(1) a) $0'16 + 0'25 + 0'21 + 0'12 + 0'26 = 1$ ✓ Se trata de una función distribución de probabilidad.

b) $P[X = \text{par}] = 0'16 + 0'12 + 0'26 = \boxed{0'54}$

c) $E[X] = 0 \cdot 0'16 + 3 \cdot 0'25 + 5 \cdot 0'21 + 6 \cdot 0'12 + 10 \cdot 0'26 = \boxed{5'12}$

$\sigma = \sqrt{0^2 \cdot 0'16 + 3^2 \cdot 0'25 + 5^2 \cdot 0'21 + 6^2 \cdot 0'12 + 10^2 \cdot 0'26 - 5'12^2} = \boxed{3'4067}$

(2)

X	p
6	4/40
4	12/40
-3	24/40

X = 'premio recibido (en euros)'

$E[X] = 6 \cdot \frac{4}{40} + 4 \cdot \frac{12}{40} + (-3) \cdot \frac{24}{40} = \frac{24 + 48 - 72}{40} = \boxed{0}$

Es un juego equilibrado. Jugando muchas veces ni ganaríamos ni perderíamos dinero, por término medio. Jugaríamos por el gusto de jugar.

(3)

SUMA	0	1	2
0	—	1	2
1	1	—	3
2	2	3	—

X	p
1	2/6
2	2/6
3	2/6

$E[X] = 1 \cdot \frac{2}{6} + 2 \cdot \frac{2}{6} + 3 \cdot \frac{2}{6} = \frac{12}{6} = \boxed{2}$

$\sigma = \sqrt{1^2 \cdot \frac{2}{6} + 2^2 \cdot \frac{2}{6} + 3^2 \cdot \frac{2}{6} - 2^2} = \boxed{0'8165}$

(4) a) $0'1 + a + 0'3 + b = 1 \Rightarrow \boxed{a + b = 0'6}$

b) $0 \cdot 0'1 + a \cdot 1 + 0'3 \cdot 2 + b \cdot 3 = 1'5$; $a + 3b = 0'9$

$\left.\begin{matrix} a + b = 0'6 \\ a + 3b = 0'9 \end{matrix}\right\} \rightarrow \boxed{\begin{matrix} a = 0'45 \\ b = 0'15 \end{matrix}}$

(5) a) $X = \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4), (4,1), (4,2), (4,3), (4,4)\}$

$P[(4,4)] = \frac{1}{4} \cdot \frac{1}{4} = \boxed{\frac{1}{16}}$

b)

X	p
0	9/16
1	6/16
2	1/16

$E[X] = 0 \cdot \frac{9}{16} + 1 \cdot \frac{6}{16} + 2 \cdot \frac{1}{16} = \frac{8}{16} = \boxed{0'5}$

X	1	2	3	4
1	0	0	0	1
2	0	0	0	1
3	0	0	0	1
4	1	1	1	2

(6) $B(5, 0'5)$

X = 'nº de caras'

$X = \{0, 1, 2, 3, 4, 5\}$

a) $P[X = 3] = \binom{5}{3} 0'5^3 \cdot 0'5^2 = \boxed{0'3125}$

b) $P[X \geq 1] = 1 - P[X = 0] = 1 - \binom{5}{0} 0'5^0 \cdot 0'5^5 = \boxed{0'9688}$

⑦ $B(100, 0'04)$ $X=$ 'nº de interruptores defectuosos' $X=\{0,1,2,\dots,99,100\}$

a) $E[X] = 100 \cdot 0'04 = \boxed{4}$

b) $P[X=6] = \binom{100}{6} 0'04^6 \cdot 0'96^{94} = \boxed{0'1052}$

c) $P[X \geq 1] = 1 - P[X=0] = 1 - \binom{100}{0} 0'04^0 \cdot 0'96^{100} = \boxed{0'9831}$

⑧ $B(7, 0'18)$ $X=$ 'nº de caras' $X=\{0,1,2,3,4,5,6,7\}$

a) $P[X=2] = \binom{7}{2} 0'18^2 \cdot 0'82^5 = \boxed{0'2523}$

b) $P[X \geq 2] = 1 - P[X=0] - P[X=1] = 1 - \binom{7}{0} 0'18^0 \cdot 0'82^7 - \binom{7}{1} 0'18^1 \cdot 0'82^6 = \boxed{0'3677}$

⑨ a) $P[Z \leq 0'43] = \boxed{0'6664}$

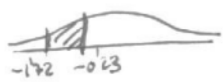
b) $P[Z \leq -1'46] = 1 - 0'9279 = \boxed{0'0721}$

c) $P[Z > 1'61] = 1 - 0'9463 = \boxed{0'0537}$

d) $P[Z > -2'06] = \boxed{0'9803}$

e) $P[0'91 < Z \leq 2'3] = 0'9893 - 0'8186 = \boxed{0'1707}$

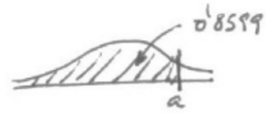
f) $P[-1'72 < Z \leq -0'23] = 0'9573 - 0'5910 = \boxed{0'3663}$



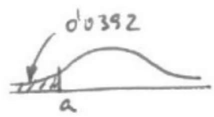
g) $P[-0'74 < Z \leq 1'5] = 0'9332 - (1 - 0'7704) = \boxed{0'7036}$

h) $P[-0'75 < Z \leq 4'1] = 1 - (1 - 0'7734) = \boxed{0'7734}$

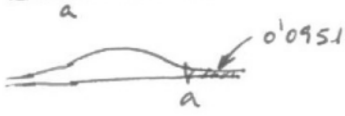
a) $P[Z \leq a] = 0'8599 \Rightarrow \boxed{a = 1'08}$



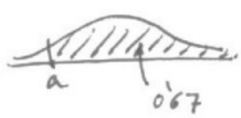
b) $P[Z \leq a] = 0'0392 \Rightarrow P[Z > a] = 0'9608 \Rightarrow \boxed{a = -1'76}$



c) $P[Z > a] = 0'0951 \Rightarrow P[Z \leq a] = 0'9049 \Rightarrow \boxed{a = 1'31}$



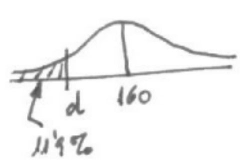
d) $P[Z > a] = 0'67 \Rightarrow \boxed{a = -0'44}$



⑩ $N(160, 20)$

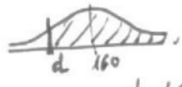
a) $P[X > 180] = P\left[Z > \frac{180-160}{20}\right] = P[Z > 1] = 1 - 0'8413 = \boxed{0'1587}$

b)



$P[X < d] = 0'119$

$P[X \geq d] = 0'881$



$P\left[Z \geq \frac{d-160}{20}\right] = 0'881 \Rightarrow \frac{d-160}{20} = -1'18 \Rightarrow \boxed{d = 136'4 \text{ cm}}$

(11) a) $N(46, 10)$

$P[X > 60] = P\left[Z > \frac{60-46}{10}\right] = P[Z > 1{,}4] = 1 - 0{,}9192 = \boxed{0{,}0808}$

b) $N(\mu, 12)$

$P[X < 60] = 0{,}85 \Rightarrow P\left[Z < \frac{60-\mu}{12}\right] = 0{,}85$

$\frac{60-\mu}{12} = 1{,}04 \Rightarrow \boxed{\mu = 47{,}52}$

c) Camino A: $P[X < 60] = 1 - 0{,}0808 = 0{,}9192$

Camino B: $P[X < 60] = 0{,}85$

Debería ir por el camino A, ya que tiene una probabilidad mayor de que necesite menos de 60 minutos en el viaje

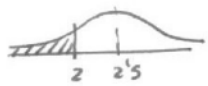
d) $B(5, 0{,}85)$ X = 'nº de días que tarda menos de una hora' $X = \{0,1,2,3,4,5\}$

(i) $P[X=5] = \binom{5}{5} 0{,}85^5 \cdot 0{,}15^0 = \boxed{0{,}4437}$

(ii) $P[X \geq 3] = \binom{5}{3} 0{,}85^3 \cdot 0{,}15^2 + \binom{5}{4} 0{,}85^4 \cdot 0{,}15^1 + \binom{5}{5} 0{,}85^5 \cdot 0{,}15^0 = \boxed{0{,}9734}$

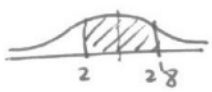
(12) a) $N(2{,}5, 0{,}3)$

$P[X < 2] = P\left[Z < \frac{2-2{,}5}{0{,}3}\right] = P[Z < -1{,}67] = 1 - 0{,}9522 = \boxed{0{,}0478}$



$P[X > 2{,}8] = P\left[Z > \frac{2{,}8-2{,}5}{0{,}3}\right] = P[Z > 1] = 1 - 0{,}8413 = \boxed{0{,}1587}$

$P[2 < X < 2{,}8] = 1 - [0{,}0478 + 0{,}1587] = \boxed{0{,}7936}$



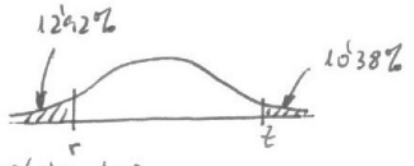
b) $B(10, 0{,}7936)$ X = 'nº de pollos con peso entre 2 Kg y 2'8 Kg'

(i) $P[X=10] = \binom{10}{10} 0{,}7936^{10} \cdot 0{,}2064^0 = \boxed{0{,}0991}$

(ii) $P[X \geq 7] = \binom{10}{7} 0{,}7936^7 \cdot 0{,}2064^3 + \binom{10}{8} 0{,}7936^8 \cdot 0{,}2064^2 + \binom{10}{9} 0{,}7936^9 \cdot 0{,}2064^1 +$

$+ \binom{10}{10} 0{,}7936^{10} \cdot 0{,}2064^0 = \boxed{0{,}8676}$

(13)



$N(6{,}84, 0{,}25)$

$P[X < t] = 1 - 0{,}1038 = 0{,}8962$

$P\left[Z < \frac{t-6{,}84}{0{,}25}\right] = 0{,}8962 \Rightarrow \frac{t-6{,}84}{0{,}25} = 1{,}26 \Rightarrow \boxed{t = 7{,}155 \text{ cm}}$

$P[X < r] = 0{,}1292$

$P\left[Z < \frac{r-6{,}84}{0{,}25}\right] = 0{,}1292 \Rightarrow P\left[Z > \frac{r-6{,}84}{0{,}25}\right] = 0{,}8708 \Rightarrow \frac{r-6{,}84}{0{,}25} = -1{,}13 \Rightarrow \boxed{r = 6{,}5575 \text{ cm}}$

(21) $X = \{1, 2, 3, 4, 5\}$

X	p
1	x
2	2x
3	3x
4	4x
5	5x
	15x

$\leftarrow 15x = 1 \Rightarrow x = 1/15 \Rightarrow$

X	p
1	1/15
2	2/15
3	3/15
4	4/15
5	5/15

$P[X = par] = \frac{2}{15} + \frac{4}{15} = \frac{6}{15} = \boxed{0'4}$

$E[x] = 1 \cdot \frac{1}{15} + 2 \cdot \frac{2}{15} + 3 \cdot \frac{3}{15} + 4 \cdot \frac{4}{15} + 5 \cdot \frac{5}{15} = \frac{55}{15} = \boxed{3'\widehat{6}}$

$\sigma = \sqrt{1^2 \cdot \frac{1}{15} + 2^2 \cdot \frac{2}{15} + 3^2 \cdot \frac{3}{15} + 4^2 \cdot \frac{4}{15} + 5^2 \cdot \frac{5}{15} - 3'\widehat{6}^2} = \boxed{1'2472}$

(22)

Y	X	P
-3€	1	1/6
-3€	2	1/6
-3€	3	1/6
-3€	4	1/6
5€	5	1/6
6€	6	1/6

$E[Y] = (-3) \cdot \frac{4}{6} + 5 \cdot \frac{1}{6} + 6 \cdot \frac{1}{6} = \boxed{-\frac{1}{6}}$

No nos interesaría mucho, porque por término medio perderíamos dinero.

(23) a]

SUMA	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$P[X=6] = \boxed{\frac{5}{36}}$

$P[X>6] = \boxed{\frac{21}{36}}$

$P[X=7 \mid X>5] = \frac{6}{30} = \boxed{\frac{1}{5}}$

b] $P[X=6] = \frac{5}{36}$; $P[X>6] = \frac{21}{36}$; $P[X<6] = \frac{10}{36}$

Y	p
+3	5/36
+1	21/36
-K	10/36

$E[Y] = 3 \cdot \frac{5}{36} + 1 \cdot \frac{21}{36} - K \cdot \frac{10}{36} = \frac{36-10K}{36}$

$E[Y]=0 \Rightarrow 36-10K=0$; $\boxed{K = 3'6}$

(24)

a] $0'3 + 0'45 + 0'2 + 0'35 = 1'3$. Es incorrecto, debe sumar 1.

b] $0'4 + K + 2K + 0'3 = 1 \Rightarrow 3K = 0'3 \Rightarrow \boxed{K = 0'1}$

c]

X	p
0	1/20
3	4/20
4	5/20
9	10/20

$P[X=0] = \boxed{\frac{1}{20}}$

$P[X>0] = 1 - \frac{1}{20} = \boxed{\frac{19}{20}}$

(25) a) $X=$ 'nº de cincos en 3 lanzamientos'

$X=\{0,1,2,3\}$

$P[X=0]=\binom{3}{0}\left(\frac{1}{6}\right)^0\cdot\left(\frac{5}{6}\right)^3=\frac{125}{216}$

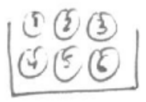
$P[X=1]=\binom{3}{1}\left(\frac{1}{6}\right)^1\cdot\left(\frac{5}{6}\right)^2=\frac{75}{216}$ Es una binomial con $n=3$, $p=1/6$

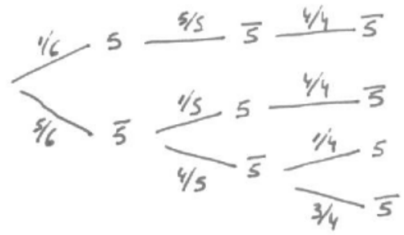
$P[X=2]=\binom{3}{2}\left(\frac{1}{6}\right)^2\cdot\left(\frac{5}{6}\right)^1=\frac{15}{216}$

$P[X=3]=\binom{3}{3}\left(\frac{1}{6}\right)^3=\frac{1}{216}$

$E[X]=n\cdot p=3\cdot\frac{1}{6}=\frac{3}{6}=\boxed{0'5}$

$\sigma=\sqrt{n\cdot p(1-p)}=\sqrt{3\cdot\frac{1}{6}\cdot\frac{5}{6}}=\boxed{0'6455}$

b) 



$X=$ 'nº de cincos en tres extracciones'

$X=\{0,1\}$

$P[X=0]=\frac{5}{6}\cdot\frac{4}{5}\cdot\frac{3}{4}=0'5$ No es binomial.

$P[X=1]=\frac{1}{6}\cdot\frac{5}{5}\cdot\frac{4}{4}+\frac{5}{6}\cdot\frac{1}{5}\cdot\frac{4}{4}+\frac{5}{6}\cdot\frac{4}{5}\cdot\frac{1}{4}=0'5$

$E[X]=0\cdot 0'5+1\cdot 0'5=\boxed{0'5}$

$\sigma=\sqrt{0^2\cdot 0'5+1^2\cdot 0'5-0'5^2}=\boxed{0'5}$

(26) $B(8,\ 0'7)$

$X=$ 'nº de fumadores mujeres' $X=\{0,1,2,3,4,5,6,7,8\}$

a) $P[X=0]=\binom{8}{0}\cdot 0'7^0\cdot 0'3^8=\boxed{6'561\cdot 10^{-5}}$

b) $P[X=4]=\binom{8}{4}\,0'7^4\cdot 0'3^4=\boxed{0'1361}$

c) $E[X]=8\cdot 0'7=\boxed{5'6}$

$\sigma=\sqrt{8\cdot 0'7\cdot 0'3}=\boxed{1'2961}$

(27) $B(3,\ 1/4)$

$X=$ 'nº de días que va en autobús' $X=\{0,1,2,3\}$

a) $E[X]=3\cdot\frac{1}{4}=\boxed{0'75}$

b) $P[X=2]=\binom{3}{2}\left(\frac{1}{4}\right)^2\cdot\left(\frac{3}{4}\right)=\boxed{\frac{9}{64}}$

c) $P[X\geq 1]=1-P[X=0]=1-\binom{3}{0}\left(\frac{1}{4}\right)^0\cdot\left(\frac{3}{4}\right)^3=1-\frac{27}{64}=\boxed{\frac{37}{64}}$

(28)

a) $P(B) = \frac{1}{4} \qquad P(R) = \frac{3}{4}$

b)

$\xrightarrow{1/4}$ B (2) ⟵ $X = 2$

$\xrightarrow{3/4}$ R (1) $\xrightarrow{1/4}$ B (2) ⟵ $X = 1+2 = 3$

R (1) $\xrightarrow{3/4}$ R (1) ⟵ $X = 1+1 = 2$

c) $P[X = 2] = \frac{1}{4} + \frac{3}{4} \cdot \frac{3}{4} = \boxed{\frac{13}{16}}$

$P[X = 3] = \frac{3}{4} \cdot \frac{1}{4} = \boxed{\frac{3}{16}}$

d)

X	p
2	13/16
3	3/16

$E[X] = 2 \cdot \frac{13}{16} + 3 \cdot \frac{3}{16} = \boxed{\frac{35}{16}}$

e)

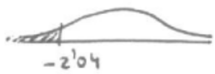
Y	P
0	13/16
10 \$	3/16

$P[\text{gane 10\$ en dos jugadas}] = \frac{13}{16} \cdot \frac{3}{16} + \frac{3}{16} \cdot \frac{13}{16} = \boxed{\frac{39}{128}}$

(29)

a) $P[Z \leq 1'47] = \boxed{0'9292}$

b) $P[Z \leq -2'04] = 1 - 0'9793 = \boxed{0'0207}$

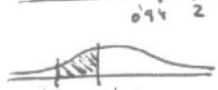


c) $P[Z > 1'65] = 1 - 0'9505 = \boxed{0'0495}$

d) $P[Z > -3'26] = \boxed{0'9994}$

e) $P[0'94 < Z \leq 2] = 0'9772 - 0'8264 = \boxed{0'1508}$

f) $P[-0'72 < Z \leq -0'35] = 0'7611 - 0'6368 = \boxed{0'1243}$

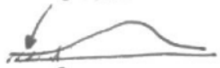


g) $P[-1 < Z \leq 1] = 0'8413 - (1 - 0'8413) = \boxed{0'6826}$

h) $P[-4'52 < Z \leq 2'1] = \boxed{0'9821}$

a) $P[Z \leq a] = 0'9649 \Rightarrow \boxed{a = 1'81}$

b) $P[Z \leq a] = 0'0322 \Rightarrow P[Z > a] = 0'9678 \Rightarrow \boxed{a = -1'85}$



c) $P[Z > a] = 0'8729 \Rightarrow \boxed{a = -1'14}$

d) $P[Z > a] = 0'3446 \Rightarrow P[Z \leq a] = 0'6554 \Rightarrow \boxed{a = 0'4}$

(30) $N(\mu, \sigma)$

$P[X<3] = 0'20 \Rightarrow P\left[Z < \frac{3-\mu}{\sigma}\right] = 0'20 \Rightarrow P\left[Z > \frac{3-\mu}{\sigma}\right] = 0'80 \Rightarrow \frac{3-\mu}{\sigma} = -0'8416$

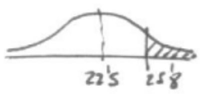
$P[X>8] = 0'10 \Rightarrow P\left[Z > \frac{8-\mu}{\sigma}\right] = 0'10 \Rightarrow P\left[Z < \frac{8-\mu}{\sigma}\right] = 0'90 \Rightarrow \frac{8-\mu}{\sigma} = 1'2815$

$$\left.\begin{aligned} 3-\mu &= -0'8416\sigma \\ 8-\mu &= 1'2815\sigma \end{aligned}\right\}$$

$5 = 2'1231\sigma \Rightarrow \sigma = \frac{5}{2'1231} = \boxed{2'3550} \Rightarrow \mu = 3 + 0'8416 \cdot 2'3550 = \boxed{4'9820}$

(31) a) $N(22'5,\ 2'2)$

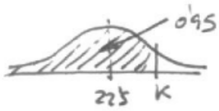
$P[X > 25'8] = P\left[Z > \frac{25'8-22'5}{2'2}\right] = P[Z > 1'5] = 1 - 0'9332 = \boxed{0'0668}$



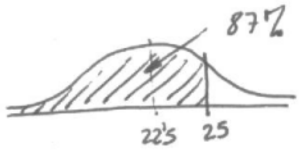
b) $P[X < K] = 0'95 \Rightarrow P\left[Z < \frac{K-22'5}{2'2}\right] = 0'95 \Rightarrow \frac{K-22'5}{2'2} = 1'6449$

$\Downarrow$

$K = 22'5 + 2'2 \cdot 1'6449 = \boxed{26'1188}$

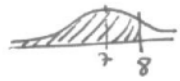


c)



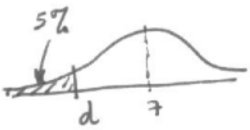
(32) $N(7,\ 0'5)$

a) $P[X<8] = P\left[Z < \frac{8-7}{0'5}\right] = P[Z<2] = \boxed{0'9772}$



$P[6<X<8] = P\left[\frac{6-7}{0'5} < X < \frac{8-7}{0'5}\right] = P[-2<Z<2] = 0'9772 - (1-0'9772) = \boxed{0'9544}$

b) $P[X<d] = 5\% = 0'05 \Rightarrow P\left[Z < \frac{d-7}{0'5}\right] = 0'05 \Rightarrow P\left[Z > \frac{d-7}{0'5}\right] = 0'95$



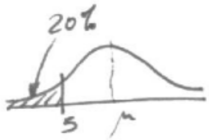
$\Downarrow$

$\frac{d-7}{0'5} = -1'6449 \Rightarrow d = 7 - 0'5 \cdot 1'6449 =$

$= \boxed{6'1776}$

c) $N(\mu,\ 0'5)$

$P[X<5] = 0'20 \Rightarrow P\left[Z < \frac{5-\mu}{0'5}\right] = 0'20 \Rightarrow P\left[Z > \frac{5-\mu}{0'5}\right] = 0'80$



$\Downarrow$

$\frac{5-\mu}{0'5} = -0'8416$

$5-\mu = -0'4208$

$\boxed{\mu = 5'4208}$

33 $N(5, 1'5)$

a) $P[X<6] = P\left[Z < \frac{6-5}{1'5}\right] = P[Z < 0'67] = \boxed{0'7475} = \boxed{74{,}75\%}$

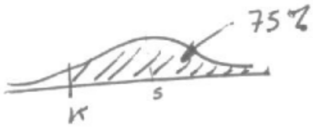
b) $P[X>K] = 0'75$

$P\left[Z > \frac{K-5}{1'5}\right] = 0'75 \Rightarrow \frac{K-5}{1'5} = -0'6745$

$\Downarrow$

$K-5 = -1'5 \cdot 0'6745$

$\boxed{K = 3'9883}$



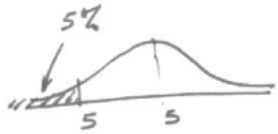
c) $P[X<K] = 0'05$

$P\left[Z < \frac{K-5}{1'5}\right] = 0'95 \Rightarrow \frac{K-5}{1'5} = -1'6449$

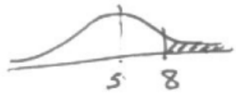
$\Downarrow$

$K-5 = -1'5 \cdot 1'6449$

$\boxed{K = 2'5327}$



d) $P[X>8] = P\left[Z > \frac{8-5}{1'5}\right] = P[Z>2] = 1 - 0'9772 = \boxed{0'0228}$



e) $B(50, 0'0228)$

$P[X=30] = \binom{50}{30} 0'0228^{30} \cdot 0'9772^{20} = \boxed{1'6256 \times 10^{-36}}$