Counting

Counting without repetition

Drum 1

6 initial elements

1 4 3 5 6 2

Drum 2

6 4 3 1

Combination

It is a subset of the 6 initial elements

*The subset can be with less than 6 elements

*Order is not important. (If you change order of selected elements then this is not a new combination)

In this example :

$$C_4^6 = \frac{6!}{4!\,(6-4)!} =$$

$$= \frac{6\times5\times4\times3\times2\times1}{4\times3\times2\times1\times(2\times1)} = \frac{30}{2} = 15 \text{ different}$$

combinations

In general:

$$C_n^m = \frac{m!}{n!\,(m-n)!}$$

Permutation

It is any arrangement of the 6 initial elements (in this example) in a definite order.

*All the initial elements are involved

*Order is important

In this example:

P(6) = 6 x 5 x 4 x 3 x 2 x 1 = 6! = 720 different permutations.

In general:

P(n) = n.(n-1).(n-2). ... .1 = n!

3 1 5 6 2 4

Variation

It is a subset of the 6 initial elements

*The subset can be with less than 6 elements

*Order is important. (If you change order of selected elements then this is a new variation)

3 4 6 1

In this example:

$$V_{6,4} = 6 \times 5 \times 4 \times 3 = 360$$
different variations
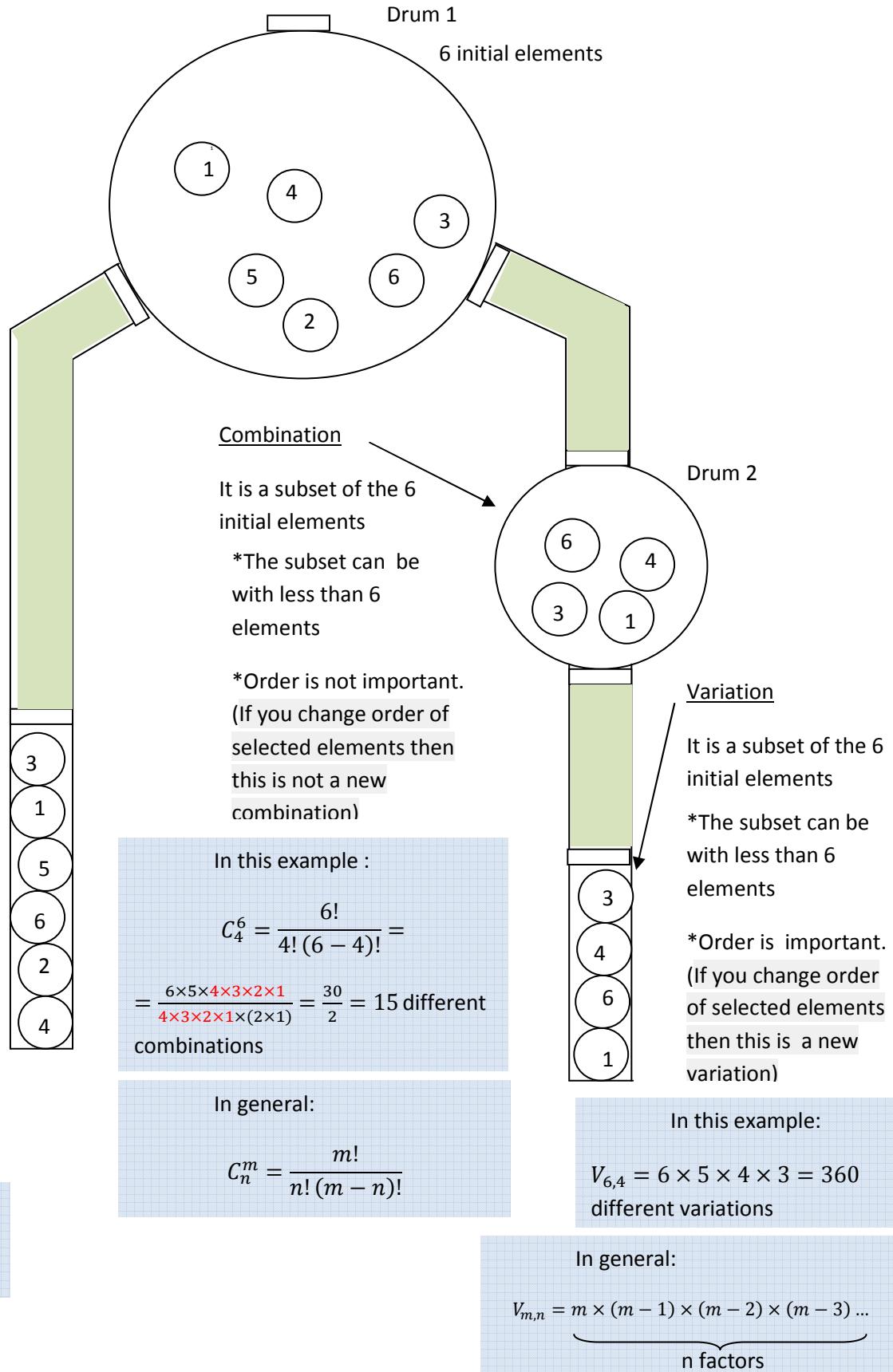
In general:

$$V_{m,n} = m \times (m-1) \times (m-2) \times (m-3) \dots$$

n factors

Exercises:

1) How many different words can we make using the letters A, B, E and L ?

2) How many 3 letter words can we make with the letters in the word LOVE?

3) How many lines can you draw using 3 non collinear (not in a single line) points A, B and C on a plane?

4) We need to form a 5 a side team in a class of 12 students. How many different teams can be formed?

5) In how many ways can you select a committee of 3 students out of 10 students?

6) How many triangles can you make using 6 non collinear points on a plane?

7) A committe including 3 boys and 4 girls is to be formed from a group of 10 boys and 12 girls. How many different committee can be formed from the group?

8) In a certain country, the car number plate is formed by 4 digits from the digits 1, 2, 3, 4, 5, 6, 7, 8 and 9 followed by 3 letters from the alphabet. How many number plates can be formed?

9) How many ways are there to select a subcommittee of 7 members from among a committee of 17?

10) Determine the total number of five-card hands that can be drawn from a deck of 52 cards.

11) There are five women and six men in a group.   From this group a committee of 4 is to be chosen. How many different ways can a committee be formed that contain at least three women?

12) In a local election, there are seven people running for three positions.   The person that has the most votes will be elected to the highest paying position.   The person with the second most votes will be elected to the second highest paying position, and likewise for the third place winner.   How many different outcomes can this election have?

13) An exacta in horse racing is when you correctly guess which horses will finish first and second.   If there are eight horses in the race, how many different possible outcomes for the exacta are there?

14) How many ways can 10 people be placed in alphabetical order according to their first names?

15) How many different ways can the letters in the word "store" be arranged?

16) A certain marathon had 50 people running for first prize, second, and third prize. How many different possible outcomes are there for the first three runners to cross the finish line?

17) how many ways can you arrange 7 different books, so that a specific book is on the third place?

18) In how many ways can you take 3 marbles out of a box with 15 different marbles?

19) In a firm are 20 workmen and 10 employees. In how many ways can you have a committee with 3 workmen and 2 employees?

20) In how many ways can you take 5 cards, with at least 2 aces, out of a game of 52 cards?

21) In how many ways can you split a group of 13 persons in 3 persons and 10 persons?

22) How many numbers consisting of 3 figures, can you make with the figures 0,1,2,3,4 ?

23) In how many ways can a supermarket manager display 5 brands of cereals in 3 spaces on a shelf?

24) How many different number-plates for cars can be made if each number-plate contains four of the digits 0 to 9 followed by a letter A to Z, assuming that

    a. no repetition of digits is allowed?
    b. repetition of digits is allowed?