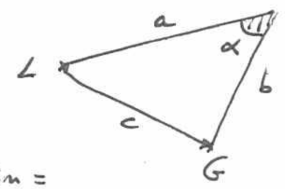
(1)

$a = 13 \text{ nudos} \cdot 24 \text{ min} =$

$= 13 \dfrac{\text{millas}}{h} \cdot 24 \text{ min} =$

$= 13 \dfrac{\text{millas}}{h} \cdot \dfrac{1851{,}85 \text{ m}}{1 \text{ milla}} \cdot \dfrac{1 h}{60 \text{ min}} \cdot 24 \text{ min} =$

$= 9629{,}62 \text{ m}$



$b = 27 \dfrac{\text{millas}}{h} \cdot \dfrac{1851{,}85 \text{ m}}{1 \text{ milla}} \cdot \dfrac{1 h}{60 \text{ min}} \cdot 24 \text{ min} = 19\,999{,}98 \text{ m}$

$c = 14 \text{ Km} = 14000 \text{ m}$

$\cos\alpha = \dfrac{a^2+b^2-c^2}{2ab} \longrightarrow \alpha = \arccos\left(\dfrac{9629{,}62^2 + 19999{,}98^2 - 14000^2}{2 \cdot 9629{,}62 \cdot 19999{,}98}\right) = \boxed{39{,}61°}$

(2) $\cos 2x + 5\cos x + 3 = 0$

$\cos^2 x - \sin^2 x + 5\cos x + 3 = 0$

$\cos^2 x - 1 + \cos^2 x + 5\cos x + 3 = 0$

$2\cos^2 x + 5\cos x + 2 = 0$

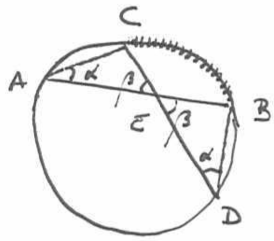
$\cos x = \begin{cases} -0'5 \Rightarrow \boxed{x = \begin{cases} 120° \\ 240° \end{cases}} \\ -2 \quad \nexists \end{cases}$

(3) $\cos\alpha = -\dfrac{5}{13} \quad (180 < \alpha < 360)$

$\sin\alpha = -\sqrt{1-\left(\frac{-5}{13}\right)^2} = -\dfrac{12}{13} \longrightarrow \operatorname{tg}\alpha = \dfrac{-12/13}{-5/13} = \dfrac{12}{5} \longrightarrow \operatorname{tg} 2\alpha = \dfrac{2\operatorname{tg}\alpha}{1-\operatorname{tg}^2\alpha} = \dfrac{2\cdot\frac{12}{5}}{1-\frac{144}{25}} = \dfrac{24/5}{-119/25} =$

$= \boxed{-\dfrac{120}{119}}$

(4)



Por el teorema del ángulo inscrito, los ángulos $\hat{A}$ y $\hat{D}$ son iguales ya que les corresponde el mismo arco.

Por lo tanto los triángulos $\widehat{ACE}$ y $\widehat{DBE}$ son semejantes.

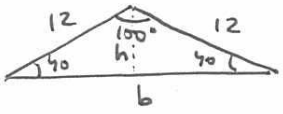
$\dfrac{AE}{ED} = \dfrac{CE}{EB} \Rightarrow CE = \dfrac{AE}{ED} \cdot EB = \dfrac{4}{7} \cdot 5 = \boxed{\dfrac{20}{7}}$

(5)

a) $\sin(\arcsin x) = x$ — Es cierto si $x \in [-1,1]$, ya que arcsen es una función inyectiva

b) $\arcsin(\sin x) = x$ — Es cierto para $x \in [-90°, 90°]$ ya que ése es el recorrido de la función arcsen

(6)



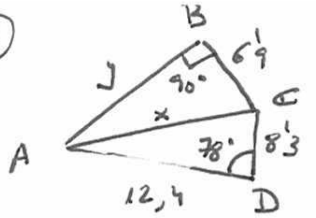
$\cos 40° = \dfrac{b/2}{12} \longrightarrow b = 24\cos 40°$

$\sin 40° = \dfrac{h}{12} \longrightarrow h = 12\sin 40°$

$P = 12+12+24\cos 40° = 42{,}383 \text{ cm}$

$A = \dfrac{24\cos 40° \cdot 12\sin 40°}{2} = 70{,}906 \text{ cm}^2$

7



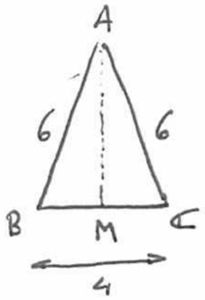
$x = \sqrt{8'3^2 + 12'4^2 - 2\cdot 8'3 \cdot 12'4 \cdot \cos 78^\circ} = 13,411$ $\qquad y = \sqrt{x^2 - 6'9^2} = 11,499$

$\text{Area } \overset{\triangle}{ABC} = \dfrac{6'9 \cdot 11,499}{2} = 39,674$

$\text{Area } \overset{\triangle}{ADC} = \dfrac{12,4 \cdot 8'3 \cdot \text{sen}78^\circ}{2} = 50,335$

$\boxed{\text{Area Total} = 90,009}$

8



$AM = \sqrt{6^2 - 2^2} = \sqrt{32} = 4\sqrt{2}$

$Tg\,30^\circ = \dfrac{PM}{2} \rightarrow PM = 2\,tg30^\circ = \dfrac{2}{\sqrt{3}}$

$AP = 4\sqrt{2} - \dfrac{2}{\sqrt{3}} = 4,502$

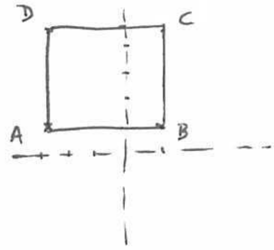
1

$M = \dfrac{A+C}{2} = (-1,3)$

$\vec{AC} = (4,4) \rightarrow \vec{BD} = (-4,4)$

$D = M + \frac{1}{2}\vec{BD} = (-1,3) + (-2,2) = (-3,5)$

$B = M - \frac{1}{2}\vec{BD} = (-1,3) - (-2,2) = (1,1)$



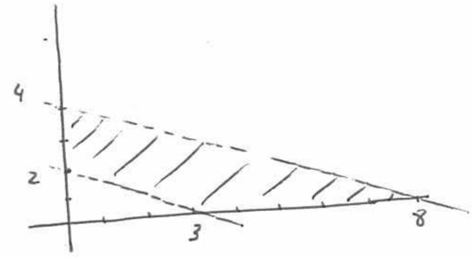
2

$2x + 3y - 6 = 0$

x	y
0	2
3	0

$x + 2y - 8 = 0$

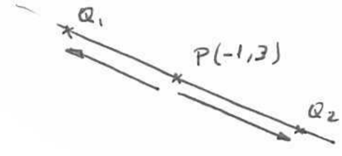
x	y
0	4
8	0



$\text{Area} = \dfrac{4\cdot 8}{2} - \dfrac{2\cdot 3}{2} = 16 - 3 = \boxed{13\ u^2}$

3

$P(-1,3)$ pertenece a $y = \dfrac{5-4x}{3}$



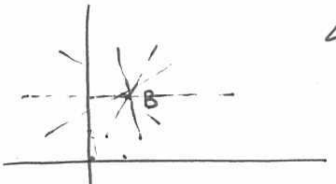
$\vec{PQ_1}$ y $\vec{PQ_2}$ son dos vectores paralelos a la recta y con módulo 5.

$y = \dfrac{5-4x}{3} \rightarrow \text{pend} = -\dfrac{4}{3} \rightarrow \vec{v} = (3,-4)$

$Q_1 = P + (3,-4) = (2,-1)$

$Q_2 = P + (-3,4) = (-4,7)$

4



Las rectas que pasan por $B(1,2)$ tendrán de ecuación:

$y - 2 = m(x-1) \quad ; \quad -mx + y + m - 2 = 0$

$d((0,0), -mx+y+m-2=0) = 2 \qquad \dfrac{|-m\cdot 0 + 0 + m - 2|}{\sqrt{(-m)^2+1^2}} = 2 \quad ; \quad \dfrac{|m-2|}{\sqrt{m^2+1}} = 2 \quad ; \quad \dfrac{|m-2|^2}{m^2+1} = 4$

$m^2 - 4m + 4 = 4m^2 + 4 \quad ; \quad 0 = 3m^2 + 4m \quad ; \quad m = \begin{cases} 0 \rightarrow \boxed{y=2} \\ -\frac{4}{3} \rightarrow \frac{4}{3}x + y - \frac{4}{3} - 2 = 0 \quad \boxed{4x + 3y - 10 = 0} \end{cases}$

(5)

Ecuación AB

$\vec{AB} = (4,3) \rightarrow m = \frac{3}{4}$

$y - 2 = \frac{3}{4}(x-1)$

$4y - 8 = 3x - 3$

$-3x + 4y - 5 = 0$

Ecuación AC

$\vec{AC} = (10,0) \rightarrow m = 0$

$y + 1 = 0(x-7)$

$y + 1 = 0$

Ecuación BC

$\vec{BC} = (6,-3) \rightarrow m = -\frac{1}{2}$

$y + 1 = -\frac{1}{2}(x-7)$

$2y + 2 = -x + 7$

$x + 2y - 5 = 0$

Ecuación AP (Bisectriz)

$$\frac{|-3x+4y-5|}{\sqrt{9+16}} = \frac{|y+1|}{\sqrt{0+1}}$$

$|-3x+4y-5| = 5|y+1|$

$-3x+4y-5 = 5y+5$

$-y = 3x + 10$

$\cancel{y = -3x - 10}$

$-3x+4y-5 = -5y-5$

$9y = 3x$

$y = \frac{x}{3}$

Coordenadas de P

$\left.\begin{array}{l} x + 2y - 5 = 0 \\ y = \frac{x}{3} \end{array}\right\}$ $x + \frac{2x}{3} - 5 = 0$

$3x + 2x - 15 = 0$

$x = 3 \rightarrow y = 1$

$\boxed{P(3,1)}$

Altura Triángulo $\widehat{APC}$

$$h = d(P, \overline{AC}) = \frac{|1+1|}{\sqrt{0^2+1^2}} = 2$$

Base Triángulo $\widehat{APC}$

$$b = |\vec{AC}| = \sqrt{100+0} = 10$$

$$\text{Area } \widehat{APC} = \frac{2 \times 10}{2} = \boxed{10\ u^2}$$

Otro procedimiento:

$\left.\begin{array}{l} \vec{AB} = (4,3) \\ \vec{AC} = (10,0) \end{array}\right\} \cos\hat{A} = \frac{(4,3)\cdot(10,0)}{\sqrt{16+9}\,\sqrt{100+0}} = \frac{40}{50} = \frac{4}{5} \Rightarrow \hat{A} = 36{,}870^\circ$

$C\hat{A}P = \frac{A}{2} = 18{,}435^\circ$

$\left.\begin{array}{l} \vec{CB} = (-6,3) \\ \vec{CA} = (-10,0) \end{array}\right\} \cos\hat{C} = \frac{(-6,3)\cdot(-10,0)}{\sqrt{36+9}\,\sqrt{100+0}} = \frac{60}{10\sqrt{45}} = \frac{6}{\sqrt{45}} \Rightarrow \hat{C} = 26{,}565^\circ$

$C\hat{P}A = 180 - 18{,}435 - 26{,}565 = 135^\circ$

$$\frac{AP}{\operatorname{sen}\hat{C}} = \frac{AC}{\operatorname{sen} C\hat{P}A} \longrightarrow AP = \frac{AC \operatorname{sen}\hat{C}}{\operatorname{sen} C\hat{P}A} = \frac{10 \operatorname{sen} 26{,}565}{\operatorname{sen} 135} = 6'325$$

$$\text{Area } \widehat{APC} = \frac{AP \cdot AC \cdot \operatorname{sen} C\hat{A}P}{2} = \frac{6'325 \cdot 10 \cdot \operatorname{sen} 18{,}435}{2} = \boxed{10\ u^2}$$