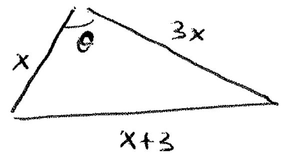
M00 P2



a) $\text{Area} = \dfrac{x\cdot 3x \,\text{sen}\theta}{2} = 2'21 \Rightarrow \boxed{\text{sen}\theta = \dfrac{4'42}{3x^2}}$

b) $(x+3)^2 = x^2 + (3x)^2 - 2\cdot x\cdot(3x)\cos\theta$

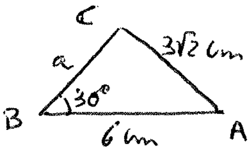
$\cancel{x^2}+6x+9 = \cancel{x^2}+9x^2 - 6x^2\cos\theta$

$6x^2\cos\theta = 9x^2-6x-9 \;;\; \cos\theta = \dfrac{9x^2-6x-9}{6x^2} = \boxed{\dfrac{3x^2-2x-3}{2x^2}}$

c) $\text{sen}^2\theta + \cos^2\theta = 1 \Rightarrow \cos^2\theta = 1-\text{sen}^2\theta \Rightarrow \left(\dfrac{3x^2-2x+3}{2x^2}\right)^2 = 1 - \left(\dfrac{4'42}{3x^2}\right)^2$ ✓

Con calculadora gráfica: $x = \begin{cases} 1'24 \to \theta = \arccos \dfrac{3\cdot 1'24^2 - 2\cdot 1'24 + 3}{2\cdot 1'24^2} = \boxed{1'86 \text{ rad}} \\ 2'94 \to \theta = \arccos \dfrac{3\cdot 2'94^2 - 2\cdot 2'94 + 3}{2\cdot 2'94^2} = \boxed{0'171 \text{ rad}} \end{cases}$

N00 P1



$(3\sqrt{2})^2 = a^2 + 6^2 - 2a\cdot 6\cos 30^\circ$

$18 = a^2 + 36 \quad 6\sqrt{3}a$

$0 = a^2 - 6\sqrt{3}a - 18 \;;\; a = \dfrac{6\sqrt{3} \pm \sqrt{36\cdot 3 - 72}}{2} = \dfrac{6\sqrt{3}\pm 6}{2} = \begin{cases} \boxed{3\sqrt{3}+3} \text{ cm} \\ \boxed{3\sqrt{3}-3} \text{ cm} \end{cases}$

M01 P1

$2\,\text{sen}x = \text{tg}x \;;\; -\frac{\pi}{2} < x < \frac{\pi}{2}$

$2\,\text{sen}x = \dfrac{\text{sen}x}{\cos x} \;;\; 2\,\text{sen}x\cos x = \text{sen}x \;;\; 2\,\text{sen}x\cos x - \text{sen}x = 0 \;;$

$\text{sen}x\cdot(2\cos x - 1) = 0 \begin{cases} \text{sen}x = 0 \Rightarrow \boxed{x = 0 \text{ rad}} \\ 2\cos x - 1 = 0 \Rightarrow \cos x = \frac{1}{2} \Rightarrow \boxed{x = \pm\frac{\pi}{3} \text{ rad}} \end{cases}$

M02 P1

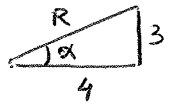
$\text{Tg}\theta + \text{ctg}\theta = 3 \;;\; \theta \in (0^\circ, 90^\circ)$

$\text{Tg}\theta + \dfrac{1}{\text{Tg}\theta} = 3 \;;\; \text{Tg}^2\theta + 1 = 3\,\text{Tg}\theta \;;\; \text{Tg}^2\theta - 3\,\text{tg}\theta + 1 = 0 \;;$

$\text{Tg}\theta = \dfrac{3\pm\sqrt{9-4}}{2} = \begin{cases} \frac{3+\sqrt{5}}{2} \Rightarrow \boxed{\theta = 20{,}9^\circ} \\ \frac{3-\sqrt{5}}{2} \Rightarrow \boxed{\theta = 69{,}1^\circ} \end{cases}$

M02 P1

a) $\underbrace{4}\cos\theta + \underbrace{3}\text{sen}\theta = R\cos(\theta - \alpha) = R\cos\theta\cos\alpha + R\,\text{sen}\theta\,\text{sen}\alpha$



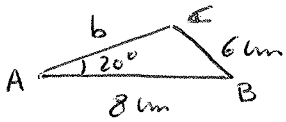
$R = \sqrt{4^2+3^2} = 5$

$\alpha = \text{arctg}\frac{3}{4} = 0'644 \text{ rad}$

$\Rightarrow 4\cos\theta + 3\,\text{sen}\theta = \boxed{5\cos(\theta - 0'644)}$

b) Máximo valor de $5\cos(\theta - 0'644)$ es 5 para $\cos(\theta - 0'644) = 1 \Rightarrow \theta - 0'644 = 0 \;;$ $\boxed{\theta = 0'644 \text{ rad}}$

N02



$6^2 = 8^2 + b^2 - 2\cdot 8\cdot b\cos 20^\circ$

$36 = 64 + b^2 - 15'035b$

$0 = b^2 - 15'035b + 28$

$b = \dfrac{15'035 \pm \sqrt{15'035^2 - 4\cdot 28}}{2} \begin{cases} 12'857 \\ 2'178 \Rightarrow \boxed{\text{Area} = 2'98 \text{ cm}^2} \end{cases}$

$\text{Area} = \dfrac{8\cdot b\cdot \text{sen}20^\circ}{2}$

También: $\dfrac{\text{sen}C}{8} = \dfrac{\text{sen}20^\circ}{6} \Rightarrow \text{sen}C = \dfrac{4\,\text{sen}20^\circ}{3} = 0'456 \Rightarrow C = \begin{cases} 27'131^\circ \\ 152'869^\circ \end{cases}$

$B = 180^\circ - 20^\circ - C \Rightarrow B = \begin{cases} 132'869^\circ \\ 7'131^\circ \longrightarrow \text{Area} = \dfrac{8\cdot 6\cdot \text{sen}B}{2} = 2'98 \text{ cm}^2 \end{cases}$ ✓

M03 P1

$\cos 2\theta = \sin^2\theta \longrightarrow \cos^2\theta - \sin^2\theta = \sin^2\theta \; ; \; \cos^2\theta = 2\sin^2\theta \; ; \; \frac{1}{2} = \mathrm{Tg}^2\theta \; ; \; \mathrm{Tg}\,\theta = \pm\sqrt{\frac{1}{2}} \; ;$

$\theta \in [0, \pi]$

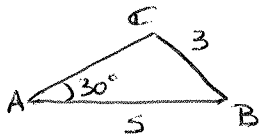
$\mathrm{Tg}\,\theta = \begin{cases} \sqrt{\frac{1}{2}} \Rightarrow \theta = \boxed{0'615 \text{ rad}} \\ -\sqrt{\frac{1}{2}} \Rightarrow \theta = -0'615 \text{ rad} + \pi \text{ rad} = \boxed{2'53 \text{ rad}} \end{cases}$

También:

$\cos^2\theta = 2\sin^2\theta \; ; \; \cos^2\theta = 2(1-\cos^2\theta) \; ; \; \cos^2\theta = 2 - 2\cos^2\theta \; ; \; 3\cos^2\theta = 2 \; ;$

$\cos^2\theta = \frac{2}{3} \; ; \; \cos\theta = \begin{cases} \sqrt{\frac{2}{3}} \Rightarrow \theta = 0'615 \text{ rad} \; ✓ \\ -\sqrt{\frac{2}{3}} \Rightarrow \theta = 2'53 \text{ rad} \; ✓ \end{cases}$

M03 P1

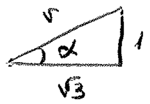


$\frac{\sin C}{5} = \frac{\sin 30°}{3} \Rightarrow \sin C = \frac{5}{6} \Rightarrow C = \begin{cases} \arcsin\frac{5}{6} = 56,4° \\ 180° - \arcsin\frac{5}{6} = 123,6° \end{cases}$

$B = 180 - 30° - C = \begin{cases} \boxed{93,6°} \\ \boxed{26,4°} \end{cases}$

N03 P2

a) $\sqrt{3}\cos\theta - \sin\theta = r\cdot\cos(\theta+\alpha) = r\cos\theta\cos\alpha - r\sin\theta\sin\alpha$



$r = \sqrt{(\sqrt{3})^2+1} = \sqrt{3+1} = \boxed{2}$

$\mathrm{Tg}\,\alpha = \frac{1}{\sqrt{3}} \Rightarrow \boxed{\alpha = \frac{\pi}{6} \text{ rad}}$

$\Rightarrow \sqrt{3}\cos\theta - \sin\theta = 2\cos\left(\theta + \frac{\pi}{6}\right)$

b) $\sqrt{3}\cos\theta - \sin\theta = 2\cos\left(\theta+\frac{\pi}{6}\right)$ toma valores entre -2 y 2

$\boxed{\sqrt{3}\cos\theta - \sin\theta \in [-2, 2]}$

c) $\sqrt{3}\cos\theta - \sin\theta = -1 \Rightarrow 2\cos\left(\theta+\frac{\pi}{6}\right) = -1 \Rightarrow \cos\left(\theta+\frac{\pi}{6}\right) = -\frac{1}{2} \Rightarrow$

$\Rightarrow \theta + \frac{\pi}{6} = \begin{cases} \frac{2\pi}{3} \Rightarrow \theta = \frac{2\pi}{3} - \frac{\pi}{6} = \boxed{\frac{\pi}{2} \text{ rad}} \\ \frac{4\pi}{3} \Rightarrow \theta = \frac{4\pi}{3} - \frac{\pi}{6} = \boxed{\frac{7\pi}{6} \text{ rad}} \end{cases}$

N03 P2

$\frac{\sin 4\theta\cdot(1-\cos 2\theta)}{\cos 2\theta\,(1-\cos 4\theta)} = \frac{2\sin 2\theta\,\cancel{\cos 2\theta}\cdot(1-\cos 2\theta)}{\cancel{\cos 2\theta}\,(1-(\cos^2 2\theta - \sin^2 2\theta))} = \frac{2\sin 2\theta\,(1-(\cos^2\theta-\sin^2\theta))}{1-\cos^2 2\theta + \sin^2 2\theta} =$

$= \frac{2\sin 2\theta\,(1-\cos^2\theta+\sin^2\theta)}{\sin^2 2\theta + \sin^2 2\theta} = \frac{\cancel{2\sin 2\theta}\cdot(\sin^2\theta+\sin^2\theta)}{\cancel{2}\sin^{\cancel{2}} 2\theta} = \frac{\cancel{2}\sin^2\theta}{\cancel{2}\sin\theta\cos\theta} = \frac{\sin\theta}{\cos\theta} = \mathrm{Tg}\,\theta \; ✓$

M04 P1

Arco = Angulo x Radio

$\widehat{AB} = \theta\cdot 1 = \theta$

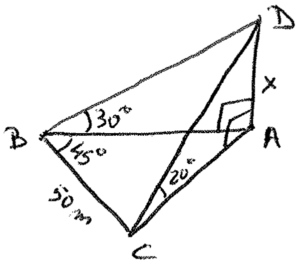
$\overline{OB_1} = 1\cdot\cos\theta = \cos\theta \Rightarrow \widehat{A_1B_1} = \theta\cdot\cos\theta$

$\overline{OB_2} = \cos\theta\cdot\cos\theta = \cos^2\theta \Rightarrow \widehat{A_2B_2} = \theta\cdot\cos^2\theta$

$\widehat{A_nB_n} = \theta\cdot\cos^n\theta$

$S_\infty = \theta + \theta\cos\theta + \theta\cos^2\theta + \cdots = \boxed{\frac{\theta}{1-\cos\theta}}$

M04 P1



$$\operatorname{tg} 20° = \frac{x}{\overline{AC}} \Rightarrow \overline{AC} = \frac{x}{\operatorname{tg}20°} = \operatorname{ctg}20° \cdot x$$

$$\operatorname{tg} 30° = \frac{x}{\overline{AB}} \Rightarrow \overline{AB} = \frac{x}{\operatorname{tg}30°} = \operatorname{ctg}30° \cdot x = \sqrt{3}x$$

$$\overline{AC}^2 = \overline{AB}^2 + \overline{BC}^2 - 2\,\overline{AB}\cdot\overline{BC}\cdot\cos 45°$$

$$\operatorname{ctg}^2 20° \cdot x^2 = (\sqrt{3}x)^2 + 50^2 - \cancel{2}\cdot 50\cdot\sqrt{3}x\cdot\frac{\sqrt{2}}{\cancel{2}}$$

$$(\operatorname{ctg}^2 20° - 3)x^2 + 50\sqrt{6}\,x - 2500 = 0$$

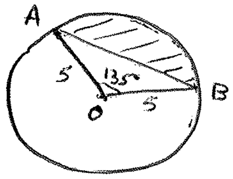
$$x = \frac{-50\sqrt{6} \pm \sqrt{2500\cdot 6 + 4(\operatorname{ctg}^2 20 - 3)\cdot 2500}}{2(\operatorname{ctg}^2 20° - 3)} = \begin{cases} \boxed{13'6} \\ \cancel{-40'37} \end{cases}$$

M04 P2

$$\cos(A+B) + \cos(A-B) = 2\cos A\cos B$$

$$\cos(A+B)+\cos(A-B) = \cos A\cos B - \cancel{\operatorname{sen}A\operatorname{sen}B} + \cos A\cos B + \cancel{\operatorname{sen}A\operatorname{sen}B} = 2\cos A\cos B \;\checkmark$$

N04 P1



$$\text{Area } \overset{\triangle}{OAB} = \frac{5\cdot 5 \operatorname{sen} 135°}{2} = \frac{25\sqrt{2}}{4}$$

$$\text{Area } \overset{\frown}{OAB} = \pi\cdot 5^2\cdot\frac{135}{360} = \frac{75\pi}{8}$$

$$\text{Area Sombreada} = \frac{75\pi}{8} - \frac{25\sqrt{2}}{4} = \boxed{20'6 \text{ cm}^2}$$

N04 P2

$$\operatorname{tg} 2A = \frac{2\operatorname{tg}A}{1-\operatorname{tg}^2 A}$$

$$\operatorname{tg}2A = \frac{\operatorname{sen}2A}{\cos 2A} = \frac{\operatorname{sen}A\cos A + \cos A\operatorname{sen}A}{\cos A\cos A - \operatorname{sen}A\operatorname{sen}A} = \frac{2\operatorname{sen}A\cos A}{\cos^2 A - \operatorname{sen}^2 A} = \frac{\frac{2\operatorname{sen}A\cancel{\cos A}}{\cos^{\cancel{2}} A}}{\frac{\cancel{\cos^2 A}}{\cancel{\cos^2 A}} - \frac{\operatorname{sen}^2 A}{\cos^2 A}} = \frac{2\operatorname{tg}A}{1-\operatorname{tg}^2 A} \;\checkmark$$

M05 P1

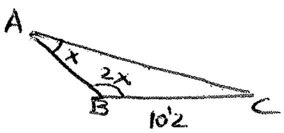
$a\operatorname{sen}4x + b\operatorname{sen}2x = 0$; $a\cdot 2\operatorname{sen}2x\cos 2x + b\cdot\operatorname{sen}2x = 0$; $\operatorname{sen}2x\,(2a\cos 2x + b) = 0 \Rightarrow$

$\Rightarrow$ $\operatorname{sin}2x = 0 \Rightarrow 2x = \begin{cases} 0 \Rightarrow \cancel{x=0} \\ \pi \Rightarrow \cancel{x=\pi/2} \end{cases}$ porque $0 < x < \pi/2$

$2a\cos 2x + b = 0$; $2a(\cos^2 x - \operatorname{sen}^2 x) + b = 0$; $2a(\cos^2 x - (1-\cos^2 x)) + b = 0$;

$2a(2\cos^2 x - 1) + b = 0$; $4a\cos^2 x - 2a + b = 0$; $4a\cos^2 x = 2a - b$; $\boxed{\cos^2 x = \frac{2a-b}{4a}}$

M05 P1

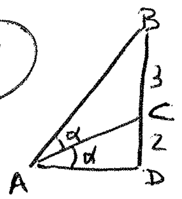


$$\frac{\overline{AC}}{\operatorname{sen}2x} = \frac{10'2}{\operatorname{sen}x} ; \quad \overline{AC} = \frac{10'2\operatorname{sen}2x}{\operatorname{sen}x} = \frac{10'2\cdot 2\cancel{\operatorname{sen}x}\cos x}{\cancel{\operatorname{sen}x}} = \boxed{20'4\cos x}$$

$$\text{Area} = \frac{10'2\cdot\overline{AC}}{2}\cdot\operatorname{sen}\hat{C}$$

$$52'02\cdot\cancel{\cos x} = \frac{10'2\cdot 20'4\cancel{\cos x}}{2}\operatorname{sen}\hat{C} \Rightarrow \operatorname{sen}\hat{C} = \frac{1}{2} \Rightarrow \hat{C} = \begin{cases}\boxed{30°} \\ \cancel{150°} \text{ No puede ser obtuso}\end{cases}$$

M05 P1

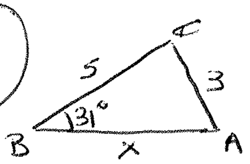


$$\left.\begin{aligned}\operatorname{tg}2\alpha &= \frac{5}{AD} \\ \operatorname{tg}\alpha &= \frac{2}{AD}\end{aligned}\right\} \quad \left.\begin{aligned} AD &= \frac{5}{\operatorname{tg}2\alpha} \\ AD &= \frac{2}{\operatorname{tg}\alpha}\end{aligned}\right\} \quad \frac{2}{\operatorname{tg}\alpha} = \frac{5}{\operatorname{tg}2\alpha} ; \quad 2\operatorname{tg}2\alpha = 5\operatorname{tg}\alpha ;$$

$$\frac{4\cancel{\operatorname{tg}\alpha}}{1-\operatorname{tg}^2\alpha} = 5\cancel{\operatorname{tg}\alpha} \;(\text{porque } \alpha\neq 0) ; \quad 4 = 5 - 5\operatorname{tg}^2\alpha ; \quad 5\operatorname{tg}^2\alpha = 1 ; \quad \operatorname{tg}^2\alpha = \frac{1}{5} ;$$

$$\operatorname{tg}\alpha = \begin{cases}\sqrt{1/5} \longrightarrow \boxed{\alpha = 24,1°} \\ \cancel{-\sqrt{1/5}} \;(\alpha \text{ es agudo})\end{cases}$$

N05
P1



$3^2 = 5^2 + x^2 - 2\cdot 5\cdot x\cdot \cos 31°$

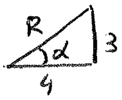
$0 = x^2 - 10\cos 31°\, x + 16$

$x = \dfrac{10\cos 31° \pm \sqrt{100\cos^2 31° - 64}}{2} = \boxed{\begin{matrix} 5'82 \text{ km} \\ 2'75 \text{ km}\end{matrix}}$

N05
P1

$4\sin\theta + 3\cos\theta = R\cos(\theta-\alpha) = R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$

$\left.\begin{matrix} 4 = R\sin\alpha \\ 3 = R\cos\alpha \end{matrix}\right\}$



$R = \sqrt{3^2+4^2} = 5$

$tg\alpha = \dfrac{3}{4} \longrightarrow \alpha = 36'87°$

$4\sin\theta + 3\cos\theta = 5\cos(\theta - 36'87°)$ toma valores entre -5 y 5.

El máximo valor de $\dfrac{1}{4\sin\theta+3\cos\theta+K}$ se dará cuando $4\sin\theta+3\cos\theta$ toma el menor valor, es decir: -5.

$\text{Max}\ \dfrac{1}{4\sin\theta+3\cos\theta+K} = 2 \Rightarrow \dfrac{1}{-5+K} = 2$; $1 = -10+2K$; $\boxed{K = \dfrac{11}{2}}$

Muestra 06
P1

$2tg^2\theta - 5\sec\theta - 10 = 0$; $\dfrac{2\sin^2\theta}{\cos^2\theta} - \dfrac{5}{\cos\theta} - 10 = 0$; $2\sin^2\theta - 5\cos\theta - 10\cos^2\theta = 0$;

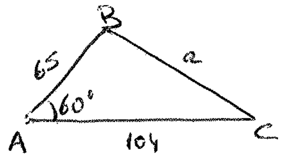
$2(1-\cos^2\theta) - 5\cos\theta - 10\cos^2\theta = 0$; $-12\cos^2\theta - 5\cos\theta + 2 = 0$

$\cos\theta = \dfrac{5\pm\sqrt{25+96}}{-24} = \dfrac{5\pm 11}{-24}$

$-\dfrac{16}{24} = -\dfrac{2}{3} \Rightarrow \boxed{\sec\theta = -\dfrac{3}{2}}$

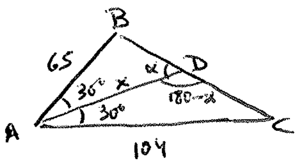
$\cancel{\dfrac{6}{24}}$ Porque $\theta \in 2°$ cuadrante

Muestra 06/08
P2



a) $a = \sqrt{65^2 + 104^2 - 2\cdot 65\cdot 104\cos 60°} = \boxed{91 \text{ m}}$

b) $\text{Area} = \dfrac{65\cdot 104\cdot \sin 60°}{2} = \dfrac{65\cdot 104\cdot \frac{\sqrt{3}}{2}}{2} = \boxed{1690\sqrt{3}\ m^2}$



c)

$\text{Area } \widehat{ABD} = \dfrac{65\cdot x\sin 30°}{2} = \dfrac{65x}{4}$ ✓

$\text{Area } \widehat{ADC} = \dfrac{x\cdot 104\cdot \sin 30°}{2} = \boxed{26x}$

$\dfrac{65x}{4} + 26x = 1690\sqrt{3} \Rightarrow \dfrac{169x}{4} = 1690\sqrt{3}$; $\boxed{x = 40\sqrt{3}\text{ m}}$ $q = 40$

d) $\text{Area } \widehat{ABD} = \dfrac{\overline{BD}\cdot x\cdot \sin\alpha}{2} = \dfrac{65x}{4}$

$\text{Area } \widehat{ADC} = \dfrac{\overline{DC}\cdot x\sin(180-\alpha)}{2} = \dfrac{104x}{4}$

Dividiendo: $\dfrac{\frac{\overline{BD}\cdot \cancel{x}\cdot \cancel{\sin\alpha}}{\cancel{2}}}{\frac{\overline{DC}\cdot \cancel{x}\cancel{\sin(180-\alpha)}}{\cancel{2}}} = \dfrac{\frac{65\cancel{x}}{\cancel{4}}}{\frac{104\cancel{x}}{\cancel{4}}} \Rightarrow \dfrac{\overline{BD}}{\overline{DC}} = \dfrac{65}{104} = \dfrac{5}{8}$ ✓

M05
TZ1
P1#8

$\text{Arco} = 2R \Rightarrow \theta = 2 \text{ rad} \longrightarrow \text{Area Triángulo} = \dfrac{R\cdot R\cdot \sin 2}{2} = \dfrac{R^2\sin 2}{2}$

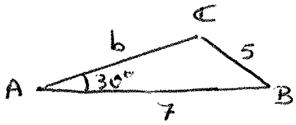
$\text{Area Sector} = \dfrac{R^2\cdot 2}{2} = R^2$

$\text{Area Sombreada} = KR^2 \Rightarrow KR^2 = R^2 - \dfrac{R^2\sin 2}{2} \Rightarrow K = \boxed{1 - \dfrac{\sin 2}{2}} = \boxed{0'545}$

N06 P1

$$\operatorname{tg}^2 2\theta = 1 \quad -\frac{\pi}{2} \le \theta \le \frac{\pi}{2} \longrightarrow \operatorname{tg} 2\theta = \begin{cases} 1 \Rightarrow 2\theta = \begin{cases} \pi/4 \longrightarrow \theta = \pi/8 \\ -3\pi/4 \longrightarrow \theta = -3\pi/8 \end{cases} \\ -1 \Rightarrow 2\theta = \begin{cases} -\pi/4 \longrightarrow \theta = -\pi/8 \\ 3\pi/4 \longrightarrow \theta = 3\pi/8 \end{cases} \end{cases}$$

N06 P1



$$5^2 = b^2 + 7^2 - 2b7\cos 30^\circ$$

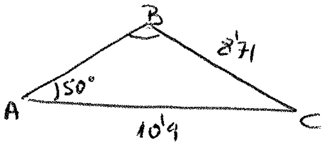
$$0 = b^2 - 7\sqrt{3}\,b + 24$$

$$b = \frac{7\sqrt{3} \pm \sqrt{147-96}}{2} = \frac{7\sqrt{3} \pm \sqrt{51}}{2} \begin{cases} b_1 = \frac{7\sqrt{3}+\sqrt{51}}{2} \\ b_2 = \frac{7\sqrt{3}-\sqrt{51}}{2} \end{cases}$$

$$\text{Area } \widehat{ABC} = \frac{7 \cdot b \cdot \operatorname{sen} 30^\circ}{2} = \frac{7}{4} b$$

$$\text{Area } \widehat{AB_1C} - \text{Area } \widehat{AB_2C} = \frac{7}{4} \cdot \frac{7\sqrt{3}+\sqrt{51}}{2} - \frac{7}{4} \cdot \frac{7\sqrt{3}-\sqrt{51}}{2} = \frac{7}{4} \cdot \frac{\cancel{7\sqrt{3}}+\sqrt{51}-\cancel{7\sqrt{3}}+\sqrt{51}}{2} = \boxed{\frac{7}{4}\sqrt{51}}\ \text{cm}^2$$

M06 P1



$$\frac{\operatorname{sen} B}{10'4} = \frac{\operatorname{sen} 50}{8'71} \Rightarrow \operatorname{sen} B = 0'9587 \Rightarrow B = \begin{cases} \cancel{73,47^\circ} \\ 106,53^\circ \end{cases}$$ Dicen que B es obtuso.

$$C = 180 - 50 - B = 23,47^\circ$$

$$\text{Area } \widehat{ABC} = \frac{10'4 \cdot 8'71 \cdot \operatorname{sen} 23,47^\circ}{2} = \boxed{18'9\ \text{cm}^2}$$

N06 P1

$$\operatorname{tg} A + \operatorname{tg} B + \operatorname{tg} C = \operatorname{tg} A \cdot \operatorname{tg} B \cdot \operatorname{tg} C$$

$$\operatorname{tg} A + \operatorname{tg} B + \operatorname{tg} C = \operatorname{tg} A + \operatorname{tg} B + \operatorname{tg}(180-(A+B)) = \operatorname{tg} A + \operatorname{tg} B - \operatorname{tg}(A+B) = \operatorname{tg} A + \operatorname{tg} B - \frac{\operatorname{tg} A + \operatorname{tg} B}{1 - \operatorname{tg} A \operatorname{tg} B} =$$

$$= \frac{\cancel{\operatorname{tg} A} - \operatorname{tg}^2 A \operatorname{tg} B + \cancel{\operatorname{tg} B} - \operatorname{tg} A \operatorname{tg}^2 B - \cancel{\operatorname{tg} A} - \cancel{\operatorname{tg} B}}{1 - \operatorname{tg} A \operatorname{tg} B} = \frac{-\operatorname{tg} A \cdot \operatorname{tg} B \cdot (\operatorname{tg} A + \operatorname{tg} B)}{1 - \operatorname{tg} A \operatorname{tg} B} =$$

$$= -\operatorname{tg} A \cdot \operatorname{tg} B \cdot \operatorname{tg}(A+B) = \operatorname{tg} A \cdot \operatorname{tg} B \cdot \operatorname{tg} C \quad ✓$$

N06 P2

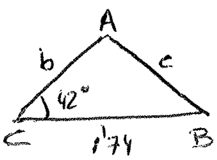
$$\left.\begin{aligned} \text{Area } \overset{\frown}{OAB} &= \frac{\pi r^2 \theta}{2\pi} = \frac{\theta r^2}{2} \\ \text{Area } \widehat{OAB} &= \frac{r^2 \operatorname{sen}\theta}{2} \end{aligned}\right\} \Rightarrow \text{Area segmento} = \frac{\theta r^2}{2} - \frac{r^2 \operatorname{sen}\theta}{2} = \frac{1}{2} r^2 (\theta - \operatorname{sen}\theta) \quad ✓$$

$$\text{Area otro segmento} = \pi r^2 - \frac{1}{2} r^2 (\theta - \operatorname{sen}\theta) = \boxed{\frac{1}{2} r^2 (2\pi - \theta + \operatorname{sen}\theta)}$$

$$\frac{\cancel{\frac{1}{2} r^2} (2\pi - \theta + \operatorname{sen}\theta)}{\cancel{\frac{1}{2} r^2} (\theta - \operatorname{sen}\theta)} = \frac{3}{2} \Rightarrow 4\pi - 2\theta + 2\operatorname{sen}\theta = 3\theta - 3\operatorname{sen}\theta \ ; \ 5\operatorname{sen}\theta = 5\theta - 4\pi \ ; \ \operatorname{sen}\theta = \theta - \frac{4\pi}{5} \quad ✓$$

$$\operatorname{sen}\theta = \theta - \frac{4\pi}{5} \Rightarrow \boxed{\theta = 2'82\ \text{rad}}$$ Hecho con calculadora gráfica

M07 P1



$$\text{Area} = 1'19 \Rightarrow \frac{b \cdot 1'74 \cdot \operatorname{sen} 42^\circ}{2} = 1'19 \Rightarrow b = \frac{2 \cdot 1'19}{1'74 \operatorname{sen} 42^\circ} = \boxed{2'04\ \text{cm}}$$

$$c = \sqrt{b^2 + 1'74^2 - 2b \cdot 1'74 \cos 42^\circ} = \boxed{1'39\ \text{cm}}$$

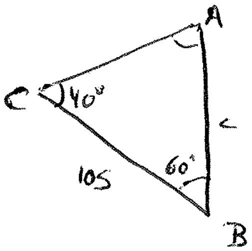
M07 P1

$$\left.\begin{aligned} \text{Area } \overset{\frown}{OAB} &= \frac{\theta \cdot 1^2}{2} = \frac{\theta}{2} \\ \text{Area } \widehat{OAB} &= \frac{1 \cdot 1 \cdot \operatorname{sen}\theta}{2} = \frac{\operatorname{sen}\theta}{2} \end{aligned}\right\} \rightarrow \text{Area segmento} = \frac{\theta}{2} - \frac{\operatorname{sen}\theta}{2} = \frac{\theta - \operatorname{sen}\theta}{2}$$

$$\text{Area } \widehat{OAB} = 3 \cdot \text{Area segmento} \Rightarrow \frac{\operatorname{sen}\theta}{\cancel{2}} = 3 \cdot \frac{\theta - \operatorname{sen}\theta}{\cancel{2}} \ ; \ \operatorname{sen}\theta = 3\theta - 3\operatorname{sen}\theta \ ; \ 4\operatorname{sen}\theta = 3\theta \ ;$$

$$4\operatorname{sen}\theta - 3\theta = 0 \Rightarrow \boxed{\theta = 1'28\ \text{rad}}$$ Hecho con calculadora gráfica.

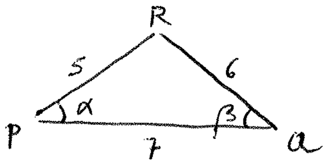
M07 P1



$\hat{A} = 180 - 40 - 60 = 80°$

$\frac{c}{\operatorname{sen} 40°} = \frac{105}{\operatorname{sen} 80°} \longrightarrow c = \frac{105 \operatorname{sen} 40°}{\operatorname{sen} 80°} = \boxed{68'5 \text{ cm}}$

N07 P1



$\cos\alpha = \frac{5^2+7^2-6^2}{2\cdot 5\cdot 7} = \frac{38}{70} \Rightarrow \alpha = 57{,}12 \longrightarrow 2\alpha = 114'24°$

$\cos\beta = \frac{6^2+7^2-5^2}{2\cdot 6\cdot 7} = \frac{60}{84} \Rightarrow \beta = 44'42 \longrightarrow 2\beta = 88'83°$

$\left.\begin{array}{l} \text{Area } \overset{\frown}{PRS} = \pi\cdot 5^2 \frac{114'24}{360} \\ \text{Area } \overset{\triangle}{PRS} = \frac{5^2\cdot \operatorname{sen} 114'24}{2} \end{array}\right\} \longrightarrow$ Area Segmento Sombreado derecho $= \frac{25\pi\cdot 114'24}{360} - \frac{25 \operatorname{sen} 114'24}{2} = 13'53$

$\left.\begin{array}{l} \text{Area } \overset{\frown}{QRS} = \frac{\pi\cdot 6^2\cdot 88'83}{360} \\ \text{Area } \overset{\triangle}{QRS} = \frac{6^2 \operatorname{sen} 88'83}{2} \end{array}\right\} \longrightarrow$ Area Segmento Sombreado izquierdo $= \frac{36\pi\cdot 88'83}{360} - \frac{36 \operatorname{sen} 88'83}{2} = 9'91$

Area Sombreada $= 13'53 + 9'91 = \boxed{23'4 \text{ u}^2}$

Muestra 08 P1

$\left.\begin{array}{l} \operatorname{tg} B = -\frac{5}{12} \\ B \text{ obtuso} \end{array}\right|$

$1 + \operatorname{tg}^2 B = \sec^2 B \longrightarrow 1 + \left(\frac{-5}{12}\right)^2 = \sec^2 B \; ; \; 1 + \frac{25}{144} = \sec^2 B \; ;$

$\sec^2 B = \frac{169}{144} \; ; \; \cos^2 B = \frac{144}{169} \; ; \; \cos B = \begin{cases} \boxed{-12/13} & \text{porque B es obtuso} \\ \cancel{12/13} \end{cases}$

$\operatorname{sen} B = \operatorname{tg} B \cdot \cos B = \frac{-5}{\cancel{12}} \cdot \frac{-\cancel{12}}{13} = \boxed{\frac{5}{13}}$

$\operatorname{sen} 2B = 2 \operatorname{sen} B \cos B = 2 \cdot \frac{5}{13} \cdot \frac{-12}{13} = \boxed{\frac{-120}{169}}$

$\cos 2B = \cos^2 B - \operatorname{sen}^2 B = \left(\frac{-12}{13}\right)^2 - \left(\frac{5}{13}\right)^2 = \frac{144-25}{169} = \boxed{\frac{119}{169}}$

Muestra 08 P1

$\operatorname{tg} 2\theta = \frac{3}{4} \; ; \; \frac{2\operatorname{tg}\theta}{1-\operatorname{tg}^2\theta} = \frac{3}{4} \; ; \; 8\operatorname{tg}\theta = 3 - 3\operatorname{tg}^2\theta \; ; \; 3\operatorname{tg}^2\theta + 8\operatorname{tg}\theta - 3 = 0$

$\operatorname{tg}\theta = \frac{-8 \pm \sqrt{64+36}}{6} = \frac{-8\pm 10}{6} \begin{cases} 2/6 = \boxed{\frac{1}{3}} \\ -\frac{18}{6} = \boxed{-3} \end{cases}$

Muestra 08 P1

$4\cos 2x - 3 \operatorname{sen} x \operatorname{cosec}^3 x + 6 = 0 \; ; \; 4(\cos^2 x - \operatorname{sen}^2 x) - 3 \operatorname{sen} x \frac{1}{\operatorname{sen}^3 x} + 6 = 0 \; ;$

$4(1 - \operatorname{sen}^2 x - \operatorname{sen}^2 x) - \frac{3}{\operatorname{sen}^2 x} + 6 = 0 \; ; \; 4 - 8\operatorname{sen}^2 x - \frac{3}{\operatorname{sen}^2 x} + 6 = 0 \; ;$

$-\frac{3}{\operatorname{sen}^2 x} = 8 \operatorname{sen}^2 x - 10 \; ; \; -3 = 8\operatorname{sen}^4 x - 10 \operatorname{sen}^2 x \; ; \; 8\operatorname{sen}^4 x - 10 \operatorname{sen}^2 x + 3 = 0$

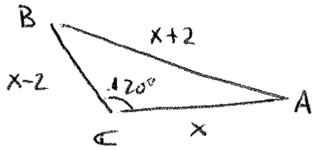
$\operatorname{sen} x = s \longrightarrow 8\cdot s^4 - 10\cdot s^2 + 3 = 0 \; ; \; s^2 = t \longrightarrow 8t^2 - 10t + 3 = 0$

$t = \frac{10 \pm \sqrt{100-96}}{16} = \frac{10\pm 2}{16} \begin{cases} \frac{12}{16} = \frac{3}{4} \to s^2 = \frac{3}{4} \Rightarrow s = \pm\frac{\sqrt{3}}{2} \longrightarrow \operatorname{sen} x = \begin{cases} \frac{\sqrt{3}}{2} \\ \cancel{-\frac{\sqrt{3}}{2}} \end{cases} \\ \frac{8}{16} = \frac{1}{2} \longrightarrow s^2 = \frac{1}{2} \Rightarrow s = \pm\sqrt{\frac{1}{2}} = \pm\frac{\sqrt{2}}{2} \longrightarrow \operatorname{sen} x = \begin{cases} \cancel{-\frac{\sqrt{2}}{2}} \\ \frac{\sqrt{2}}{2} \end{cases} \end{cases}$ porque $x \in [0, \pi]$

$\longrightarrow \operatorname{sen} x = \begin{cases} \frac{\sqrt{3}}{2} \Rightarrow \boxed{x = \pi/3 \; , \; 2\pi/3} \\ \frac{\sqrt{2}}{2} \Rightarrow \boxed{x = \pi/4 \; , \; 3\pi/4} \end{cases}$

Muestra 08
P1

$x-2 < x < x+2$, por lo que $x+2$ es el lado enfrentado al mayor ángulo.



$$(x+2)^2 = (x-2)^2 + x^2 - 2(x-2)x \cdot \cos 120^\circ$$
$$\cancel{x^2} + 4x + \cancel{4} = \cancel{x^2} - 4x + \cancel{4} + x^2 - \cancel{2}(x^2 - 2x) \frac{-1}{\cancel{2}}$$
$$4x = -4x + x^2 + x^2 - 2x$$
$$0 = 2x^2 - 10x$$
$$0 = 2x(x-5) \rightarrow \cancel{x=0} \quad \text{No estaría definido } x-2$$
$$\rightarrow \boxed{x=5}$$

$$\text{Area} = \frac{(x-2)\cdot x \,\text{sen}120^\circ}{2} = \frac{(5-2)\cdot 5 \cdot \sqrt{3}/2}{2} = \frac{15\sqrt{3}}{4} \checkmark$$

$$\frac{\text{sen} A}{x-2} = \frac{\text{sen}120^\circ}{x+2} \rightarrow \text{sen} A = \frac{3\cdot\sqrt{3}/2}{7} = \frac{3\sqrt{3}}{14}$$

$$\frac{\text{sen} B}{x} = \frac{\text{sen}120^\circ}{x+2} \rightarrow \text{sen} B = \frac{5\cdot\sqrt{3}/2}{7} = \frac{5\sqrt{3}}{14}$$

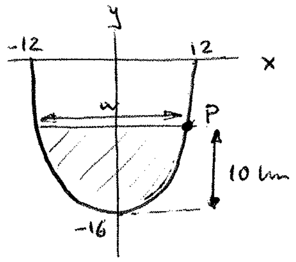
$$\text{sen} A + \text{sen} B + \text{sen} C = \frac{3\sqrt{3}}{14} + \frac{5\sqrt{3}}{14} + \frac{\sqrt{3}}{2} = \boxed{\frac{15\sqrt{3}}{14}}$$

M08
P1

$$AB = \sqrt{3^2+2^2} = \sqrt{13} \quad ; \quad AC = \sqrt{3^2+4^2} = \sqrt{9+16} = 5$$

$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \text{sen}\alpha\,\text{sen}\beta = \frac{3}{5}\cdot\frac{3}{\sqrt{13}} + \frac{4}{5}\cdot\frac{2}{\sqrt{13}} = \boxed{\frac{17}{5\sqrt{13}}} = \boxed{\frac{17\sqrt{13}}{65}}$$

M08
P1



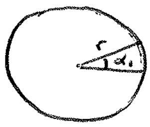
$$y = 16\sec\left(\frac{\pi x}{36}\right) - 32 = \frac{16}{\cos\left(\frac{\pi x}{36}\right)} - 32$$

El punto P tiene de coordenada y: $-16+10 = -6$

$$y=-6 \Rightarrow -6 = \frac{16}{\cos\left(\frac{\pi x}{36}\right)} - 32 \;;\; 26 = \frac{16}{\cos\left(\frac{\pi x}{36}\right)} \;;\; \cos\left(\frac{\pi x}{36}\right) = \frac{16}{26} = \frac{8}{13}$$

$$\frac{\pi x}{36} = \arccos\left(\frac{8}{13}\right) \Rightarrow x = \frac{36}{\pi}\arccos\left(\frac{8}{13}\right) \Rightarrow w = 2x = \boxed{\frac{72}{\pi}\arccos\frac{8}{13} \text{ km}}$$

M08
P1



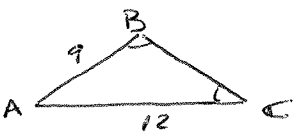
$$\text{Area}_{\text{Sector}_1} = \pi r^2 \cdot \frac{\alpha_1}{2\pi} = \frac{r^2}{2}\alpha_1$$

Area sectores: $\frac{r^2}{2}\alpha_1, \frac{r^2}{2}\alpha_2, \dots, \frac{r^2}{2}\alpha_{12}$

$$S_{12} = \frac{r^2}{2}\alpha_1 + \frac{r^2}{2}\alpha_2 + \dots + \frac{r^2}{2}\alpha_{12} = \frac{r^2}{2}(\alpha_1+\alpha_2+\dots+\alpha_{12}) = \frac{r^2}{2}\cdot 2\pi = r^2\pi$$

$$\left.\begin{array}{l} S_{12} = \pi r^2 \\ \alpha_{12} = 2\alpha_1 \end{array}\right\} \rightarrow \left.\begin{array}{l} \pi r^2 = \frac{\frac{r^2}{2}\alpha_1 + \frac{r^2}{2}\alpha_{12}}{2}\cdot 12 \\ \alpha_{12} = 2\alpha_1 \end{array}\right\} \rightarrow \pi \cancel{r^2} = \frac{\cancel{r^2}}{2}\frac{\alpha_1 + 2\alpha_1}{2}\cdot 12 \;;\; \pi = \frac{36\alpha_1}{4} \Rightarrow \boxed{\alpha_1 = \frac{\pi}{9} \text{ rad}}$$

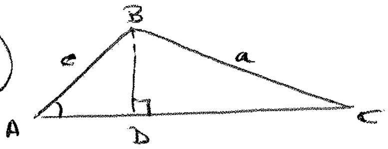
M08
P1



$$B = 2C \;;\; \frac{\text{sen} B}{12} = \frac{\text{sen} C}{9} \;;\; \frac{\text{sen} 2C}{12} = \frac{\text{sen} C}{9} \;;$$

$$\frac{2\,\cancel{\text{sen} C}\cos C}{12} = \frac{\cancel{\text{sen} C}}{9} \Rightarrow \boxed{\cos C = \frac{2}{3}}$$

M08 P2



$\cos\hat{A} = \frac{\overline{AD}}{c} \rightarrow \overline{AD} = c\cos\hat{A}$

$\overline{CD} = \overline{AC} - \overline{AD} = b - c\cos\hat{A}$ ✓

b) $\left.\begin{array}{l}\overline{BD}^2 = c^2 - \overline{AD}^2 \\ \overline{BD}^2 = a^2 - \overline{CD}^2\end{array}\right\}$ $c^2 - \overline{AD}^2 = a^2 - \overline{CD}^2$; $c^2 - c^2\cos^2 A = a^2 - (b - c\cos A)^2$;

$c^2 - \cancel{c^2\cos^2 A} = a^2 - b^2 + 2bc\cos A - \cancel{c^2\cos^2 A} \Rightarrow \boxed{a^2 = b^2 + c^2 - 2bc\cos A}$ ✓

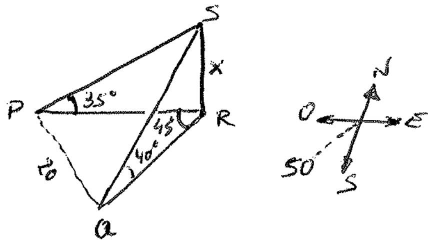
c) $b^2 = a^2 + c^2 - 2ac\cos\hat{B}$

$b^2 = a^2 + c^2 - 2ac\cos 60^\circ = a^2 + c^2 - 2ac\cdot\frac{1}{2} = a^2 + c^2 - ac$; $0 = c^2 - ac + (a^2 - b^2)$

$c = \frac{a \pm\sqrt{a^2 - 4(a^2-b^2)}}{2} = \frac{a\pm\sqrt{a^2 - 4a^2 + 4b^2}}{2} = \frac{a\pm\sqrt{4b^2 - 3a^2}}{2} =$

$= \frac{a}{2} \pm \sqrt{\frac{4b^2 - 3a^2}{4}} = \boxed{\frac{a}{2} \pm \sqrt{b^2 - \frac{3}{4}a^2}}$ ✓

M08 P2



$\text{ctg}\,35^\circ = \frac{PQ}{x} \Rightarrow PQ = x\cdot\text{ctg}\,35^\circ$

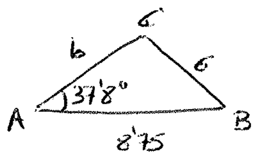
$\text{ctg}\,40^\circ = \frac{QR}{x} \Rightarrow QR = x\,\text{ctg}\,40^\circ$

$20^2 = (x\,\text{ctg}\,35^\circ)^2 + (x\,\text{ctg}\,40^\circ)^2 - 2x\,\text{ctg}\,35^\circ\,\text{ctg}\,40^\circ\cos 45^\circ$

$400 = x^2\left[\text{ctg}^2 35 + \text{ctg}^2 40 - \sqrt{2}\,\text{ctg}\,35^\circ\,\text{ctg}\,40^\circ\right]$

$x = \sqrt{\frac{400}{\text{ctg}^2 35 + \text{ctg}^2 40 - \sqrt{2}\,\text{ctg}\,35^\circ\,\text{ctg}\,40^\circ}} = \boxed{19'5\text{ m}}$

M08 P2



$6^2 = b^2 + 8'75^2 - 2b\cdot 8'75\cdot\cos 37'8$

$0 = b^2 - 13'83b + 40'5625 \Rightarrow b = \boxed{\begin{array}{l} 9'60 \\ 4'22\end{array}}$

N08 P1 a) $\text{tg}\,x + \text{ctg}\,x = 2\,\text{cosec}\,2x$

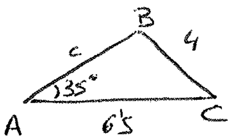
$\text{tg}\,x + \text{ctg}\,x = \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{\sin^2 x + \cos^2 x}{\cos x\sin x} = \frac{1}{\cos x\sin x} = \frac{2}{2\sin x\cos x} = \frac{2}{\sin 2x} = 2\,\text{cosec}\,2x$ ✓

e) $\text{cosec}\,2x = 1'5\,\text{tg}\,x - 0'5 \rightarrow 2\,\text{cosec}\,2x = 3\,\text{tg}\,x - 1$; $\text{tg}\,x + \text{ctg}\,x = 3\,\text{tg}\,x - 1$;

$\frac{1}{\text{tg}\,x} = 2\,\text{tg}\,x - 1$; $1 = 2\,\text{tg}^2 x - \text{tg}\,x$; $0 = 2\,\text{tg}^2 x - \text{tg}\,x - 1$

$\text{tg}\,x = \frac{1\pm\sqrt{1+8}}{4} = \frac{1\pm 3}{4}$ $\begin{array}{l} 1 \Rightarrow \boxed{x = \pi/4} \\ \cancel{-\frac{1}{2}} \quad \text{porque } 0 \le x \le \pi/2\end{array}$

N08 P2



$\frac{\sin B}{6'5} = \frac{\sin 35^\circ}{4}$; $\sin B = \frac{6'5\cdot\sin 35^\circ}{4} = 0'932 \Rightarrow B = \boxed{\begin{array}{l} 68'76^\circ \\ 111'24^\circ\end{array}}$

$\frac{c}{\sin C} = \frac{4}{\sin 35^\circ}$; $c = \frac{4\sin C}{\sin 35^\circ} = \frac{4\sin(180 - 35 - B)}{\sin 35^\circ} = \frac{4\sin(145 - B)}{\sin 35^\circ}$

Si $B = 68'76^\circ \longrightarrow$ $\boxed{c = 6'77\text{ cm}}$

Si $B = 111'24^\circ \longrightarrow$ $\boxed{c = 3'88\text{ cm}}$

M09 P1

a) $\operatorname{arctg}\left(\frac{1}{2}\right) + \operatorname{arctg}\left(\frac{1}{3}\right) = \frac{\pi}{4}$

$\operatorname{tg}\left(\operatorname{arctg}\left(\frac{1}{2}\right) + \operatorname{arctg}\left(\frac{1}{3}\right)\right) = \operatorname{tg}\frac{\pi}{4}$; $\dfrac{\operatorname{tg}(\operatorname{arctg}\frac{1}{2}) + \operatorname{tg}(\operatorname{arctg}\frac{1}{3})}{1-\operatorname{tg}(\operatorname{arctg}\frac{1}{2})\cdot\operatorname{tg}(\operatorname{arctg}\frac{1}{3})} = 1$;

$\dfrac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\cdot\frac{1}{3}} = 1$; $\dfrac{\frac{5}{6}}{1-\frac{1}{6}} = 1$; $\dfrac{\frac{5}{6}}{\frac{5}{6}} = 1$; $1 = 1$ ✓

b) $\operatorname{arctg} 2 + \operatorname{arctg} 3 = \alpha$

$\operatorname{tg}(\operatorname{arctg} 2 + \operatorname{arctg} 3) = \operatorname{tg}\alpha$; $\dfrac{\operatorname{tg}(\operatorname{arctg} 2) + \operatorname{tg}(\operatorname{arctg} 3)}{1-\operatorname{tg}(\operatorname{arctg} 2)\cdot\operatorname{tg}(\operatorname{arctg} 3)} = \operatorname{tg}\alpha$;

$\dfrac{2+3}{1-2\cdot 3} = \operatorname{tg}\alpha$; $\operatorname{tg}\alpha = \dfrac{5}{-5} = -1 \Rightarrow \boxed{\alpha = 3\pi/4}$

También: Aplicando que los ángulos complementarios tienen cruzados los valores de seno y coseno y por lo tanto tienen inversas sus tangentes: $\operatorname{arctg} 2 = \frac{\pi}{2} - \operatorname{arctg}\frac{1}{2}$; $\operatorname{arctg} 3 = \frac{\pi}{2} - \operatorname{arctg}\frac{1}{3}$

$\operatorname{arctg} 2 + \operatorname{arctg} 3 = \frac{\pi}{2} - \operatorname{arctg}\frac{1}{2} + \frac{\pi}{2} - \operatorname{arctg}\frac{1}{3} = \pi - \left(\operatorname{arctg}\frac{1}{2} + \operatorname{arctg}\frac{1}{3}\right) = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$ ✓

M09 P1

Area B $= \dfrac{r^2\alpha}{2}$

Area A $= \dfrac{R^2\alpha}{2} - \dfrac{r^2\alpha}{2} = \dfrac{(R^2-r^2)\alpha}{2}$

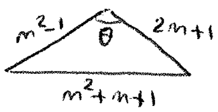
$\dfrac{A}{B} = \dfrac{2}{1} \Rightarrow \dfrac{\frac{(R^2-r^2)\cancel{\alpha}}{\cancel{2}}}{\frac{r^2\cancel{\alpha}}{\cancel{2}}} = 2$; $R^2 - r^2 = 2r^2$; $R^2 = 3r^2$; $\dfrac{R^2}{r^2} = 3$; $\boxed{\dfrac{R}{r} = \sqrt{3}}$

M09 P1

a) $m > 1 \Rightarrow m+1 > 0 \Rightarrow m+1 > -1 \Rightarrow m^2+m+1 > m^2-1$ ✓

$m > 1 \Rightarrow m^2 > m \Rightarrow m^2+m+1 > m+m+1 \Rightarrow m^2+m+1 > 2m+1$ ✓

b) El mayor ángulo: θ, estará enfrentado al mayor lado: m^2+m+1



$(m^2+m+1)^2 = (m^2-1)^2 + (2m+1)^2 - 2(m^2-1)(2m+1)\cdot\cos\theta$

$\cancel{m^4} + m^2 + \cancel{1} + 2m^3 + 2m^2 + 2m = \cancel{m^4} - 2m^2 + 1 + 4m^2 + 4m + \cancel{1} - 2(2m^3+m^2-2m-1)\cos\theta$;

$2(2m^3+m^2-2m-1)\cos\theta = -2m^3 - m^2 - 2m + 1$; $\cos\theta = \dfrac{-(2m^3+m^2+\cancel{2m}-1)}{2(2m^3+m^2+\cancel{2m}-1)} = -\dfrac{1}{2} \Rightarrow \boxed{\theta = 120^\circ}$ ✓

N09 P2

a) Area Sombreada $= \underbrace{\dfrac{4\cdot 4\sin x}{2}}_{\text{Area } \widehat{OPQ}} - \underbrace{\dfrac{2^2 x}{2}}_{\text{Area sector}} = 8\sin x - 2x$ ✓

M10
P1

a) $\operatorname{sen} 2nx = \operatorname{sen}[(2n+1)x - x] = \operatorname{sen}(2n+1)x \cos x - \cos(2n+1)x \operatorname{sen} x$ ✓

b) $\cos x + \cos 3x + \cdots + \cos(2n-1)x = \dfrac{\operatorname{sen} 2nx}{2\operatorname{sen} x}$

$\underline{n=1}$: $\cos x = \dfrac{\operatorname{sen} 2x}{2\operatorname{sen} x} = \dfrac{\cancel{2\operatorname{sen} x}\cos x}{\cancel{2\operatorname{sen} x}} = \cos x$ ✓

$\underline{n=k}$: Suponiendo cierto que: $\cos x + \cos 3x + \cdots + \cos(2k-1)x = \dfrac{\operatorname{sen} 2kx}{2\operatorname{sen} x}$ vamos a demostrar que: $\cos x + \cos 3x + \cdots + \cos(2(k+1)-1)x = \dfrac{\operatorname{sen} 2(k+1)x}{2\operatorname{sen} x}$

$$\cos x + \cos 3x + \cdots + \cos(2(k+1)-1)x = \frac{\operatorname{sen} 2kx}{2\operatorname{sen} x} + \cos(2k+1)x = \frac{\operatorname{sen} 2kx + \cos(2k+1)x \cdot 2\operatorname{sen} x}{2\operatorname{sen} x} =$$

$$= \frac{\operatorname{sen}(2k+1)x \cdot \cos x - \cos(2k+1)x \cdot \operatorname{sen} x + 2\cos(2k+1)x \operatorname{sen} x}{2\operatorname{sen} x} = \frac{\operatorname{sen}(2k+1)x \cdot \cos x + \cos(2k+1)x \cdot \operatorname{sen} x}{2\operatorname{sen} x} =$$

$$= \frac{\operatorname{sen}((2k+1)x + x)}{2\operatorname{sen} x} = \frac{\operatorname{sen}(2k+1+1)x}{2\operatorname{sen} x} = \frac{\operatorname{sen}(2k+2)x}{2\operatorname{sen} x} = \frac{\operatorname{sen} 2(k+1)x}{2\operatorname{sen} x}$$ ✓

c) $\cos x + \cos 3x = \frac{1}{2} \longrightarrow \underline{n=2}$: $\cos x + \cos 3x = \dfrac{\operatorname{sen} 4x}{2\operatorname{sen} x}$

$\dfrac{\operatorname{sen} 4x}{\cancel{2}\operatorname{sen} x} = \dfrac{1}{\cancel{2}}$; $\operatorname{sen} 4x = \operatorname{sen} x$

$\to 4x = x \Rightarrow x = 0$

$\to 4x = \pi - x \Rightarrow x = \pi/5$

$\to 4x = x + 2\pi \Rightarrow x = \frac{2\pi}{3}$

$\to 4x = 3\pi - x \Rightarrow x = \frac{3\pi}{5}$

También: $\operatorname{sen} 4x = \operatorname{sen} x$; $2\operatorname{sen} 2x \cos 2x = \operatorname{sen} x$; $4\operatorname{sen} x \cos x(\cos^2 x - \operatorname{sen}^2 x) = \operatorname{sen} x$;

$4\operatorname{sen} x \cos x(\cos^2 x - 1 + \cos^2 x) = \operatorname{sen} x$; $4\operatorname{sen} x \cos x(2\cos^2 x - 1) - \operatorname{sen} x = 0$;

$8\operatorname{sen} x \cos^3 x - 4\operatorname{sen} x \cos x - \operatorname{sen} x = 0$

$\operatorname{sen} x(8\cos^3 x - 4\cos x - 1) = 0$

$\to \operatorname{sen} x = 0 \Rightarrow x = \boxed{0}$ ✓, $\cancel{\pi}$

$\to 8\cos^3 x - 4\cos x - 1 = 0$

$\cos x = t$: $8t^3 - 4t - 1 = 0$

	8	0	−4	−1
−1/2		−4	2	1
	8	−4	−2	0

$8t^2 - 4t - 2 = 0$

$t = \dfrac{4 \pm \sqrt{16+64}}{16} = \dfrac{4 \pm \sqrt{80}}{16} = \dfrac{1 \pm \sqrt{5}}{4}$

$\cos x = \dfrac{1+\sqrt{5}}{4} \Rightarrow x = 36° = \boxed{\frac{\pi}{5} \text{ rad}}$ ✓ ; $144° = \boxed{\frac{2\pi}{3} \text{ rad}}$ ✓

$\cos x = \dfrac{1-\sqrt{5}}{4} \Rightarrow x = 108° = \boxed{\frac{3\pi}{5} \text{ rad}}$ ✓ ; $\cancel{252°}$

M10
P1

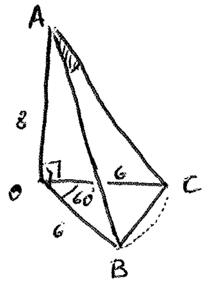
$\operatorname{sen}\left(x + \frac{\pi}{3}\right) = 2\operatorname{sen} x \operatorname{sen} \frac{\pi}{3}$

$\operatorname{sen} x \cos \frac{\pi}{3} + \cos x \operatorname{sen} \frac{\pi}{3} = 2\operatorname{sen} x \operatorname{sen} \pi/3$; $\operatorname{sen} x \cdot \frac{1}{2} + \cos x \cdot \frac{\sqrt{3}}{2} = 2\operatorname{sen} x \cdot \frac{\sqrt{3}}{2}$; $\operatorname{sen} x + \sqrt{3}\cos x = 2\sqrt{3}\operatorname{sen} x$;

$(1 - 2\sqrt{3})\operatorname{sen} x = -\sqrt{3}\cos x$; $\operatorname{tg} x = \dfrac{-\sqrt{3}}{1-2\sqrt{3}} = \dfrac{-\sqrt{3}(1+2\sqrt{3})}{(1-2\sqrt{3})(1+2\sqrt{3})} = \dfrac{-\sqrt{3}-6}{1-12} = \dfrac{6+\sqrt{3}}{11}$;

$\boxed{11 \operatorname{tg} x = 6 + \sqrt{3}}$ ✓ $a = 6$, $b = 1$

M10 P2

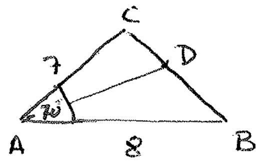


$AB = \sqrt{8^2+6^2} = 10$

$BC = \sqrt{6^2+6^2-2\cdot 6\cdot 6\cdot \cos 60^\circ} = 6$

$\cos B\hat{A}C = \dfrac{10^2+10^2-6^2}{2\cdot 10\cdot 10} = \dfrac{41}{50} \Rightarrow \boxed{B\hat{A}C = 34'9^\circ}$

M10 P2



$BC = \sqrt{7^2+8^2-2\cdot 7\cdot 8\cdot \cos 70^\circ} = 8'643 \Rightarrow \begin{cases} BD = \frac{2}{3}BC = 5'762 \\ DC = \frac{1}{3}BC = 2'881 \end{cases}$

$\dfrac{\sin \sigma}{8} = \dfrac{\sin 70^\circ}{8'643} \Rightarrow \sin \sigma = \dfrac{8\sin 70^\circ}{8'643} = 0'870 \Rightarrow \sigma = \begin{cases} 60,44^\circ \\ \cancel{119,56^\circ} \end{cases}$

porque con 70° supera 180°

$AD = \sqrt{AC^2 + DC^2 - 2\cdot AC\cdot DC\cdot \cos 60'44^\circ} =$

$= \sqrt{7^2 + 2'881^2 - 2\cdot 7\cdot 2'881\cdot \cos 60'44^\circ} = \boxed{6'12 \text{ cm}}$

M10 P2

$\text{Area Sector}_1 = \dfrac{2^2\theta}{2} = 2\theta$

a) $S_{ab} = \pi\cdot 2^2 = \dfrac{2\theta}{1-K} \Rightarrow \theta = 2\pi(1-K)$ ✓

b) Perimetro sector con ángulo α $= r + r + r\alpha = r(\alpha+2)$

$\left.\begin{array}{l}\text{Area Sector}_3 = \text{Area Sector}_1 \cdot K^2 = 2\theta K^2 \\ \text{Area Sector}_3 = \dfrac{2^2\theta_3}{2} = 2\theta_3\end{array}\right\}$ $2\theta_3 = 2\theta K^2 \rightarrow \theta_3 = \theta K^2 = 2\pi(1-K)K^2$

$\text{Perimetro sector}_1 = 2\cdot(\theta+2) = 2(2\pi(1-K)+2) = 4(\pi - \pi K + 1)$

$\text{Perimetro sector}_3 = 2(\theta_3+2) = 2(2\pi(1-K)K^2+2) = 4(\pi K^2 - \pi K^3 + 1)$

$P_3 = \dfrac{P_1}{2} \Rightarrow \cancel{4}(\pi K^2 - \pi K^3 + 1) = \dfrac{\cancel{4}(\pi - \pi K + 1)}{2}$; $2\pi K^2 - 2\pi K^3 + 2 = \pi - \pi K + 1$;

$0 = 2\pi K^3 - 2\pi K^2 - \pi K + \pi - 1$ Utilizando la calculadora gráfica:

$K = \boxed{0'456} \Rightarrow \theta = \boxed{3'42 \text{ rad}}$

N10 P2

$\text{Area} = \dfrac{6\cdot 5 \sin B}{2}$; $10 = 15\sin B \Rightarrow \boxed{\sin B = \frac{2}{3}}$

$\sin B = \dfrac{2}{3} \Rightarrow B = \begin{cases} 41'81^\circ \Rightarrow AC = \sqrt{6^2+5^2-2\cdot 6\cdot 5\cos 41'81^\circ} = \boxed{4'03 \text{ cm}} \\ 138'19^\circ \Rightarrow AC = \sqrt{6^2+5^2-2\cdot 6\cdot 5\cos 138'19^\circ} = \boxed{10'28 \text{ cm}} \end{cases}$

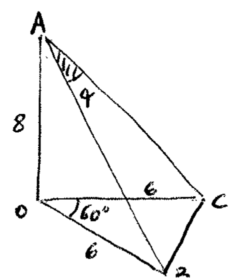
M11 P1

a) $\dfrac{\sin 2\theta}{1+\cos 2\theta} = \dfrac{2\sin\theta\cos\theta}{1+\cos^2\theta - \sin^2\theta} = \dfrac{2\sin\theta\cos\theta}{\cancel{1}+\cos^2\theta - \cancel{1}+\cos^2\theta} = \dfrac{\cancel{2}\sin\theta\cancel{\cos\theta}}{\cancel{2}\cos^{\cancel{2}}\theta} = \dfrac{\sin\theta}{\cos\theta} = \text{Tg}\,\theta$ ✓

b) $\cot g\dfrac{\pi}{8} = \dfrac{1}{\text{tg}\,\pi/8} = \dfrac{1+\cos 2\pi/8}{\sin 2\pi/8} = \dfrac{1+\cos \pi/4}{\sin \pi/4} = \dfrac{1+\sqrt{2}/2}{\sqrt{2}/2} = \dfrac{2+\sqrt{2}}{\sqrt{2}} = \dfrac{(2+\sqrt{2})\sqrt{2}}{\sqrt{2}\sqrt{2}} = \dfrac{2\sqrt{2}+2}{2} =$

$= \boxed{1+\sqrt{2}}$ $a=1$ $b=1$

M10 P2



$O\hat{B}C$ es un Triángulo equilátero, porque es isósceles y el ángulo $B\hat{O}C$ es de 60°. Por lo tanto: $BC = 6$ m

En el Triángulo $A\hat{B}C$: $\overline{BC}^2 = \overline{AB}^2 + \overline{AC}^2 - 2\overline{AB}\cdot\overline{AC}\cos\alpha$

$6^2 = 10^2 + 10^2 - 2\cdot 10\cdot 10\cos\alpha \Rightarrow \cos\alpha = 0'82$

$\boxed{\alpha = 34,92^\circ}$

M11 P2

$$\frac{\sin C}{7} = \frac{\sin 40^\circ}{5} \;;\; \sin C = \frac{7\sin 40^\circ}{5} = 0'8999 \Rightarrow C = 64{,}15^\circ$$

$$\cancel{5^2} = \cancel{5^2} + CD^2 - 2\cdot 5\cdot CD\cdot \cos 64{,}15^\circ \;;\; 0 = CD\,(CD - 10\cos 64{,}15^\circ) \begin{cases} \cancel{CD = 0} \\ CD = 10\cos 64'15 = \boxed{4'36} \end{cases}$$

M11 P2

$$\cos A\hat{C}B = \frac{7^2+7^2-9^2}{2\cdot 7\cdot 7} = \frac{17}{98} \Rightarrow A\hat{C}B = 1'396 \text{ rad}$$

$$\cos A\hat{D}B = \frac{5^2+5^2-9^2}{2\cdot 5\cdot 5} = \frac{-31}{50} \Rightarrow A\hat{D}B = 2'240 \text{ rad}$$

Area sombreada derecho = Area Sector CAB − Area Triángulo CAB =

$$= \frac{7^2\cdot 1'396}{2} - \frac{7\cdot 7\sin 1'396}{2} = 10'084$$

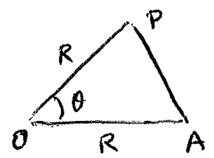
Area Sombreada izquierda = Area Sector DAB − Area Triángulo DAB =

$$= \frac{5^2\cdot 2'240}{2} - \frac{5\cdot 5\cdot \sin 2'240}{2} = 18'187$$

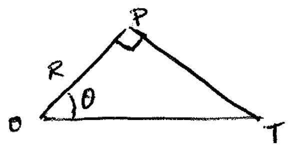
Area Sombreada $= 10'084 + 18'187 = \boxed{28'3 \text{ cm}^2}$

M11 P1

a)

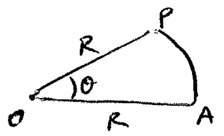


$$\text{Area } \widehat{AOP} = \frac{R\cdot R\sin\theta}{2} = \frac{R^2\sin\theta}{2}$$



$$\tan\theta = \frac{PT}{R} \rightarrow PT = R\cdot \tan\theta$$

$$\text{Area } \widehat{POT} = \frac{OP\cdot OT}{2} = \frac{R\cdot R\tan\theta}{2} = \frac{R^2\tan\theta}{2}$$

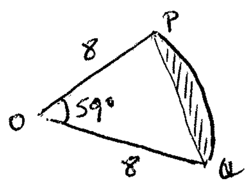


$$\text{Area } \widehat{AOP} = \frac{R^2\theta}{2}$$

$$\text{Area } \widehat{AOP} < \text{Area } \widehat{AOP} < \text{Area } \widehat{POT}$$

$$\frac{\cancel{R^2}\sin\theta}{\cancel{2}} < \frac{\cancel{R^2}\theta}{\cancel{2}} < \frac{\cancel{R^2}\tan\theta}{\cancel{2}} \Rightarrow \boxed{\sin\theta < \theta < \tan\theta}$$

M11 P2

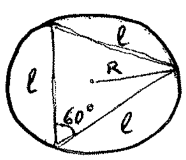


$$\text{Area } \widehat{POQ} = \frac{59^\circ\cdot\frac{\pi}{180^\circ}\cdot 8^2}{2} = \frac{472\pi}{45} \text{ cm}^2$$

$$\text{Area } \widehat{POQ} = \frac{8\cdot 8\sin 59^\circ}{2} = 32\sin 59^\circ \text{ cm}^2$$

$$\text{Area sombreada} = \frac{472\pi}{45} - 32\sin 59^\circ = \boxed{5{,}52 \text{ cm}^2}$$

M11 P2



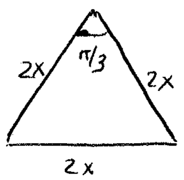
$$\frac{l}{\sin 60^\circ} = 2R \Rightarrow l = 2R\frac{\sqrt3}{2} = R\cdot\sqrt3$$

$$P = 3l = 3R\sqrt3$$

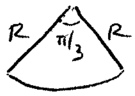
$$A = \frac{l\cdot l\cdot \sin 60^\circ}{2} = \frac{l^2\sqrt3}{4} = \frac{(R\cdot\sqrt3)^2\cdot\sqrt3}{4} = \frac{3\sqrt3 R^2}{4}$$

$$\frac{P}{A} = \frac{\cancel{3}\cancel{R}\cancel{\sqrt3}}{\frac{\cancel{3}\cancel{\sqrt3}R^{\cancel{2}}}{4}} = \boxed{\frac{4}{R}} \quad k=4$$

N11 P1



$$\text{Area Triangulo} = \frac{2x \cdot 2x \operatorname{sen}\frac{\pi}{3}}{2} = \frac{4x^2\frac{\sqrt{3}}{2}}{2} = x^2\sqrt{3}$$



$$\text{Area Sector} = \frac{R^2\,\pi/3}{2} = \frac{\pi R^2}{6}$$

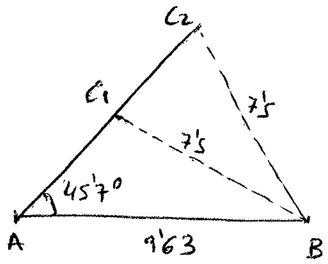
$$\text{Area Sector} = \frac{1}{2}\,\text{Area Triangulo} \Rightarrow \frac{\pi R^2}{6} = \frac{x^2\sqrt{3}}{2}\ ;\ R^2 = \frac{3\sqrt{3}x^2}{\pi} \Rightarrow \boxed{R = x\sqrt{\frac{3\sqrt{3}}{\pi}}}$$

M12 P1

$$\frac{\cos A + \operatorname{sen} A}{\cos A - \operatorname{sen} A} = \frac{(\cos A + \operatorname{sen} A)^2}{(\cos A - \operatorname{sen} A)(\cos A + \operatorname{sen} A)} = \frac{\cos^2 A + 2\cos A \operatorname{sen} A + \operatorname{sen}^2 A}{\cos^2 A - \operatorname{sen}^2 A} = \frac{1 + 2\operatorname{sen} A \cos A}{\cos 2A} =$$

$$= \frac{1 + \operatorname{sen} 2A}{\cos 2A} = \frac{1}{\cos 2A} + \frac{\operatorname{sen} 2A}{\cos 2A} = \boxed{\sec 2A + \operatorname{Tg} 2A}$$

M12 P2



$$\overline{BC}^2 = \overline{AC}^2 + \overline{AB}^2 - 2\overline{AC}\cdot\overline{AB}\cdot\cos 45'7^\circ$$

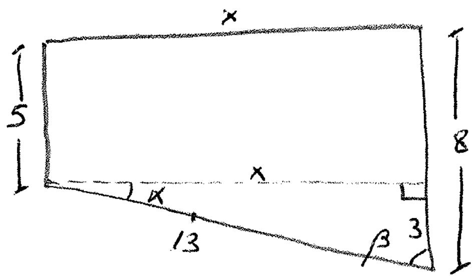
$$7'5^2 = \overline{AC}^2 + 9'63^2 - 2\,\overline{AC}\cdot 9'63\cdot\cos 45'7^\circ$$

$$0 = \overline{AC}^2 - 13{,}45\,\overline{AC} + 36{,}4869$$

$$\overline{AC} = \frac{13{,}45 \pm \sqrt{13'45^2 - 4\cdot 36{,}4869}}{2} = \boxed{\begin{matrix} 9{,}68\text{ cm} \\ 3{,}77\text{ cm}\end{matrix}}$$

M12 P2

$$x = \sqrt{13^2 - 3^2} = \sqrt{160}\text{ cm}$$

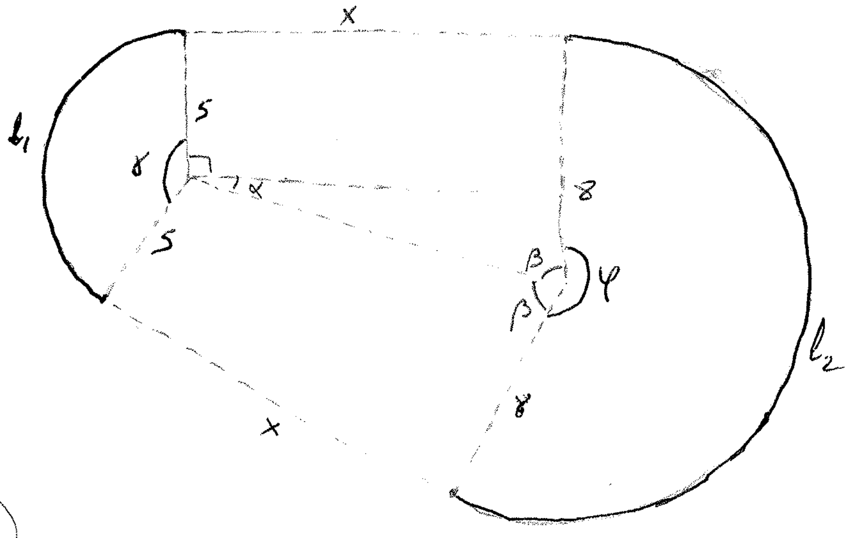


$$\alpha = \operatorname{sen}^{-1}\frac{3}{13} = 13{,}34^\circ$$

$$\beta = \cos^{-1}\frac{3}{13} = 76.66^\circ$$

$$\gamma = 360 - 2\times(90+\alpha) = 153{,}32^\circ$$

$$\varphi = 360 - 2\beta = 206'68^\circ$$



360° —— $2\pi\cdot 5$

γ —— l_1

$$l_1 = \frac{10\pi\cdot 153'32^\circ}{360^\circ} = 13'3797\text{ cm}$$

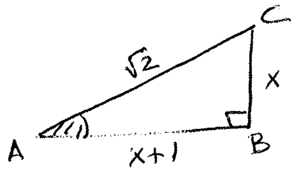
360° —— $2\pi\cdot 8$

φ —— l_2

$$l_2 = \frac{16\pi\cdot 206'68^\circ}{360^\circ} = 28'8580\text{ cm}$$

$$\text{longitud cuerda} = 2\times\sqrt{160} + 13'3797 + 28'8580 = \boxed{67'5\text{ cm}}$$

M12
P1



Llamo: $x = BC$

Como $x+1 > x \Rightarrow \hat{C} > \hat{A}$

a) $\cos A = \dfrac{x+1}{\sqrt{2}}$ | $\cos A - \operatorname{sen} A = \dfrac{x+1}{\sqrt{2}} - \dfrac{x}{\sqrt{2}} = \dfrac{1}{\sqrt{2}}$ ✓

$\operatorname{sen} A = \dfrac{x}{\sqrt{2}}$

b) $(\cos A - \operatorname{sen} A)^2 = \left(\dfrac{1}{\sqrt{2}}\right)^2$

$\cos^2 A - 2\operatorname{sen} A \cos A + \operatorname{sen}^2 A = \dfrac{1}{2}$; $1 - \operatorname{sen}(2A) = \dfrac{1}{2}$; $\operatorname{sen}(2A) = \dfrac{1}{2}$

$2A = \begin{cases} 30^\circ + 360^\circ N \\ 150^\circ + 360^\circ N \end{cases}$; $A = \begin{cases} 15^\circ + 180^\circ N \\ 75^\circ + 180^\circ N \end{cases}$; $\boxed{A =} \begin{cases} \boxed{15^\circ \Rightarrow C = 75^\circ} \\ \cancel{75^\circ \Rightarrow C = 15^\circ} \end{cases}$ Porque $\hat{C} \not> \hat{A}$

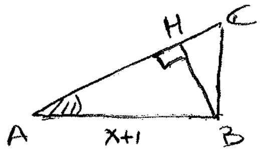
c) $x^2 + (x+1)^2 = (\sqrt{2})^2$

$x^2 + x^2 + 2x + 1 = 2$; $2x^2 + 2x - 1 = 0$

$x = \dfrac{-2 \pm \sqrt{4+8}}{4} = \dfrac{-2 \pm 2\sqrt{3}}{4} = \dfrac{-1 \pm \sqrt{3}}{2}$ → $\dfrac{-1+\sqrt{3}}{2}$; $\cancel{\dfrac{-1-\sqrt{3}}{2}}$ No puede ser negativo.

$\operatorname{sen} A = \dfrac{x}{\sqrt{2}} = \dfrac{\frac{-1+\sqrt{3}}{2}}{\sqrt{2}} = \dfrac{-1+\sqrt{3}}{2\sqrt{2}} = \dfrac{(-1+\sqrt{3})\sqrt{2}}{2\sqrt{2}\sqrt{2}} = \boxed{\dfrac{-\sqrt{2}+\sqrt{6}}{4}}$ ✓

d)



$\operatorname{sen} A = \dfrac{BH}{x+1} \Rightarrow BH = (x+1)\operatorname{sen} A = \left(\dfrac{-1+\sqrt{3}}{2} + 1\right) \cdot \dfrac{\sqrt{6}-\sqrt{2}}{4} =$

$= \dfrac{\sqrt{3}+1}{2} \cdot \dfrac{\sqrt{6}-\sqrt{2}}{4} = \dfrac{\sqrt{18} - \sqrt{6} + \sqrt{6} - \sqrt{2}}{8} =$

$= \dfrac{3\sqrt{2} - \sqrt{2}}{8} = \dfrac{2\sqrt{2}}{8} = \boxed{\dfrac{\sqrt{2}}{4}}$

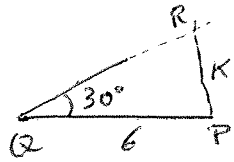
N12
P1#1

$\cos\alpha = -\dfrac{3}{4}$ $\left(\pi/2 < \alpha < \pi\right)$

$\operatorname{sen}\alpha = \pm\sqrt{1 - \left(-\dfrac{3}{4}\right)^2} = \pm\sqrt{1 - \dfrac{9}{16}} = \dfrac{\pm\sqrt{7}}{4} \longrightarrow \dfrac{+\sqrt{7}}{4}$ porque $\pi/2 < \alpha < \pi$

$\operatorname{sen} 2\alpha = 2\operatorname{sen}\alpha\cos\alpha = 2 \cdot \dfrac{\sqrt{7}}{4} \cdot \left(-\dfrac{3}{4}\right) = \boxed{-\dfrac{3\sqrt{7}}{8}}$

N12
P1#7



$K^2 = 6^2 + \overline{QR}^2 - 12\,\overline{QR}\cos 30^\circ = 6^2 + \overline{QR}^2 - 12\,\overline{QR} \cdot \dfrac{\sqrt{3}}{2} = 36 + \overline{QR}^2 - 6\sqrt{3}\,\overline{QR}$

$0 = \overline{QR}^2 - 6\sqrt{3}\,\overline{QR} + (36 - K^2)$

$\overline{QR} = \dfrac{6\sqrt{3} \pm \sqrt{(6\sqrt{3})^2 - 4(36-K^2)}}{2} = \dfrac{6\sqrt{3} \pm \sqrt{108 - 144 + 4K^2}}{2} = \dfrac{6\sqrt{3} \pm \sqrt{4K^2 - 36}}{2}$

a) $K = 4 \longrightarrow \overline{QR} = \dfrac{6\sqrt{3} \pm \sqrt{4 \cdot 4^2 - 36}}{2} = \dfrac{6\sqrt{3} \pm \sqrt{28}}{2} = \boxed{\dfrac{6\sqrt{3} \pm 2\sqrt{7}}{2}} = \boxed{3\sqrt{3} \pm \sqrt{7}}$

b) Para que tenga alguna solución: $4K^2 - 36 \geq 0$; $K^2 \geq 9$; $K \in \cancel{(-\infty, -3]} \cup [3, +\infty)$ ← K no puede ser negativo

Para que únicamente tenga una solución: $6\sqrt{3} - \sqrt{4K^2 - 36} < 0$

$6\sqrt{3} < \sqrt{4K^2 - 36}$; $(6\sqrt{3})^2 < 4K^2 - 36$; $108 < 4K^2 - 36$; $144 < 4K^2$; $K^2 > 36 \Rightarrow$

$\Rightarrow K \in \cancel{(-\infty, -6)} \cup (6, +\infty)$, como $6 > 3 \Rightarrow \boxed{K \in (6, +\infty) \cup \{3\}}$

$K=3$ lo solución porque sólo tendríamos una solución de $\overline{QR}$

M13 P1#11a TZ1

a) $\cos\left(\frac{\pi}{6}+x\right) = \cos\frac{\pi}{6}\cdot\cos x - \sin\frac{\pi}{6}\sin x = \boxed{\frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x}$

b) $2\cdot\cos\left(\frac{\pi}{6}+x\right) = \sqrt{3}\cos x - \sin x$

$\sqrt{3}\cos x - \sin x = 1 \Rightarrow 2\cos\left(\frac{\pi}{6}+x\right)=1$; $\cos\left(\frac{\pi}{6}+x\right)=\frac{1}{2}$

$\frac{\pi}{6}+x = \begin{cases} \pi/3 \longrightarrow x = \boxed{\pi/6} \\ \frac{5\pi}{3} \longrightarrow x = \frac{9\pi}{6} = \boxed{\frac{3\pi}{2}} \end{cases}$

M13 P1#11b TZ1

$P(x) = 2x^3 - x^2 - 2x + 1$

i) $P(1) = 2-1-2+1 = 0$ ✓

ii)
$$\begin{array}{r|rrrr} & 2 & -1 & -2 & 1 \\ 1 & & 2 & 1 & -1 \\ \hline & 2 & 1 & -1 & \boxed{0} \end{array}$$

$2x^2+x-1=0 \qquad x = \frac{-1\pm\sqrt{1+8}}{4} = \frac{-1\pm 3}{4} = \begin{cases} 1/2 \\ -1 \end{cases}$

$P(x)=0 \Rightarrow \boxed{x = \begin{cases} 1 \\ 1/2 \\ -1 \end{cases}}$

iii) $\sin 2\theta\cos\theta + \sin^2\theta = 2\sin\theta\cos\theta\cos\theta + \sin^2\theta = 2\sin\theta\cos^2\theta + \sin^2\theta =$

$= 2\sin\theta(1-\sin^2\theta) + \sin^2\theta = 2\sin\theta - 2\sin^3\theta + \sin^2\theta =$

$= \boxed{-2\sin^3\theta + \sin^2\theta + 2\sin\theta}$

iv) $\sin 2\theta\cos\theta + \sin^2\theta = 1 \Rightarrow -2\sin^3\theta + \sin^2\theta + 2\sin\theta = 1$

$0 = 2\sin^3\theta - \sin^2\theta - 2\sin\theta + 1$

$\sin\theta = \begin{cases} 1 \longrightarrow \theta = \pi/2 \\ 1/2 \longrightarrow \theta = \begin{cases} \pi/6 \\ 5\pi/6 \end{cases} \\ -1 \longrightarrow \theta = 3\pi/2 \end{cases}$

M13 TZ2 P1#10

a) $\arctan\left(\frac{1}{5}\right) + \arctan\left(\frac{1}{8}\right) = \arctan\left(\frac{1}{p}\right) \Rightarrow$

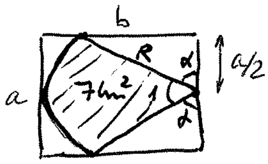
$\Rightarrow \frac{1}{p} = \tan\left[\arctan\left(\frac{1}{5}\right) + \arctan\left(\frac{1}{8}\right)\right] = \frac{\tan\arctan\left(\frac{1}{5}\right) + \tan\left(\arctan\frac{1}{8}\right)}{1-\tan\arctan\left(\frac{1}{5}\right)\tan\left(\arctan\frac{1}{8}\right)} =$

$= \frac{1/5+1/8}{1-\frac{1}{5}\cdot\frac{1}{8}} = \frac{13/40}{39/40} = \frac{1}{3} \Rightarrow \boxed{p=3}$

b) $x = \arctan(1/2) + \arctan(1/5) + \arctan(1/8) = \arctan(1/2) + \arctan(1/3)$

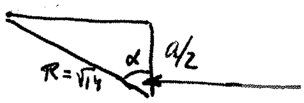
$\tan x = \frac{1/2+1/3}{1-\frac{1}{2}\frac{1}{3}} = \frac{5/6}{5/6} = 1 \Rightarrow \boxed{x = \pi/4}$

M13 TZ1 P2#5



$7 = \frac{R^2\cdot 1}{2} \Rightarrow R = \sqrt{14}$ cm

Obviamente, $b = \sqrt{14} \simeq \boxed{3'7 \text{ cm}}$



$2\alpha + 1 = \pi \rightarrow \alpha = \frac{\pi-1}{2}$

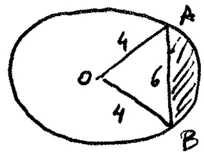
$\cos\left(\frac{\pi-1}{2}\right) = \frac{a/2}{\sqrt{14}} \Rightarrow a = 2\sqrt{14}\cos\left(\frac{\pi-1}{2}\right) = \boxed{3'6 \text{ cm}}$

M13 TZ2 P2#1

a) $6^2 = 4^2+4^2-2\cdot4\cdot4\cos A\hat{O}B$

$36 = 16+16-32\cos A\hat{O}B$

$\cos A\hat{O}B = \frac{-4}{32} \Rightarrow \quad A\hat{O}B = \boxed{1'696 \text{ rad}}$



b) Área Sombreada = Área Sector − Área Triángulo =

$= \frac{4^2\cdot 1'696}{2} - \frac{4^2 \sin 1'696}{2} = 8(1'696 - \sin 1'696) = \boxed{5'63 \text{ cm}^2}$

M13 TZ2 P2#6

a) $3\cos^2 x - 8\cos x + 4 = 0$

$\cos x = \frac{8\pm\sqrt{64-48}}{6} = \frac{8\pm4}{6} = \begin{cases} 2 \; * \\ 2/3 \end{cases} \longrightarrow \boxed{x = \begin{cases} 132^\circ \\ 48^\circ \end{cases}}$

b) $3\sec^2 x - 8\sec^2 x + 4 = 0$

$\sec^2 x = \begin{cases} 2 \longrightarrow \boxed{\sec x = \pm\sqrt{2}} \\ -2/3 \; * \end{cases}$

N13 P1#8

$\sin(x+y)\sin(x-y) = (\sin x\cos y + \cos x \sin y)(\sin x \cos y - \cos x \sin y) =$

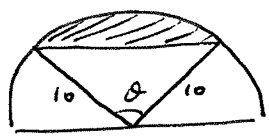
$= \sin^2 x\cos^2 y - \cos^2 x \sin^2 y =$

$= \sin^2 x(1-\sin^2 y) - (1-\sin^2 x)\sin^2 y =$

$= \sin^2 x - \cancel{\sin^2 x \sin^2 y} - \sin^2 y + \cancel{\sin^2 x \sin^2 y} = \sin^2 x - \sin^2 y$ ✓

N13 P2#8

Area Sombreada = Area Sector − Area Triángulo =



$= \frac{10^2\theta}{2} - \frac{10\cdot10\cdot\sin\theta}{2} =$

$= 50\theta - 50\sin\theta$ ✓

Área Sombreada + Área no sombreada $= \frac{\pi\cdot 10^2}{2} = 50\pi$

$50\theta - 50\sin\theta + 2(50\theta - 50\sin\theta) = 50\pi$

$150\theta - 150\sin\theta = 50\pi \quad ; \quad 3\theta - 3\sin\theta = \pi \Longrightarrow \theta = \boxed{1'969 \text{ rad}}$ Resuelto con calculadora gráfica

Muestra 14 P1#1

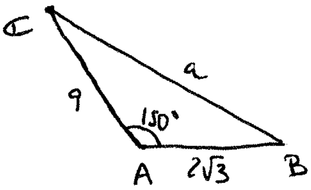
$\cos\theta = 1/3$

a) $\sin\theta = +\sqrt{1-(1/3)^2} = \sqrt{8/3} = \boxed{\frac{2\sqrt{2}}{3}} \longrightarrow \tan\theta = \frac{2\sqrt{3}/3}{1/3} = 2\sqrt{2}$

b) $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\cdot2\sqrt{2}}{1-(2\sqrt{2})^2} = \frac{4\sqrt{2}}{1-8} = \boxed{-\frac{4\sqrt{2}}{7}}$

c) $\cos(\theta/2) = \sqrt{\frac{1+\cos\theta}{2}} = \sqrt{\frac{1+1/3}{2}} = \sqrt{\frac{4}{6}} = \boxed{\frac{\sqrt{6}}{3}}$

Muestra 14
P1 #6

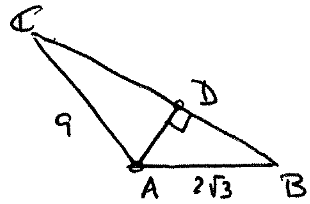


a) $a^2 = 9^2 + (2\sqrt{3})^2 - 2\cdot 9\cdot 2\sqrt{3}\cos 150°$

$a^2 = 81 + 12 - 36\sqrt{3}\cdot\frac{-\sqrt{3}}{2}$

$a^2 = 93 + 54$

$a^2 = 147$; $a = \sqrt{147} = \boxed{7\sqrt{3}}$

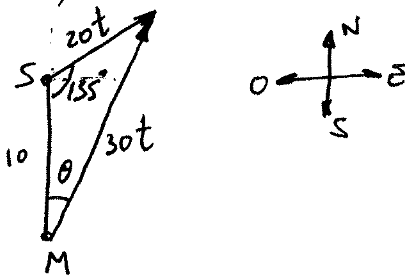


b) $\frac{9}{\sin B} = \frac{7\sqrt{3}}{\sin 150}$

$\sin B = \frac{9\sin 150°}{7\sqrt{3}} = \frac{9\cdot 1/2}{7\sqrt{3}} = \frac{9}{14\sqrt{3}}$

$\sin B = \frac{AD}{2\sqrt{3}} \Rightarrow AD = \sin B\cdot 2\sqrt{3} = \frac{9}{14\cancel{\sqrt{3}}}\cdot 2\cancel{\sqrt{3}} = \boxed{\frac{9}{7}}$

Muestra 14
P2 #7



b) $(30t)^2 = 10^2 + (20t)^2 - 2\cdot 10\cdot 20t\cdot\cos 135°$

$900t^2 = 100 + 400t^2 - 400t\cdot\frac{-\sqrt{2}}{2}$

$500t^2 - 200\sqrt{2}t - 100 = 0$

$5t^2 - 2\sqrt{2}t - 1 = 0$

$t = \frac{2\sqrt{2} \pm \sqrt{8+20}}{2} = \frac{2\sqrt{2}\pm\sqrt{28}}{2}$ → $\boxed{0'81199\ldots}$ ⇒ $\boxed{12:49\ h}$

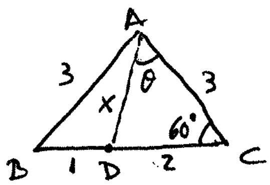
→ ✱

a) $\frac{20\cancel{t}}{\sin\theta} = \frac{30\cancel{t}}{\sin 135} \Rightarrow \sin\theta = \frac{2\cdot\sin 135}{30} = \frac{\cancel{2}\sqrt{2}/\cancel{2}}{30} = \frac{\sqrt{2}}{30} \Rightarrow \boxed{\theta = 2'81°}$

También

b) $\frac{3\cancel{0}t}{\sin 135°} = \frac{1\cancel{0}}{\sin(180-135-\theta)} \Rightarrow t = \frac{\sin 135°}{3\sin(16'9)} = 0'81199\ldots$ ✓

M14
TZ1
P1 #7



$x^2 = 2^2 + 3^2 - 2\cdot 2\cdot 3\cos 60°$

$x^2 = 4 + 9 - 12\cdot\frac{1}{2} = 7$

$x = \sqrt{7}$

$2^2 = 3^2 + (\sqrt{7})^2 - 2\cdot 3\cdot\sqrt{7}\cdot\cos\theta$

$4 = 9 + 7 - 6\sqrt{7}\cos\theta$; $6\sqrt{7}\cos\theta = 12$; $\boxed{\cos\theta = \frac{2}{\sqrt{7}}}$

M14
TZ1
P1 #10

$\sin x + \cos x = \frac{2}{3} \rightarrow (\sin x + \cos x)^2 = \frac{4}{9}$; $\sin^2 x + 2\sin x\cos x + \cos^2 x = \frac{4}{9}$;

$1 + \sin 2x = \frac{4}{9} \Rightarrow \sin 2x = -\frac{5}{9} \rightarrow \cos 2x = \pm\sqrt{1 - \left(-\frac{5}{9}\right)^2} = \pm\sqrt{\frac{81-25}{81}} = \pm\frac{\sqrt{56}}{9}$

$\cos 4x = \cos 2(2x) = \cos^2(2x) - \sin^2(2x) = \left(\pm\frac{\sqrt{56}}{9}\right)^2 - \left(-\frac{5}{9}\right)^2 = \frac{56}{81} - \frac{25}{81} = \boxed{\frac{31}{81}}$

M14 TZ2 P1#5b

$$|\cos\tfrac{x}{4}| = \tfrac{1}{2} \Rightarrow \cos\tfrac{x}{4} = \begin{cases} 1/2 \longrightarrow \frac{x}{4} = \begin{cases} \pi/3 \longrightarrow x = \frac{4\pi}{3} \\ 5\pi/3 \longrightarrow x = \frac{20\pi}{3} \end{cases} \\ -1/2 \longrightarrow \frac{x}{4} = \begin{cases} 2\pi/3 \longrightarrow x = \frac{8\pi}{3} \\ 4\pi/3 \longrightarrow x = \frac{16\pi}{3} \end{cases} \end{cases}$$

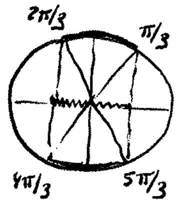
M14 TZ2 P1#9

$\sin x, \sin 2x, 4\sin x\cos^2 x, \ldots$

a) $\dfrac{\sin 2x}{\sin x} = \dfrac{2\cancel{\sin x}\cos x}{\cancel{\sin x}} = 2\cos x$

$\dfrac{4\sin x\cos^2 x}{\sin 2x} = \dfrac{4\cancel{\sin x}\cancel{\cos^2 x}}{2\cancel{\sin x}\cancel{\cos x}} = 2\cos x$ ✓ $\Rightarrow r = \boxed{2\cos x}$

b) $|r|<1 \Rightarrow |2\cos x|<1 \Rightarrow |\cos x| < \frac{1}{2}$



$\boxed{x \in \left(\frac{\pi}{3}, \frac{2\pi}{3}\right)} \cup \cancel{\left(\frac{4\pi}{3}, \frac{5\pi}{3}\right)}$ ⟵ Dice que $-\frac{\pi}{2} < x < \frac{\pi}{2}$

c) $x = \arccos\frac{1}{4} \Rightarrow \cos x = \frac{1}{4} \longrightarrow r = 2\cdot\frac{1}{4} = \frac{1}{2}$

$\longrightarrow \sin x = +\sqrt{1-(1/4)^2} = \frac{\sqrt{15}}{4}$

porque $x>0$

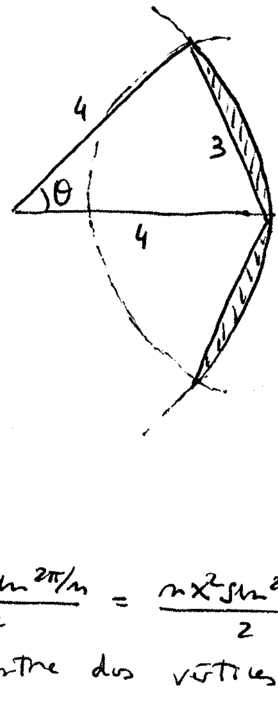
$S_\infty = \dfrac{\sin x}{1-r} = \dfrac{\sqrt{15}/4}{1-1/2} = \dfrac{\sqrt{15}}{2}$ ✓

M14 TZ2 P2#4

a) $3^2 = 4^2 + 4^2 - 2\cdot4\cdot4\cos\theta$

$9 = 16+16-32\cos\theta$

$\cos\theta = \frac{23}{32} \Rightarrow \theta \simeq 44° = \boxed{0'769 \text{ rad}}$



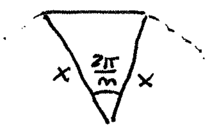
b) Area Sombreada =

$= 2\cdot(\text{Area Sector} - \text{Area Triángulo}) =$

$= 2\left(\frac{4^2\cdot 0'769}{2} - \frac{4^2\sin 0'769}{2}\right) = \boxed{1'18 \text{ cm}^2}$

N14 P2#9

a)



Area Polígono $= n\cdot$ Area Triángulo $= n\cdot\dfrac{x\cdot x\sin 2\pi/n}{2} = \dfrac{nx^2\sin 2\pi/n}{2}$

Para n par, la máxima distancia es entre dos vértices opuestos, es decir, un diámetro : $2x$

$C = \dfrac{\frac{4nx^2\sin 2\pi/n}{2}}{\pi\cdot(2x)^2} = \dfrac{4nx^2\sin 2\pi/n}{8\pi x^2} = \dfrac{n\sin 2\pi/n}{2\pi}$ ✓

b) n par : $C > 0'99$; $\dfrac{n\sin 2\pi/n}{2\pi} > 0'99 \Rightarrow \boxed{n=26}$

n impar : $C > 0'99$; $\dfrac{n\sin 2\pi/n}{\pi(1+\cos\pi/n)} > 0'99 \Rightarrow \boxed{n=21}$

(Resuelto con calculadora Gráfica)

c) Como se alcanzan antes grandes valores de compactibilidad siendo n impar que siendo par, no es una buena medida.

N14
P2#14

a) $3\,\text{sen}\,B + 4\cos C = 6 \longrightarrow 9\,\text{sen}^2 B + 24\,\text{sen}\,B\cos C + 16\cos^2 C = 36$

$4\,\text{sen}\,C + 3\cos B = 1 \longrightarrow + \; 16\,\text{sen}^2 C + 24\,\text{sen}\,C\cos B + 9\cos^2 B = 1$

$$9(\text{sen}^2 B + \cos^2 B) + 16(\text{sen}^2 C + \cos^2 C) + 24(\text{sen}\,B\cos C + \cos B\,\text{sen}\,C) = 37$$

$$9 + 16 + 24\,\text{sen}(B+C) = 37$$

$$24\,\text{sen}(B+C) = 12$$

$$\text{sen}(B+C) = \frac{1}{2} \quad ✓$$

b) $\text{sen}(B+C) = \frac{1}{2} \Rightarrow B + C = 30° \Rightarrow A = 150°$

$\phantom{\text{sen}(B+C) = \frac{1}{2} \Rightarrow}\; B + C = 150° \Rightarrow A = 30°$

$A = 150°$ no es posible porque B (y C) sería menor que 30°:

$3\,\text{sen}\,B + 4\cos C < 3\cdot\frac{1}{2} + 4\cdot 1 = 5'5$

No podría ser $3\,\text{sen}\,B + 4\cos C = 6$

M15
TZ2
P1#3

$\text{tg}\,x + \text{tg}\,2x = 0 \quad (0° \le x < 360°)$

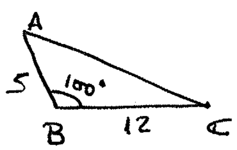
$\text{tg}\,x + \dfrac{2\,\text{tg}\,x}{1 - \text{tg}^2 x} = 0$

$\text{tg}\,x = t \qquad t + \dfrac{2t}{1-t^2} = 0 \;;\; t - t^3 + 2t = 0 \;;\; 3t - t^3 = 0$

$t(3 - t^2) = 0$
- $t = 0 \Rightarrow x = \begin{cases} 0° \\ 180° \end{cases}$
- $t = \sqrt{3} \Rightarrow x = \begin{cases} 60° \\ 240° \end{cases}$
- $t = -\sqrt{3} \Rightarrow x = \begin{cases} 120° \\ 300° \end{cases}$

M15
TZ2
P2#1

a) $\text{Area} = \dfrac{5\cdot 12\cdot \text{sen}\,100}{2} = \boxed{29'5\ \text{cm}^2}$



b) $AC^2 = 5^2 + 12^2 - 2\cdot 5\cdot 12\cos 100° \Rightarrow AC = \boxed{13'8\ \text{cm}}$