1

a) $\sin 75^\circ = \sin(45^\circ+30^\circ) = \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ = \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{1}{2} = \boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$

b) $\sin 15^\circ = \sin(45^\circ-30^\circ) = \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ = \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2} = \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

c) $\operatorname{tg} 135^\circ = \operatorname{tg}(180^\circ-45^\circ) = \frac{\operatorname{tg}180^\circ - \operatorname{tg}45^\circ}{1+\operatorname{tg}180^\circ\cdot\operatorname{tg}45^\circ} = \frac{0-1}{1+0\cdot 1} = \frac{-1}{1} = \boxed{-1}$

d) $\operatorname{Tg} 285^\circ = \operatorname{Tg}(-75^\circ) = -\operatorname{Tg} 75^\circ = -\frac{\sin 75^\circ}{\cos 75^\circ} = -\frac{\sin 75^\circ}{\sin 15^\circ} = -\frac{\frac{\sqrt{6}+\sqrt{2}}{4}}{\frac{\sqrt{6}-\sqrt{2}}{4}} = -\frac{\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}} =$

$= -\frac{(\sqrt{6}+\sqrt{2})(\sqrt{6}+\sqrt{2})}{(\sqrt{6}-\sqrt{2})(\sqrt{6}+\sqrt{2})} = -\frac{(\sqrt{6})^2+2\sqrt{6}\sqrt{2}+(\sqrt{2})^2}{(\sqrt{6})^2-(\sqrt{2})^2} = -\frac{6+2\sqrt{12}+2}{6-2} = -\frac{8+4\sqrt{3}}{4} = \boxed{-2-\sqrt{3}}$

2

a) $\cos(3x) = \cos(2x+x) = \cos 2x \cos x - \sin 2x \sin x = (\cos^2 x - \sin^2 x)\cos x - 2\sin x\cos x\cdot\sin x =$

$= (\cos^2 x - 1 + \cos^2 x)\cos x - 2\sin^2 x\cos x = (2\cos^2 x - 1)\cos x - 2(1-\cos^2 x)\cos x =$

$= (2\cos^2 x - 1 - 2 + 2\cos^2 x)\cos x = \boxed{(4\cos^2 x - 3)\cos x} = \boxed{4\cos^3 x - 3\cos x}$

b) $\cos(4x) = \cos 2(2x) = \cos^2(2x) - \sin^2(2x) = (\cos^2 x - \sin^2 x)^2 - (2\sin x\cos x)^2 =$

$= (\cos^2 x - 1 + \cos^2 x)^2 - 4\sin^2 x\cos^2 x = (2\cos^2 x - 1)^2 - 4(1-\cos^2 x)\cos^2 x =$

$= 4\cos^4 x - 4\cos^2 x + 1 - 4\cos^2 x + 4\cos^4 x = \boxed{8\cos^4 x - 8\cos^2 x + 1}$

3

a) $\sin a = \frac{3}{5} \rightarrow \cos a = -\sqrt{1-\frac{9}{25}} = -\frac{4}{5} \rightarrow \operatorname{tg} a = \frac{3/5}{-4/5} = -\frac{3}{4}$

$\cos b = \frac{5}{13} \rightarrow \sin b = -\sqrt{1-\frac{25}{169}} = -\frac{12}{13} \rightarrow \operatorname{tg} b = \frac{-12/13}{5/13} = -\frac{12}{5}$

$\sin(a+b) = \sin a\cdot\cos b + \cos a\cdot\sin b = \frac{3}{5}\cdot\frac{5}{13} + \frac{-4}{5}\cdot\frac{-12}{13} = \boxed{\frac{63}{65}} = 0'969$

$\operatorname{tg}(a-b) = \frac{\operatorname{tg} a - \operatorname{tg} b}{1+\operatorname{tg} a\cdot\operatorname{tg} b} = \frac{-3/4 + 12/5}{1+\frac{-3}{4}\cdot\frac{-12}{5}} = \boxed{\frac{33}{56}} = 0'589$

$\cos(2a+b) = \cos 2a\cos b - \sin 2a \sin b = (\cos^2 a - \sin^2 a)\cdot\cos b - 2\sin a\cos a\sin b =$

$= \left(\frac{16}{25} - \frac{9}{25}\right)\cdot\frac{5}{13} - 2\cdot\frac{3}{5}\cdot\frac{-4}{5}\cdot\frac{-12}{13} = \boxed{-\frac{253}{325}} = -0'779$

b) $\sin a = \frac{3}{5} \rightarrow a = 143'13^\circ$
$\cos b = \frac{5}{13} \rightarrow b = 292'62^\circ$
$\rightarrow$ $\sin(a+b) = \sin(143'13^\circ + 292'62^\circ) = 0'969$
$\operatorname{tg}(a-b) = \operatorname{tg}(143'13^\circ - 292'62^\circ) = 0'589$
$\cos(2a+b) = \cos(2\cdot 143'13^\circ + 292'62^\circ) = -0'779$

4

a) $\operatorname{Tg}\alpha = -\frac{40}{9} \rightarrow 1+\left(\frac{-40}{9}\right)^2 = \sec^2\alpha$; $1+\frac{1600}{81} = \sec^2\alpha$; $\sec^2\alpha = \frac{1681}{81}$;

$\sec\alpha = -\sqrt{\frac{1681}{81}} = -\frac{41}{9} \Rightarrow \cos\alpha = -\frac{9}{41} \rightarrow \sin\alpha = \operatorname{Tg}\alpha\cdot\cos\alpha = \frac{-40}{9}\cdot\frac{-9}{41} = \frac{40}{41}$

$\sin(2\alpha) = 2\sin\alpha\cos\alpha = 2\cdot\frac{40}{41}\cdot\frac{-9}{41} = \boxed{\frac{-720}{1681}} \simeq -0'428$

b) $\operatorname{tg}\alpha = -\frac{40}{9} \rightarrow \alpha = -77'32 + 180 = 102'68^\circ \longrightarrow \sin(2\cdot 102'68^\circ) = \boxed{-0'428}$

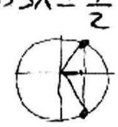
(5) $\operatorname{arcTg}(1/2) + \operatorname{arcTg}(1/3) = \pi/4$

$$\operatorname{Tg}\left(\operatorname{arcTg}(1/2) + \operatorname{arcTg}(1/3)\right) = \frac{\operatorname{Tg}(\operatorname{arcTg}(1/2)) + \operatorname{Tg}(\operatorname{arcTg}(1/3))}{1 - \operatorname{Tg}(\operatorname{arcTg}(1/2)) \cdot \operatorname{Tg}(\operatorname{arcTg}(1/3))} = \frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\cdot\frac{1}{3}} = \frac{\frac{5}{6}}{1-1/6} = \frac{5/6}{5/6} = 1$$

Cualquier arcTgx Toma valores entre $-\pi/2$ y $\pi/2$, por lo Tanto, la suma $\operatorname{arcTg}(1/2) + \operatorname{arcTg}(1/3)$ Tomará algún valor entre $-\pi$ y π. En ese intervalo, el único ángulo con Tangente igual a 1 es 45°.

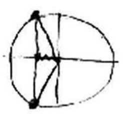
Por lo Tanto: $\boxed{\operatorname{arcTg}(1/2) + \operatorname{arcTg}(1/3) = \pi/4}$ ✓

(6) a) $\cos 3X = \frac{1}{2} \Rightarrow 3X = \begin{cases} 60° + N\cdot 360° \\ 300° + N\cdot 360° \end{cases}$; $X = \begin{cases} 20° + N\cdot 120° \\ 100° + N\cdot 120° \end{cases}$ $\boxed{X = \begin{cases} 20° \\ 140° \\ 260° \end{cases} \quad X = \begin{cases} 100° \\ 220° \\ 340° \end{cases}}$



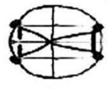
b) $\operatorname{Sen}X \cdot \cos X = 0 \Rightarrow \begin{cases} \operatorname{Sen}X = 0 \\ \cos X = 0 \end{cases}$ $\boxed{X = \begin{cases} 0° \\ 180° \end{cases} \quad X = \begin{cases} 90° \\ 270° \end{cases}}$

c) $\cos(2X - \pi) = -\frac{1}{2} \Rightarrow 2X - 180° = \begin{cases} 120° + N\cdot 360° \\ 240° + N\cdot 360° \end{cases}$; $2X = \begin{cases} 300° + N\cdot 360° \\ 420° + N\cdot 360° \end{cases}$; $X = \begin{cases} 150° + N\cdot 180° \\ 210° + N\cdot 180° \end{cases}$ $\boxed{X = \begin{cases} 150° \\ 330° \end{cases} \quad X = \begin{cases} 210° \\ 30° \end{cases}}$



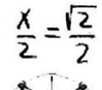
d) $\operatorname{Sen}X + \cos X = 0$; $\operatorname{Sen}X = -\cos X$; $\frac{\operatorname{Sen}X}{\cos X} = -1$; $\operatorname{Tg}X = -1 \Rightarrow$ $\boxed{X = \begin{cases} 135° \\ 315° \end{cases}}$

e) $\cos^2 X = \frac{3}{4}$; $\cos X = \pm\frac{\sqrt{3}}{2}$ $X = \begin{cases} 30° \\ 150° \\ 210° \\ 330° \end{cases}$



f) $\operatorname{Tg}2X = 1 \Rightarrow 2X = \begin{cases} 45° + N\cdot 360° \\ 225° + N\cdot 360° \end{cases}$; $2X = 45 + N\cdot 180°$; $X = 22{,}5° + N\cdot 90°$; $\boxed{X = \begin{cases} 22{,}5° \\ 112{,}5° \\ 202{,}5° \\ 292{,}5° \end{cases}}$

g) $\operatorname{Sen}\frac{X}{2} = \frac{\sqrt{2}}{2} \Rightarrow \frac{X}{2} = \begin{cases} 45° + N\cdot 360° \\ 135° + N\cdot 360° \end{cases}$; $X = \begin{cases} 90° + N\cdot 720° \\ 270° + N\cdot 720° \end{cases}$; $\boxed{X = \begin{cases} 90° \\ 270° \end{cases}}$



h) $\operatorname{ctg}X + \frac{\operatorname{Sen}X}{1+\cos X} = 2$; $\frac{\cos X}{\operatorname{Sen}X} + \frac{\operatorname{Sen}X}{1+\cos X} = 2$; $\frac{\cos X + \cos^2 X + \operatorname{Sen}^2 X}{\operatorname{Sen}X(1+\cos X)} = 2$; $\frac{\cancel{1+\cos X}}{\operatorname{Sen}X\,\cancel{(1+\cos X)}} = 2$;

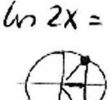
$\frac{1}{\operatorname{Sen}X} = 2$; $\operatorname{Sen}X = \frac{1}{2} \Rightarrow$ $\boxed{X = \begin{cases} 30° \\ 150° \end{cases}}$

i] $\sin(\pi - 3x) = -\frac{\sqrt{2}}{2} \Rightarrow 180° - 3x = \begin{cases} 225° + N\cdot 360° \\ 315° + N\cdot 360° \end{cases} \; ; \; -3x = \begin{cases} 45° + N\cdot 360° \\ 135° + N\cdot 360° \end{cases} \; ;$

$x = \begin{cases} -15° + N\cdot 120° \\ -45° + N\cdot 120° \end{cases}$ $\boxed{x = \begin{cases} 105° \\ 225° \\ 345° \end{cases} \quad x = \begin{cases} 75° \\ 195° \\ 315° \end{cases}}$

7 a] $\cos^2 x - \sin^2 x = \frac{1}{2}$

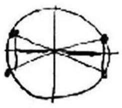
$\cos 2x = \frac{1}{2} \Rightarrow 2x = \begin{cases} 60° + N\cdot 360° \\ 300° + N\cdot 360° \end{cases} \; ; \; x = \begin{cases} 30° + N\cdot 180° \\ 150° + N\cdot 180° \end{cases} \; ; \boxed{x = \begin{cases} 30° \\ 210° \\ 150° \\ 330° \end{cases}} \; ; \boxed{x = \begin{cases} \pi/6 \\ 7\pi/6 \\ 5\pi/6 \\ 11\pi/6 \end{cases}}$



También:

$\cos^2 x - (1 - \cos^2 x) = \frac{1}{2} \; ; \; 2\cos^2 x - 1 = \frac{1}{2} \; ; \; 2\cos^2 x = \frac{3}{2} \; ; \; \cos^2 x = \frac{3}{4} \; ;$

$\cos x = \pm \frac{\sqrt{3}}{2} \Rightarrow x = \begin{cases} 30° + N\cdot 360° \\ 150° + N\cdot 360° \\ 210° + N\cdot 360° \\ 330° + N\cdot 360° \end{cases} \; ; \; x = \begin{cases} 30° + N\cdot 180° \\ 150° + N\cdot 180° \end{cases}$ ✓



b] $\sin x - \cos x = \frac{\sqrt{2}}{2}$

$(\sin x - \cos x)^2 = \left(\frac{\sqrt{2}}{2}\right)^2 \; ; \; \sin^2 x - 2\sin x \cos x + \cos^2 x = \frac{1}{2} \; ; \; 1 - 2\sin x \cos x = \frac{1}{2} \; ;$

$2\sin x \cos x = \frac{1}{2} \; ; \; \sin 2x = \frac{1}{2} \Rightarrow 2x = \begin{cases} 30° + N\cdot 360° \\ 150° + N\cdot 360° \end{cases} \; ; \; x = \begin{cases} 15° + N\cdot 180° \\ 75° + N\cdot 180° \end{cases} \; ; \; x = \begin{cases} 15° \\ 195° \\ 75° \\ 255° \end{cases}$

Como elevamos al cuadrado, hay que comprobar:

$\sin 15° - \cos 15° = -\frac{\sqrt{2}}{2} \neq \frac{\sqrt{2}}{2}$

$\sin 195° - \cos 195° = \frac{\sqrt{2}}{2}$ ✓

$\sin 75° - \cos 75° = \frac{\sqrt{2}}{2}$ ✓

$\sin 255° - \cos 255° = -\frac{\sqrt{2}}{2} \neq \frac{\sqrt{2}}{2}$

$\boxed{x = \begin{cases} 75° \\ 195° \end{cases}} \; ; \; \boxed{x = \begin{cases} 5\pi/12 \\ 13\pi/12 \end{cases}}$

c] $\operatorname{tg} x \cdot \sec x = \sqrt{2}$

$\frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} = \sqrt{2} \; ; \; \sin x = \sqrt{2}\cos^2 x \; ; \; \sin x = \sqrt{2}(1 - \sin^2 x) \; ; \; \sqrt{2}\sin^2 x + \sin x - \sqrt{2} = 0$

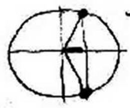
$\sin x = \frac{-1 \pm \sqrt{1 + 4(\sqrt{2})^2}}{2\sqrt{2}} = \frac{-1 \pm 3}{2\sqrt{2}} = \begin{cases} \frac{-4}{2\sqrt{2}} = \cancel{-\frac{2}{\sqrt{2}}} \\ \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \end{cases}$ Porque es menor que -1.

$\Rightarrow \boxed{x = \begin{cases} 225° \\ 135° \end{cases}} \; ; \; \boxed{x = \begin{cases} 5\pi/4 \\ 3\pi/4 \end{cases}}$

d) $3\cos X = 2\sec X - 5$

$3\cos X = \dfrac{2}{\cos X} - 5$; $3\cos^2 X = 2 - 5\cos X$; $3\cos^2 X + 5\cos X - 2 = 0$

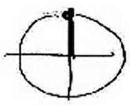
$\cos X = \dfrac{-5 \pm \sqrt{25+24}}{6} = \dfrac{-5 \pm 7}{6} \begin{cases} \dfrac{2}{6} = \dfrac{1}{3} \Rightarrow \boxed{x = \begin{cases} 71^\circ \\ 289^\circ \end{cases}} ; \boxed{x = \begin{cases} 1{,}23 \text{ rad} \\ 5{,}05 \text{ rad} \end{cases}} \\ \cancel{-2} \end{cases}$



e) $\log_2 \cos x + 1 = \log_2 \operatorname{cosec} x$

$\log_2 \cos x - \log_2 \operatorname{cosec} x = -1$; $\log_2 \dfrac{\cos x}{\operatorname{cosec} x} = -1$; $\dfrac{\cos x}{\operatorname{cosec} x} = 2^{-1}$; $\dfrac{\cos x}{\operatorname{cosec} x} = \dfrac{1}{2}$;

$2\cos X = \operatorname{cosec} X$; $2\cos X = \dfrac{1}{\sin X}$; $2\sin X\cos X = 1$; $\sin 2X = 1 \Rightarrow$

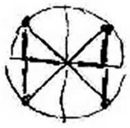


$\Rightarrow 2X = 90^\circ + N\cdot 360^\circ$; $x = 45^\circ + N\cdot 180^\circ$; $\boxed{x = \begin{cases} 45^\circ \\ \cancel{225^\circ} \end{cases}}$; $\boxed{x = \pi/4 \text{ rad}}$

No sirve

f) $\operatorname{tg}^2 2x = 1$

$\operatorname{tg} 2x = \pm 1$; $2X = \begin{cases} 45^\circ + N\cdot 360^\circ \\ 225^\circ + N\cdot 360^\circ \\ 135^\circ + N\cdot 360^\circ \\ 315^\circ + N\cdot 360^\circ \end{cases}$; $x = \begin{cases} 22{,}5^\circ + N\cdot 180^\circ \\ 112{,}5^\circ + N\cdot 180^\circ \\ 67{,}5^\circ + N\cdot 180^\circ \\ 157{,}5^\circ + N\cdot 180^\circ \end{cases}$; $\boxed{x = \begin{cases} 22{,}5^\circ \\ 202{,}5^\circ \\ 112{,}5^\circ \\ 292{,}5^\circ \\ 67{,}5^\circ \\ 247{,}5^\circ \\ 157{,}5^\circ \\ 337{,}5^\circ \end{cases}}$; $\boxed{x = \begin{cases} \pi/8 \\ 9\pi/8 \\ 5\pi/8 \\ 13\pi/8 \\ 3\pi/8 \\ 11\pi/8 \\ 7\pi/8 \\ 15\pi/8 \end{cases}}$ rad



g) $\cos^2\frac{x}{2} - \sin^2\frac{x}{2} = \sin x$; $\cos 2\frac{x}{2} = \sin x$; $\cos x = \sin x$; $1 = \operatorname{tg} x \Rightarrow x = \begin{cases} 45^\circ = \boxed{\pi/4} \\ 225^\circ = \boxed{5\pi/4} \end{cases}$ rad

h) $\cos(2x) + \sin x = 4\sin^2 x$

$\cos^2 x - \sin^2 x + \sin x = 4\sin^2 x$; $1 - \sin^2 x - \sin^2 x + \sin x = 4\sin^2 x$; $-6\sin^2 x + \sin x + 1 = 0$;

$\sin x = \dfrac{-1 \pm \sqrt{1+24}}{-12} = \dfrac{-1 \pm 5}{-12} \begin{cases} \dfrac{-4}{12} = -\dfrac{1}{3} \Rightarrow x = \begin{cases} 340'53^\circ = \boxed{1'89\pi} \\ 199'47^\circ = \boxed{1'11\pi} \end{cases} \\ \dfrac{-6}{-12} = \dfrac{1}{2} \Rightarrow x = \begin{cases} 30^\circ = \boxed{\pi/6} \\ 150^\circ = \boxed{5\pi/6} \end{cases} \end{cases}$

i) $\operatorname{tg}(2x) = -\operatorname{tg} x$

$\dfrac{2\operatorname{tg} x}{1 - \operatorname{tg}^2 x} = -\operatorname{tg} x$; $2\operatorname{tg} x = -\operatorname{tg} x + \operatorname{tg}^3 x$; $3\operatorname{tg} x - \operatorname{tg}^3 x = 0$; $\operatorname{tg} x\cdot(3 - \operatorname{tg}^2 x) = 0$

$\operatorname{tg} x\cdot(3-\operatorname{tg}^2 x) = 0 \begin{cases} \to \operatorname{tg} x = 0 \to x = \boxed{\begin{cases} 0 \\ \pi \end{cases}} \\ \to \operatorname{tg}^2 x = 3 ; \operatorname{tg} x = \begin{cases} \to \sqrt{3} \to x = \begin{cases} 60^\circ = \boxed{\pi/3} \\ 240^\circ = \boxed{4\pi/3} \end{cases} \\ \to -\sqrt{3} \to x = \begin{cases} 120^\circ = \boxed{2\pi/3} \\ 300^\circ = \boxed{5\pi/3} \end{cases} \end{cases} \end{cases}$

(8) a) $4\cos(2x) - 3\sin x \cos\sec^3 x + 6 = 0$

$4(\cos^2 x - \sin^2 x) - 3\sin x \frac{1}{\sin^3 x} + 6 = 0$; $4(1-2\sin^2 x) - \frac{3\cancel{\sin x}}{\sin^{\cancel{3}\,2} x} + 6 = 0$

$4 - 8\sin^2 x - \frac{3}{\sin^2 x} + 6 = 0$; $10 - 8\sin^2 x - \frac{3}{\sin^2 x} = 0$

$t = \sin x$ $\quad 10 - 8t^2 - \frac{3}{t^2} = 0$; $10t^2 - 8t^4 - 3 = 0$; $8t^4 - 10t^2 + 3 = 0$ ✓

b) $t^2 = \frac{10 \pm \sqrt{100-96}}{16} = \frac{10 \pm 2}{16}$ $\begin{cases} \frac{12}{16} = \frac{3}{4} \Rightarrow t = \frac{\pm\sqrt{3}}{2} \to \sin x = \frac{\pm\sqrt{3}}{2} \\ \frac{8}{16} = \frac{1}{2} \Rightarrow t = \pm\frac{\sqrt{2}}{2} \to \sin x = \frac{\pm\sqrt{2}}{2} \end{cases}$ $\boxed{x = \begin{cases} \pi/3 \\ 2\pi/3 \\ \pi/4 \\ 3\pi/4 \end{cases}}$ $x \in [0,\pi)$

(9) a) $\frac{\sin(2\alpha)}{1+\cos(2\alpha)} = \frac{2\sin\alpha\cos\alpha}{1+\cos^2\alpha - \sin^2\alpha} = \frac{2\sin\alpha\cos\alpha}{\cancel{1}+\cos^2\alpha - \cancel{1}+\cos^2\alpha} = \frac{2\sin\alpha\cancel{\cos\alpha}}{2\cos^2\alpha} = \operatorname{tg}\alpha$ ✓

b) $\operatorname{ctg}(\pi/8) = \frac{1}{\operatorname{tg}(\pi/8)} = \frac{1}{\frac{\sin(2\pi/8)}{1+\cos(2\pi/8)}} = \frac{1+\cos(\pi/4)}{\sin(\pi/4)} = \frac{1+\sqrt{2}/2}{\sqrt{2}/2} = \frac{2+\sqrt{2}}{\sqrt{2}} =$

$= \frac{(2+\sqrt{2})\sqrt{2}}{(\sqrt{2})^2} = \frac{2\sqrt{2}+2}{2} = \boxed{1+\sqrt{2}}$ $\quad a=1$, $b=1$

(10) a) He listado en una hoja de cálculo ángulos, de grado en grado, de 0° a 360° poniendo a su derecha el valor de $5\cos\alpha - 12\sin\alpha$. El menor valor resulta ser $-12'9997$ para un ángulo de 113° mientras que el mayor valor es $12'9997$ para 293°.
Con el mismo programa de hoja de cálculo he representado estos valores, resultando puntos que perfilan una sinusoide.

b) $5\cos x - 12\sin x = A\cos(x+\alpha)$

$5\cos x - 12\sin x = A\cos x\cos\alpha - A\sin x\sin\alpha$

$5 = A\cos\alpha \to A = \frac{5}{\cos\alpha}$

$12 = A\sin\alpha \to 12 = \frac{5}{\cos\alpha}\cdot\sin\alpha$; $\frac{12}{5} = \operatorname{tg}\alpha \Rightarrow \boxed{\alpha = 67'38°}$

$25 = A^2\cos^2\alpha$
$+\ 144 = A^2\sin^2\alpha$
$169 = A^2 \Rightarrow \boxed{A = 13}$

$\boxed{5\cos x - 12\sin x = 13\cdot\cos(x + 67'38°)}$

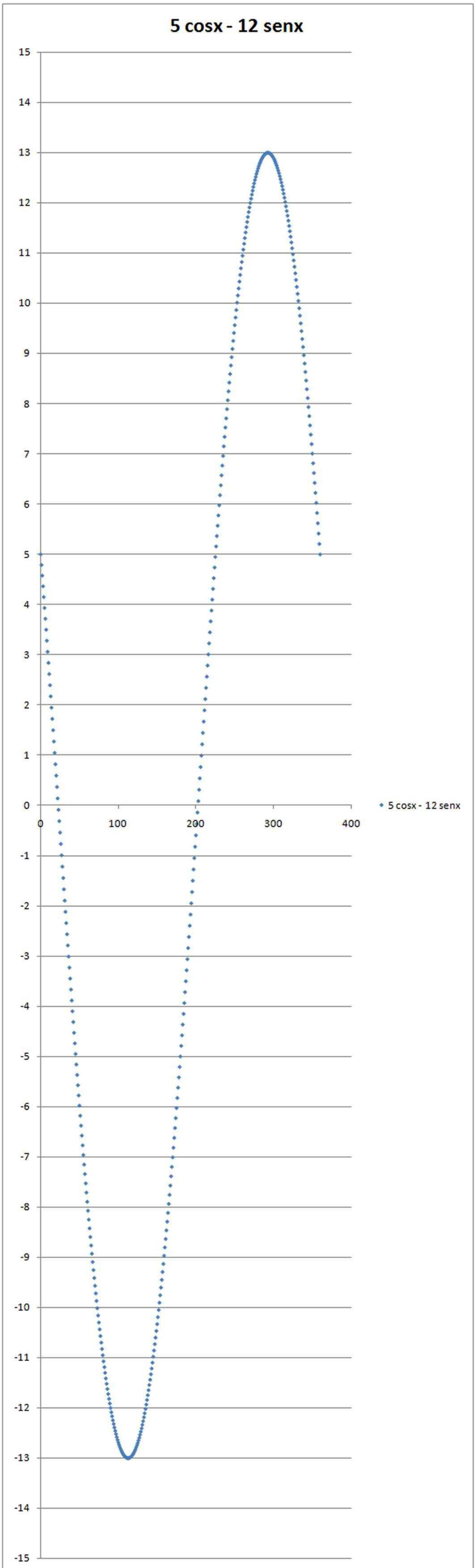
El mayor valor de $5\cos x - 12\sin x$ será 13, para $x + 67'38° = 0° \Rightarrow x = -67'38° \equiv \boxed{293°}$

El menor valor de $5\cos x - 12\sin x$ será -13, para $x + 67'38° = 180° \Rightarrow \boxed{x = 113°}$

c) $5\cos x - 12\sin x = -2 \Rightarrow 13\cdot\cos(x+67'38°) = -2 \Rightarrow \cos(x+67'38°) = -\frac{2}{13} \Rightarrow$

$\Rightarrow x + 67'38° = \begin{cases} 98'85° \\ 261'15° \end{cases} \Rightarrow \boxed{x = \begin{cases} 31° \\ 194° \end{cases}}$

x	5 cosx - 12 senx	x	5 cosx - 12 senx	x	5 cosx - 12 senx	x	5 cosx - 12 senx
0	5,0000	90	-12,0000	180	-5,0000	270	12,0000
1	4,7898	91	-12,0854	181	-4,7898	271	12,0854
2	4,5782	92	-12,1672	182	-4,5782	272	12,1672
3	4,3651	93	-12,2452	183	-4,3651	273	12,2452
4	4,1507	94	-12,3196	184	-4,1507	274	12,3196
5	3,9351	95	-12,3901	185	-3,9351	275	12,3901
6	3,7183	96	-12,4569	186	-3,7183	276	12,4569
7	3,5003	97	-12,5199	187	-3,5003	277	12,5199
8	3,2813	98	-12,5791	188	-3,2813	278	12,5791
9	3,0612	99	-12,6344	189	-3,0612	279	12,6344
10	2,8403	100	-12,6859	190	-2,8403	280	12,6859
11	2,6184	101	-12,7336	191	-2,6184	281	12,7336
12	2,3958	102	-12,7773	192	-2,3958	282	12,7773
13	2,1724	103	-12,8172	193	-2,1724	283	12,8172
14	1,9484	104	-12,8532	194	-1,9484	284	12,8532
15	1,7238	105	-12,8852	195	-1,7238	285	12,8852
16	1,4987	106	-12,9133	196	-1,4987	286	12,9133
17	1,2731	107	-12,9375	197	-1,2731	287	12,9375
18	1,0471	108	-12,9578	198	-1,0471	288	12,9578
19	0,8208	109	-12,9741	199	-0,8208	289	12,9741
20	0,5942	110	-12,9864	200	-0,5942	290	12,9864
21	0,3675	111	-12,9948	201	-0,3675	291	12,9948
22	0,1406	112	-12,9992	202	-0,1406	292	12,9992
23	-0,0862	113	-12,9997	203	0,0862	293	12,9997
24	-0,3131	114	-12,9962	204	0,3131	294	12,9962
25	-0,5399	115	-12,9888	205	0,5399	295	12,9888
26	-0,7665	116	-12,9774	206	0,7665	296	12,9774
27	-0,9929	117	-12,9620	207	0,9929	297	12,9620
28	-1,2189	118	-12,9427	208	1,2189	298	12,9427
29	-1,4446	119	-12,9195	209	1,4446	299	12,9195
30	-1,6699	120	-12,8923	210	1,6699	300	12,8923
31	-1,8946	121	-12,8612	211	1,8946	301	12,8612
32	-2,1188	122	-12,8262	212	2,1188	302	12,8262
33	-2,3423	123	-12,7872	213	2,3423	303	12,7872
34	-2,5651	124	-12,7444	214	2,5651	304	12,7444
35	-2,7872	125	-12,6977	215	2,7872	305	12,6977
36	-3,0083	126	-12,6471	216	3,0083	306	12,6471
37	-3,2286	127	-12,5927	217	3,2286	307	12,5927
38	-3,4479	128	-12,5344	218	3,4479	308	12,5344
39	-3,6661	129	-12,4724	219	3,6661	309	12,4724
40	-3,8832	130	-12,4065	220	3,8832	310	12,4065
41	-4,0992	131	-12,3368	221	4,0992	311	12,3368
42	-4,3138	132	-12,2634	222	4,3138	312	12,2634
43	-4,5272	133	-12,1862	223	4,5272	313	12,1862
44	-4,7392	134	-12,1054	224	4,7392	314	12,1054
45	-4,9497	135	-12,0208	225	4,9497	315	12,0208
46	-5,1588	136	-11,9326	226	5,1588	316	11,9326
47	-5,3663	137	-11,8407	227	5,3663	317	11,8407
48	-5,5721	138	-11,7453	228	5,5721	318	11,7453
49	-5,7762	139	-11,6463	229	5,7762	319	11,6463
50	-5,9786	140	-11,5437	230	5,9786	320	11,5437
51	-6,1791	141	-11,4376	231	6,1791	321	11,4376
52	-6,3778	142	-11,3280	232	6,3778	322	11,3280
53	-6,5746	143	-11,2150	233	6,5746	323	11,2150
54	-6,7693	144	-11,0985	234	6,7693	324	11,0985
55	-6,9619	145	-10,9787	235	6,9619	325	10,9787
56	-7,1525	146	-10,8555	236	7,1525	326	10,8555
57	-7,3409	147	-10,7290	237	7,3409	327	10,7290
58	-7,5270	148	-10,5993	238	7,5270	328	10,5993
59	-7,7108	149	-10,4663	239	7,7108	329	10,4663
60	-7,8923	150	-10,3301	240	7,8923	330	10,3301
61	-8,0714	151	-10,1908	241	8,0714	331	10,1908
62	-8,2480	152	-10,0484	242	8,2480	332	10,0484
63	-8,4221	153	-9,9029	243	8,4221	333	9,9029
64	-8,5937	154	-9,7544	244	8,5937	334	9,7544
65	-8,7626	155	-9,6030	245	8,7626	335	9,6030
66	-8,9289	156	-9,4486	246	8,9289	336	9,4486
67	-9,0924	157	-9,2913	247	9,0924	337	9,2913
68	-9,2532	158	-9,1312	248	9,2532	338	9,1312
69	-9,4111	159	-8,9683	249	9,4111	339	8,9683
70	-9,5662	160	-8,8027	250	9,5662	340	8,8027
71	-9,7184	161	-8,6344	251	9,7184	341	8,6344
72	-9,8676	162	-8,4635	252	9,8676	342	8,4635
73	-10,0138	163	-8,2900	253	10,0138	343	8,2900
74	-10,1570	164	-8,1140	254	10,1570	344	8,1140
75	-10,2970	165	-7,9355	255	10,2970	345	7,9355
76	-10,4339	166	-7,7545	256	10,4339	346	7,7545
77	-10,5677	167	-7,5713	257	10,5677	347	7,5713
78	-10,6982	168	-7,3857	258	10,6982	348	7,3857
79	-10,8255	169	-7,1978	259	10,8255	349	7,1978
80	-10,9495	170	-7,0078	260	10,9495	350	7,0078
81	-11,0701	171	-6,8157	261	11,0701	351	6,8157
82	-11,1874	172	-6,6214	262	11,1874	352	6,6214
83	-11,3012	173	-6,4252	263	11,3012	353	6,4252
84	-11,4116	174	-6,2270	264	11,4116	354	6,2270
85	-11,5186	175	-6,0268	265	11,5186	355	6,0268
86	-11,6220	176	-5,8249	266	11,6220	356	5,8249
87	-11,7219	177	-5,6212	267	11,7219	357	5,6212
88	-11,8182	178	-5,4157	268	11,8182	358	5,4157
89	-11,9109	179	-5,2087	269	11,9109	359	5,2087
90	-12,0000	180	-5,0000	270	12,0000	360	5,0000



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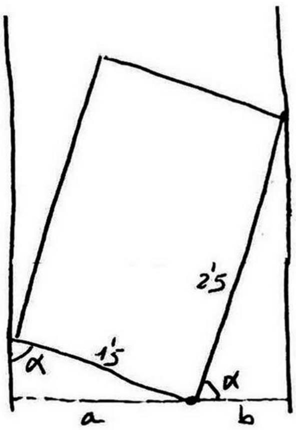

$$\operatorname{sen}\alpha = \frac{a}{1{,}5} \longrightarrow a = 1{,}5\operatorname{sen}\alpha$$

$$+$$

$$\cos\alpha = \frac{b}{2{,}5} \longrightarrow b = 2{,}5\cos\alpha$$

$$2 = 1{,}5\operatorname{sen}\alpha + 2{,}5\cos\alpha$$

$$4 = 3\operatorname{sen}\alpha + 5\cos\alpha \longrightarrow \cos\alpha = \frac{4-3\operatorname{sen}\alpha}{5}$$

$$\operatorname{sen}^2\alpha + \cos^2\alpha = 1$$

$$\operatorname{sen}^2\alpha + \left(\frac{4-3\operatorname{sen}\alpha}{5}\right)^2 = 1$$

$$\operatorname{sen}^2\alpha + \frac{16-24\operatorname{sen}\alpha+9\operatorname{sen}^2\alpha}{25} = 1 \quad ; \quad 25\operatorname{sen}^2\alpha + 16 - 24\operatorname{sen}\alpha + 9\operatorname{sen}^2\alpha = 25 \quad ;$$

$$34\operatorname{sen}^2\alpha - 24\operatorname{sen}\alpha - 9 = 0 \quad ; \operatorname{sen}\alpha = \frac{24 \pm \sqrt{576+1224}}{68} = \begin{cases} 0{,}98 \Rightarrow \boxed{\alpha = 77{,}65^\circ} = \boxed{77^\circ 38' 59{,}64''} \\ \qquad\quad\ \cancel{102{,}35} \\ -0{,}27 \Rightarrow \alpha = \begin{cases} \cancel{344{,}28} \\ \cancel{164{,}28} \end{cases} \end{cases}$$

Los ángulos son: $\boxed{\alpha = 77{,}65^\circ}$ y $\boxed{90-\alpha = 12{,}35^\circ}$

12 $$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \operatorname{sen}\alpha\operatorname{sen}\beta = \frac{3}{\sqrt{3^2+4^2}}\cdot\frac{3}{\sqrt{3^2+2^2}} + \frac{4}{\sqrt{3^2+4^2}}\cdot\frac{2}{\sqrt{3^2+2^2}} =$$

$$= \frac{9+8}{5\sqrt{13}} = \boxed{\frac{17}{5\sqrt{13}}}$$