(1) $y = \frac{3\cos x}{2-\operatorname{sen} x} \qquad 0 \le x \le 2\pi$

Dominio $= [0, 2\pi]$ (ya que $\operatorname{sen} x \neq 2 \ \forall x$)

$$y' = \frac{-3\operatorname{sen} x(2-\operatorname{sen} x) + 3\cos x \cos x}{(2-\operatorname{sen} x)^2} = 3\,\frac{-2\operatorname{sen} x + \operatorname{sen}^2 x + \cos^2 x}{(2-\operatorname{sen} x)^2} = 3\,\frac{1-2\operatorname{sen} x}{(2-\operatorname{sen} x)^2}$$

$y' = 0 \implies \operatorname{sen} x = \frac{1}{2} \longrightarrow \begin{matrix} x = 30^\circ \\ x = 150^\circ \end{matrix}$

$$\begin{array}{ccccccc} 0 & & 30^\circ & & 150^\circ & & 360^\circ \\ & + \nearrow & & - \searrow & & + \nearrow & \end{array}$$

signo $y' = \frac{3(1-2\operatorname{sen} x)}{(2-\operatorname{sen} x)^2}$

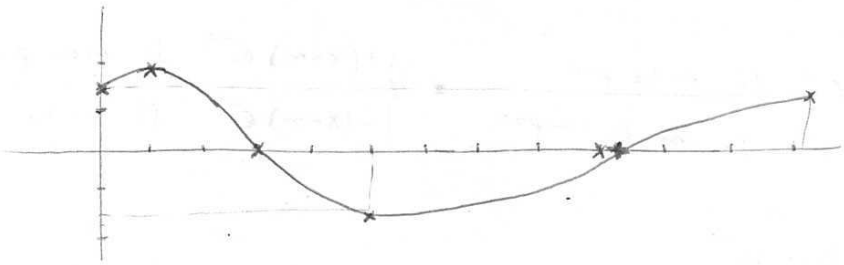
Máximo en $x = 30^\circ \longrightarrow y = \frac{3\ \sqrt{3}/2}{2-1/2} = \sqrt{3}$ $\qquad (\pi/6, \sqrt{3})$

Mínimo en $x = 150^\circ \longrightarrow y = \frac{3\ -\sqrt{3}/2}{2-1/2} = -\sqrt{3}$ $\qquad (5\pi/6, -\sqrt{3})$

$x = 2\pi \implies y = 3/2 \qquad (2\pi, 3/2)$

$x = 0 \implies y = 3/2 \qquad (0, 3/2)$

$y = 0 \implies \cos x = 0 \longrightarrow x = 90^\circ \quad x = 270^\circ \qquad (\pi/2, 0) \qquad (3\pi/2, 0)$



(2) $y = \frac{(x+1)^2}{x-1}$

Dominio $= \mathbb{R} - \{1\}$ $\qquad \lim_{x\to 1} f(x) = \infty \begin{cases} \lim_{x\to 1^-} f(x) = -\infty \\ \lim_{x\to 1^+} f(x) = +\infty \end{cases}$

$$y' = \frac{2(x+1)(x-1) - (x+1)^2}{(x-1)^2} = \frac{(x+1)(2x-2-x-1)}{(x-1)^2} = \frac{(x+1)(x-3)}{(x-1)^2}$$

$y' = 0 \implies x = \begin{matrix} -1 \\ 3 \end{matrix}$

$$\begin{array}{ccccccccc} & + \nearrow & -1 & - \searrow & 1 & - \searrow & 3 & + \nearrow & \end{array}$$

signo y'

Máximo en $x = -1 \longrightarrow y = 0 \qquad (-1, 0)$

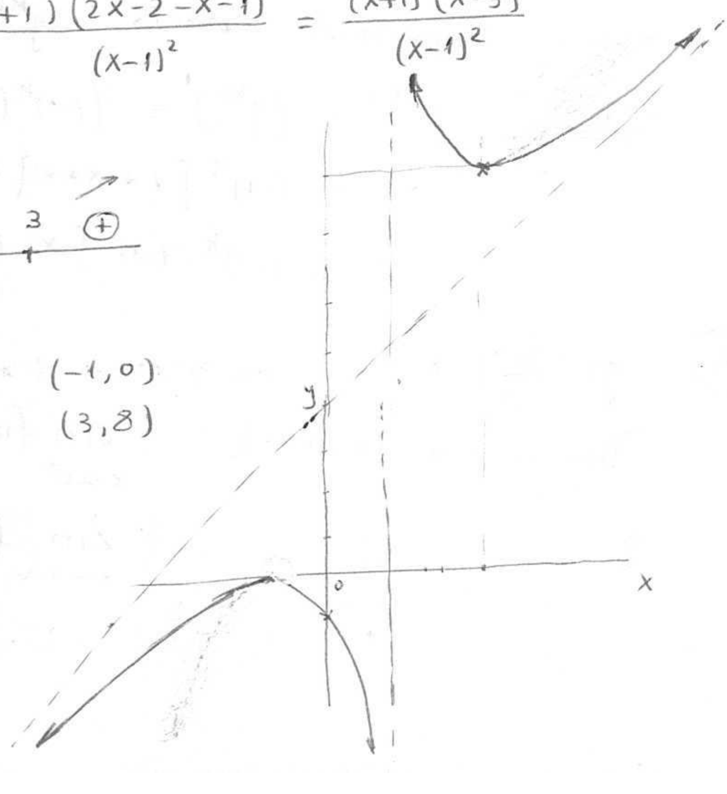
Mínimo en $x = 3 \longrightarrow y = 8 \qquad (3, 8)$

$\lim_{x\to -\infty} f(x) = -\infty$

$\lim_{x\to +\infty} f(x) = +\infty$

$x = 0 \implies y = -1 \qquad (0, -1)$

$y = 0 \implies x = -1 \qquad (-1, 0)$



NOTA :

$$\begin{array}{r|l} x^2+2x+1 & x-1 \\ \underline{-x^2+x} & x+3 \\ 3x+1 & \\ \underline{-3x+3} & \\ 4 & \end{array}$$

$$y = \frac{(x+1)^2}{x-1} = x+3+\frac{4}{x-1}$$

Eso significa que cuando $x \to +\infty$ y $x \to -\infty$ la curva $y = \frac{(x+1)^2}{x-1}$ se aproxima progresivamente a la recta $y=x+3$

Es decir, que tiene la asíntota oblicua $y = x+3$.

(3) $y = xe^{-x}$

$y' = e^{-x} + xe^{-x}\cdot(-1) = (1-x)e^{-x}$

$y'' = -1\cdot e^{-x} + (1-x)\,e^{-x}\cdot(-1) = (x-2)e^{-x}$

$y''' = e^{-x} + (x-2)\,e^{-x}\cdot(-1) = (3-x)e^{-x}$

$$y^{n)} = \begin{cases} (x-n)e^{-x} & \text{si } n \text{ es par} \\ (n-x)e^{-x} & \text{si } n \text{ es impar} \end{cases} = \begin{cases} +(x-n)e^{-x} & \text{si } n \text{ es par} \\ -(x-n)e^{-x} & \text{si } n \text{ es impar} \end{cases} =$$

$$= \left\{ (-1)^n\cdot(x-n)e^{-x} \right.$$

Demostración :

$\underline{n=1}$: $y^{1)} = (-1)^1(x-1)e^{-x} = (-x+1)e^{-x} = (1-x)e^{-x}$ ✓

$\underline{n=k}$: Suponiendo cierto $y^{k)} = (-1)^k(x-k)e^{-x}$

$\underline{n=k+1}$: $y^{k+1)} = \left(y^{k)}\right)' = \left((-1)^k(x-k)e^{-x}\right)' = (-1)^k\left[e^{-x} + (x-k)e^{-x}(-1)\right] =$

$= (-1)^k\left[1-x+k\right]e^{-x} = (-1)^k\left[(k+1)-x\right]e^{-x} =$

$= (-1)^k\cdot(-1)\left[x-(k+1)\right]e^{-x} = (-1)^{k+1}\left(x-(k+1)\right)e^{-x}$ ✓

(4) $y = \frac{\ln x}{x}$

Dominio $= (0, +\infty)$

$\lim_{x\to 0^+} f(x) = \frac{-\infty}{+0} = -\infty$

$\lim_{x\to +\infty} f(x) = \frac{+\infty}{+\infty} = 0$ (Ya que $\ln x \ll x$ para $x \to +\infty$)

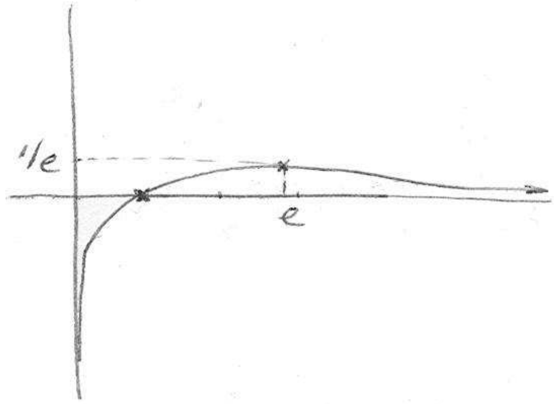
$$y' = \frac{\frac{1}{x}x - \ln x}{x^2} = \frac{1 - \ln x}{x^2}$$

$y' = 0 \Longrightarrow \ln x = 1 \rightarrow x = e \rightarrow y = 1/e$

$0 \quad \oplus \quad e \quad \ominus$

signo de y'

Máximo en $x = e$ $\quad (e, 1/e)$



$x = 0 \rightarrow$ No corta al eje y

$y = 0 \rightarrow x = 1 \quad (1,0)$

d) $\frac{\ln x}{x} = \frac{1}{2}\ln 2$

$2\ln x = x \ln 2$

$\ln x^2 = \ln 2^x$

$x^2 = 2^x \Longrightarrow \boxed{x = 2}$, $\boxed{x = 4}$

e) Para $y > \frac{1}{e} \Longrightarrow$ No hay puntos de curva

Para $y = \frac{1}{e} \Longrightarrow$ Hay 1 sólo punto

Para $0 < y < \frac{1}{e} \Longrightarrow$ Hay 2 puntos de curva

Para $y \leq 0 \Longrightarrow$ Hay 1 punto.

Luego $\ln x = \frac{1}{4}x \Longrightarrow$ Dos puntos $\Longrightarrow$ Dos raíces

$\ln x = 4x \Longrightarrow$ Ningún punto $\Longrightarrow$ Sin solución.

(5) $\frac{x - y}{x + y} = \frac{x^2}{y} + 1$

$\cancel{xy} - y^2 = x^3 + x^2y + \cancel{xy} + y^2$

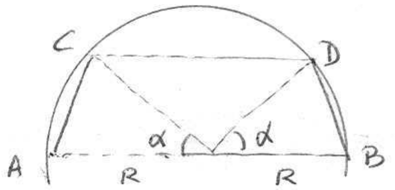
$0 = x^3 + 2y^2 + x^2y$

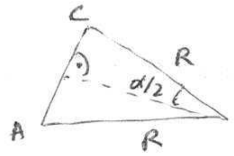
$0 = 3x^2 + 4yy' + 2xy + x^2y'$

$-3x^2 - 2xy = (4y + x^2)y'$

$$\boxed{y' = -\frac{3x^2 + 2xy}{x^2 + 4y}}$$

6

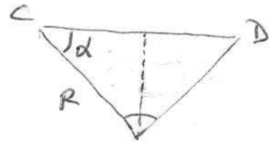




$$\operatorname{sen}\left(\frac{\alpha}{2}\right) = \frac{\overline{AC}/2}{R} \Rightarrow$$

$$\Rightarrow \overline{AC} = 2R\operatorname{sen}\left(\frac{\alpha}{2}\right)$$

$$\overline{BD} = 2R\operatorname{sen}\left(\frac{\alpha}{2}\right)$$



$$\cos\alpha = \frac{\overline{CD}/2}{R} \Rightarrow \overline{CD} = 2R\cos\alpha$$

$$\text{Perímetro} = 2R + 4R\operatorname{sen}\left(\frac{\alpha}{2}\right) + 2R\cos\alpha$$

$$\frac{dP}{d\alpha} = P' = 4R\cos\left(\frac{\alpha}{2}\right)\frac{1}{2} - 2R\operatorname{sen}\alpha = 2R\cos\left(\frac{\alpha}{2}\right) - 2R\operatorname{sen}\alpha$$

$$P' = 0 \Rightarrow 2R\cos\left(\frac{\alpha}{2}\right) = 2R\operatorname{sen}\alpha \Rightarrow \cos\left(\frac{\alpha}{2}\right) = \operatorname{sen}\alpha \Rightarrow$$

$$\Rightarrow \cos\left(\frac{\alpha}{2}\right) = 2\operatorname{sen}\left(\frac{\alpha}{2}\right)\cos\left(\frac{\alpha}{2}\right) \begin{cases} \nearrow \cos\left(\frac{\alpha}{2}\right) = 0 \Rightarrow \\ \searrow 1 = 2\operatorname{sen}\left(\frac{\alpha}{2}\right) \Rightarrow \end{cases}$$

$$\Rightarrow \begin{cases} \frac{\alpha}{2} = 270^\circ \rightarrow \alpha = \cancel{540}^\circ \\ \frac{\alpha}{2} = 90^\circ \Rightarrow \alpha = \cancel{180}^\circ \end{cases}$$

$$\Rightarrow \operatorname{sen}\left(\frac{\alpha}{2}\right) = \frac{1}{2} \Rightarrow \begin{cases} \frac{\alpha}{2} = 30^\circ \Rightarrow \alpha = 60^\circ \\ \alpha/2 = 150^\circ \Rightarrow \alpha = \cancel{300}^\circ \end{cases}$$

$\nearrow$ $\oplus$ 60° $\ominus$ $\searrow$

signo de $P' = 2R\cos\left(\frac{\alpha}{2}\right) - 2R\operatorname{sen}\alpha$

Máximo para $\boxed{\alpha = 60^\circ}$

7

$$C(x) = \begin{cases} 5x & \text{si } 0 < x \leq 10 \\ \sqrt{ax^2 + 500} & \text{si } x > 10 \end{cases}$$

$$\left.\begin{array}{l} \lim\limits_{x \to 10^-} C(x) = 50 \\ \lim\limits_{x \to 10^+} C(x) = \sqrt{100a + 500} \end{array}\right\} \Rightarrow \sqrt{100a+500} = 50 \Rightarrow 100a = 2500 - 500 \Rightarrow$$

$$\Rightarrow \boxed{a = 20}$$

$$\lim_{x \to +\infty} \frac{C(x)}{x} = \lim_{x \to +\infty} \frac{\sqrt{20x^2 + 500}}{x} = \lim_{x \to +\infty} \sqrt{20 + \frac{500}{x^2}} = \boxed{\sqrt{20}}$$

(8) $y = x^5 - 5x^4 + 5x^3$

$y' = 5x^4 - 20x^3 + 15x^2 = 5x^2(x^2 - 4x + 3)$

$y' = 0 \Longrightarrow \boxed{x = 0}$ $\qquad (0,0)$

$\qquad\quad \longrightarrow x^2 - 4x + 3 = 0 \longrightarrow \boxed{x = 1}$ $\qquad (1, 1)$

$\qquad\qquad\qquad\qquad\qquad\quad \boxed{x = 3}$ $\qquad (3, -27)$

(9) $f(x) = x^3 + 6x^2 + 15x - 23$

$\lim\limits_{x \to -\infty} f(x) = -\infty \qquad \lim\limits_{x \to +\infty} f(x) = +\infty$

$f'(x) = 3x^2 + 12x + 15$

$f'(x) = 0 \Longrightarrow 3x^2 + 12x + 15 = 0$

$x^2 + 4x + 5 = 0 \longrightarrow$ No tiene raíces reales.

$\nearrow$ (+)

signo $f'(x)$.

La función es siempre creciente, luego pasa de $-\infty$ al $+\infty$ atravesando el eje X una sola vez.

Luego $x^3 + 6x^2 + 15x - 23 = 0$ tiene una única raíz real.

(10) $y = x^5 - 5x - 1$ $\qquad \lim\limits_{x \to -\infty} f(x) = -\infty$

$y' = 5x^4 - 5$ $\qquad \lim\limits_{x \to +\infty} f(x) = +\infty$

$y' = 0 \Longrightarrow x^4 = 1 \Longrightarrow x = \pm 1$

$\nearrow$ (+) $\quad -1 \quad$ $\searrow$ (−) $\quad 1 \quad$ $\nearrow$ (+)

signo $y' = 5x^4 - 5$

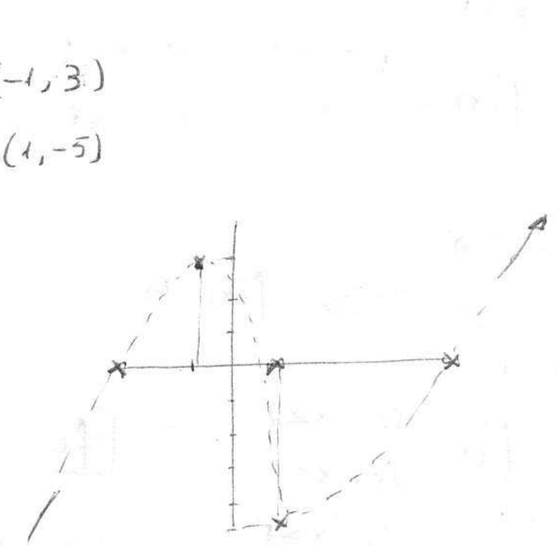
Máximo en $x = -1 \longrightarrow y = 3$ $\qquad (-1, 3)$

Mínimo en $x = 1 \longrightarrow y = -5$ $\qquad (1, -5)$

Al crecer desde $-\infty$ hasta el máximo corta al eje X una vez

Al pasar del máximo al mínimo corta al eje X por 2ª vez

Al crecer desde el mínimo al $+\infty$ cortará al eje X por 3ª y última vez.



(11) $f'(x) = 2x$

$g'(x) = 3x^2$

$f'(x) = g'(x) \Longrightarrow 3x^2 = 2x \Longrightarrow x(3x-2) = 0 \begin{cases} x = 0 \\ x = 2/3 \end{cases}$

$\oplus \quad 0 \quad \ominus \quad 2/3 \quad \oplus$

signo de $f'(x) - g'(x)$

Luego: $f'(x) > g'(x)$ en $(-\infty, 0) \cup (2/3, +\infty)$

$f'(x) = g'(x)$ en $x = 0$, $x = 2/3$

$f'(x) < g'(x)$ en $(0, 2/3)$

(12) $xy^2 + x^2y = 2$

$y^2 + x2yy' + 2xy + x^2y' = 0$

$y'(2xy + x^2) = -y^2 - 2xy \qquad y' = -\dfrac{y^2+2xy}{x^2+2xy}$

$P(1,1) \longrightarrow$ pendiente recta tangente $= -\dfrac{1+2}{1+2} = -1 \longrightarrow$ pendiente recta normal $= -\dfrac{1}{-1} = 1$

Ecuación Normal: $y - 1 = 1(x-1) \Longrightarrow \boxed{y = x}$

(13) $f(x) = \begin{cases} 5x^2 - 2x - 11 & x \in (-\infty, 1) \\ -\dfrac{8}{x} & x \in [1, +\infty) \end{cases}$

$5x^2 - 2x - 11$ está definido $\forall x \in \mathbb{R}$

$-\dfrac{8}{x}$ no está definido en $x = 0$, pero $0 \notin [1, +\infty)$

Luego: Dominio $= \mathbb{R}$

$\lim\limits_{x \to 1^-} f(x) = 5 - 2 - 11 = -8$

$\lim\limits_{x \to 1^+} f(x) = -\dfrac{8}{1} = -8$

$\longrightarrow f(x)$ continua en $x = 1 \Longrightarrow \boxed{f(x) \text{ continua en } \mathbb{R}}$

$f'(x) = \begin{cases} 10x - 2 & \text{si } x < 1 \\ \dfrac{8}{x^2} & \text{si } x > 1 \end{cases}$ Veamos en $x = 1$

$\lim\limits_{x \to 1^-} f'(x) = 10 - 2 = 8$

$\lim\limits_{x \to 1^+} f'(x) = \dfrac{8}{1} = 8$

$\longrightarrow f(x)$ es derivable en $x = 1$

$f'(x) = \begin{cases} 10x - 2 & \text{si } x < 1 \\ 8 & \text{si } x = 1 \\ \dfrac{8}{x^2} & \text{si } x > 1 \end{cases}$

$\boxed{f(x) \text{ derivable en } \mathbb{R}}$

(14)

$$f(x) = \begin{cases} x+1 & \text{si } x \leq -1 \\ 1-x^2 & \text{si } -1 < x \leq 2 \\ \dfrac{1}{x-3} & \text{si } x > 2 \end{cases} \qquad \text{Dominio} = \mathbb{R} - \{3\}$$

$\lim_{x \to -1^-} f(x) = -1+1 = 0$
$\lim_{x \to -1^+} f(x) = 1-1 = 0$ $\Rightarrow$ f(x) continua en $x=-1$

$\lim_{x \to 2^-} f(x) = 1-4 = -3$
$\lim_{x \to 2^+} f(x) = \dfrac{1}{2-3} = -1$ $\Rightarrow$ f(x) no es continua en $x=2$

$\boxed{f(x) \text{ continua en } \mathbb{R} - \{2,3\}}$

$$f'(x) = \begin{cases} 1 & \text{si } x < -1 \\ -2x & \text{si } -1 < x < 2 \\ \dfrac{-1}{(x-3)^2} & \text{si } x > 2 \end{cases}$$

$\lim_{x \to -1^-} f'(x) = 1$
$\lim_{x \to -1^+} f(x) = 2$ $\Rightarrow$ f(x) no es derivable en $x=-1$

Como f(x) no es continua en $x=2$ y $x=3$ $\Longrightarrow$ f(x) no es derivable en $x=2$ y $x=3$

f(x) derivable en $\mathbb{R} - \{-1, 2, 3\}$

(15)

$$f(x) = \begin{cases} x^2+2x-1 & \text{si } x < 0 \\ ax+b & \text{si } 0 \leq x < 1 \\ 2 & \text{si } x \geq 1 \end{cases}$$

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) \Longrightarrow \boxed{-1 = b}$

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) \Longrightarrow a+b = 2 \Longrightarrow \boxed{a = 3}$

(16)

$y = 1 - \sqrt[5]{x^4} = 1 - x^{4/5}$ Dominio $= \mathbb{R}$

$y' = -\dfrac{4}{5} x^{\frac{4}{5}-1} = -\dfrac{4}{5} x^{-1/5} = \dfrac{-4}{5\sqrt[5]{x}}$ No está definida en $x=0$

$+$ 0 $-$

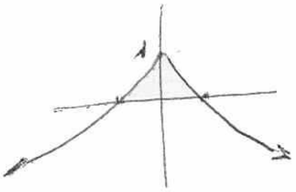
signo de y'

f(x) es creciente en $(-\infty, 0)$
f(x) es decreciente en $(0, +\infty)$

Sin embargo $f'(0)$ no es nula, de hecho no está definida.

El máximo absoluto es en $x=0 \longrightarrow (0, 1)$

La gráfica es:



(17) $V = \pi R^2 H = 160 \longrightarrow H = \dfrac{160}{\pi R^2}$

$S = 2\pi R^2 + 2\pi R H$

$S = 2\pi R^2 + 2\pi R \dfrac{160}{\pi R^2} = 2\pi R^2 + \dfrac{320}{R}$

$S' = 4\pi R - \dfrac{320}{R^2}$

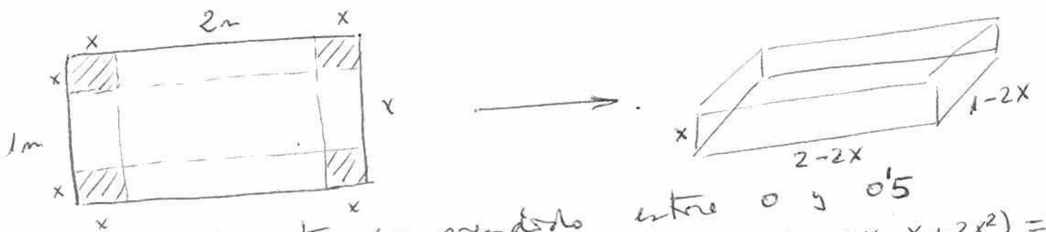
$S' = 0 \implies 4\pi R = \dfrac{320}{R^2}$; $R^3 = \dfrac{80}{\pi}$ $\quad R = \sqrt[3]{\dfrac{80}{\pi}}$

$\searrow \ominus \quad \sqrt[3]{\dfrac{80}{\pi}} \quad \nearrow \oplus$

signo de $S' = 4\pi R - \dfrac{320}{R^2}$

Superficie mínima para $\boxed{R = \sqrt[3]{\dfrac{80}{\pi}}} \longrightarrow H = \dfrac{160}{\pi \sqrt[3]{\dfrac{6400}{\pi^2}}} = \dfrac{160}{\sqrt[3]{6400\pi}} = \boxed{\dfrac{4}{\sqrt[3]{\pi}}}$

(18)



x debe de estar comprendido entre 0 y 0'5

$V = x(2-2x)(1-2x) = 2x(1-x)(1-2x) = 2x(1-2x-x+2x^2) =$

$= 2x - 6x^2 + 4x^3$

$V' = 2 - 12x + 12x^2 = 2(6x^2 - 6x + 1)$

$V' = 0 \implies x = \dfrac{6 \pm \sqrt{36-24}}{12} = \dfrac{6 \pm 2\sqrt{3}}{12} = \dfrac{3 \pm \sqrt{3}}{4}$ → $\dfrac{3-\sqrt{3}}{4}$; → $\cancel{\dfrac{3+\sqrt{3}}{4}}$ (Es mayor que 0'5)

$0 \quad \nearrow \oplus \quad \dfrac{3-\sqrt{3}}{4} \quad \searrow \ominus \quad 0'5$

signo de $V' = (6x^2 - 6x + 1) \cdot 2$

Volumen máximo para $\boxed{x = \dfrac{3-\sqrt{3}}{4}}$

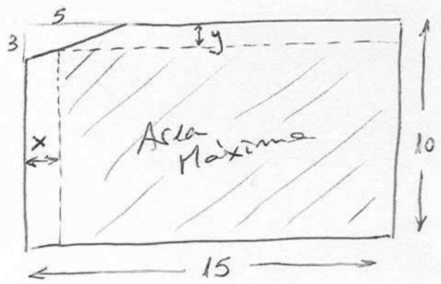
Dimensiones de la caja: $\dfrac{3-\sqrt{3}}{4}$

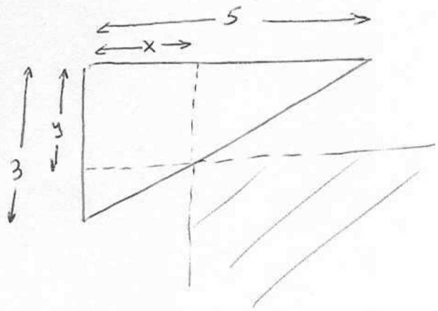
$2 - 2 \cdot \dfrac{3-\sqrt{3}}{4} = 2 - \dfrac{3-\sqrt{3}}{2} = \dfrac{1+\sqrt{3}}{2}$

$1 - 2\dfrac{3-\sqrt{3}}{4} = 1 - \dfrac{3-\sqrt{3}}{2} = \dfrac{-1+\sqrt{3}}{2}$

$V_{MAX} = \dfrac{3-\sqrt{3}}{4} \cdot \dfrac{1+\sqrt{3}}{2} \cdot \dfrac{-1+\sqrt{3}}{2} = \dfrac{(3-\sqrt{3})(\sqrt{3}^2 - 1^2)}{16} = \dfrac{(3-\sqrt{3}) \cdot 2}{16} = \boxed{\dfrac{3-\sqrt{3}}{8}}$

(19)





$$\frac{5}{3} = \frac{x}{3-y} \Rightarrow 15-5y = 3x$$

$$y = \frac{15-3x}{5}$$

$$\text{Área} = (15-x)\cdot(10-y) = (15-x)\left(10-\frac{15-3x}{5}\right) = (15-x)\left(7+\frac{3x}{5}\right) \qquad \text{Dominio} = [0,5]$$

$$A' = -\left(7+\frac{3x}{5}\right) + (15-x)\frac{3}{5} = -7-\frac{3x}{5}+9-\frac{3x}{5} = 2-\frac{6x}{5} = \frac{10-6x}{5}$$

$$A' = 0 \Rightarrow \frac{10-6x}{5} = 0 \Rightarrow x = \frac{5}{3}$$

$$0 \quad (+) \quad 5/3 \quad (-) \quad 5$$

Signo $A' = \dfrac{10-6x}{5}$

Área máxima para $x = \frac{5}{3} \rightarrow y = 2$

$A_{MAX} = \frac{320}{3}$ cm^3

Dimensiones definitivas: $\frac{40}{3}$ cm $\times$ 8 cm

(20)

$$y = ax^3+bx^2+cx+d \quad , \quad y' = 3ax^2+2bx+c \quad , \quad y'' = 6ax+2b$$

$P(2,1) \Rightarrow 1 = 8a+4b+2c+d$

Inflexión en $x=2 \Rightarrow 0 = 12a+2b$

Recta tangente horizontal en $x=2$ $\Rightarrow 0 = 12a+4b+c$

$P(0,0) \Rightarrow \boxed{0 = d}$

$$\left.\begin{aligned} 8a+4b+2c &= 1 \\ 6a+b &= 0 \\ 12a+4b+c &= 0 \end{aligned}\right\}$$

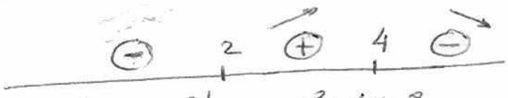
$$a = \frac{\begin{vmatrix} 1 & 4 & 2 \\ 0 & 1 & 0 \\ 0 & 4 & 1 \end{vmatrix}}{\begin{vmatrix} 8 & 4 & 2 \\ 6 & 1 & 0 \\ 12 & 4 & 1 \end{vmatrix}} = \frac{1}{8} \qquad b = \frac{\begin{vmatrix} 8 & 1 & 2 \\ 6 & 0 & 0 \\ 12 & 0 & 1 \end{vmatrix}}{8} = \frac{-6}{8} \qquad c = \frac{\begin{vmatrix} 8 & 4 & 1 \\ 6 & 1 & 0 \\ 12 & 4 & 0 \end{vmatrix}}{8} = \frac{12}{8}$$

$$\boxed{y = \frac{1}{8}x^3 - \frac{6}{8}x^2 + \frac{12}{8}x}$$

(21) $f(x) = -\frac{1}{3}\left(x^3 - 9x^2 + 24x - 48\right)$

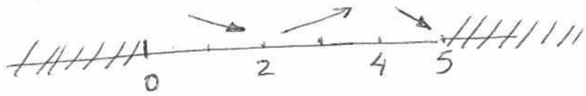
$f'(x) = -\frac{1}{3}\left(3x^2 - 18x + 24\right) = -x^2 + 6x - 8$

$f'(x) = 0 \Rightarrow -x^2 + 6x - 8 = 0 \Rightarrow x = \begin{cases} 2 \\ 4 \end{cases}$



Signo $f'(x) = -x^2 + 6x - 8$

a) Entre 1987 y 1992 corresponde a $x \in [0, 5]$



El mayor nº de miembros podría estar en $x = 0$ o en $x = 4$

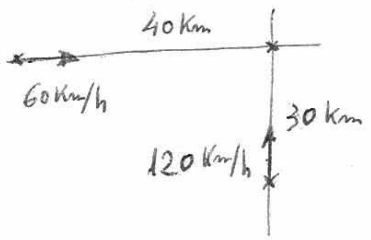
$x = 0 \longrightarrow f(0) = 16$

$x = 4 \longrightarrow f(4) = \frac{32}{3}$

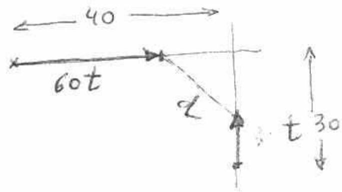
El máximo nº de miembros fue en 1987, su fundación, con 16 miembros

$\lim_{x \to +\infty} f(x) = -\infty$ Luego habrá algún valor de x en el que $y = 0$, luego se quedará sin miembros.

(22)



Al cabo de t horas:



$$d = \sqrt{(40 - 60t)^2 + (30 - 120t)^2} = \sqrt{400(2-3t)^2 + 900(1-4t)^2}$$

$$d' = \frac{800(2-3t)\cdot(-3) + 1800(1-4t)(-4)}{2\sqrt{400(2-3t)^2 + 900(1-4t)^2}} = 100\,\frac{-12(2-3t) - 36(1-4t)}{\sqrt{400(2-3t)^2 + 900(1-4t)^2}}$$

$d' = 0 \Rightarrow -24 + 36t - 36 + 144t = 0$; $180t = 60 \Rightarrow t = \frac{60}{180} = \frac{1}{3}\,h$

Signo $d' = 100\,\frac{-60 + 180t}{\sqrt{400(2-3t)^2 + 900(1-4t)^2}}$

Mínima distancia para $t = \frac{1}{3}\,h$ = 20 minutos.

La distancia más pequeña es:

$$d = \sqrt{\left(40 - 60\,\frac{1}{3}\right)^2 + \left(30 - 120\,\frac{1}{3}\right)^2} = \sqrt{400 + 100} = \boxed{\sqrt{500}\ \text{Km}}$$

(23)

a) $$\lim_{x\to 0} \frac{(1-\cos x)\sin x}{x^2} = \frac{0}{0} = \lim_{x\to 0} \frac{+\sin x \sin x + (1-\cos x)\cos x}{2x} =$$

$$= \lim_{x\to 0} \frac{\sin^2 x + \cos x - \cos^2 x}{2x} = \frac{0}{0} = \lim_{x\to 0} \frac{2\sin x\cos x - \sin x + 2\cos x \sin x}{2} = \boxed{0}$$

b) $$\lim_{x\to 0^+} x\ln x = \lim_{x\to 0^+} \frac{\ln x}{1/x} = \frac{-\infty}{+\infty} = \lim_{x\to 0^+} \frac{1/x}{-1/x^2} = \lim_{x\to 0^+}(-x) = \boxed{0}$$

c) $$\lim_{x\to +\infty} \frac{x^2}{e^x} = \frac{+\infty}{+\infty} = \lim_{x\to +\infty} \frac{2x}{e^x} = \frac{+\infty}{+\infty} = \lim_{x\to +\infty} \frac{2}{e^x} = \frac{2}{+\infty} = \boxed{0}$$

* d) $$\lim_{x\to -\infty} \frac{e^x}{x} = \frac{+0}{-\infty} = \boxed{0}$$

e) $$\lim_{x\to +\infty} \frac{\ln x}{e^x} = \frac{+\infty}{+\infty} = \lim_{x\to +\infty} \frac{1/x}{e^x} = \frac{+0}{+\infty} = \boxed{0}$$

f) $$\lim_{x\to +\infty} \frac{\ln x}{\sin x} = \frac{+\infty}{[-1,1]} = \boxed{\infty}$$

g) $$\lim_{x\to +\infty} \left(\frac{3x-1}{3x+4}\right)^{2x} = 1^{+\infty}$$

$$m = \lim_{x\to +\infty} \left(\frac{3x-1}{3x+4}\right)^{2x} \quad ; \quad \ln m = \lim_{x\to +\infty} 2x \ln \frac{3x-1}{3x+4} = \lim_{x\to +\infty} \frac{\ln \frac{3x-1}{3x+4}}{1/2x} =$$

$$= \frac{\ln 1}{1/\infty} = \frac{0}{0} = \lim_{x\to +\infty} \frac{\ln(3x-1) - \ln(3x+4)}{1/2x} = \lim_{x\to +\infty} \frac{\frac{3}{3x-1} - \frac{3}{3x+4}}{-\frac{1}{2x^2}} =$$

$$= \lim_{x\to +\infty} \frac{\frac{(9x+12)-(9x-3)}{(3x-1)(3x+4)}}{-1/2x^2} = \lim_{x\to +\infty} \frac{30x^2}{-(3x-1)(3x+4)} = -\frac{30}{9} = -\frac{10}{3}$$

$$\ln m = -\frac{10}{3} \qquad \boxed{m = e^{-10/3}}$$

h) $$\lim_{x\to +\infty} \left(\frac{1-6x}{5-2x}\right)^{x} = 3^{+\infty} = +\infty$$

i) $$\lim_{x\to 0^+} x^{\sin x} = 0^0 \qquad m = \lim_{x\to 0^+} x^{\sin x}$$

$$\ln m = \lim_{x\to 0^+} \sin x \ln x = \lim_{x\to 0^+} \frac{\ln x}{1/\sin x} = \frac{\ln 0}{1/0} = \frac{-\infty}{+\infty} =$$

$$= \lim_{x\to 0^+} \frac{1/x}{-\frac{\cos x}{\sin^2 x}} = \lim_{x\to 0^+} \frac{-\sin^2 x}{x\cos x} = \frac{0}{0} = \lim_{x\to 0^+} \frac{-2\sin x\cos x}{\cos x - x\sin x} =$$

$$= \frac{0}{1} = 0 \Longrightarrow \ln m = 0 \Longrightarrow \boxed{m = 1}$$

j] $\lim\limits_{x\to 1^+}\left(\frac{1}{x-1}\right)^{\ln x} = \left(\frac{1}{0^+}\right)^{\ln 1} = (+\infty)^0$

$m = \lim\limits_{x\to 1^+}\left(\frac{1}{x-1}\right)^{\ln x}$

$$\ln m = \lim_{x\to 1^+} \ln\left(\frac{1}{x-1}\right)^{\ln x} = \lim_{x\to 1^+} \ln x \cdot \ln\left(\frac{1}{x-1}\right) = \lim_{x\to 1^+} \frac{\ln\left(\frac{1}{x-1}\right)}{\frac{1}{\ln x}} =$$

$$= \lim_{x\to 1^+} \frac{\ln 1 - \ln(x-1)}{\frac{1}{\ln x}} = \lim_{x\to 1^+} \frac{-\ln(x-1)}{\frac{1}{\ln x}} = \left(\frac{-\ln 0}{\frac{1}{\ln 1}} = \frac{+\infty}{\frac{1}{0}} = \frac{+\infty}{+\infty}\right) =$$

$$= \lim_{x\to 1^+} \frac{-\frac{1}{x-1}}{\frac{-1/x}{\ln^2 x}} = \lim_{x\to 1^+} \frac{x\ln^2 x}{x-1} = \left(\frac{0}{0}\right) = \lim_{x\to 1^+} \frac{\ln^2 x + x\cdot 2\ln x\,\frac{1}{x}}{1} =$$

$$= \lim_{x\to 1^+} \left(\ln^2 x + 2\ln x\right) = 0$$

$\ln m = 0 \implies m = 1 \implies \boxed{\lim\limits_{x\to 1^+}\left(\frac{1}{x-1}\right)^{\ln x} = 1}$