1 \times 2)$

$$\begin{pmatrix} a & b & c \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \end{pmatrix}$$

$\boxed{a = 0}$, $2a + b = 0$ $\longrightarrow \boxed{b = 0}$

$$\boxed{B = \begin{pmatrix} 0 & 0 & c \end{pmatrix}, \ c \in \mathbb{R}}$$

SEPT 01) $A \cdot B \cdot A = C$

Si A es $m \times m$, $\boxed{B \text{ será } m \times m}$

$ABA = C \Rightarrow B = A^{-1} C A^{-1} = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 3 & 2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}^{-1} =$

$= \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix} \cdot \begin{pmatrix} 3 & 2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ -6 & -3 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix} =$

$= \boxed{\begin{pmatrix} -1 & 2 \\ 0 & -3 \end{pmatrix}}$

JUN 02) a) $C(A+X)B = I$

$A + X = C^{-1} I B^{-1}$

$X = \boxed{C^{-1} B^{-1} - A}$

b) $B = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \rightarrow B^{-1} = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$

$C = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \rightarrow C^{-1} = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$

$$X = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 3 & 4 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix} - \begin{pmatrix} 3 & 4 \\ 1 & 2 \end{pmatrix} = \boxed{\begin{pmatrix} -2 & -5 \\ -2 & 0 \end{pmatrix}}$$

SEPT 02

$$\begin{vmatrix} 2a & a & a & a \\ a & 2a & a & a \\ a & a & 2a & a \\ a & a & a & 2a \end{vmatrix} = a^4 \begin{vmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{vmatrix} = a^4 \begin{vmatrix} 2 & 1 & 1 & 1 \\ -1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ -3 & -1 & -1 & 0 \end{vmatrix} = -a^4 \begin{vmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ -3 & -1 & -1 \end{vmatrix} =$$

$\boxed{\begin{array}{l} F2 \to F2 - F1 \\ F3 \to F3 - F1 \\ F4 \to F4 - 2 \cdot F1 \end{array}}$

$$= -a^4(-3-1-1) = \boxed{5a^4}$$

$$M = \begin{pmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{pmatrix} \qquad |M| = 5$$

$m_{11} = \begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix} = 4$

$m_{12} = -\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix} = -1$

$m_{13} = +\begin{vmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 2 \end{vmatrix} = -1$

$m_{14} = -\begin{vmatrix} 1 & 2 & 1 \\ 1 & 1 & 2 \\ 1 & 1 & 1 \end{vmatrix} = -1$

$m_{21} = -\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix} = -1$

$m_{22} = \begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix} = 4$

$m_{23} = -\begin{vmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 2 \end{vmatrix} = -1$

$m_{24} = \begin{vmatrix} 2 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & 1 & 1 \end{vmatrix} = -1$

$m_{31} = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 2 \end{vmatrix} = -1$

$m_{32} = -\begin{vmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 2 \end{vmatrix} = -1$

$m_{33} = \begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix} = 4$

$m_{34} = -\begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{vmatrix} = -1$

$m_{41} = -\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{vmatrix} = -1$

$m_{42} = \begin{vmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 2 & 1 \end{vmatrix} = -1$

$m_{43} = -\begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{vmatrix} = -1$

$m_{44} = \begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix} = 4$

$$\boxed{M^{-1} = \begin{pmatrix} 4/5 & -1/5 & -1/5 & -1/5 \\ -1/5 & 4/5 & -1/5 & -1/5 \\ -1/5 & -1/5 & 4/5 & -1/5 \\ -1/5 & -1/5 & -1/5 & 4/5 \end{pmatrix}}$$

SEPT 02 a) $AC = I \Rightarrow C = A^{-1} = \begin{pmatrix} -1/7 & 3/7 \\ 3/7 & -2/7 \end{pmatrix}$

$BD = I \Rightarrow D = B^{-1} = \begin{pmatrix} 1 & 0 \\ 1/5 & -1/5 \end{pmatrix}$

b) $C^{-1} = (A^{-1})^{-1} = A = \begin{pmatrix} 2 & 3 \\ 3 & 1 \end{pmatrix}$

$D^{-1} = (B^{-1})^{-1} = B = \begin{pmatrix} 1 & 0 \\ 1 & -5 \end{pmatrix}$

$$\left[\begin{pmatrix} 2 & 3 \\ 3 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 1 & -5 \end{pmatrix}\right] \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 3 \\ 2 & 6 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \longrightarrow \left.\begin{array}{l} x + 3y = 1 \\ 2x + 6y = 2 \end{array}\right\}$$

$M = \begin{pmatrix} 1 & 3 \\ 2 & 6 \end{pmatrix} \quad |M| = 0 \Rightarrow$ rango $M = 1$

$M^+ = \begin{pmatrix} 1 & 3 & 1 \\ 2 & 6 & 2 \end{pmatrix} \quad \begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix} = 0 \Rightarrow$ rango $M^+ = 1$

$n = 2$

$\Rightarrow$ $\boxed{\text{S. Compatible Indeterminado}}$

$\boxed{\begin{array}{l} x = 1 - 3y \\ y \in \mathbb{R} \end{array}}$ Son las infinitas soluciones.

JUN 03

a) $(B-C)\cdot A = 0$; A no singular $\Longrightarrow (B-C) = 0\cdot A^{-1} \Rightarrow B-C=0 \Rightarrow \boxed{B=C}$

b) $XA=0$; A no singular $\Longrightarrow \boxed{X=0}$ Sería la única solución, siempre que A sea no singular.

Si A es singular, no se podría deducir como única solución $X=0$.

$$|A| = \begin{vmatrix} 2 & -6 \\ -1 & 3 \end{vmatrix} = 0$$, A es singular.

$$\begin{pmatrix} & \\ & \end{pmatrix} \cdot \begin{pmatrix} 2 & -6 \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

Habría infinitas soluciones para X. Por ejemplo: $X = \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix}$; $X = \begin{pmatrix} 1 & 2 \\ -3 & -6 \end{pmatrix}$; ⋮

SEPT 03

a) $$|A| = \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{vmatrix} = 1 \longrightarrow \text{Existe } A^{-1}$$

$A_{11} = \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} = 0$; $A_{21} = -\begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0$; $A_{31} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$

$A_{12} = -\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = 1$; $A_{22} = \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} = 0$; $A_{32} = -\begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0$

$A_{13} = \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} = 0$; $A_{23} = -\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = 1$; $A_{33} = \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} = 0$

$$\boxed{A^{-1} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}}$$

b) $$A^2 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$A^3 = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I$$

$A^4 = A^3 \cdot A = I\cdot A = A$

$A^5 = A^4 \cdot A = A\cdot A = A^2$

Es decir:

$$A = A^4 = A^7 = \text{-----} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} = A^n \quad (\text{con } n = \dot{3}+1)$$

$$A^2 = A^5 = A^8 = \text{-----} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} = A^n \quad (\text{con } n = \dot{3}+2)$$

$$A^3 = A^6 = A^9 = \text{-----} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = A^n \quad (\text{con } n = \dot{3})$$

JUN 04

$$|A| = \begin{vmatrix} 1 & 0 & 2 \\ -2 & 1 & x \\ 1 & x & 0 \end{vmatrix} = -4x - 2 - x^2$$

a)

$$-x^2 - 4x - 2 = 0$$

$$x = \frac{4 \pm \sqrt{16-8}}{-2} = \frac{4 \pm 2\sqrt{2}}{-2} \begin{cases} -2+\sqrt{2} \\ -2-\sqrt{2} \end{cases}$$

$\boxed{\text{Para } x \neq -2 \pm \sqrt{2} \quad A \text{ tendrá inversa}}$

b)

$$A = \begin{pmatrix} 1 & 0 & 2 \\ -2 & 1 & -1 \\ 1 & -1 & 0 \end{pmatrix} \qquad |A| = 1$$

$$A_{11} = \begin{vmatrix} 1 & -1 \\ -1 & 0 \end{vmatrix} = -1 \qquad A_{21} = -\begin{vmatrix} 0 & 2 \\ -1 & 0 \end{vmatrix} = -2 \qquad A_{31} = \begin{vmatrix} 0 & 2 \\ 1 & -1 \end{vmatrix} = -2$$

$$A_{12} = -\begin{vmatrix} -2 & -1 \\ 1 & 0 \end{vmatrix} = -1 \qquad A_{22} = \begin{vmatrix} 1 & 2 \\ 1 & 0 \end{vmatrix} = -2 \qquad A_{32} = -\begin{vmatrix} 1 & 2 \\ -2 & -1 \end{vmatrix} = -3$$

$$A_{13} = \begin{vmatrix} -2 & 1 \\ 1 & -1 \end{vmatrix} = 1 \qquad A_{23} = -\begin{vmatrix} 1 & 0 \\ 1 & -1 \end{vmatrix} = +1 \qquad A_{33} = \begin{vmatrix} 1 & 0 \\ -2 & 1 \end{vmatrix} = 1$$

$$\boxed{A^{-1} = \begin{pmatrix} -1 & -2 & -2 \\ -1 & -2 & -3 \\ 1 & 1 & 1 \end{pmatrix}}$$

c)

$$A \cdot B = C \cdot D$$

$$(3\times3)\cdot(3\times2) = (3\times2)\cdot(2\times2)$$

$\boxed{B \text{ debe ser } 3\times2}$

$$B = A^{-1}CD = \begin{pmatrix} -1 & -2 & -2 \\ -1 & -2 & -3 \\ 1 & 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} -1 & -2 & -2 \\ -1 & -2 & -3 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} = \boxed{\begin{pmatrix} -1 & -2 \\ -1 & -2 \\ 1 & 1 \end{pmatrix}}$$

SEPT 04

$$|A| = \begin{vmatrix} m & 2 & 6 \\ 2 & m & 4 \\ 2 & m & 6 \end{vmatrix} = 6m^2 + \cancel{12m} + 16 - \cancel{12m} - 24 - 4m^2 = 2m^2 - 8$$

a)

$$2m^2 - 8 = 0 \quad ; \quad m^2 = 4 \quad \boxed{m = \pm 2}$$

- Si $\left.\begin{matrix} m \neq 2 \\ m \neq -2 \end{matrix}\right\} \Rightarrow \boxed{\text{rango } A = 3}$
- Si $m = 2 \longrightarrow A = \begin{pmatrix} 2 & 2 & 6 \\ 2 & 2 & 4 \\ 2 & 2 & 6 \end{pmatrix} \qquad \begin{vmatrix} 2 & 6 \\ 2 & 4 \end{vmatrix} = 8 - 12 = -4 \Rightarrow \boxed{\text{rango } A = 2}$
- Si $m = -2 \longrightarrow A = \begin{pmatrix} -2 & 2 & 6 \\ 2 & -2 & 4 \\ 2 & -2 & 6 \end{pmatrix} \qquad \begin{vmatrix} 2 & 6 \\ -2 & 4 \end{vmatrix} = 8 + 12 = 20 \Rightarrow \boxed{\text{rango } A = 2}$

b)

$$A \cdot X = B$$

$$(3\times3)(3\times2) = (3\times2)$$

$\boxed{X \text{ debe ser } 3\times2}$

c)

$$A \cdot X = B \Rightarrow X = A^{-1}B$$

$m = 0$: $A = \begin{pmatrix} 0 & 2 & 6 \\ 2 & 0 & 4 \\ 2 & 0 & 6 \end{pmatrix} \qquad |A| = -8$

$$A_{11} = \begin{vmatrix} 0 & 4 \\ 0 & 6 \end{vmatrix} = 0 \qquad A_{21} = -\begin{vmatrix} 2 & 6 \\ 0 & 6 \end{vmatrix} = -12 \qquad A_{31} = \begin{vmatrix} 2 & 6 \\ 0 & 4 \end{vmatrix} = 8$$

$$A_{12} = -\begin{vmatrix} 2 & 4 \\ 2 & 6 \end{vmatrix} = -4 \qquad A_{22} = \begin{vmatrix} 0 & 6 \\ 2 & 6 \end{vmatrix} = -12 \qquad A_{32} = \begin{vmatrix} 0 & 6 \\ 2 & 4 \end{vmatrix} = -12$$

$$A_{13} = \begin{vmatrix} 2 & 0 \\ 2 & 0 \end{vmatrix} = 0 \qquad A_{23} = -\begin{vmatrix} 0 & 2 \\ 2 & 0 \end{vmatrix} = 4 \qquad A_{33} = \begin{vmatrix} 0 & 2 \\ 2 & 0 \end{vmatrix} = -4$$

$$\boxed{A^{-1} = \begin{pmatrix} 0 & 3/2 & -1 \\ 1/2 & 3/2 & -3/2 \\ 0 & -1/2 & 1/2 \end{pmatrix}}$$

$$X = A^{-1}\cdot B = \begin{pmatrix} 0 & 3/2 & -1 \\ 1/2 & 3/2 & -3/2 \\ 0 & -1/2 & 1/2 \end{pmatrix} \cdot \begin{pmatrix} 2 & 2 \\ 1 & 0 \\ -1 & 2 \end{pmatrix} = \boxed{\begin{pmatrix} 5/2 & -2 \\ 4 & -2 \\ -1 & 1 \end{pmatrix}}$$

JUN 05

$$\begin{vmatrix} 0 & 1 & x \\ x & x & 1 \\ -x & 1 & x \end{vmatrix} = \cancel{x^2} - x + x^3 - \cancel{x^2} = x^3 - x$$

$$x^3 - x = 0 \quad ; \quad +x(x^2-1) = 0 \Longrightarrow \boxed{\begin{matrix} x=0 \\ x=1 \\ x=-1 \end{matrix}}$$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & x & 1 \\ 1 & 1 & x^2 \end{vmatrix} = x^3 + \cancel{1} + 1 - x - x^2 - \cancel{1} = x^3 - x^2 - x + 1$$

$$\begin{array}{r|rrrr} & 1 & -1 & -1 & 1 \\ 1 & & 1 & 0 & -1 \\ \hline & 1 & 0 & -1 & \boxed{0} \\ 1 & & 1 & 1 & \\ \hline & 1 & 1 & \boxed{0} & \\ -1 & & -1 & & \\ \hline & 1 & \boxed{0} & & \end{array}$$

$$\boxed{\begin{matrix} x = 1 \text{ (raíz doble)} \\ x = -1 \end{matrix}}$$

SEPT 05

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = K$$

$$\begin{vmatrix} d & 2e & f \\ a & 2b & c \\ g & 2h & i \end{vmatrix} \overset{2}{=} 2\begin{vmatrix} d & e & f \\ a & b & c \\ g & h & i \end{vmatrix} = -2\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = \boxed{-2K}$$

$$\begin{vmatrix} a+b & b & 2c \\ d+e & e & 2f \\ g+h & h & 2i \end{vmatrix} = \begin{vmatrix} a & b & 2c \\ d & e & 2f \\ g & h & 2i \end{vmatrix} + \begin{vmatrix} b & b & 2c \\ e & e & 2f \\ h & h & 2i \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} + 0 = \boxed{2K}$$

2

iguales

JUN 06

a) $$|A| = \begin{vmatrix} 1 & 0 & -1 \\ 0 & x & 3 \\ 4 & 1 & -x \end{vmatrix} = -x^2 + 4x - 3$$

$$-x^2 + 4x - 3 = 0 \rightarrow x = \begin{cases} 1 \\ 3 \end{cases}$$

$\boxed{A \text{ tiene inversa } \forall x \in \mathbb{R} - \{1, 3\}}$

b) $$x = 2 \rightarrow A = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 2 & 3 \\ 4 & 1 & -2 \end{pmatrix}$$

$$|A| = -4 + 8 - 3 = 1$$

$A_{11} = \begin{vmatrix} 2 & 3 \\ 1 & -2 \end{vmatrix} = -7$ $\quad A_{21} = -\begin{vmatrix} 0 & -1 \\ 1 & -2 \end{vmatrix} = -1$ $\quad A_{31} = \begin{vmatrix} 0 & -1 \\ 2 & 3 \end{vmatrix} = 2$

$A_{12} = -\begin{vmatrix} 0 & 3 \\ 4 & -2 \end{vmatrix} = 12$ $\quad A_{22} = \begin{vmatrix} 1 & -1 \\ 4 & -2 \end{vmatrix} = 2$ $\quad A_{32} = -\begin{vmatrix} 1 & -1 \\ 0 & 3 \end{vmatrix} = -3$

$A_{13} = \begin{vmatrix} 0 & 2 \\ 4 & 1 \end{vmatrix} = -8$ $\quad A_{23} = -\begin{vmatrix} 1 & 0 \\ 4 & 1 \end{vmatrix} = -1$ $\quad A_{33} = \begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix} = 2$

$$\boxed{A^{-1} = \begin{pmatrix} -7 & -1 & 2 \\ 12 & 2 & -3 \\ -8 & -1 & 2 \end{pmatrix}}$$

c) $$x = 5 \rightarrow A = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 5 & 3 \\ 4 & 1 & -5 \end{pmatrix} \rightarrow |A| = -25 + 20 - 3 = -8$$

$$|bA| = b^3 \cdot |A| = -8b^3$$

$$-8b^3 = 1 \Rightarrow b^3 = \frac{-1}{8} \qquad \boxed{b = -\frac{1}{2}}$$

SEPT 06

a) $B \cdot A = \begin{pmatrix} K & 0 & -1 \\ 1 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 2 & K \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} K & -1 \\ 3 & K+2 \end{pmatrix}$

$|B \cdot A| = K^2+2K+3$

$K^2+2K+3=0 \longrightarrow K = \frac{-2\pm\sqrt{4-12}}{2} = \frac{-2\pm 2i\sqrt{2}}{2} = -1\pm i\sqrt{2}$

$\boxed{\text{Entonces } B\cdot A \text{ tiene inversa } \forall x\in\mathbb{R}}$

b) $A \cdot B = \begin{pmatrix} 1 & 0 \\ 2 & K \\ 0 & 1 \end{pmatrix} \begin{pmatrix} K & 0 & -1 \\ 1 & 1 & 2 \end{pmatrix} = \begin{pmatrix} K & 0 & -1 \\ 3K & K & 2K-2 \\ 1 & 1 & 2 \end{pmatrix}$

$|A\cdot B| = 2K^2-3K+K-2K^2+2K = 0$

$\boxed{\text{Entonces } A\cdot B \text{ no tiene inversa para ningún } x\in\mathbb{R}}$

JUN 07

a) $A = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 2 \\ 1 & a & 1 \end{pmatrix}$

$\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -1 \neq 0 \longrightarrow r(A) \geq 2$

$\begin{vmatrix} 0 & 1 & 2 \\ 1 & 0 & 2 \\ 1 & a & 1 \end{vmatrix} = 2a+2-1 = 2a+1$

$2a+1=0 \longrightarrow a=-1/2$

$\boxed{\begin{array}{l} \text{Si } a=-1/2 \Rightarrow r(A)=2 \\ \text{Si } a\neq -1/2 \Rightarrow r(A)=3 \end{array}}$

$B = \begin{pmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 2 & 2 \\ 1 & a & 1 & 1+a \end{pmatrix}$

Para $a\neq -1/2 \Rightarrow r(B)=3$ (Hecho con A)

Para $a=-1/2 \Rightarrow B = \begin{pmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 2 & 2 \\ 1 & -1/2 & 1 & 1/2 \end{pmatrix}$

$\begin{vmatrix} 0 & 1 & 3 \\ 1 & 0 & 2 \\ 1 & -1/2 & 1/2 \end{vmatrix} = -\frac{3}{2}+2-\frac{1}{2} = 0$

$\boxed{\begin{array}{l} \text{Si } a=-1/2 \Rightarrow r(B)=2 \\ \text{Si } a\neq -1/2 \Rightarrow r(B)=3 \end{array}}$

b) $a=-1 \longrightarrow A = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 2 \\ 1 & -1 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 2 & 2 \\ 1 & -1 & 1 & 0 \end{pmatrix}$

$A\cdot X = B \Longrightarrow \boxed{X = A^{-1}\cdot B}$ (Como $a=-1\neq -1/2 \Rightarrow \exists A^{-1}$)

$|A| = -2+2-1=-1$

$A_{11} = \begin{vmatrix} 0 & 2 \\ -1 & 1 \end{vmatrix} = 2 \quad A_{12} = -\begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} = 1 \quad A_{13} = \begin{vmatrix} 1 & 0 \\ 1 & -1 \end{vmatrix} = -1$

$A_{21} = -\begin{vmatrix} 1 & 2 \\ -1 & 1 \end{vmatrix} = -3 \quad A_{22} = \begin{vmatrix} 0 & 2 \\ 1 & 1 \end{vmatrix} = -2 \quad A_{23} = -\begin{vmatrix} 0 & 1 \\ 1 & -1 \end{vmatrix} = 1$

$A_{31} = \begin{vmatrix} 1 & 2 \\ 0 & 2 \end{vmatrix} = 2 \quad A_{32} = -\begin{vmatrix} 0 & 2 \\ 1 & 2 \end{vmatrix} = 2 \quad A_{33} = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -1$

$\Rightarrow A^{-1} = \begin{pmatrix} -2 & 3 & -2 \\ -1 & 2 & -2 \\ 1 & -1 & 1 \end{pmatrix}$

$X = \begin{pmatrix} -2 & 3 & -2 \\ -1 & 2 & -2 \\ 1 & -1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 2 & 2 \\ 1 & -1 & 1 & 0 \end{pmatrix} = \boxed{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}}$

SEPT 07

$$A = \begin{pmatrix} -1 & -2 & -2 \\ 1 & 2 & 1 \\ 0 & -1 & -1 \end{pmatrix}$$

a)

$$A^2 = A \cdot A = \begin{pmatrix} -1 & 0 & 2 \\ 1 & 1 & -1 \\ -1 & -1 & 0 \end{pmatrix}$$

$$A^3 = A^2 \cdot A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I \implies \boxed{A^3 - I = 0}$$

b)

$$A^{13} = A^3 \cdot A^3 \cdot A^3 \cdot A^3 \cdot A = A = \boxed{\begin{pmatrix} -1 & -2 & -2 \\ 1 & 2 & 1 \\ 0 & -1 & -1 \end{pmatrix}}$$

NOTA: $13 \,\underline{|\,3}$, cociente 4, resto 1 $\implies A^{13} = A^{12} \cdot A = (A^3)^4 \cdot A$

c)

$$A^2X + I = A \longrightarrow A \cdot A^2 X + A \cdot I = A \cdot A \longrightarrow A^3X + A = A^2 \longrightarrow IX + A = A^2 \implies$$

$$\implies X = A^2 - A = \begin{pmatrix} -1 & 0 & 2 \\ 1 & 1 & -1 \\ -1 & -1 & 0 \end{pmatrix} - \begin{pmatrix} -1 & -2 & -2 \\ 1 & 2 & 1 \\ 0 & -1 & -1 \end{pmatrix} = \boxed{\begin{pmatrix} 0 & 2 & 4 \\ 0 & -1 & -2 \\ -1 & 0 & 1 \end{pmatrix}}$$

JUN 08

$$A = \begin{pmatrix} x & y & x \\ y & 0 & y \\ 1 & z & z \end{pmatrix} \qquad B = (a \quad 2 \quad 3) \qquad C = (4, \ 0, \ 2)$$

a)

$$|A| = \begin{vmatrix} x & y & x \\ y & 0 & y \\ 1 & z & z \end{vmatrix} = \cancel{xyz} + y^2 - y^2z - \cancel{xyz} = y^2(1-z)$$

Para $\left.\begin{array}{l} x \in \mathbb{R} \\ y \in \mathbb{R}-\{0\} \\ z \in \mathbb{R}-\{1\} \end{array}\right\}$ existe A^{-1}. $\boxed{\text{Para } y=0 \text{ ó } z=1 \ \nexists A^{-1}}$

b)

$B \cdot A = C$

$$(a \quad 2 \quad 3)\begin{pmatrix} x & y & x \\ y & 0 & y \\ 1 & z & z \end{pmatrix} = (4 \quad 0 \quad 2)$$

$$\left.\begin{array}{r} ax + 2y + 3 = 4 \\ ay + 3z = 0 \\ ax + 2y + 3z = 2 \end{array}\right\} \longrightarrow \left\{\begin{array}{r} ax + 2y \quad = 1 \\ ay + 3z = 0 \\ ax + 2y + 3z = 2 \end{array}\right.$$

$$M = \begin{pmatrix} a & 2 & 0 \\ 0 & a & 3 \\ a & 2 & 3 \end{pmatrix} \qquad \begin{vmatrix} 2 & 0 \\ a & 3 \end{vmatrix} = 6 \neq 0 \implies r(M) \geq 2$$

$$|M| = 3a^2 + \cancel{6a} - \cancel{6a} = 3a^2$$

- $\boxed{\text{Si } a \neq 0} \implies r(M) = 3 \implies \boxed{\text{El sistema tiene solución única}}$
- $\boxed{\text{Si } a = 0} \implies M = \begin{pmatrix} 0 & 2 & 0 \\ 0 & 0 & 3 \\ 0 & 2 & 3 \end{pmatrix} \quad r(M) = 2$

$$M^* = \begin{pmatrix} 0 & 2 & 0 & 1 \\ 0 & 0 & 3 & 0 \\ 0 & 2 & 3 & 2 \end{pmatrix} \qquad \begin{vmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ 2 & 3 & 2 \end{vmatrix} = 12 - 6 = 6 \neq 0 \implies r(M^*) = 3 \Bigg\} \implies$$

$\implies \boxed{\text{El sistema no tiene solución}}$

c) Para $a \neq 0$:

$$x = \frac{\begin{vmatrix} 1 & 2 & 0 \\ 0 & a & 3 \\ 2 & 2 & 3 \end{vmatrix}}{3a^2} = \frac{3a + 12 - 6}{3a^2} = \frac{3a+6}{3a^2} = \boxed{\frac{a+2}{a}}$$

$$y = \frac{\begin{vmatrix} a & 1 & 0 \\ 0 & 0 & 3 \\ a & 2 & 3 \end{vmatrix}}{3a^2} = \frac{3a - 6a}{3a^2} = \frac{-3a}{3a^2} = \boxed{-\frac{1}{a}}$$

$$z = \frac{\begin{vmatrix} a & 2 & 1 \\ 0 & a & 0 \\ a & 2 & 2 \end{vmatrix}}{3a^2} = \frac{2a^2 - a^2}{3a^2} = \frac{a^2}{3a^2} = \boxed{\frac{1}{3}}$$

SEPT 08

$A^2=6A-9I$

a) $A^4 = A^2\cdot A^2 = (6A-9I)\cdot(6A-9I) = 36A^2 - 54A - 54A - 81I =$

$= 36(6A-9I) - 108A - 81I = 216A - 324I - 108A - 81I =$

$= \boxed{108A - 405I}$

b) $B = \begin{pmatrix} 1 & 3 & 1 \\ -2 & 6 & 1 \\ 2 & -3 & 2 \end{pmatrix}$

$B^2 = \begin{pmatrix} 1 & 3 & 1 \\ -2 & 6 & 1 \\ 2 & -3 & 2 \end{pmatrix}\begin{pmatrix} 1 & 3 & 1 \\ -2 & 6 & 1 \\ 2 & -3 & 2 \end{pmatrix} = \begin{pmatrix} -3 & 18 & 6 \\ -12 & 27 & 6 \\ 12 & -18 & 3 \end{pmatrix}$ ✓

$6B-9I = \begin{pmatrix} 6 & 18 & 6 \\ -12 & 36 & 6 \\ 12 & -18 & 12 \end{pmatrix} - \begin{pmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{pmatrix} = \begin{pmatrix} -3 & 18 & 6 \\ -12 & 27 & 6 \\ 12 & -18 & 3 \end{pmatrix}$ ✓

$\boxed{B^2 = 6B-9I}$

c) $|B| = \begin{vmatrix} 1 & 3 & 1 \\ -2 & 6 & 1 \\ 2 & -3 & 2 \end{vmatrix} = 12+6+6-\cancel{12}+\cancel{12}+3 = 27$

$B_{11} = \begin{vmatrix} 6 & 1 \\ -3 & 2 \end{vmatrix} = 15$; $B_{21} = -\begin{vmatrix} 3 & 1 \\ -3 & 2 \end{vmatrix} = -9$; $B_{31} = \begin{vmatrix} 3 & 1 \\ 6 & 1 \end{vmatrix} = -3$

$B_{12} = -\begin{vmatrix} -2 & 1 \\ 2 & 2 \end{vmatrix} = 6$; $B_{22} = \begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix} = 0$; $B_{32} = -\begin{vmatrix} 1 & 1 \\ -2 & 1 \end{vmatrix} = -3$

$B_{13} = \begin{vmatrix} -2 & 6 \\ 2 & -3 \end{vmatrix} = -6$; $B_{23} = -\begin{vmatrix} 1 & 3 \\ 2 & -3 \end{vmatrix} = 9$; $B_{33} = \begin{vmatrix} 1 & 3 \\ -2 & 6 \end{vmatrix} = 12$

$B^{-1} = \begin{pmatrix} 15/27 & -9/27 & -3/27 \\ 6/27 & 0 & -3/27 \\ -6/27 & 9/27 & 12/27 \end{pmatrix} = \boxed{\begin{pmatrix} 5/9 & -1/3 & -1/9 \\ 2/9 & 0 & -1/9 \\ -2/9 & 1/3 & 4/9 \end{pmatrix}}$

JUN 09

a) $P = \begin{pmatrix} 0 & 1 & 3 \\ 2-a & 1 & a \\ 3 & 3 & a \end{pmatrix}$ $\qquad \begin{vmatrix} 0 & 3 \\ 3 & a \end{vmatrix} = -9 \neq 0 \Rightarrow r(P) \geq 2$

$|P| = \begin{vmatrix} 0 & 1 & 3 \\ 2-a & 1 & a \\ 3 & 3 & a \end{vmatrix} = 18-9a+3a-9-2a+a^2 = a^2-8a+9$

$a^2-8a-9=0$

$a = \dfrac{8\pm\sqrt{64+36}}{2} = \dfrac{8\pm10}{2}$ → 9, → -1

$$\boxed{\begin{array}{l} \bullet\ \text{Si } a=9 \rightarrow \text{rango}(P)=2 \\ \bullet\ \text{Si } a=-1 \rightarrow \text{rango}(P)=2 \\ \bullet\ \text{Si } \left.\begin{array}{l} a\neq 9 \\ a\neq -1 \end{array}\right\} \rightarrow \text{rango}(P)=3 \end{array}}$$

b) $a=1$: $\quad PX=Q$

Como existe P^{-1}, $\quad \boxed{X = P^{-1}\cdot Q}$

Veamos la inversa de P: $\qquad P = \begin{pmatrix} 0 & 1 & 3 \\ 1 & 1 & 1 \\ 3 & 3 & 1 \end{pmatrix}$

$|P| = 1^2 - 8\cdot 1 + 9 = 2$

$P_{11} = \begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} = -2$ $\quad P_{21} = -\begin{vmatrix} 1 & 3 \\ 3 & 1 \end{vmatrix} = 8$ $\quad P_{31} = \begin{vmatrix} 1 & 3 \\ 1 & 1 \end{vmatrix} = -2$

$P_{12} = -\begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} = 2$ $\quad P_{22} = \begin{vmatrix} 0 & 3 \\ 3 & 1 \end{vmatrix} = -9$ $\quad P_{32} = -\begin{vmatrix} 0 & 3 \\ 1 & 1 \end{vmatrix} = 3$

$P_{13} = \begin{vmatrix} 1 & 1 \\ 3 & 3 \end{vmatrix} = 0$ $\quad P_{23} = -\begin{vmatrix} 0 & 1 \\ 3 & 3 \end{vmatrix} = 3$ $\quad P_{33} = \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} = -1$

$$P^{-1} = \frac{1}{2} \cdot \begin{pmatrix} -2 & 8 & -2 \\ 2 & -9 & 3 \\ 0 & 3 & -1 \end{pmatrix}$$

$$X = P^{-1}Q = \frac{1}{2}\begin{pmatrix} -2 & 8 & -2 \\ 2 & -9 & 3 \\ 0 & 3 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & -1 \\ 1 & 1 & 2 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} -4 & 14 & -14 \\ 5 & -15 & 17 \\ -1 & 5 & -5 \end{pmatrix} = \boxed{\begin{pmatrix} -2 & 7 & -7 \\ 5/2 & -15/2 & 17/2 \\ -1/2 & 5/2 & -5/2 \end{pmatrix}}$$

SEPT 09

a) $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & m & 1 \\ m & 1 & 1 \end{pmatrix}$

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & m & 1 \\ m & 1 & 1 \end{vmatrix} = m + 1 + m - m^2 - 1 - 1 = -m^2 + 2m - 1$$

$-m^2 + 2m - 1 = 0 \quad ; \quad m = 1$

$\boxed{\text{Si: } m \neq 1 \text{ existe } A^{-1}}$

b) $\underline{m = 2}$ $\quad A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{pmatrix}$

$|A| = -4 + 4 - 1 = -1$

$A_{11} = \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = 1$ $\quad A_{21} = -\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 0$ $\quad A_{31} = \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = -1$

$A_{12} = -\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = 1$ $\quad A_{22} = \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = -1$ $\quad A_{32} = -\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 0$

$A_{13} = \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} = -3$ $\quad A_{23} = -\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = 1$ $\quad A_{33} = \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = 1$

$$\boxed{A^{-1} = \begin{pmatrix} -1 & 0 & 1 \\ -1 & 1 & 0 \\ 3 & -1 & -1 \end{pmatrix}}$$

c) $m = 2$, como $\exists A^{-1} \Rightarrow$ $\quad X = A^{-1} \cdot B$

$$X = \begin{pmatrix} -1 & 0 & 1 \\ -1 & 1 & 0 \\ 3 & -1 & -1 \end{pmatrix}\begin{pmatrix} -4 \\ 1 \\ 4 \end{pmatrix} = \boxed{\begin{pmatrix} 8 \\ 5 \\ -17 \end{pmatrix}}$$

JUNIO 10 fase General

a) $A - mI = \begin{pmatrix} 1 & 2 & 0 \\ 2 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} - m\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1-m & 2 & 0 \\ 2 & 1-m & 1 \\ 0 & 0 & 1-m \end{pmatrix}$

$$|A - mI| = \begin{vmatrix} 1-m & 2 & 0 \\ 2 & 1-m & 1 \\ 0 & 0 & 1-m \end{vmatrix} = (1-m)^3 - 4(1-m) = (1-m)\left[(1-m)^2 - 4\right] = (1-m)(m^2 - 2m - 3)$$

$|A - mI| = 0 \rightarrow (1-m)(m^2 - 2m - 3) = 0$ $\rightarrow 1 - m = 0 \Rightarrow m = 1$; $\rightarrow m^2 - 2m - 3 = 0 \Rightarrow m = -1,\ 3$

$\boxed{\text{Para } m = 1,\ m = -1,\ m = 3,\ A - mI \text{ no tiene inversa}}$

b) $B = A - mI = \begin{pmatrix} 1-m & 2 & 0 \\ 2 & 1-m & 1 \\ 0 & 0 & 1-m \end{pmatrix}$

$|B| = (1-m)(m^2 - 2m - 3)$

$B_{11} = \begin{vmatrix} 1-m & 1 \\ 0 & 1-m \end{vmatrix} = (1-m)^2$ $\quad B_{21} = -\begin{vmatrix} 2 & 0 \\ 0 & 1-m \end{vmatrix} = -2(1-m)$ $\quad B_{31} = \begin{vmatrix} 2 & 0 \\ 1-m & 1 \end{vmatrix} = 2$

$B_{12} = -\begin{vmatrix} 2 & 1 \\ 0 & 1-m \end{vmatrix} = -2(1-m)$ $\quad B_{22} = \begin{vmatrix} 1-m & 0 \\ 0 & 1-m \end{vmatrix} = (1-m)^2$ $\quad B_{32} = -\begin{vmatrix} 1-m & 0 \\ 2 & 1 \end{vmatrix} = -(1-m)$

$B_{13} = \begin{vmatrix} 2 & 1-m \\ 0 & 0 \end{vmatrix} = 0$ $\quad B_{23} = -\begin{vmatrix} 1-m & 2 \\ 0 & 0 \end{vmatrix} = 0$ $\quad B_{33} = \begin{vmatrix} 1-m & 2 \\ 2 & 1-m \end{vmatrix} = (1-m)^2 - 4 = m^2 - 2m - 3$

$$\boxed{(A - mI)^{-1} = \frac{1}{(1-m)(m^2 - 2m - 3)} \cdot \begin{pmatrix} (1-m)^2 & -2(1-m) & 2 \\ -2(1-m) & (1-m)^2 & -(1-m) \\ 0 & 0 & m^2 - 2m - 3 \end{pmatrix}}$$

JUN 10 fase específica

a) $|A| = \begin{vmatrix} -x & 1 & 1 \\ 1 & -x & 1 \\ 1 & 1 & -x \end{vmatrix} = -x^3+1+1+x+x+x = -x^3+3x+2$

$-x^3+3x+2=0$

	-1	0	3	2
-1		1	-1	-2
	-1	1	2	0
-1		1	-2	
	-1	2	0	
2		-2		
	-1	0		

$x = \begin{cases} -1 \text{ (doble)} \\ 2 \end{cases}$

b) • $\boxed{\text{Si } \begin{matrix} x \neq -1 \\ x \neq 2 \end{matrix} \Big\} \rightarrow \text{rango}(A) = 3}$

• Si $x=-1$: $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \rightarrow$ obviamente : $\boxed{\text{si } x=-1 \rightarrow \text{rango}(A)=1}$

• Si $x=2$: $A = \begin{pmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{pmatrix}$ $\begin{vmatrix} -2 & 1 \\ 1 & -2 \end{vmatrix} = 3 \neq 0$, $\begin{vmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{vmatrix} = 0$ $\Big\} \Rightarrow \boxed{\text{si } x=2 \rightarrow \text{rango}(A)=2}$

SEPT 10 fase general

a) $|A| = \begin{vmatrix} 1 & m & 0 \\ 0 & 1 & m \\ 1 & 1 & -2 \end{vmatrix} = -2+m^2-m = m^2-m-2$

b) $m^2-m-2=0$

$m = \dfrac{1 \pm \sqrt{1+8}}{2} = \dfrac{1 \pm 3}{2} \begin{matrix} \nearrow 2 \\ \searrow -1 \end{matrix}$

$\boxed{\text{Si } \begin{matrix} m \neq 2 \\ m \neq -1 \end{matrix} \Big\} \Rightarrow A \text{ tiene inversa}}$

c) $m=1$: $A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{pmatrix}$

$|A| = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{vmatrix} = -2+1-1 = -2$

$A_{11} = \begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix} = -3$ $\quad A_{21} = -\begin{vmatrix} 1 & 0 \\ 1 & -2 \end{vmatrix} = 2$ $\quad A_{31} = \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} = 1$

$A_{12} = -\begin{vmatrix} 0 & 1 \\ 1 & -2 \end{vmatrix} = 1$ $\quad A_{22} = \begin{vmatrix} 1 & 0 \\ 1 & -2 \end{vmatrix} = -2$ $\quad A_{32} = -\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = -1$

$A_{13} = \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} = -1$ $\quad A_{23} = -\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 0$ $\quad A_{33} = \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} = 1$

$$\boxed{A^{-1} = \frac{1}{-2} \begin{pmatrix} -3 & 2 & 1 \\ 1 & -2 & -1 \\ -1 & 0 & 1 \end{pmatrix}}$$

SEPT 10 fase específica

a) $|M| = \begin{vmatrix} 0 & 2 & 1 \\ 1 & 1 & 1 \\ -1 & -2 & -2 \end{vmatrix} = -2-2+1+4 = 1 \neq 0 \Rightarrow \exists M^{-1}$

$M_{11} = \begin{vmatrix} 1 & 1 \\ -2 & -2 \end{vmatrix} = 0$ $\quad M_{21} = -\begin{vmatrix} 2 & 1 \\ -2 & -2 \end{vmatrix} = 2$ $\quad M_{31} = \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = 1$

$M_{12} = -\begin{vmatrix} 1 & 1 \\ -1 & -2 \end{vmatrix} = 1$ $\quad M_{22} = \begin{vmatrix} 0 & 1 \\ -1 & -2 \end{vmatrix} = 1$ $\quad M_{32} = -\begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} = 1$

$M_{13} = \begin{vmatrix} 1 & 1 \\ -1 & -2 \end{vmatrix} = -1$ $\quad M_{23} = -\begin{vmatrix} 0 & 2 \\ -1 & -2 \end{vmatrix} = -2$ $\quad M_{33} = \begin{vmatrix} 0 & 2 \\ 1 & 1 \end{vmatrix} = -2$

$$\boxed{M^{-1} = \begin{pmatrix} 0 & 2 & 1 \\ 1 & 1 & 1 \\ -1 & -2 & -2 \end{pmatrix}}$$

b) $XM + M = 2M^2$; $XM = 2M^2 - M$; $X = (2M^2-M)M^{-1} = 2M - I$

$X = 2 \cdot \begin{pmatrix} 0 & 2 & 1 \\ 1 & 1 & 1 \\ -1 & -2 & -2 \end{pmatrix} - \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \boxed{\begin{pmatrix} -1 & 4 & 2 \\ 2 & 1 & 2 \\ -2 & -4 & -5 \end{pmatrix}}$

JUN 11 fase general

a) $|A| = \begin{vmatrix} a+1 & 2 & a+1 \\ 0 & a-1 & 1-a \\ 1 & 1 & a \end{vmatrix} = (a+1)(a-1)a + 2(1-a) - (a+1)(a-1) - (a+1)(1-a) =$

$(1-a = -(a-1))$

$= (a+1)(a-1)a - 2(a-1) - \cancel{(a+1)(a-1)} + \cancel{(a+1)(a-1)} =$

$= (a-1)\cdot\left((a+1)a-2\right) = (a-1)(a^2+a-2) = (a-1)(a-1)(a+2) = \boxed{(a-1)^2(a+2)}$

$|A| = 0 \Rightarrow (a-1)^2(a+2) = 0 \; ; \; a = \begin{cases} 1 \\ 2 \end{cases}$

$\boxed{A \text{ tiene inversa para } \begin{cases} a \neq 1 \\ a \neq 2 \end{cases}}$

b) $\underline{a=0}$: $A = \begin{pmatrix} 1 & 2 & 1 \\ 0 & -1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

$|A| = \begin{vmatrix} 1 & 2 & 1 \\ 0 & -1 & 1 \\ 1 & 1 & 0 \end{vmatrix} = 2+1-1 = 2 \neq 0 \Rightarrow \exists A^{-1}$

$A_{11} = \begin{vmatrix} -1 & 1 \\ 1 & 0 \end{vmatrix} = -1$ | $A_{21} = -\begin{vmatrix} 2 & 1 \\ 1 & 0 \end{vmatrix} = 1$ | $A_{31} = \begin{vmatrix} 2 & 1 \\ -1 & 1 \end{vmatrix} = 3$

$A_{12} = -\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = 1$ | $A_{22} = \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} = -1$ | $A_{32} = -\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} = -1$

$A_{13} = \begin{vmatrix} 0 & -1 \\ 1 & 1 \end{vmatrix} = +1$ | $A_{23} = -\begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} = 1$ | $A_{33} = \begin{vmatrix} 1 & 2 \\ 0 & -1 \end{vmatrix} = -1$

$A^{-1} = \frac{1}{2}\begin{pmatrix} -1 & 1 & 3 \\ 1 & -1 & -1 \\ +1 & 1 & -1 \end{pmatrix} = \boxed{\begin{pmatrix} -1/2 & 1/2 & 3/2 \\ 1/2 & -1/2 & -1/2 \\ 1/2 & 1/2 & -1/2 \end{pmatrix}}$

JUN 11 fase específica

a) $|A| = \begin{vmatrix} 1 & a & 1 \\ 1 & 1 & 0 \\ 1 & a & 0 \end{vmatrix} = a-1$

$|A| = 0 \Rightarrow a = 1$

$\boxed{A \text{ tiene inversa para } a \neq 1}$

b) $A_{11} = \begin{vmatrix} 1 & 0 \\ a & 0 \end{vmatrix} = 0$ | $A_{21} = -\begin{vmatrix} a & 1 \\ a & 0 \end{vmatrix} = a$ | $A_{31} = \begin{vmatrix} a & 1 \\ 1 & 0 \end{vmatrix} = -1$

$A_{12} = -\begin{vmatrix} 1 & 0 \\ 1 & 0 \end{vmatrix} = 0$ | $A_{22} = \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} = -1$ | $A_{32} = -\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} = 1$

$A_{13} = \begin{vmatrix} 1 & 1 \\ 1 & a \end{vmatrix} = a-1$ | $A_{23} = -\begin{vmatrix} 1 & a \\ 1 & a \end{vmatrix} = 0$ | $A_{33} = \begin{vmatrix} 1 & a \\ 1 & 1 \end{vmatrix} = 1-a$

$A^{-1} = \frac{1}{a-1}\begin{pmatrix} 0 & a & -1 \\ 0 & -1 & 1 \\ a-1 & 0 & 1-a \end{pmatrix} = \boxed{\begin{pmatrix} 0 & a/a-1 & -1/a-1 \\ 0 & -1/a-1 & 1/a-1 \\ 1 & 0 & 1 \end{pmatrix}}$ (para $a \neq 1$)

Julio 11 fase específica

$AX = B \Rightarrow X = A^{-1}B$ si es que existe A^{-1}.

$|A| = \begin{vmatrix} 2 & 1 & -1 \\ 0 & 1 & 2 \\ 1 & 0 & 1 \end{vmatrix} = 2+2+1 = 5 \neq 0 \Rightarrow \exists A^{-1}$ ✓

$A_{11} = \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} = 1$ | $A_{21} = -\begin{vmatrix} 1 & -1 \\ 0 & 1 \end{vmatrix} = -1$ | $A_{31} = \begin{vmatrix} 1 & -1 \\ 1 & 2 \end{vmatrix} = 3$

$A_{12} = -\begin{vmatrix} 0 & 2 \\ 1 & 1 \end{vmatrix} = 2$ | $A_{22} = \begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} = 3$ | $A_{32} = -\begin{vmatrix} 2 & -1 \\ 0 & 2 \end{vmatrix} = -4$

$A_{13} = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -1$ | $A_{23} = -\begin{vmatrix} 2 & 1 \\ 1 & 0 \end{vmatrix} = 1$ | $A_{33} = \begin{vmatrix} 2 & 1 \\ 0 & 1 \end{vmatrix} = 2$

$\boxed{A^{-1} = \frac{1}{5}\begin{pmatrix} 1 & -1 & 3 \\ 2 & 3 & -4 \\ -1 & 1 & 2 \end{pmatrix}}$

$X = \frac{1}{5}\begin{pmatrix} 1 & -1 & 3 \\ 2 & 3 & -4 \\ -1 & 1 & 2 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 & -1 \\ 0 & 1 & 1 \\ 1 & 2 & 0 \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 4 & 6 & -2 \\ -2 & -3 & 1 \\ 1 & 4 & 2 \end{pmatrix} = \boxed{\begin{pmatrix} 4/5 & 6/5 & -2/5 \\ -2/5 & -3/5 & 1/5 \\ 1/5 & 4/5 & 2/5 \end{pmatrix}}$

JUN 12 fase específica

a) $|A| = \begin{vmatrix} a-1 & 2 & a-1 \\ 0 & a+1 & -1-a \\ 1 & 1 & a \end{vmatrix} = (a-1)(a+1)a + 2(-1-a) - (a+1)(a-1) - (a-1)(-1-a) =$

$-1-a = -(a+1)$

$= (a-1)(a+1)a - 2(a+1) - (a+1)(a-1) + (a-1)(a+1) =$

$= (a+1)\left((a-1)a-2\right) = (a+1)(a^2-a-2) = (a+1)(a+1)(a-2) = (a+1)^2(a-2)$

$|A| = 0 \Rightarrow (a+1)^2(a-2) = 0 \;;\; a = \begin{cases} -1 \\ 2 \end{cases}$

$\boxed{\text{A tiene inversa para } \begin{matrix} a \neq -1 \\ a \neq 2 \end{matrix}}$

b) $a=0$: $A = \begin{pmatrix} -1 & 2 & -1 \\ 0 & 1 & -1 \\ 1 & 1 & 0 \end{pmatrix}$

$|A| = \begin{vmatrix} -1 & 2 & -1 \\ 0 & 1 & -1 \\ 1 & 1 & 0 \end{vmatrix} = -2+1-1 = -2 \neq 0 \Rightarrow \exists A^{-1}$

$A_{11} = \begin{vmatrix} 1 & -1 \\ 1 & 0 \end{vmatrix} = 1 \quad A_{21} = -\begin{vmatrix} 2 & -1 \\ 1 & 0 \end{vmatrix} = -1 \quad A_{31} = \begin{vmatrix} -1 & 2 \\ 0 & 1 \end{vmatrix} = -1$

$A_{12} = -\begin{vmatrix} 0 & -1 \\ 1 & 0 \end{vmatrix} = -1 \quad A_{22} = \begin{vmatrix} -1 & -1 \\ 1 & 0 \end{vmatrix} = 1 \quad A_{32} = -\begin{vmatrix} -1 & -1 \\ 0 & -1 \end{vmatrix} = -1$

$A_{13} = \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} = -1 \quad A_{23} = -\begin{vmatrix} -1 & 2 \\ 1 & 1 \end{vmatrix} = 3 \quad A_{33} = \begin{vmatrix} -1 & 2 \\ 0 & 1 \end{vmatrix} = -1$

$A^{-1} = \frac{1}{-2}\begin{pmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & 3 & -1 \end{pmatrix} = \boxed{\begin{pmatrix} -1/2 & 1/2 & 1/2 \\ 1/2 & -1/2 & 1/2 \\ 1/2 & -3/2 & 1/2 \end{pmatrix}}$

Julio 12 fase general

$A = \begin{pmatrix} x & b & c-4 \\ a & x & 3 \\ b & c & x \end{pmatrix}$

a) $A^t = -A \Rightarrow \begin{cases} x=0 \\ b=-a \\ c-4=-b \\ x=0 \\ 3=-c \\ x=0 \end{cases} \Rightarrow \boxed{x=0}$

$a+b=0 \longrightarrow a=-b=\boxed{-7}$

$b+c=4 \rightarrow b=4-c=\boxed{7}$

$\boxed{c=-3}$

b) $\begin{matrix} a=1 \\ b=1 \\ c=1 \end{matrix} \Bigg| \rightarrow A = \begin{pmatrix} x & 1 & -3 \\ 1 & x & 3 \\ 1 & 1 & x \end{pmatrix}$

$|A| = \begin{vmatrix} x & 1 & -3 \\ 1 & x & 3 \\ 1 & 1 & x \end{vmatrix} = x^3 \cancel{-3} \cancel{+3} \cancel{+3x} -x \cancel{-3x} = x^3-x = x(x^2-1) = x(x+1)(x-1)$

$|A| = 0 \rightarrow x = \begin{cases} 0 \\ 1 \\ -1 \end{cases}$

- $\boxed{\text{Si } \begin{matrix} x\neq 0 \\ x \neq 1 \\ x \neq -1 \end{matrix} \Rightarrow \text{rango}(A) = 3}$
- $x=0$: $A = \begin{pmatrix} 0 & 1 & -3 \\ 1 & 0 & 3 \\ 1 & 1 & 0 \end{pmatrix}$

$|A| = 0 - 3 + 3 = 0 \Rightarrow \text{rango}(A) < 3$

$\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -1 \neq 0 \Rightarrow \text{rango}(A) = 2$

$\boxed{\text{Si } x=0 \Rightarrow \text{rango}(A) = 2}$

• $x=1$: $A=\begin{pmatrix}1 & 1 & -3\\ 1 & 1 & 3\\ 1 & 1 & 1\end{pmatrix}$

$|A|=\begin{vmatrix}1 & 1 & -3\\ 1 & 1 & 3\\ 1 & 1 & 1\end{vmatrix}=\cancel{1}\cancel{-3}\cancel{+3}+\cancel{3}\cancel{-1}-\cancel{3}=0 \rightarrow \text{rango}(A)<3$

$\begin{vmatrix}1 & -3\\ 1 & 3\end{vmatrix}=3+3=6\neq 0 \Rightarrow \text{rango}(A)=2$

$\boxed{\text{Si } x=1 \Rightarrow \text{rango}(A)=2}$

• Si $x=-1$: $A=\begin{pmatrix}-1 & 1 & -3\\ 1 & -1 & 3\\ 1 & 1 & -1\end{pmatrix}$

$|A|=\begin{vmatrix}-1 & 1 & -3\\ 1 & -1 & 3\\ 1 & 1 & -1\end{vmatrix}=\cancel{-1}\cancel{-3}\cancel{+3}\cancel{-3}\cancel{+1}\cancel{+3}=0 \rightarrow \text{rango}(A)<3$

$\begin{vmatrix}-1 & 3\\ 1 & -1\end{vmatrix}=1-3=-2\neq 0 \Rightarrow \text{rango}(A)=2$

$\boxed{\text{Si } x=-1 \Rightarrow \text{rango}(A)=2}$

c) $\left.\begin{matrix}a=0\\ b=0\\ c=0\end{matrix}\right| \rightarrow A=\begin{pmatrix}x & 0 & -4\\ 0 & x & 3\\ 0 & 0 & x\end{pmatrix}$

$A+A^t=\begin{pmatrix}x & 0 & -4\\ 0 & x & 3\\ 0 & 0 & x\end{pmatrix}+\begin{pmatrix}x & 0 & 0\\ 0 & x & 0\\ -4 & 3 & x\end{pmatrix}=\begin{pmatrix}2x & 0 & -4\\ 0 & 2x & 3\\ -4 & 3 & 2x\end{pmatrix}$

$|A+A^t|=\begin{vmatrix}2x & 0 & -4\\ 0 & 2x & 3\\ -4 & 3 & 2x\end{vmatrix}=8x^3-32x-18x=8x^3-50x=2x(4x^2-25)$

$|A+A^t|=0 \Rightarrow 2x(4x^2-25)=0 \Rightarrow \boxed{x \begin{cases}0\\ \pm 5/2\end{cases}}$

Julio 12 fase específica

a) $|A|=\begin{vmatrix}2 & 1 & -1\\ 0 & 1 & 2\\ 0 & 0 & 1\end{vmatrix}=2\neq 0 \Rightarrow \exists A^{-1}$

$A_{11}=\begin{vmatrix}1 & 2\\ 0 & 1\end{vmatrix}=1$ | $A_{21}=-\begin{vmatrix}1 & -1\\ 0 & 1\end{vmatrix}=-1$ | $A_{31}=\begin{vmatrix}1 & -1\\ 1 & 2\end{vmatrix}=3$

$A_{12}=-\begin{vmatrix}0 & 2\\ 0 & 1\end{vmatrix}=0$ | $A_{22}=\begin{vmatrix}2 & -1\\ 0 & 1\end{vmatrix}=2$ | $A_{32}=-\begin{vmatrix}2 & -1\\ 0 & 2\end{vmatrix}=-4$

$A_{13}=\begin{vmatrix}0 & 1\\ 0 & 0\end{vmatrix}=0$ | $A_{23}=-\begin{vmatrix}2 & 1\\ 0 & 0\end{vmatrix}=0$ | $A_{33}=\begin{vmatrix}2 & 1\\ 0 & 1\end{vmatrix}=2$

$\boxed{A^{-1}=\frac{1}{2}\begin{pmatrix}1 & -1 & 3\\ 0 & 2 & -4\\ 0 & 0 & 2\end{pmatrix}}$

b) $XA=B \Rightarrow X=B\cdot A^{-1}=\begin{pmatrix}1 & -1 & 0\\ 2 & 0 & 3\end{pmatrix}\cdot\frac{1}{2}\begin{pmatrix}1 & -1 & 3\\ 0 & 2 & -4\\ 0 & 0 & 2\end{pmatrix}=$

$=\frac{1}{2}\begin{pmatrix}1 & -3 & 7\\ 2 & -2 & 12\end{pmatrix}=\boxed{\begin{pmatrix}1/2 & -3/2 & 7/2\\ 1 & -1 & 6\end{pmatrix}}$

Julio 12 fase específica

a) $|A|=\begin{vmatrix}-x & 1 & 0\\ 0 & -x & 1\\ c & b & a-x\end{vmatrix}=x^2(a-x)+c+bx=\boxed{-x^3+ax^2+bx+c}$

b) $c=0 \rightarrow p(x)=-x^3+ax^2+bx$

$-x^3+ax^2+bx=0$; $x(-x^2+ax+b)=0 \begin{cases}\boxed{x=0}\\ -x^2+ax+b=0\end{cases}$

$x=\boxed{\dfrac{a\pm\sqrt{a^2+4b}}{-2a}}$

Junio 13 fase General

$$A=\begin{pmatrix}1&a&1\\1&1&0\\1&0&0\end{pmatrix}$$

$$M=A^2-A^t=\begin{pmatrix}1&a&1\\1&1&0\\1&0&0\end{pmatrix}\begin{pmatrix}1&a&1\\1&1&0\\1&0&0\end{pmatrix}-\begin{pmatrix}1&1&1\\a&1&0\\1&0&0\end{pmatrix}=$$

$$=\begin{pmatrix}a+2&2a&1\\2&a+1&1\\1&a&1\end{pmatrix}-\begin{pmatrix}1&1&1\\a&1&0\\1&0&0\end{pmatrix}=\begin{pmatrix}a+1&2a-1&0\\2-a&a&1\\0&a&1\end{pmatrix}$$

$$|M|=\begin{vmatrix}a+1&2a-1&0\\2-a&a&1\\0&a&1\end{vmatrix}=a(\cancel{a+1})-(2-a)(2a-1)-a(\cancel{a+1})=(a-2)(2a-1)$$

$$|M|=0 \Rightarrow \boxed{a=\begin{cases}2\\1/2\end{cases}}$$

- Si $\left.\begin{matrix}a\neq 2\\a\neq 1/2\end{matrix}\right| \Rightarrow r(M)=3$
- Si $a=2$: $M=\begin{pmatrix}3&3&0\\0&2&1\\0&2&1\end{pmatrix}$ $\quad\begin{vmatrix}3&3\\0&2\end{vmatrix}=6\neq0 \Rightarrow r(M)=2$
- Si $a=1/2$: $M=\begin{pmatrix}3/2&0&0\\3/2&1/2&1\\0&1/2&1\end{pmatrix}$ $\quad\begin{vmatrix}3/2&0\\3/2&1/2\end{vmatrix}=\frac{3}{4}\neq0 \Rightarrow r(M)=2$

JUN 13 fase Específica

$$|A|=\begin{vmatrix}1&a&1\\1-a&1&2\\a&a^2&-1\end{vmatrix}=-1+a^2(1-a)+2a^2-a+a(1-a)-2a^2=$$

$$=-1+\cancel{a^2}-a^3+\cancel{2a^2}-\cancel{a}+\cancel{a}-\cancel{a^2}-\cancel{2a^2}=-1-a^3$$

$$|A|=0 \Rightarrow -1-a^3=0\ ;\quad a^3=-1\ ;\quad a=\sqrt[3]{-1}=-1$$

a) • Si $a=-1$, no existe A^{-1}

• Si $a\neq-1$, sí existe A^{-1}

b) $a=0$ $\quad A=\begin{pmatrix}1&0&1\\1&1&2\\0&0&-1\end{pmatrix}$

$$|A|=\begin{vmatrix}1&0&1\\1&1&2\\0&0&-1\end{vmatrix}=-1$$

$A_{11}=\begin{vmatrix}1&2\\0&-1\end{vmatrix}=-1$ $\quad A_{12}=-\begin{vmatrix}1&2\\0&-1\end{vmatrix}=1$ $\quad A_{13}=\begin{vmatrix}1&1\\0&0\end{vmatrix}=0$

$A_{21}=-\begin{vmatrix}0&1\\0&-1\end{vmatrix}=0$ $\quad A_{22}=\begin{vmatrix}1&1\\0&-1\end{vmatrix}=-1$ $\quad A_{23}=-\begin{vmatrix}1&0\\0&0\end{vmatrix}=0$

$A_{31}=\begin{vmatrix}0&1\\1&2\end{vmatrix}=-1$ $\quad A_{32}=-\begin{vmatrix}1&1\\1&2\end{vmatrix}=-1$ $\quad A_{33}=\begin{vmatrix}1&0\\1&1\end{vmatrix}=1$

$$A^{-1}=\frac{1}{-1}\begin{pmatrix}-1&0&-1\\1&-1&-1\\0&0&1\end{pmatrix}=\boxed{\begin{pmatrix}1&0&1\\-1&1&1\\0&0&-1\end{pmatrix}}$$

JUL 13 fase General

$$|A| = \begin{vmatrix} -a & 2 & 0 \\ 1 & -1-a & 0 \\ 0 & 0 & 1-a \end{vmatrix} = -a(-1-a)(1-a) - 2(1-a) = (1-a)(a+a^2-2)$$

$|A| = 0 \to 1-a=0 \;;\; a=1$

$\to a^2+a-2=0 \;;\; a = 1, -2$

JUL 13 fase Específica

$$A - x I_3 = \begin{pmatrix} 0 & 2 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - x \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -x & 2 & 0 \\ 1 & -1-x & 0 \\ 0 & 0 & 1-x \end{pmatrix}$$

$$p(x) = |A - x I_3| = \begin{vmatrix} -x & 2 & 0 \\ 1 & -1-x & 0 \\ 0 & 0 & 1-x \end{vmatrix} = -x(-1-x)(1-x) - 2(1-x) =$$

$$= (1-x)(+x+x^2-2) = -(x-1)(x^2+x-2)$$

$p(x) = 0 \Rightarrow \to x-1=0 \;;\; x=1$

$\to x^2+x-2=0 \;;\; x = 1, -2$ $\Rightarrow \boxed{p(x) = -(x-1)^2(x+2)}$

JUN 14 fase Específica

a) $$|A| = \begin{vmatrix} \sec\theta & \operatorname{tg}\theta & 0 \\ \operatorname{tg}\theta & \sec\theta & 0 \\ 0 & 0 & -1 \end{vmatrix} = -\sec^2\theta + \operatorname{tg}^2\theta = -\frac{1}{\cos^2\theta} + \frac{\operatorname{sen}^2\theta}{\cos^2\theta} =$$

$$= \frac{\operatorname{sen}^2\theta - 1}{\cos^2\theta} = \frac{-(1-\operatorname{sen}^2\theta)}{\cos^2\theta} = \frac{-\cos^2\theta}{\cos^2\theta} = -1$$

- $|A| \neq 0$ para cualquier θ en que esté definida la matriz. Por lo que existirá A^{-1} para $\theta \neq \pm\pi/2$.
- Para $\theta = \pm\pi/2$ no es que se anule el determinante, es que no está definida la matriz (ni su inversa, claro)

JUL 14 fase General

$$A - x I_2 = \begin{pmatrix} a & b \\ c & d \end{pmatrix} - x \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} a-x & b \\ c & d-x \end{pmatrix}$$

$$|A - x I_2| = \begin{vmatrix} a-x & b \\ c & d-x \end{vmatrix} = (a-x)(d-x) - bc = ad - ax - dx + x^2 - bc =$$

$$= x^2 - (a+d)x + (ad-bc)$$

Traza $(A) = a+d$ ✓

$\det(A) = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$ ✓

JUL 14 fase Específica

a) $$|A| = \begin{vmatrix} 1 & 1 & -1 \\ 1 & 0 & 2 \\ 0 & 2 & -1 \end{vmatrix} = -2 + 1 - 4 = \boxed{-5}$$

b) $$|3A| = 3^3 \cdot |A| = 27 \cdot (-5) = \boxed{-135}$$

También: $$3A = 3 \cdot \begin{pmatrix} 1 & 1 & -1 \\ 1 & 0 & 2 \\ 0 & 2 & -1 \end{pmatrix} = \begin{pmatrix} 3 & 3 & -3 \\ 3 & 0 & 6 \\ 0 & 6 & -3 \end{pmatrix}$$

$$|3A| = \begin{vmatrix} 3 & 3 & -3 \\ 3 & 0 & 6 \\ 0 & 6 & -3 \end{vmatrix} = -54 + 27 - 108 = -135 \checkmark$$

c) $$|(3A)^3| = |(3A)\cdot(3A)\cdot(3A)| =$$

$$= |3A| \cdot |3A| \cdot |3A| = (-135)^3 = \boxed{-2\,460\,375}$$

Junio 15
fase
General

a) $|A| = \begin{vmatrix} a & b & c \\ a & x & c \\ a & b & x \end{vmatrix} = ax^2 + \cancel{abc} + abc - acx - abx - \cancel{abc} =$

$= ax^2 - (ab+ac)x + abc =$

$= a\cdot\left(x^2 - (b+c)x + bc\right)$

$|A| = 0 \Rightarrow x^2 - (b+c)x + bc = 0$

$x = \dfrac{b+c \pm \sqrt{(b+c)^2 - 4bc}}{2} = \dfrac{b+c \pm \sqrt{b^2 + 2bc + c^2 - 4bc}}{2} =$

$= \dfrac{b+c \pm \sqrt{b^2 - 2bc + c^2}}{2} = \dfrac{b+c \pm \sqrt{(b-c)^2}}{2} =$

$= \dfrac{b+c \pm (b-c)}{2} = \boxed{\begin{matrix} b \\ c \end{matrix}}$

También: Es obvio que $|A|$ es un polinomio de 2º grado, luego tendrá dos raíces.
Para $x = b$, la 2ª columna es proporcional a la 1ª columna, por lo que el determinante será 0.
Lo mismo sucede para $x = c$ con la 1ª y 3ª columnas.
Por lo tanto, las raíces son $x = b$, $x = c$ ✓

b) $\left.\begin{matrix} x = 1 \\ b = c = 2 \end{matrix}\right\} \longrightarrow |A| = a\cdot\left(1^2 - (2+2)\cdot 1 + 2\cdot 2\right) = a$

$\boxed{A \text{ tiene inversa si } a \neq 0}$

c) $\left.\begin{matrix} x = 0 \\ b = c = a = 1 \end{matrix}\right\} \longrightarrow A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

$|A| = 1 + 1 - 1 = 1 \neq 0 \longrightarrow$ Tiene inversa.

$A_{11} = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -1$ $\quad A_{12} = -\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} = 1$ $\quad A_{13} = \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} = 1$

$A_{21} = -\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} = 1$ $\quad A_{22} = \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} = -1$ $\quad A_{23} = -\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 0$

$A_{31} = \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} = 1$ $\quad A_{32} = -\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 0$ $\quad A_{33} = \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} = -1$

$$\boxed{A^{-1} = \begin{pmatrix} -1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{pmatrix}}$$

Junio 15
fase
Específica

a) $A^2 = I_3 \Rightarrow |A^2| = |I_3| \Rightarrow |A|^2 = 1 \Rightarrow |A| = \pm 1 \neq 0 \Rightarrow$ Existe A^{-1}

$A^2 = I_3 \Rightarrow A\cdot A = I_3 \Rightarrow A$ es inversa de sí misma. $\boxed{A^{-1} = A}$

b) A^n dependerá de la paridad del exponente n:

- Si n es par: $n = 2m \longrightarrow A^n = A^{2m} = (A^2)^m = (I_3)^m = \boxed{I_3}$
- Si n es impar: $n = 2m+1 \longrightarrow A^n = A^{2m+1} = A^{2m}\cdot A = I_3\cdot A = \boxed{A}$

c) $A^2 = \begin{pmatrix} 1 & 1 & 1 \\ 0 & a & 0 \\ 0 & 0 & a \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 & 1 \\ 0 & a & 0 \\ 0 & 0 & a \end{pmatrix} = \begin{pmatrix} 1 & 1+a & 1+a \\ 0 & a^2 & 0 \\ 0 & 0 & a^2 \end{pmatrix}$

$A^2 = I_3 \Rightarrow \left.\begin{matrix} 1+a=0 \rightarrow a=-1 \\ a^2=1 \rightarrow a = \pm 1 \end{matrix}\right\} \Rightarrow \boxed{a=-1}$

Julio 15 fase General

a) $A = \begin{pmatrix} 1 & -1 & -a \\ a & 3 & 1 \\ -2 & a & 2a \end{pmatrix}$

$|A| = \begin{vmatrix} 1 & -1 & -a \\ a & 3 & 1 \\ -2 & a & 2a \end{vmatrix} = \cancel{6a} - a^3 + 2 - \cancel{6a} + 2a^2 - a = -a^3 + 2a^2 - a + 2$

$-a^3 + 2a^2 - a + 2 = 0$

	-1	2	-1	2
2		-2	0	-2
	-1	0	-1	$\boxed{0}$

$-a^2 - 1 = 0$; $a^2 = -1$ Absurdo

$\boxed{\text{A tendrá inversa para } a \neq 2}$

b) $\underline{a=0}$ $\quad A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 3 & 1 \\ -2 & 0 & 0 \end{pmatrix}$

$|A| = \begin{vmatrix} 1 & -1 & 0 \\ 0 & 3 & 1 \\ -2 & 0 & 0 \end{vmatrix} = 2$

$A_{11} = \begin{vmatrix} 3 & 1 \\ 0 & 0 \end{vmatrix} = 0 \quad A_{12} = -\begin{vmatrix} 0 & 1 \\ -2 & 0 \end{vmatrix} = -2 \quad A_{13} = \begin{vmatrix} 0 & 3 \\ -2 & 0 \end{vmatrix} = 6$

$A_{21} = -\begin{vmatrix} -1 & 0 \\ 0 & 0 \end{vmatrix} = 0 \quad A_{22} = \begin{vmatrix} 1 & 0 \\ -2 & 0 \end{vmatrix} = 0 \quad A_{23} = -\begin{vmatrix} 1 & -1 \\ -2 & 0 \end{vmatrix} = 2$

$A_{31} = \begin{vmatrix} -1 & 0 \\ 3 & 1 \end{vmatrix} = -1 \quad A_{32} = -\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = -1 \quad A_{33} = \begin{vmatrix} 1 & -1 \\ 0 & 3 \end{vmatrix} = 3$

$$\boxed{A^{-1} = \frac{1}{2} \begin{pmatrix} 0 & 0 & -1 \\ -2 & 0 & -1 \\ 6 & 2 & 3 \end{pmatrix}}$$

Julio 15 fase Específica

a) $A^{-1} = A^t \Rightarrow |A^{-1}| = |A^t| \Rightarrow \frac{1}{|A|} = |A| \Rightarrow |A|^2 = 1 \Rightarrow \boxed{|A| = \pm 1}$

b) $|A^n|$ dependerá de la paridad de n:

- Si n es par: $n = 2m$ $\quad |A^n| = |A^{2m}| = |A|^{2m} = (\pm 1)^{2m} = \boxed{1}$
- Si n es impar: $n = 2m+1$ $\quad |A^n| = |A^{2m+1}| = |A^{2m} \cdot A| =$
  $= |A|^{2m} \cdot |A| = (\pm 1)^{2m} \cdot (\pm 1) =$
  $= 1 \cdot (\pm 1) = \boxed{\pm 1}$

c) $|A^{-1}| = \frac{1}{|A|} = \frac{1}{\pm 1} = \boxed{\pm 1}$

Junio 16
fase
General

a) $|A| = \begin{vmatrix} a+b & a & a \\ a & a+b & a \\ a & a & a+b \end{vmatrix} = (a+b)^3 + a^3 + a^3 - a^2(a+b) - a^2(a+b) - a^2(a+b) =$

$= (a+b)^3 + 2a^3 - 3a^2(a+b) = \cancel{a^3} + \cancel{3a^2b} + 3ab^2 + b^3 + \cancel{2a^3} - \cancel{3a^3} - \cancel{3a^2b} =$

$= 3ab^2 + b^3 = b^2(3a+b)$

- Si $\left.\begin{matrix} b \neq 0 \\ 3a+b \neq 0 \end{matrix}\right| \Rightarrow \boxed{\text{rango } A = 3}$ por ser $|A| \neq 0$
- Si $b = 0 \Rightarrow A = \begin{pmatrix} a & a & a \\ a & a & a \\ a & a & a \end{pmatrix} \Rightarrow$

  $\Rightarrow$ Si $a=0 \rightarrow A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow \boxed{r(A) = 0}$

  Si $a \neq 0 \rightarrow A = a\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \Rightarrow \boxed{r(A) = 1}$

  Por tener 3 filas iguales.
- Si $3a+b=0 \Rightarrow b = -3a \rightarrow A = \begin{pmatrix} a-3a & a & a \\ a & a-3a & a \\ a & a & a-3a \end{pmatrix} = \begin{pmatrix} -2a & a & a \\ a & -2a & a \\ a & a & -2a \end{pmatrix} \Rightarrow$

  $\Rightarrow$ Si $a=0 \rightarrow A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow \boxed{r(A) = 0}$ Caso idéntico al anterior

  Si $a \neq 0 \rightarrow A = -2a\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \Rightarrow \boxed{r(A) = 1}$

Resumen:

$b \neq 0$, $3a+b \neq 0$	$\Rightarrow$	$r(A) = 3$
$a = 0$, $b = 0$	$\Rightarrow$	$r(A) = 0$
$a \neq 0$, $b = 0$	$\Rightarrow$	$r(A) = 1$
$a \neq 0$, $3a+b=0$	$\Rightarrow$	$r(A) = 1$

Junio 16
fase
Específica

a) $|A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & c \\ 1 & 8 & c \end{vmatrix} = 2c + 8 + \cancel{c} - 2 - \cancel{c} - 8c = \boxed{6-6c}$

b) $6 - 6c = 0 \Rightarrow \boxed{c = 1}$

Julio 16
fase
General

$A = \begin{pmatrix} x & b & c \\ a & x & 1 \\ b & c & x \end{pmatrix}$

a) A simétrica $\Rightarrow \left.\begin{matrix} a = b \\ b = c \\ c = 1 \end{matrix}\right\} \Rightarrow \boxed{\begin{matrix} a=1 \\ b=1 \\ c=1 \end{matrix}}$

b) $A = \begin{pmatrix} x & 1 & 1 \\ 1 & x & 1 \\ 1 & 1 & x \end{pmatrix}$

$|A| = \begin{vmatrix} x & 1 & 1 \\ 1 & x & 1 \\ 1 & 1 & x \end{vmatrix} = x^3 + 1 + 1 - x - x - x = x^3 - 3x + 2$

$x^3 - 3x + 2 = 0$

$$\begin{array}{r|rrrr} & 1 & 0 & -3 & 2 \\ 1 & & 1 & 1 & -2 \\ \hline & 1 & 1 & -2 & \boxed{0} \\ 1 & & 1 & 2 & \\ \hline & 1 & 2 & \boxed{0} & \\ -2 & & -2 & & \\ \hline & 1 & \boxed{0} & & \end{array}$$

$\boxed{\text{Existirá } A^{-1} \text{ para } \begin{matrix} x \neq 1 \\ x \neq -2 \end{matrix}}$

Julio 16 fase específica

a) $a=1$, $b=2$: $p(x) = \begin{vmatrix} x & 0 & c \\ -1 & x & 2 \\ 0 & -1 & 1 \end{vmatrix} = x^2+c+2x = x^2+2x+c$

$p(x)=0 \Rightarrow x^2+2x+c=0$

$x = \frac{-2 \pm \sqrt{4-4c}}{2}$

Tendrá una raíz doble si $4-4c=0 \Rightarrow \boxed{c=1}$

b) $x = \frac{-2 \pm 0}{2} \Rightarrow \boxed{x=-1}$

Modelo 17

Es el mismo de Junio 2011 fase general.