① $x^2+y^2-4x-6y+4=0$

$x=0 \longrightarrow y^2-6y+4=0 \qquad y=\frac{6\pm\sqrt{36-16}}{2}=\frac{6\pm\sqrt{20}}{2}=3\pm\sqrt{5} \qquad \boxed{(0,\ 3\pm\sqrt{5})}$

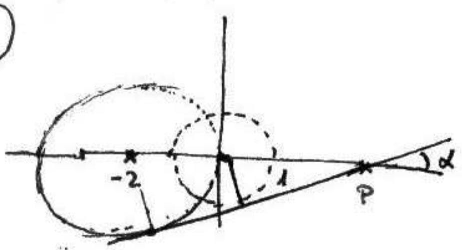
$2x+2yy'-4-6y'=0 \longrightarrow (2y-6)y'=4-2x \longrightarrow y'=\frac{4-2x}{2y-6}=\frac{2-x}{y-3}$

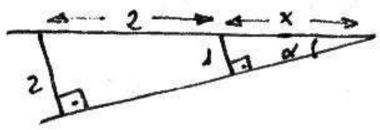
$(0,3+\sqrt{5}) \longrightarrow y'=\frac{2}{\sqrt{5}} \longrightarrow \vec{v}=(\sqrt{5},2)$

$(0,3-\sqrt{5}) \longrightarrow y'=\frac{2}{-\sqrt{5}} \longrightarrow \vec{w}=(-\sqrt{5},2)$

$\cos\alpha=\frac{|(\sqrt{5},2)\cdot(-\sqrt{5},2)|}{\sqrt{5+4}\ \sqrt{5+4}}=\frac{|-5+4|}{9}=\left|-\frac{1}{9}\right|=\frac{1}{9} \Rightarrow \boxed{\alpha=83{,}62^\circ}$

②





$\frac{1}{x}=\frac{2}{2+x} \longrightarrow 2+x=2x \Rightarrow 2=x \longrightarrow P(2,0)$

$\operatorname{Sen}\alpha=\frac{1}{2} \Rightarrow \boxed{\alpha=30^\circ} \longrightarrow y-0=\operatorname{Tg}30^\circ\cdot(x-2)$

$\boxed{y=\frac{x-2}{\sqrt{3}}}$

③ $y=\frac{2x+5}{x+3}$

$y'=\frac{2(x+3)-(2x+5)}{(x+3)^2}=\frac{1}{(x+3)^2}$

$y'=\frac{1}{9} \Rightarrow \frac{1}{9}=\frac{1}{(x+3)^2} \Rightarrow (x+3)^2=9 \Rightarrow x+3=\pm 3 \begin{cases} x=0 \\ x=-6 \end{cases}$

$x=0 \begin{cases} y=\frac{5}{3} \\ y'=\frac{1}{9} \end{cases} \qquad \boxed{y-\frac{5}{3}=\frac{1}{9}x}$

$x=-6 \begin{cases} y=-\frac{7}{3} \\ y'=\frac{1}{9} \end{cases} \qquad \boxed{y+\frac{7}{3}=\frac{1}{9}(x+6)}$

④ $y=\frac{x^2+11}{2x+1} \qquad y'=\frac{2x(2x+1)-(x^2+11)\cdot 2}{(2x+1)^2}=\frac{2x^2+2x-22}{(2x+1)^2}$

$x=2 \begin{cases} y=3 \\ y'=-\frac{10}{25}=-\frac{2}{5} \end{cases} \qquad \boxed{y-3=\frac{5}{2}(x-2)}$

(5) $3y^2+5xy=13$

$6yy'+5y+5xy'=0 \longrightarrow y'(6y+5x)=-5y \longrightarrow y'=\dfrac{-5y}{6y+5x}$

$y=1 \rightarrow 3+5x=13 \longrightarrow x=2$

$\rightarrow y'=\dfrac{-5}{6+10}=-\dfrac{5}{16}$

$\boxed{y-1=-\dfrac{5}{16}(x-2)}$

(6) $2x^2+2y^2+12x-16y+45=0$

$4x+4yy'+12-16y'=0 \longrightarrow y'(4y-16)=-12-4x \longrightarrow y'=\dfrac{-12-4x}{4y-16}=\dfrac{3+x}{4-y}$

$P(-1'5,4'5) \Rightarrow y'=\dfrac{3-1'5}{4-4'5}=\dfrac{1'5}{-0'5}=-3$

$y-4'5=-3(x+1'5)$

$\boxed{y=-3x}$ Recta tangente que pasa por $(0,0)$

$P(x_0,y_0) \longrightarrow y-y_0=\dfrac{3+x_0}{4-y_0}(x-x_0)$

Si debe pasar por el origen: $-y_0=-\dfrac{3+x_0}{4-y_0}x_0$

$-4y_0+y_0^2=-3x_0-x_0^2$

$x_0^2+y_0^2+3x_0-4y_0=0$

$$\left.\begin{aligned}2x_0^2+2y_0^2+12x_0-16y_0+45&=0\\ x_0^2+y_0^2+3x_0-4y_0&=0\end{aligned}\right\} \longrightarrow \begin{aligned}&2x_0^2+2y_0^2+12x_0-16y_0+45=0\\ -\;&2x_0^2+2y_0^2+6x_0-8y_0=0\\ \hline &6x_0-8y_0+45=0 \rightarrow y_0=\frac{6x_0+45}{8}\end{aligned}$$

$x_0^2+\left(\dfrac{6x_0+45}{8}\right)^2+3x_0-4\,\dfrac{6x_0+45}{8}=0 \longrightarrow x_0^2+\dfrac{36x_0^2+540x_0+2025}{64}+3x_0-\dfrac{24x_0+180}{8}=0$

$\rightarrow 64x_0^2+36x_0^2+540x_0+2025+192x_0-192x_0-1440=0$

$100x_0^2+540x_0+585=0$

$20x_0^2+108x_0+117=0 \longrightarrow x_0=\dfrac{-108\pm\sqrt{11\,664-9360}}{40}=\dfrac{-108\pm48}{40}$ $\rightarrow -\dfrac{3}{2}$ (Es el punto $P(-1'5,4'5)$) $\rightarrow \boxed{-\dfrac{39}{10}}$ ✓

$x_0=-\dfrac{39}{10} \rightarrow y_0=\dfrac{-6\,\frac{39}{10}+45}{8}=\dfrac{27}{10} \Rightarrow y-\dfrac{27}{10}=\dfrac{3-\frac{39}{10}}{4-27/10}\left(x+\dfrac{39}{10}\right) \Rightarrow \boxed{y=-\dfrac{9}{13}x}$

(7) $y = x^3(4-x)$

$y' = 3x^2(4-x) + x^3\cdot(-1) = 12x^2 - 3x^3 - x^3 = 12x^2 - 4x^3 = 4x^2(3-x)$

$x=4 \begin{cases} y=0 \\ y'=-64 \end{cases}$ $\boxed{y = \frac{1}{64}(x-4)}$

(8) $y = x^3 - 3x^2 + 5x + 1 \longrightarrow y' = 3x^2 - 6x + 5$

Pend $=5 \Rightarrow 3x^2-6x+5=5 \Rightarrow 3x^2-6x=0 \Rightarrow 3x(x-2)=0 \begin{cases} x=0 \\ x=2 \end{cases}$

$x=0 \begin{cases} y=1 \\ y'=5 \end{cases}$ $\boxed{y-1=5x}$

$x=2 \begin{cases} y=7 \\ y'=5 \end{cases}$ $\boxed{y-7=5(x-2)}$

Pend $=1 \Rightarrow 3x^2-6x+5=1 \Rightarrow 3x^2-6x+4=0$

$x = \dfrac{6 \pm \sqrt{36-48}}{6}$ $\boxed{\text{No hay solución}}$

Pend $=m \Rightarrow 3x^2-6x+5=m \Rightarrow 3x^2-6x+(5-m)=0$

$x = \dfrac{6 \pm \sqrt{36-12(5-m)}}{6} = \dfrac{6 \pm \sqrt{12m-24}}{6}$

Si $12m-24>0 \longrightarrow$ hay dos rectas tangentes

Si $12m-24<0 \longrightarrow$ No hay ninguna recta tangente

Si $12m-24=0 \Rightarrow \boxed{m=2}$ hay una única recta tangente.

(9) $x^2+y^2+mx+ny+p=0$

$A(0,0) \Rightarrow \boxed{p=0}$

$B(1,2) \Rightarrow 1+4+m+2n+p=0$

$C(2,1) \Rightarrow 4+1+2m+n+p=0$

$\longrightarrow \begin{cases} m+2n=-5 \\ 2m+n=-5 \end{cases} \Rightarrow \boxed{\begin{array}{l} m=-\frac{5}{3} \\ n=-\frac{5}{3} \end{array}}$

$x^2+y^2-\frac{5}{3}x-\frac{5}{3}y=0$

$2x+2yy'-\frac{5}{3}-\frac{5}{3}y'=0 \longrightarrow y'\left(2y-\frac{5}{3}\right)=\frac{5}{3}-2x \rightarrow y' = \dfrac{\frac{5}{3}-2x}{2y-\frac{5}{3}} = \dfrac{5-6x}{6y-5}$

$A(0,0) \rightarrow y' = -1 \qquad \boxed{y = -x}$

$B(1,2) \rightarrow y' = \dfrac{-1}{7} \qquad \boxed{y - 2 = -\frac{1}{7}(x-1)}$

$C(2,1) \rightarrow y' = \dfrac{-7}{1} \qquad \boxed{y - 1 = -7(x-2)}$

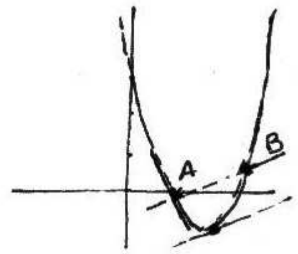
(10) $y = 2x^2 - 5x + 3 \qquad y' = 4x - 5$

$A(1,0) \longrightarrow y' = -1 \qquad y = +1(x-1) \rightarrow \boxed{y = x - 1}$

$\left.\begin{array}{l} y = 2x^2 - 5x + 3 \\ y = x - 1 \end{array}\right\} \quad \begin{array}{l} 2x^2 - 5x + 3 = x - 1 \\ 2x^2 - 6x + 4 = 0 \longrightarrow x = \begin{cases} 1 \rightarrow A(1,0) \\ 2 \rightarrow y = 1 \quad \boxed{B(2,1)} \end{cases} \end{array}$

$\vec{AB} = (1,1) \rightarrow$ pend $= 1$

$1 = 4x - 5 \Rightarrow x = \dfrac{3}{2} \begin{cases} y = 0 \\ y' = 1 \end{cases} \qquad \boxed{y = x - \frac{3}{2}}$



(11) $xy^2 + x^2 y = 2$

$y^2 + 2xyy' + 2xy + x^2 y' = 0$

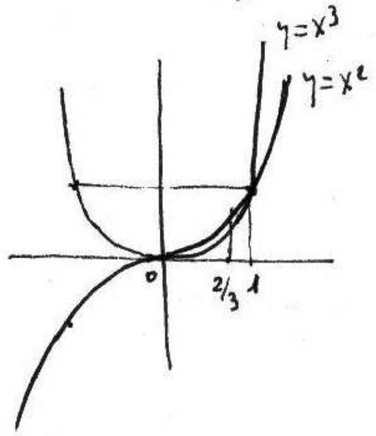
$(2xy + x^2)y' = -y^2 - 2xy \rightarrow y' = \dfrac{-y^2 - 2xy}{2xy + x^2}$

$P(1,1) \rightarrow y' = \dfrac{-1-2}{2+1} = \boxed{-1} \rightarrow y - 1 = 1(x-1) \rightarrow \boxed{y = x}$

(12) $\left.\begin{array}{l} f(x) = x^2 \longrightarrow f'(x) = 2x \\ g(x) = x^3 \longrightarrow g'(x) = 3x^2 \end{array}\right\} \quad f'(x) - g'(x) = 2x - 3x^2 = x(2-3x)$

$\ominus \quad 0 \quad \oplus \quad 2/3 \quad \ominus$

Signo $f'(x) - g'(x) = x(2-3x)$



$f'(x) > g'(x)$ en $x \in (0, \frac{2}{3})$

$f'(x) < g'(x)$ en $x \in (-\infty, 0) \cup (\frac{2}{3}, +\infty)$

$f'(x) = g'(x)$ en $x = 0$ y $x = \frac{2}{3}$

(13) $3x^2-2xy+y^2=9$

$$6x-2y-2xy'+2yy'=0 \longrightarrow y'=\frac{2y-6x}{2y-2x}=\frac{y-3x}{y-x}$$

$$P(2,3) \longrightarrow y'=\frac{3-6}{3-2}=-3 \qquad \boxed{y-3=-3(x-2)}$$

(14) $x^2y^3-x^3y^2-12=0$

$$2xy^3+3x^2y^2y'-3x^2y^2-2x^3yy'=0$$

$$y'=\frac{3x^2y^2-2xy^3}{3x^2y^2-2x^3y}=\frac{3xy-2y^2}{3xy-2x^2}$$

$$P(-1,2) \longrightarrow y'=\frac{-6-8}{-6-2}=\frac{-14}{-8}=\frac{7}{4}$$

$$\boxed{y-2=\frac{7}{4}(x+1)}$$

(15) $y=3x-\operatorname{tg}x$

$$y'=3-(1+\operatorname{tg}^2x)=2-\operatorname{tg}^2x$$

$$y=x-2 \Rightarrow \text{pend}=1 \longrightarrow 1=2-\operatorname{tg}^2x \Rightarrow \operatorname{tg}^2x=1$$

$$\operatorname{tg}x=\pm1 \qquad \boxed{x=\frac{\pi}{4}+k\cdot\frac{\pi}{2}}$$

$k=0,\pm1,\pm2,\dots$