Álgebra en exámenes BI - NM

Nov 01

The transformation $\boldsymbol{R}$ is represented by the matrix $\boldsymbol{R} = \begin{pmatrix} 0.28 & 0.96 \\ 0.96 & -0.28 \end{pmatrix}$

(a) Calculate the determinant of this matrix.

(b) Show that the matrix is its own inverse.

Nov 01

An unknown 2×2 matrix, X, satisfies the matrix equation

$$\begin{pmatrix} 2 & -1 \\ 5 & 3 \end{pmatrix} \mathbf{X} = \begin{pmatrix} 4 & -4 \\ -1 & 12 \end{pmatrix}.$$

(a) Write down the inverse of the matrix $\begin{pmatrix} 2 & -1 \\ 5 & 3 \end{pmatrix}$.

(b) Hence, express X as a product of two matrices.

(c) Evaluate X.

Mayo 03

Las matrices $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{X}$ están dadas por

$$\boldsymbol{A} = \begin{pmatrix} 3 & 1 \\ -5 & 6 \end{pmatrix}, \boldsymbol{B} = \begin{pmatrix} 4 & 8 \\ 0 & -3 \end{pmatrix}, \boldsymbol{X} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \text{ donde } a, b, c \text{ y } d \in \mathbb{Q}.$$

Suponiendo que $\boldsymbol{AX} + \boldsymbol{X} = \boldsymbol{B}$, halle los valores **exactos** de a, b, c y d.

Mayo 05

Sean $\boldsymbol{C} = \begin{pmatrix} -2 & 4 \\ 1 & 7 \end{pmatrix}$ y $\boldsymbol{D} = \begin{pmatrix} 5 & 2 \\ -1 & a \end{pmatrix}$.

La matriz de orden 2×2 $\boldsymbol{Q}$ es tal que $3\boldsymbol{Q} = 2\boldsymbol{C} - \boldsymbol{D}$

(a) Halle $\boldsymbol{Q}$.

(b) Halle $\boldsymbol{CD}$.

(c) Halle $\boldsymbol{D}^{-1}$.

Nov 05

Matrices $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ are defined by

$$\boldsymbol{A}=\begin{pmatrix}5 & 1\\ 7 & 2\end{pmatrix} \quad \boldsymbol{B}=\begin{pmatrix}2 & 4\\ -3 & 15\end{pmatrix} \quad \boldsymbol{C}=\begin{pmatrix}9 & -7\\ 8 & 2\end{pmatrix}.$$

Let $\boldsymbol{X}$ be an unknown 2×2 matrix satisfying the equation

$$\boldsymbol{AX}+\boldsymbol{B}=\boldsymbol{C}.$$

This equation may be solved for $\boldsymbol{X}$ by rewriting it in the form

$$\boldsymbol{X}=\boldsymbol{A}^{-1}\boldsymbol{D}$$

where $\boldsymbol{D}$ is a 2×2 matrix.

(a) Write down $\boldsymbol{A}^{-1}$.

(b) Find $\boldsymbol{D}$.

(c) Find $\boldsymbol{X}$.

Mayo 06

Sea S_n la suma de los primeros n términos de la serie aritmética $2+4+6+\dots$.

(a) Halle

(i) S_4;

(ii) S_{100}.

Sea $\boldsymbol{M}=\begin{pmatrix}1 & 2\\ 0 & 1\end{pmatrix}$.

(b) (i) Halle $\boldsymbol{M}^2$.

(ii) Compruebe que $\boldsymbol{M}^3=\begin{pmatrix}1 & 6\\ 0 & 1\end{pmatrix}$.

Se puede suponer ahora que $\boldsymbol{M}^n=\begin{pmatrix}1 & 2n\\ 0 & 1\end{pmatrix}$, para $n\geq 4$. La suma $\boldsymbol{T}_n$ se define por

$$\boldsymbol{T}_n=\boldsymbol{M}^1+\boldsymbol{M}^2+\boldsymbol{M}^3+\dots+\boldsymbol{M}^n.$$

(c) (i) Escriba $\boldsymbol{M}^4$.

(ii) Halle $\boldsymbol{T}_4$.

(d) Utilizando los resultados obtenidos en la parte (a) (ii), halle $\boldsymbol{T}_{100}$.

Mayo 06

Let $\boldsymbol{A} = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 2 & 0 & 1 \end{pmatrix}$, $\boldsymbol{B} = \begin{pmatrix} 18 \\ 23 \\ 13 \end{pmatrix}$ and $\boldsymbol{X} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$.

(a) Write down the inverse matrix $\boldsymbol{A}^{-1}$.

(b) Consider the equation $\boldsymbol{AX} = \boldsymbol{B}$.

(i) Express $\boldsymbol{X}$ in terms of $\boldsymbol{A}^{-1}$ and $\boldsymbol{B}$.

(ii) **Hence**, solve for $\boldsymbol{X}$.

Mayo 06

(a) Let $\begin{pmatrix} b & 3 \\ 7 & 8 \end{pmatrix} + \begin{pmatrix} 9 & 5 \\ -2 & 7 \end{pmatrix} = \begin{pmatrix} 4 & 8 \\ a & 15 \end{pmatrix}$.

(i) Write down the value of a.

(ii) Find the value of b.

(b) Let $3\begin{pmatrix} -4 & 8 \\ 2 & 1 \end{pmatrix} - 5\begin{pmatrix} 2 & 0 \\ q & -4 \end{pmatrix} = \begin{pmatrix} -22 & 24 \\ -9 & 23 \end{pmatrix}$.

Find the value of q.

Mayo 06

La matriz $\boldsymbol{A} = \begin{pmatrix} 1 & 2 & 0 \\ -3 & 1 & -1 \\ 2 & -2 & 1 \end{pmatrix}$ tiene una inversa $\boldsymbol{A}^{-1} = \begin{pmatrix} -1 & -2 & -2 \\ 1 & 1 & 1 \\ a & 6 & b \end{pmatrix}$.

(a) Escriba el valor de

(i) a;

(ii) b.

Considere las siguientes ecuaciones simultáneas

$$\begin{aligned} x + 2y &= 7 \\ -3x + y - z &= 10 \\ 2x - 2y + z &= -12 \end{aligned}$$

(b) Escriba estas ecuaciones como una ecuación matricial.

(c) Resuelva la ecuación matricial.

Muestra 06

Let $A=\begin{pmatrix} a & b \\ c & 0 \end{pmatrix}$ and $B=\begin{pmatrix} 1 & 0 \\ d & e \end{pmatrix}$. Giving your answers in terms of *a, b, c, d* and *e*,

(a) write down $\boldsymbol{A}+\boldsymbol{B}$;

(b) find $\boldsymbol{AB}$.

Nov 06 P1

Let $A=\begin{pmatrix} 3 & 2 \\ k & 4 \end{pmatrix}$ and $B=\begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix}$. Find, in terms of *k*,

(a) $2\boldsymbol{A}-\boldsymbol{B}$;

(b) $\det(2\boldsymbol{A}-\boldsymbol{B})$.

Mayo 07 P1

(a) Write down the inverse of the matrix $A=\begin{pmatrix} 1 & -3 & 0 \\ 2 & 0 & 1 \\ 4 & 1 & 3 \end{pmatrix}$.

(b) Hence or otherwise solve

$$\begin{aligned} x-3y &= 1 \\ 2x+z &= 2 \\ 4x+y+3z &= -1 \end{aligned}$$

Mayo 07 P1

Sea $A=\begin{pmatrix} 1 & x & -1 \\ 3 & 1 & 4 \end{pmatrix}$ y $B=\begin{pmatrix} 3 \\ x \\ 2 \end{pmatrix}$.

(a) Halle $\boldsymbol{AB}$.

(b) La matriz $C=\begin{pmatrix} 20 \\ 28 \end{pmatrix}$ y $2\boldsymbol{AB}=\boldsymbol{C}$. Halle el valor de x.

Nov 07 P2

Let $A=\begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$.

(a) Find

(i) $\boldsymbol{A}^{-1}$;

(ii) $\boldsymbol{A}^{2}$.

Let $B=\begin{pmatrix} p & 2 \\ 0 & q \end{pmatrix}$.

(b) Given that $2\boldsymbol{A}+\boldsymbol{B}=\begin{pmatrix} 2 & 6 \\ 4 & 3 \end{pmatrix}$, find the value of *p* and of *q*.

(c) Hence find $\boldsymbol{A}^{-1}\boldsymbol{B}$.

(d) Let $\boldsymbol{X}$ be a 2×2 matrix such that $\boldsymbol{AX}=\boldsymbol{B}$. Find $\boldsymbol{X}$.

Mayo 08 P1

Let $\boldsymbol{M}=\begin{pmatrix}2 & 1\\ 2 & -1\end{pmatrix}$.

(a) Write down the determinant of $\boldsymbol{M}$.

(b) Write down $\boldsymbol{M}^{-1}$.

(c) **Hence** solve $\boldsymbol{M}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}4\\ 8\end{pmatrix}$.

Mayo 08 P1

Let $\boldsymbol{A}=\begin{pmatrix}1 & 2\\ 3 & -1\end{pmatrix}$ and $\boldsymbol{B}=\begin{pmatrix}3 & 0\\ -2 & 1\end{pmatrix}$.

Find

(a) $\boldsymbol{A}+\boldsymbol{B}$;

(b) $-3\boldsymbol{A}$;

(c) $\boldsymbol{AB}$.

Muestra 06/08 P2

(a) Write down the inverse of the matrix $\boldsymbol{A}=\begin{pmatrix}1 & -3 & 1\\ 2 & 2 & -1\\ 1 & -5 & 3\end{pmatrix}$.

(b) **Hence** solve the simultaneous equations

$$\begin{aligned} x - 3y + z &= 1\\ 2x + 2y - z &= 2\\ x - 5y + 3z &= 3 \end{aligned}$$

Muestra 08 P1

(a) Given $\boldsymbol{A}=\begin{pmatrix}7 & 8\\ 2 & 3\end{pmatrix}$, find $\boldsymbol{A}^{-1}$.

(b) **Hence**, solve the system of simultaneous equations.

$$\begin{aligned} 7x+8y &= 1\\ 2x+3y &= 1 \end{aligned}$$

Muestra 08 P1

Let $\boldsymbol{M}=\begin{pmatrix}2 & -1\\ -3 & 4\end{pmatrix}$, and $\boldsymbol{O}=\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}$. Given that $\boldsymbol{M}^2-6\boldsymbol{M}+k\boldsymbol{I}=\boldsymbol{O}$, find k.

Muestra 08 P1

Let $A = \begin{pmatrix} 1 & -2 \\ 0 & 3 \end{pmatrix}$.

(a) Find A^2.

(b) Let $B = \begin{pmatrix} -3 & 4 \\ 2 & 1 \end{pmatrix}$. Solve the matrix equation $3X + A = B$.

Nov 08 P2

Let $A = \begin{pmatrix} 1 & -1 & 3 \\ 2 & 1 & 1 \\ 0 & 2 & -2 \end{pmatrix}$.

(a) Write down A^{-1}.

The matrix B satisfies the equation $\left(I - \frac{1}{2}B\right)^{-1} = A$, where I is the 3×3 identity matrix.

(b) (i) Show that $B = -2(A^{-1} - I)$.

(ii) Find B.

(iii) Write down $\det B$.

(iv) **Hence**, explain why B^{-1} exists.

Let $BX = C$, where $X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ and $C = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$.

(c) (i) Find X.

(ii) Write down a system of equations whose solution is represented by X.

Nov 08 P1

Let $A = \begin{pmatrix} 1 & -2 \\ 3 & p \end{pmatrix}$ and $B = \begin{pmatrix} -2 & 1 \\ q & \frac{1}{2} \end{pmatrix}$.

(a) Find AB in terms of p and q.

(b) Matrix B is the inverse of matrix A. Find the value of p and of q.

Mayo 09
P1

A matrix $\boldsymbol{M}$ has **inverse** $\boldsymbol{M}^{-1} = \begin{pmatrix} 5 & 0 \\ 1 & 2 \end{pmatrix}$.

(a) Find $\boldsymbol{M}$.

(b) Solve the matrix equation $\boldsymbol{MX} = \boldsymbol{B}$, where $\boldsymbol{B} = \begin{pmatrix} 1 \\ 7 \end{pmatrix}$ and $\boldsymbol{X} = \begin{pmatrix} x \\ y \end{pmatrix}$.

Mayo 09
P1

Let $\boldsymbol{A} = \begin{pmatrix} 5 & 1 \\ 6 & 2 \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} 2 & -1 \\ -6 & 5 \end{pmatrix}$.

(a) (i) Find $\boldsymbol{AB}$.

(ii) Write down the inverse of $\boldsymbol{A}$.

Let $\boldsymbol{X} = \begin{pmatrix} x \\ y \end{pmatrix}$ and $\boldsymbol{C} = \begin{pmatrix} 8 \\ -4 \end{pmatrix}$.

(b) Solve the matrix equation $\boldsymbol{AX} = \boldsymbol{C}$.

Nov 09
P2

Let $\boldsymbol{A} = \begin{pmatrix} 3 & 0 & 1 \\ 2 & -3 & 0 \\ 4 & -2 & 1 \end{pmatrix}$.

(a) Write down $\boldsymbol{A}^{-1}$.

(b) Let $\boldsymbol{B}$ be a 3×3 matrix. Given that $\boldsymbol{AB} + \begin{pmatrix} -3 & 2 & 1 \\ 5 & 3 & 4 \\ -9 & 2 & 10 \end{pmatrix} = \begin{pmatrix} 7 & 6 & -7 \\ 6 & 5 & -8 \\ 1 & 7 & -5 \end{pmatrix}$, find $\boldsymbol{B}$.

Mayo 10
P1

Sean $\boldsymbol{A} = \begin{pmatrix} 1 & -2 \\ 3 & 4 \end{pmatrix}$ y $\boldsymbol{B} = \begin{pmatrix} -5 \\ 5 \end{pmatrix}$.

(a) Halle $\boldsymbol{AB}$.

(b) Resuelva $\boldsymbol{A}^{-1}\boldsymbol{X} = \boldsymbol{B}$.

Nov 10 P1#7

Sea $A=\begin{pmatrix} 9e^x & e^x \\ e^x & e^{3x} \end{pmatrix}$.

(a) Halle una expresión para $\det A$.

(b) Halle el valor de x para el cual A no tiene inversa. Exprese la respuesta de la forma $a\ln b$, donde $a, b \in \mathbb{Z}$.

Mayo 11 P1#7

Sea $A=\begin{pmatrix} 3 & x \\ -2 & -3 \end{pmatrix}$.

(a) Halle el valor de x para el cual no existe A^{-1}.

(b) Sabiendo que $A = A^{-1}$, halle x.

Mayo 11 P2#4 (resolver con la calculadora gráfica)

El siguiente sistema de ecuaciones lineales se puede escribir como una ecuación matricial del tipo $MX = N$.

$$\begin{aligned} x+6y-3z&=-1\\ 4x+2y-4z&=12\\ x+y+5z&=15 \end{aligned}$$

(a) Escriba las matrices M y N.

(b) Resuelva la ecuación **matricial** $MX = N$.

(c) A partir de lo anterior, escriba la solución del sistema de ecuaciones lineales.

Nov 11 P1#5

Sean $P=\begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix}$ y $Q=\begin{pmatrix} 4 & -2 \\ -10 & 6 \end{pmatrix}$.

(a) Halle PQ.

(b) A partir de lo anterior o de cualquier otro modo, escriba P^{-1}.

Mayo 12 P2#6 (resolver con la calculadora gráfica)

Sean $A=\begin{pmatrix} -1 & -1 & 2 \\ -1 & 0 & 1 \\ 2 & 1 & -2 \end{pmatrix}$ y $B=\begin{pmatrix} 1 & 0 & 2 \\ 0 & -1 & 0 \\ 3 & 1 & 2 \end{pmatrix}$.

(a) Escriba A^{-1}.

(b) Sea C una matriz de orden 3×3 tal que $ACA^{-1} = B$. Halle C.

Nov 12 P1#1

Sean $A=\begin{pmatrix} 0 & 3 \\ -2 & 4 \end{pmatrix}$ y $B=\begin{pmatrix} -1 & 0 \\ 5 & 1 \end{pmatrix}$.

(a) Halle AB.

(b) Sabiendo que $X - 2A = B$, halle X.

Nov 12 P2#2 (resolver con la calculadora gráfica)

Sea $\boldsymbol{A}=\begin{pmatrix}2&2&1\\3&1&0\\4&2&1\end{pmatrix}$.

(a) Escriba $\boldsymbol{A}^{-1}$.

(b) A partir de lo anterior o de cualquier otro modo, halle $\boldsymbol{B}$, sabiendo que $\boldsymbol{AB}=\begin{pmatrix}-1&6&-1\\5&-1&3\\5&2&7\end{pmatrix}$.

Mayo 13 P1#2

Sean $\boldsymbol{A}=\begin{pmatrix}1&2\\3&0\end{pmatrix}$, $\boldsymbol{B}=\begin{pmatrix}2&1\\1&q\end{pmatrix}$, $\boldsymbol{C}=\begin{pmatrix}p&-1\\6&3\end{pmatrix}$, de modo que $\boldsymbol{AB}=\boldsymbol{C}$.

(a) Halle el valor de p.

(b) Halle el valor de q.

Mayo 13 P2#1 (resolver con la calculadora gráfica)

Sean $\boldsymbol{A}-\begin{pmatrix}1&0&-1\\0&1&1\\-1&1&1\end{pmatrix}$ y $\boldsymbol{B}=\begin{pmatrix}1\\-1\\3\end{pmatrix}$.

(a) Escriba A^{-1}.

(b) Resuelva $\boldsymbol{AX}=\boldsymbol{B}$.

Nov 13 P1#2

Sean $\boldsymbol{A}=\begin{pmatrix}1&2\\3&0\end{pmatrix}$, $\boldsymbol{B}=\begin{pmatrix}2&1\\1&3\end{pmatrix}$ y $\boldsymbol{C}=\begin{pmatrix}2&-4\\0&1\end{pmatrix}$.

(a) Halle $\boldsymbol{AB}$.

(b) Halle $\det(\boldsymbol{AB}+\boldsymbol{C})$.

Nov 13 P2#1 (resolver con la calculadora gráfica)

Sean $\boldsymbol{A}=\begin{pmatrix}0&5&4\\1&2&1\\2&2&0\end{pmatrix}$ y $\boldsymbol{B}=\begin{pmatrix}11\\7\\10\end{pmatrix}$.

(a) Escriba $\boldsymbol{A}^{-1}$.

(b) A partir de lo anterior o de cualquier otro modo, resuelva la ecuación $\boldsymbol{AX}=\boldsymbol{B}$.