(15) $$\boxed{1^3+2^3+3^3+\cancel{4^3}+\dots+n^3 = \frac{n^2(n+1)^2}{4} \qquad n\in\mathbb{N}}$$

Para $n=1$ $\qquad 1 = \frac{1^2 \cdot 2^2}{4} = 1$ ok

$n=2$ $\qquad \underset{\overset{\parallel}{9}}{1^3+2^3} = \frac{2^2\cdot 3^2}{4} = 9$ OK

$n=3$ $\qquad 1^3+2^3+3^3 = \frac{3^2\cdot(4)^2}{4} = \frac{3^2\cdot 2^2\cdot 1\cancel{2^2}}{\cancel{4}_1}$

$1+8+27 = 36$? ok.

Sup q. es cierto para $n-1$:

$$1^3+2^3+\dots+(n-1)^3 = \frac{(n-1)^2\cdot n^2}{4}$$

$$1^3+2^3+\dots+(n-1)^3+n^3 = \frac{(n-1)^2\cdot n^2}{4} + n^3 =$$

$$= \frac{(n-1)^2\cdot n^2+4n^3}{4} = \frac{(n^2-2n+1)\cdot n^2+4n^3}{4} =$$

$$= \frac{n^4-2n^3+n^2+4n^3}{4} = \frac{n^4+2n^3+n^2}{4} = \frac{n^2(n^2+2n+1)}{4}$$

$$= \frac{(n+1)^2\cdot n^2}{4}$$

16

$$\boxed{\left(1+\frac{1}{1}\right)\cdot\left(1+\frac{1}{2}\right)\cdot\left(1+\frac{1}{3}\right)\cdot\ldots\cdot\left(1+\frac{1}{n}\right)=n+1}$$

$\underline{n=1}$

$$\left(1+\frac{1}{1}\right)=\cancel{n}1+1$$

$$2=2$$

$n=2$

$$\left(1+\frac{1}{1}\right)\cdot\left(1+\frac{1}{2}\right)=3$$

$$\|$$

$$2\left(1+\frac{1}{2}\right)$$

$$\|$$

$$2+1$$

$n=3$

$$\left(1+\frac{1}{1}\right)\cdot\left(1+\frac{1}{2}\right)\cdot\left(1+\frac{1}{3}\right)=3+1$$

$$2\cdot\left(1+\frac{1}{2}\right)\cdot\left(1+\frac{1}{3}\right)=2\left(1+\cancel{2}\left(\frac{1}{2}+\frac{1}{3}\right)+\frac{1}{6}\right)$$

$$=2\left(1+\frac{3+2}{6}+\frac{1}{6}\right)=2\left(\frac{6}{6}+\frac{5}{6}+\frac{1}{6}\right)=2\cdot2=\underline{\underline{4}}$$

Sup. cierto para $n-1$:

$$\left(1+\frac{1}{1}\right)\cdot\left(1+\frac{1}{2}\right)\cdot\left(1+\frac{1}{3}\right)\cdot\ldots\cdot\left(1+\frac{1}{n-1}\right)=n-1+1=n$$

Veamos para n:

$$\left(1+\frac{1}{1}\right)\cdot\left(1+\frac{1}{2}\right)\cdot\left(1+\frac{1}{3}\right)\cdot\ldots\cdot\left(1+\frac{1}{n-1}\right)\cdot\left(1+\frac{1}{n}\right)=n\cdot\left(1+\frac{1}{n}\right)=$$

$$=n+1$$

OK.

$$\boxed{⑰ \quad 2^{2n} - 3n - 1 \qquad \text{Divisible por } 9.}$$

$n = 1$

$n = 2$ ok.

Sup. cierto para $n-1$

$$2^{2(n-1)} - 3(n-1) - 1 = 9 \cdot k$$

$$\| $$

$$4^{n-1} - 3n + 3 - 1 = 4^{n-1} - 3n + 2 = 9k.$$

Veamos que: $2^{2n} - 3n - 1$ es divisible por 9.

$$2^{2n} - 3n - 1 = 4^{n} - 3n - 1 = 4 \cdot 4^{n-1} - 3n - 1 =$$

$$= 4 \cdot \left[4^{n-1} - 3n + 2 + 3n - 2 \right] - 3n - 1 =$$

$$= 4 \cdot \left[4^{n-1} - 3n + 2 \right] + 4 \cdot 3n - 4 \cdot 2 - 3n - 1 =$$

$$= 4 \cdot 9k + 12n - 8 - 3n - 1 = 4 \cdot 9k + 9n - 9 =$$

$$= 9 \left[4k + n - 1 \right] \qquad \text{Divisible por } 9.$$

92W. P.1

V.2009

Demuestra que: $\displaystyle\sum_{r=1}^{n} r\cdot(r!) = (n+1)! - 1 \qquad n\in\mathbb{N}$

Para $n=1$ $\qquad 1\cdot 1! = 2! - 1$ $\qquad$ ok.

$n=2$ $\qquad 1\cdot 1! + 2\cdot 2! = 3! - 1 = 3\cdot 2 - 1 = 5$

$$\underbrace{1\cdot 1! + 2\cdot 2!}_{=\,1+4\,=\,5}$$

ok

Supongamos cierto para $n-1$:

$$\sum_{r=1}^{n-1} r\cdot(r!) = n! - 1$$

Veamos q. se cumple para n:

$$\sum_{r=1}^{n} r\cdot(r!) = \sum_{r=1}^{n-1} r\cdot(r!) + n\cdot(n!) = n! - 1 \cancel{1} + n\cdot(n!)$$

$$= \cancel{n!(n-1) = n!\cdot n - n!}$$

$$= n! + n\cdot(n!) - 1 = n!(1+n) - 1 = (n+1)! - 1$$

ok