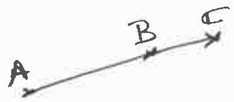
JUN 94

a)



$\vec{AB} = (0,1,0)$

$\vec{AC} = (0,6,a-1)$

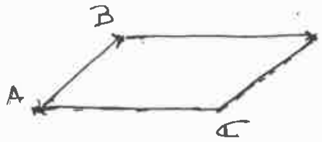
A, B, C alineados $\Rightarrow \vec{AB} \parallel \vec{AC} \Rightarrow \frac{0}{0} = \frac{1}{6} = \frac{0}{a-1} \Rightarrow \boxed{a=1}$

También:

A, B, C alineados $\Rightarrow \vec{AB} \wedge \vec{AC} = (0,0,0) \Rightarrow \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 1 & 0 \\ 0 & 6 & a-1 \end{vmatrix} = (0,0,0) \Rightarrow$

$\Rightarrow (a-1, 0, 0) = (0,0,0) \Rightarrow \boxed{a=1}$

b)

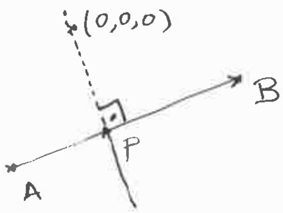


$\text{Area} = 3 \Rightarrow |\vec{AB} \wedge \vec{AC}| = 3 \Rightarrow$

$\Rightarrow |(a-1, 0, 0)| = 3 \Rightarrow \sqrt{(a-1)^2} = 3 \Rightarrow$

$\Rightarrow (a-1)^2 = 9 \Rightarrow a-1 = \pm 3 \Rightarrow \boxed{a=4}$ $\boxed{a=-2}$

c)



$P = (1+0\cdot r, 0+1\cdot r, 1+0r) = (1, r, 1)$

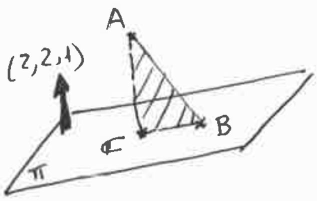
$\vec{OP} = (1, r, 1)$

$\vec{OP} \perp \vec{AB} \Rightarrow (1,r,1)\cdot(0,1,0) = 0 \Rightarrow$

$\Rightarrow r = 0 \Rightarrow \vec{OP} = (1,0,1)$

$$\boxed{\begin{aligned} x &= r \\ y &= 0 \\ z &= r \end{aligned}}$$

SEPT 94



$B \in \pi \Rightarrow 2\cdot 2 + 2\cdot 1 + a - 3 = 0 \Rightarrow$

$\Rightarrow 4 + 2 + a - 3 = 0 \Rightarrow \boxed{a=-3}$

$$\left.\begin{aligned} x &= 1+2r \\ y &= 2r \\ z &= 2+r \\ 2x&+2y+z-3=0 \end{aligned}\right\} \Rightarrow 2(1+2r) + 2\cdot 2r + 2+r-3 = 0$$

$2 + 4r + 4r + 2 + r - 3 = 0$

$9r = -1$

$\boxed{r = -\frac{1}{9}} \Rightarrow C = \left(1-\frac{2}{9}, -\frac{2}{9}, 2-\frac{1}{9}\right)$

$\boxed{C = \left(\frac{7}{9}, -\frac{2}{9}, \frac{17}{9}\right)}$

$A(1,0,2)$

$B(2,1,-3)$

$C(7/9, -2/9, 17/9)$

$\text{Area} = \frac{1}{2}|\vec{AC} \wedge \vec{AB}| = \frac{1}{2}\left|\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2/9 & -2/9 & -1/9 \\ 1 & 1 & -5 \end{vmatrix}\right| = \frac{1}{2}\left|\frac{1}{9}\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & -2 & -1 \\ 1 & 1 & -5 \end{vmatrix}\right| =$

$= \frac{1}{18}|(11, -11, 0)| = \frac{1}{18}\sqrt{121+121} = \boxed{\frac{11\sqrt{2}}{18} \text{ u.s.}}$

$\vec{AC} = (-2/9, -2/9, -1/9)$

$\vec{AB} = (1, 1, -5)$

$\cos\hat{A} = \frac{-2/9 - 2/9 + 5/9}{\sqrt{4/81+4/81+1/81}\sqrt{1+1+25}} = \frac{1/9}{\sqrt{3}} = \frac{1}{9\sqrt{3}} \Rightarrow \boxed{\hat{A} = 89°} \Rightarrow \boxed{\hat{B} = 1°}$

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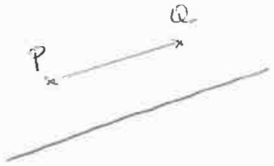


i) $\vec{PQ} = (6, b-2, c)$

$x-1 = 2y = 2z+2 \rightarrow x = 1+2y$, $z = -1+y$

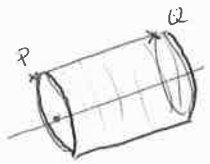
$\left.\begin{array}{l} x = 1+2r \\ y = r \\ z = -1+r \end{array}\right\} \longrightarrow \vec{v} = (2,1,1)$

$\vec{PQ} \parallel \vec{v} \Rightarrow \dfrac{6}{2} = \dfrac{b-2}{1} = \dfrac{c}{1} \Longrightarrow \boxed{\begin{array}{l} b=5 \\ c=3 \end{array}}$

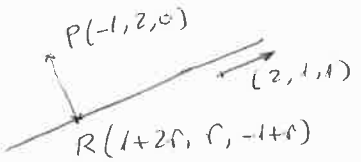
ii) $\vec{PQ} = (6,3,3)$

$d(P,Q) = |\vec{PQ}| = \sqrt{36+9+9} = \boxed{\sqrt{54}}$

iii)



Radio = distancia de P (ó Q) a la recta
Altura = $\sqrt{54}$



$\vec{PR} = (2+2r, r-2, -1+r)$

$\vec{PR} \perp (2,1,1) \Rightarrow 4+4r+r-2-1+r = 0$

$6r+1 = 0$

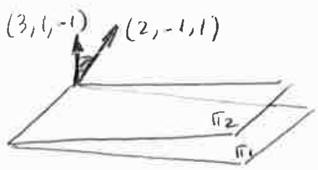
$r = -\frac{1}{6} \Rightarrow R = \left(\frac{2}{3}, -\frac{1}{6}, -\frac{7}{6}\right)$

$\Downarrow$

$\text{Radio} = |\vec{PR}| = \sqrt{\left(\frac{5}{3}\right)^2 + \left(\frac{-13}{6}\right)^2 + \left(\frac{-7}{6}\right)^2} = \sqrt{\frac{318}{36}} = \sqrt{\frac{53}{6}}$

$\text{Volumen} = \pi R^2 H = \pi \frac{53}{6} \cdot \sqrt{54} = \boxed{203,93 \text{ u.v.}}$

SEPT 95



$\cos\alpha = \dfrac{(3,1,-1)\cdot(2,-1,1)}{\sqrt{9+1+1}\ \sqrt{4+1+1}} = \dfrac{4}{\sqrt{66}} \Longrightarrow \boxed{\alpha = 61°}$

Al formar un ángulo de 61°, los planos no son paralelos, luego se cortan sobre una recta.

$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 1 & -1 \\ 2 & -1 & 1 \end{vmatrix} = (0, -5, -5)$

La recta estará contenida en ambos planos, luego será perpendicular común a los dos vectores normales, luego será paralela a su producto vectorial.

JUN 96

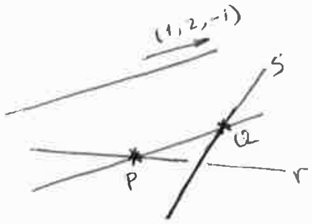
$r \equiv x = y = z \longrightarrow \left.\begin{matrix} x = r \\ y = r \\ z = r \end{matrix}\right\}$

$s \equiv \dfrac{x-1}{1} = \dfrac{y-2}{2} = \dfrac{z}{2} \longrightarrow \left.\begin{matrix} x = 1+s \\ y = 2+2s \\ z = 2s \end{matrix}\right\}$

$\left\{ \begin{matrix} r = 1+s \\ r = 2+2s \\ r = 2s \end{matrix} \right\} \longrightarrow \left. \begin{matrix} r - s = 1 \\ r - 2s = 2 \\ r - 2s = 0 \end{matrix} \right\}$

$M = \begin{pmatrix} 1 & -1 \\ 1 & -2 \\ 1 & -2 \end{pmatrix}$ $\boxed{\text{rango } M = 2}$ → Sistema Incomp. → **Las rectas se cruzan**

$M^+ = \begin{pmatrix} 1 & -1 & 1 \\ 1 & -2 & 2 \\ 1 & -2 & 0 \end{pmatrix}$ $\begin{vmatrix} 1 & -1 & 1 \\ 1 & -2 & 2 \\ 1 & -2 & 0 \end{vmatrix} = 2 \Rightarrow \boxed{\text{rango } M^+ = 3}$



$P = (r, r, r)$
$Q = (1+s, 2+2s, 2s)$ $\Big| \longrightarrow \vec{PQ} = (1+s-r,\ 2+2s-r,\ 2s-r)$

$\vec{PQ} \parallel (1,2,-1) \Rightarrow \dfrac{1+s-r}{1} = \dfrac{2+2s-r}{2} = \dfrac{2s-r}{-1}$

$\left. \begin{matrix} -2-2s+r = 4s-2r \\ 2+2s-2r = 2+2s-r \end{matrix} \right\} \longrightarrow \left. \begin{matrix} -6s+3r = 2 \\ -r = 0 \end{matrix} \right\} \Rightarrow$

$\Rightarrow \boxed{\begin{matrix} r = 0 \\ s = -\frac{1}{3} \end{matrix}} \Rightarrow$ $P = (0,0,0)$
$Q = \left(1 - \frac{1}{3}, 2 - \frac{2}{3}, -\frac{2}{3}\right) = \left(\frac{2}{3}, \frac{4}{3}, -\frac{2}{3}\right)$

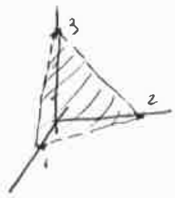
$$\boxed{\left. \begin{matrix} x = 0 + \lambda \\ y = 0 + 2\lambda \\ z = 0 - \lambda \end{matrix} \right\}}$$

SEPT 96

$A \cdot X = B$

orden $A = 3\times3 \Rightarrow$ rango $A = \begin{cases} \cancel{0} \text{ (lo excluye el enunciado)} \\ 1 \\ 2 \\ 3 \end{cases}$

- Si rango $A = 3$ habría una única solución. ABSURDO, ya que conocemos tres
- Si rango $A = 2$, las soluciones formarían una recta, ABSURDO, ya que las tres soluciones que conocemos no están alineadas.
- Si rango $A = 1$, las soluciones formarían un plano:

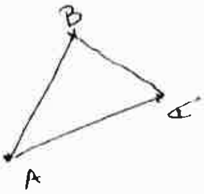


Por lo tanto las soluciones forman un plano que pasa por los puntos $A(1,0,0)$ $B(0,2,0)$ $C(0,0,3)$

$\vec{AB} = (-1,2,0)$ $\vec{AB} \wedge \vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 2 & 0 \\ -1 & 0 & 3 \end{vmatrix} = (6,3,2) \rightarrow$ $6x + 3y + 2z + D = 0$
$6 + 0 + 0 + D = 0$ $\boxed{D = -6}$
$\vec{AC} = (-1,0,3)$

$$\boxed{6x + 3y + 2z - 6 = 0}$$

SEPT 96



$\vec{AB} = (-1,-2,-1)$

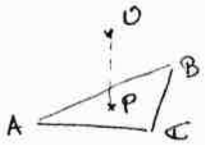
$\vec{AC} = (1,2,-1)$

Al no ser paralelos, los 3 puntos no estarán alineados, luego formarán un triángulo.

$$\text{Area} = \frac{1}{2}\left|\vec{AB}\wedge\vec{AC}\right| = \frac{1}{2}\left|\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & -2 & -1 \\ 1 & 2 & -1 \end{vmatrix}\right| = \frac{1}{2}\left|(4,-2,0)\right| = \frac{1}{2}\sqrt{16+4} = \frac{\sqrt{20}}{2} = \boxed{\sqrt{5}\ us}$$

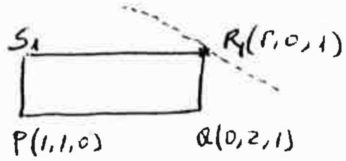
Plano $\overset{\triangle}{ABC}$: $4x-2y+D=0$ $4-2+D=0 \Rightarrow \boxed{D=-2}$

$\boxed{4x-2y-2=0}$



$$\left.\begin{array}{l} 4x-2y-2=0 \\ x=4r \\ y=-2r \\ z=0 \end{array}\right\} \Rightarrow 16r+4r-2=0 \Rightarrow r=\frac{1}{10} \Rightarrow \boxed{P=\left(\frac{2}{5},\frac{-1}{5},0\right)}$$

JUN 97 Hay dos posibles planteamientos:

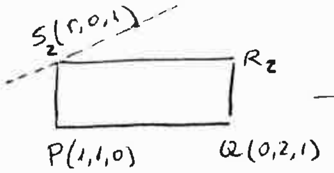


$\longrightarrow$ $\vec{PQ} = (-1,1,1)$, $\vec{QR_1} = (r,-2,0)$ | $\vec{PQ}\perp\vec{QR_1} \Rightarrow -r-2=0$

$\boxed{r=-2}$

$\boxed{R_1(-2,0,1)}$

$S_1 = P+\vec{QR_1} = (1,1,0)+(-2,-2,0) = \boxed{(-1,-1,0)}$



$\longrightarrow$ $\vec{PQ} = (-1,1,1)$, $\vec{PS_2} = (r-1,-1,1)$ | $\vec{PQ}\perp\vec{PS_2} \Rightarrow 1-r-1+1=0$

$\boxed{r=1}$

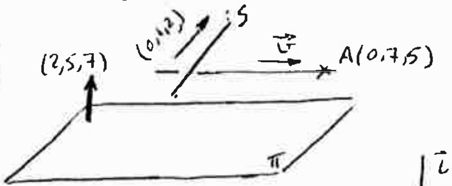
$\boxed{S_2(1,0,1)}$

$R_2 = Q+\vec{PS_2} = (0,2,1)+(0,-1,1) = \boxed{(0,1,2)}$

Habría sólo una solución si la recta r fuese paralela a $\vec{PQ}$.

SEPT 97

i)



$\vec{v}$ debe ser perpendicular a (2,5,7) y a (0,1,2):

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 5 & 7 \\ 0 & 1 & 2 \end{vmatrix} = (3,-4,2)$$

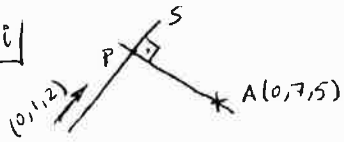
$$\boxed{\begin{array}{l} x=3r \\ y=7-4r \\ z=5+2r \end{array}}$$

ii)

$$\left.\begin{array}{l} 2x+5y+7z+3=0 \\ \left.\begin{array}{l} x=0 \\ y=\lambda+1 \\ z=2\lambda \end{array}\right\} \end{array}\right\} \quad 0+5(\lambda+1)+7\cdot 2\lambda+3=0 \ ;\ 5\lambda+5+14\lambda+3=0 \quad \boxed{\lambda=-\frac{8}{19}}$$

La recta s corta al plano π en el punto $\left(0,\frac{11}{19},\frac{-16}{19}\right)$

iii)



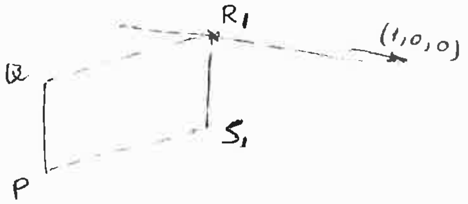
$P=(0,\lambda+1,2\lambda)$

$\vec{AP} = (0,\lambda-6,5-2\lambda)$

$\vec{AP}\perp(0,1,2) \Rightarrow \lambda-6+10-4\lambda=0$ $\boxed{\lambda=\frac{4}{3}} \Rightarrow \vec{AP} = \left(0,-\frac{14}{3},\frac{7}{3}\right)$

$$\boxed{\left.\begin{array}{l} x=0 \\ y=7-2r \\ z=5+r \end{array}\right\}}$$

JUN98



$R_1 = (x, 0, 1)$

$\vec{PQ} \perp \vec{QR_1} \Rightarrow (-1,1,1) \perp (x,-2,0) \Rightarrow$

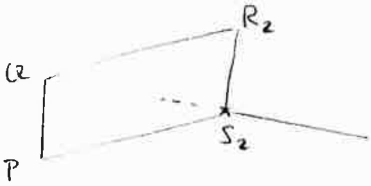
$\Rightarrow -x-2=0$

$x=-2$

$\boxed{R_1(-2,0,1)}$

$S_1 = P + \vec{QR} = (1,1,0) + (-2,-2,0) = \boxed{(-1,-1,0)}$

Hay otra solución si es el vértice S_2 el que pertenece a la recta:



$\vec{PQ} \perp \vec{PS_2} \Rightarrow (-1,1,1)\cdot(x-1,-1,1) = 0$

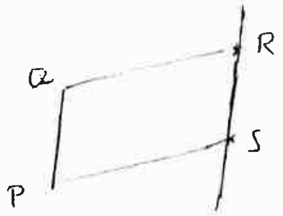
$-x+1-1+1=0$

$x=1$

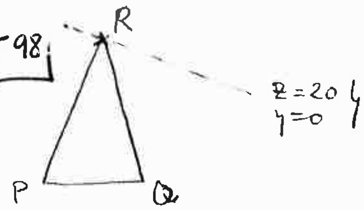
$\boxed{S_2(0,-1,1)}$

$R_2 = Q + \vec{PS_2} = (0,2,1) + (-1,-2,1) = \boxed{(-1,0,2)}$

c) Si la recta fuese paralela a $\vec{PQ}$, la solución sería única:



SEPT98



$R = (x, 0, 20)$

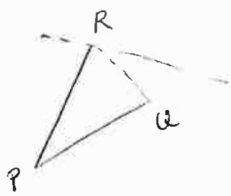
$|\vec{PR}| = |\vec{QR}| \Rightarrow \sqrt{(x-2)^2 + 20^2} = \sqrt{x^2 + (-4)^2 + 18^2}$

$x^2 - 4x + 4 + 400 = x^2 + 16 + 324$

$-4x = -64$

$x = 16 \Rightarrow \boxed{R(16,0,20)}$

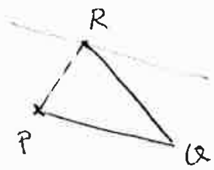
Podría haber más soluciones según qué vértice es el del ángulo desigual:



$|\vec{PR}| = |\vec{PQ}| \Rightarrow \sqrt{(x-2)^2 + 20^2} = \sqrt{(-2)^2 + 4^2 + 2^2}$

$(x-2)^2 + 400 = 24$

$(x-2)^2 = -876$ $\boxed{\text{Sin solución}}$

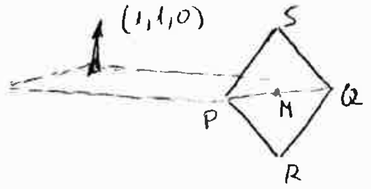


$|\vec{QR}| = |\vec{QP}| \Rightarrow \sqrt{x^2 + (-4)^2 + 18^2} = \sqrt{2^2 + (-4)^2 + (-2)^2}$

$x^2 + 340 = 24$

$x^2 = -316$ $\boxed{\text{Sin solución}}$

JUN99 P y Q verifican la ecuación $x+y=0$, luego el dibujo es:



$\vec{RS}$ es paralelo a $(1,1,0)$ $\Rightarrow$ $\vec{RS} = (t,t,0)$

$|\vec{RS}| = |\vec{PQ}|$ $\Rightarrow$ $\sqrt{t^2+t^2} = \sqrt{(-2)^2+4^2+2^2}$

$2t^2 = 24$ $\Rightarrow$ $t = \pm\sqrt{12}$ $\Rightarrow$ $\vec{RS} = (\pm\sqrt{12}, \pm\sqrt{12}, 0)$

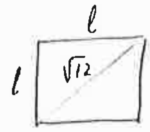
$M = \frac{P+Q}{2} = (2,-2,2)$

$R = M + \frac{1}{2}(\sqrt{12},\sqrt{12},0) = (2+\frac{1}{2}\sqrt{12}, -2+\frac{1}{2}\sqrt{12}, 2) = \boxed{(2+\sqrt{3}, -2+\sqrt{3}, 2)}$

$S = M - \frac{1}{2}(\sqrt{12},\sqrt{12},0) = \boxed{(2-\sqrt{3}, -2-\sqrt{3}, 2)}$

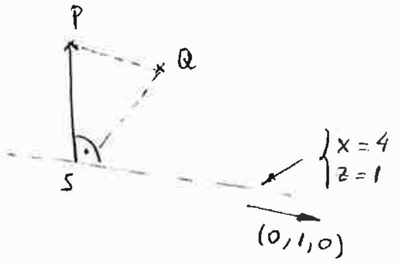
b) $\boxed{\begin{cases} x = 2+r \\ y = -2+r \\ z = 2 \end{cases}}$

c) $|\vec{PQ}| = \sqrt{12}$ $\Rightarrow$



$12 = l^2+l^2$ $\Rightarrow$ $l = \sqrt{6}$ $\Rightarrow$ $\boxed{\text{Perímetro} = 4\sqrt{6}}$

SEPT99



$S = (4, y, 1)$

$\vec{PS} = (4, y-1, 1)$

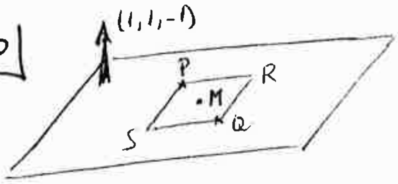
$\vec{PS} \perp (0,1,0)$ $\Rightarrow$ $(4,y-1,1)\cdot(0,1,0) = 0$

$y = 1$

$\boxed{S(4,1,1)}$

b) $\text{Area} = \frac{1}{2}\left|\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & 0 & 1 \\ -1 & 0 & 1 \end{vmatrix}\right| = \frac{1}{2}|(0,-5,0)| = \boxed{\frac{5}{2} \text{ u.s.}}$

JUN00



$\vec{RS}$ es perpendicular a $\vec{PQ} = (2,-2,2)$
$\vec{RS}$ " " " $(1,1,-1)$ $\Rightarrow$

$\Rightarrow$ $\vec{RS}$ es paralelo a $\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -2 & 2 \\ 1 & 1 & -1 \end{vmatrix} = (0,4,4) \Rightarrow \vec{RS} = (0,t,t)$

$|\vec{RS}| = |\vec{PQ}| \Rightarrow \sqrt{t^2+t^2} = \sqrt{4+4+4} \Rightarrow 2t^2 = 12 \Rightarrow t = \pm\sqrt{6} \Rightarrow$

$\Rightarrow \vec{RS} = (0, \pm\sqrt{6}, \pm\sqrt{6})$

$M = \frac{P+Q}{2} = (1,3,3)$

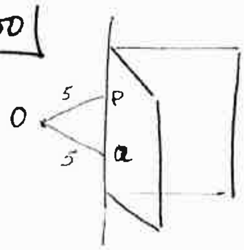
$R = M + \frac{1}{2}\vec{RS} = (1,3,3) + (0, \frac{1}{2}\sqrt{6}, \frac{1}{2}\sqrt{6}) = \boxed{(1, 3+\frac{1}{2}\sqrt{6}, 3+\frac{1}{2}\sqrt{6})}$

$S = M - \frac{1}{2}\vec{RS} = (1,3,3) - (0, \frac{1}{2}\sqrt{6}, \frac{1}{2}\sqrt{6}) = \boxed{(1, 3-\frac{1}{2}\sqrt{6}, 3-\frac{1}{2}\sqrt{6})}$

b)

$$\boxed{\left.\begin{array}{l} x = 2r \\ y = -2r \\ z = 2r \end{array}\right\}}$$

SEPT00



$\left.\begin{array}{l} x - y = -1-z \\ 6x-3y = 6-10z \end{array}\right\}$

$x = \dfrac{\begin{vmatrix} -1-z & -1 \\ 6-10z & -3 \end{vmatrix}}{\begin{vmatrix} 1 & -1 \\ 6 & -3 \end{vmatrix}} = \dfrac{3+3z+6-10z}{3} = \dfrac{9-7z}{3}$

$y = \dfrac{\begin{vmatrix} 1 & -1-z \\ 6 & 6-10z \end{vmatrix}}{3} = \dfrac{6-10z+6+6z}{3} = \dfrac{12-4z}{3}$

$P\left(\frac{9-7z}{3}, \frac{12-4z}{3}, z\right)$

$|\vec{OP}| = 5 \Rightarrow \sqrt{\frac{(9-7z)^2}{9} + \frac{(12-4z)^2}{9} + z^2} = 5 \rightarrow 81 - 126z + 49z^2 + 144 - 96z + 16z^2 + 9z^2 = 225$

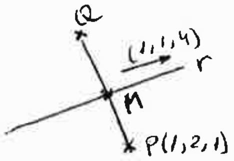
$74z^2 - 222z = 0$

$z = 0 \Rightarrow \boxed{P(3,4,0)}$

$z = 3 \Rightarrow \boxed{Q(-4,0,3)}$

$M = \frac{P+Q}{2} = \boxed{\left(-\frac{1}{2}, 2, \frac{3}{2}\right)}$

JUN 01



a)

$M = (-1+r, 2+r, 3+4r)$ | $\vec{PM} = (-2+r, r, 2+4r)$

$P = (1,2,1)$

$\vec{PM} \perp (1,1,4) \Rightarrow (-2+r, r, 2+4r)\cdot(1,1,4) = 0 \Rightarrow -2+r+r+8+16r = 0 \Rightarrow$

$\Rightarrow r = -\frac{6}{18} = -\frac{1}{3} \Rightarrow M = (-4/3, 5/3, 5/3)$

$\vec{PM} = \left(-\frac{7}{3}, -\frac{1}{3}, \frac{2}{3}\right) \rightarrow \begin{cases} x = 1-7r \\ y = 2-r \\ z = 1+2r \end{cases}$

b) $Q = M + \vec{PM} = \left(-\frac{4}{3}, \frac{5}{3}, \frac{5}{3}\right) + \left(-\frac{7}{3}, -\frac{1}{3}, \frac{2}{3}\right) = \boxed{\left(-\frac{11}{3}, \frac{4}{3}, \frac{7}{3}\right)}$

SEPT 01 $2mx + (m+1)y - 3(m-1)z + m + 4 = 0$

a) $P(1,-1,2) \Rightarrow 2m - (m+1) - 6(m-1) + m + 4 = 0$

$2m - m - 1 - 6m + 6 + m + 4 = 0$

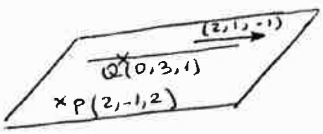
$-4m + 9 = 0 \qquad m = \frac{9}{4}$

$m = \frac{9}{4} \Rightarrow \frac{18}{4}x + \frac{13}{4}y - \frac{15}{4}z + \frac{25}{4} = 0 \rightarrow \boxed{18x + 13y - 15z + 25 = 0}$

b) $\begin{cases} x + 3z - 1 = 0 \\ y - 5z + 2 = 0 \end{cases} \longrightarrow \begin{cases} x = 1 - 3z \\ y = -2 + 5z \end{cases} \rightarrow \begin{cases} x = 1 - 3r \\ y = -2 + 5r \\ z = r \end{cases}$

$(-3, 5, 1) \parallel (2m, m+1, 3-3m) \Rightarrow$

JUN 02



$\vec{PQ} = (-2, 4, -1)$

$\begin{vmatrix} x & y-3 & z-1 \\ -2 & 4 & -1 \\ 2 & 1 & -1 \end{vmatrix} = 0$

$-4x - 2(z-1) - 2(y-3) - 8(z-1) - 2(y-3) + x = 0$

$-3x - 4(y-3) - 10(z-1) = 0$

$-3x - 4y + 12 - 10z + 10 = 0 \qquad \boxed{-3x - 4y - 10z + 22 = 0}$

$d = \frac{|0 - 4 - 0 + 22|}{\sqrt{9 + 16 + 100}} = \boxed{\frac{18}{\sqrt{125}}}$

SEPT 02

$$\pi \equiv \begin{cases} x = 2+t \\ y = s \\ z = 1-2s+2t \end{cases}$$

$$\begin{vmatrix} x-2 & y & z-1 \\ 0 & 1 & -2 \\ 1 & 0 & 2 \end{vmatrix} = 0$$

$$2(x-2) - 2y - (z-1) = 0$$

$$\boxed{2x - 2y - z - 3 = 0}$$

a)

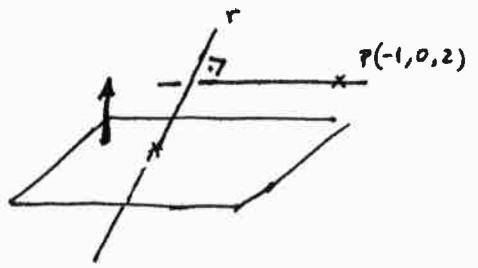
$$\left. \begin{array}{l} 2x-2y-z-3=0 \\ \left. \begin{array}{l} x = 2\lambda \\ y = -1+3\lambda \\ z = \lambda \end{array} \right\} \end{array} \right\}$$

$$4\lambda + 2 - 6\lambda - \lambda - 3 = 0$$

$$-3\lambda = 1$$

$$\boxed{\lambda = -\frac{1}{3}} \Rightarrow$$ La recta corta al plano en el punto $\boxed{\begin{array}{l} x = -2/3 \\ y = -2 \\ z = -1/3 \end{array}}$

b)



$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -2 & -1 \\ 2 & 3 & 1 \end{vmatrix} = (1, -4, 10)$$

$$\boxed{\begin{cases} x = -1 + \lambda \\ y = -4\lambda \\ z = 2 + 10\lambda \end{cases}}$$

JUN 03

$$\left. \begin{array}{l} 2x+3y+z=2 \\ x+y-z=1 \end{array} \right\}$$

a) Al no tener vectores normales proporcionales, se cortarán sobre una recta:

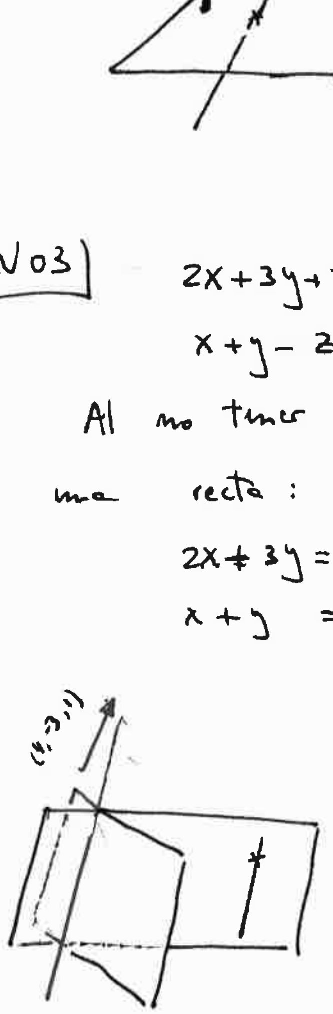
$$\left. \begin{array}{l} 2x + 3y = 2-z \\ x + y = 1+z \end{array} \right\}$$

$$x = \frac{\begin{vmatrix} 2-z & 3 \\ 1+z & 1 \end{vmatrix}}{\begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix}} = \frac{2-z-3-3z}{2-3} = 1+4z$$

$$y = \frac{\begin{vmatrix} 2 & 2-z \\ 1 & 1+z \end{vmatrix}}{2-3} = \frac{2+2z-2+z}{-1} = -3z$$

$$\boxed{\begin{cases} x = 1+4\lambda \\ y = -3\lambda \\ z = \lambda \end{cases}}$$

b)



$$\boxed{\begin{cases} x = 5+4\lambda \\ y = -3-3\lambda \\ z = 1+\lambda \end{cases}}$$

SEPT 03

$$\left. \begin{array}{l} \left. \begin{array}{l} x = 2+3r \\ y = \lambda + r \\ z = -2r \end{array} \right\} \\ \left. \begin{array}{l} x = -2-s \\ y = 1+2s \\ z = 3+3s \end{array} \right\} \end{array} \right\} \rightarrow \left. \begin{array}{l} 2+3r = -2-s \\ \lambda + r = 1+2s \\ -2r = 3+3s \end{array} \right\} \quad \left. \begin{array}{l} 3r+s = -4 \\ r-2s = 1-\lambda \\ -2r-3s = 3 \end{array} \right\}$$

Las rectas no tienen vectores proporcionales luego son secantes o se cruzan, pero ni paralelas, ni coincidentes.

$$M^+ = \begin{pmatrix} 3 & 1 & | & -4 \\ 1 & -2 & | & 1-\lambda \\ -2 & -3 & | & 3 \end{pmatrix}$$

$\begin{vmatrix} 3 & 1 \\ 1 & -2 \end{vmatrix} = -7 \neq 0 \Rightarrow$ rango $M = 2$

$\begin{vmatrix} 3 & 1 & -4 \\ 1 & -2 & 1-\lambda \\ -2 & -3 & 3 \end{vmatrix} = -18 + 12 - 2 + 2\lambda + 16 - 3 + 9 - 9\lambda = 14 - 7\lambda$

- Si $\lambda \neq 2 \Rightarrow$ rango $M^+ = 3 \Rightarrow$ Sistema Incompatible $\Rightarrow$ **Las rectas se cruzan**
- Si $\lambda = 2 \Rightarrow$ rango $M = 2 \Rightarrow$ S. Compat. Determ. $\Rightarrow$ **Las rectas son secantes, se cortan en 1 punto**

b) $\lambda = 2$: $\left.\begin{matrix} 3r + s = -4 \\ r - 2s = -1 \end{matrix}\right\}$

$r = \dfrac{\begin{vmatrix} -4 & 1 \\ -1 & -2 \end{vmatrix}}{\begin{vmatrix} 3 & 1 \\ 1 & -2 \end{vmatrix}} = \dfrac{9}{-7} \Rightarrow \begin{cases} x = 2 - \frac{27}{7} = -\frac{13}{7} \\ y = 2 - \frac{9}{7} = \frac{5}{7} \\ z = \frac{18}{7} \end{cases}$

$s = \dfrac{\begin{vmatrix} 3 & -4 \\ 1 & -1 \end{vmatrix}}{\begin{vmatrix} 3 & 1 \\ 1 & -2 \end{vmatrix}} = \dfrac{1}{-7} \Rightarrow \begin{cases} x = -2 + \frac{1}{7} = -\frac{13}{7} \\ y = 1 - \frac{2}{7} = \frac{5}{7} \\ z = 3 - \frac{3}{7} = \frac{18}{7} \end{cases}$

$\Rightarrow P\left(\frac{-13}{7}, \frac{5}{7}, \frac{18}{7}\right)$

JUN 04 a) $A(1,-1,0)$, $B(1,0,-1)$, $C(0,1,-1)$

$\vec{AB} = (0,1,-1)$

$\vec{AC} = (-1,2,-1)$

$\begin{vmatrix} x-1 & y+1 & z \\ 0 & 1 & -1 \\ -1 & 2 & -1 \end{vmatrix} = 0$

$x - 1 + y + 1 + z = 0$

$\pi \equiv \boxed{x + y + z = 0}$

b) $1 = d\left(A'(1,-1,a),\ x+y+z=0\right)$

$1 = \dfrac{|1-1+a|}{\sqrt{1+1+1}} \rightarrow \sqrt{3} = |a| \Rightarrow \boxed{a = \pm\sqrt{3}}$

c) $\pi' \equiv x + y + z + D = 0$, ya que es paralelo a π

$A'(1,-1,1) \in \pi' \Rightarrow 1 - 1 + 1 + D = 0 \Rightarrow D = -1 \Rightarrow \pi' \equiv \boxed{x + y + z - 1 = 0}$

Volumen = Area Base · Altura.

Area $\widehat{ABC} = \dfrac{|\vec{AB} \times \vec{AC}|}{2} = \dfrac{|(1,1,1)|}{2} = \dfrac{\sqrt{3}}{2}$

Altura $= d\left(A'(1,-1,1),\ x+y+z=0\right) = \dfrac{|1-1+1|}{\sqrt{3}} = \dfrac{1}{\sqrt{3}}$

Volumen $= \dfrac{\sqrt{3}}{2} \cdot \dfrac{1}{\sqrt{3}} = \boxed{\dfrac{1}{2}\ u.v}$

SEPT 04

$\pi: ax+2y-4z=b$

$r: \frac{x-3}{4} = \frac{y-1}{-4} = \frac{z+3}{1}$

a) $a=1$:

$$\left.\begin{array}{l} x+2y-4z=b \\ x=3+4\lambda \\ y=1-4\lambda \\ z=-3+\lambda \end{array}\right\} \quad 3+4\lambda+2-8\lambda+12-4\lambda=b$$

$-8\lambda = b-17$

$\boxed{\lambda = \frac{b-17}{-8}}$ Para cualquier valor de b, la recta corta al plano en un punto.

b) $a=1$: $(3,1,-3)$ ya pertenece a la recta.

$(3,1,-3) \in \pi \Rightarrow 3+2+12=b \quad \boxed{b=17}$

c)

$$\left.\begin{array}{l} ax+2y-4z=b \\ x=3+4\lambda \\ y=1-4\lambda \\ z=-3+\lambda \end{array}\right\} \longrightarrow 3a+4a\lambda+2-8\lambda+12-4\lambda=b$$

$(4a-12)\lambda = b-14-3a$

La recta estará contenida en el plano si la ecuación tiene infinitas soluciones : $0\cdot\lambda=0$

$$\left.\begin{array}{l} 4a-12=0 \\ b-14-3a=0 \end{array}\right\} \quad \boxed{a=3} \quad b=14+9=23 \quad \boxed{b=23}$$

SEPT 04 **a)** $A(-1,1,0)$, $B(0,1,1)$ | $\vec{AB}=(1,0,1)$

$$\boxed{\left.\begin{array}{l} x=-1+\lambda \\ y=1 \\ z=\lambda \end{array}\right\}}$$

b) $x+z+D=0$

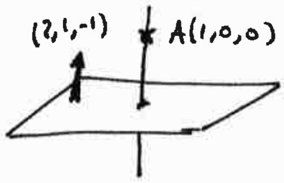
$-1+0+D=0$

$D=1$

$\boxed{x+z+1=0}$

c) $d=\frac{|0+1+1|}{\sqrt{1+0+1}} = \frac{2}{\sqrt{2}} = \boxed{\sqrt{2}}$

JUN 05 **a)**



$$\boxed{\left.\begin{array}{l} x=1+2\lambda \\ y=\lambda \\ z=-\lambda \end{array}\right\}}$$

b) π' será paralelo a π : $2x+y-z+D=0$

$2+0-0+D=0$

$D=-2$

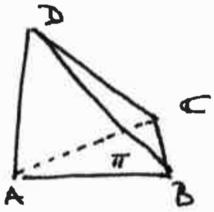
$\boxed{2x+y-z-2=0}$

c) $d = d(A(1,0,0),\ 2x+y-z-1=0) = \dfrac{|2+0-0-1|}{\sqrt{4+1+1}} = \boxed{\dfrac{1}{\sqrt{6}}}$

SEPT 05

a) $\vec{AB} = (-3,2,0)$ $\quad \vec{AB}\times\vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -3 & 2 & 0 \\ -3 & 0 & 6 \end{vmatrix} = (12,18,6)$

$\vec{AC} = (-3,0,6)$



$\text{Area } \overset{\triangle}{ABC} = \dfrac{|\vec{AB}\times\vec{AC}|}{2} = \dfrac{\sqrt{12^2+18^2+6^2}}{2} =$

$= \dfrac{6\sqrt{14}}{2} = \boxed{3\sqrt{14}}$

b) $\begin{vmatrix} x-3 & y & z \\ -3 & 2 & 0 \\ -3 & 0 & 6 \end{vmatrix} = 0$

$12(x-3) + 18y + 6z = 0$

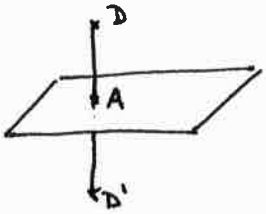
$12x + 18y + 6z - 36 = 0$

$\pi \equiv \boxed{2x + 3y + z - 6 = 0}$

c) $\vec{AD} = (a-3, 3, 1)$

$\vec{AD} \perp \pi \Rightarrow \vec{AD} \parallel (2,3,1) \Rightarrow \dfrac{a-3}{2} = \dfrac{3}{3} = \dfrac{1}{1} \Rightarrow \boxed{a=5}$

d)



$D' = D + 2\vec{DA} = (5,3,1) + 2(-2,-3,-1) = \boxed{(1,-3,-1)}$

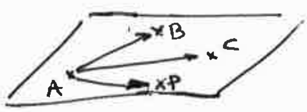
JUN 06

a) $a=2 \rightarrow$ $A(1,1,1)$, $B(2,2,b)$, $C(1,0,0)$, $P(2,0,1)$ $\quad \vec{AC} = (0,-1,-1)$, $\vec{AP} = (1,-1,0)$ $\quad \begin{vmatrix} x-1 & y & z \\ 0 & -1 & -1 \\ 1 & -1 & 0 \end{vmatrix} = 0$

$-(x-1) - y + z = 0$ $\quad$ $-x - y + z + 1 = 0$ Es el plano que contiene a A, C y P.

$B \in \text{Plano} \rightarrow$ $\quad -2-2+b+1 = 0$ $\quad \boxed{b=3}$

Otro procedimiento: $\quad \vec{AB} = (1,1,b-1)$, $\vec{AC} = (0,-1,-1)$, $\vec{AP} = (1,-1,0)$

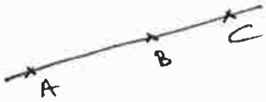


$\vec{AB}$, $\vec{AC}$ y $\vec{AP}$ deben ser coplanarios, luego su producto mixto es nulo:

$\begin{vmatrix} 1 & 1 & b-1 \\ 0 & -1 & -1 \\ 1 & -1 & 0 \end{vmatrix} = 0 \rightarrow -1 + b - 1 - 1 = 0 \Rightarrow b = 3$

b) $\vec{AB} = (a-1, 1, b-1)$

$\vec{AC} = (0, -1, -1)$



$\vec{AB}$ y $\vec{AC}$ deben ser paralelos $\Rightarrow$ $\frac{a-1}{0} = \frac{1}{-1} = \frac{b-1}{-1}$; $\frac{a-1}{0} = -1 = 1-b$

$\boxed{a=1}$ $\boxed{b=2}$

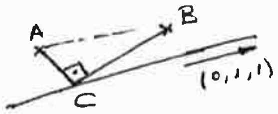
<u>otro procedimiento</u>:

El producto vectorial $\vec{AB} \times \vec{AC}$ debe ser $\vec{0}$:

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a-1 & 1 & b-1 \\ 0 & -1 & -1 \end{vmatrix} = (b-2, 1-a, 1-a) = (0,0,0) \Rightarrow \left.\begin{array}{l} b-2=0 \\ 1-a=0 \\ 1-a=0 \end{array}\right\} \Rightarrow \boxed{\begin{array}{l} b=2 \\ a=1 \end{array}}$$

SEPT 06

$A(1,1,0)$
$B(0,0,2)$
$C(1, 1+\lambda, 1+\lambda)$

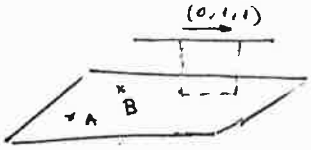


a) $\vec{CA} \perp \vec{CB} \Rightarrow \vec{CA} \cdot \vec{CB} = 0$

$(0, -\lambda, -1-\lambda) \cdot (-1, -1-\lambda, 1-\lambda) = 0$; $\lambda + \lambda^2 - 1 + \lambda - \lambda + \lambda^2 = 0$

$2\lambda^2 + \lambda - 1 = 0$; $\lambda = \frac{-1 \pm \sqrt{1+8}}{4} = \frac{-1 \pm 3}{4} <$ $1/2 \rightarrow \boxed{C(1, \frac{3}{2}, \frac{3}{2})}$; $-1 \rightarrow \boxed{C(1,0,0)}$

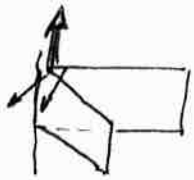
b) $\vec{AB} = (-1, -1, 2)$



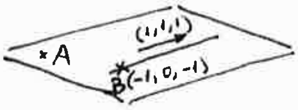
$$\begin{vmatrix} x-1 & y-1 & z \\ -1 & -1 & 2 \\ 0 & 1 & 1 \end{vmatrix} = 0 \ ; \ -3(x-1) + (y-1) - z = 0 \ ; \ \boxed{-3x + y - z + 2 = 0}$$

JUN 07 **a)** $\left.\begin{array}{l} x-y=-1 \\ y-z=1 \end{array}\right\}$ $\left.\begin{array}{l} x = y-1 \\ y \in \mathbb{R} \\ z = y-1 \end{array}\right\} \rightarrow \left.\begin{array}{l} x = -1+r \\ y = r \\ z = -1+r \end{array}\right\} \Rightarrow \boxed{\vec{u} = (1,1,1)}$

<u>También</u>: $\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{vmatrix} = \boxed{(1,1,1)}$



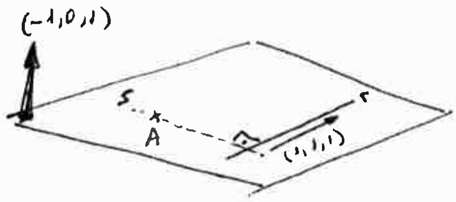
b) $\begin{vmatrix} x-1 & y-1 & z-1 \\ 1 & 1 & 1 \\ -2 & -1 & -2 \end{vmatrix} = 0$



$\left.\begin{array}{l} A(1,1,1) \\ B(-1,0,-1) \end{array}\right| \rightarrow \vec{AB} = (-2,-1,-2)$

$-(x-1) + (z-1) = 0$ $\boxed{-x + z = 0}$

c) $\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ -1 & 0 & 1 \end{vmatrix} = (1,-2,1)$



El vector director de s tiene que ser perpendicular a (1,1,1) [la recta r] y al vector normal al plano (-1,0,1)

$$\boxed{\left.\begin{array}{l} x = 1+r \\ y = 1-2r \\ z = 1+r \end{array}\right\} \equiv s}$$

SEPT 07

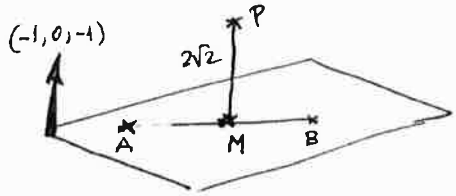
a) A(2,2,0), B(0,0,2), C(0,1,2)

$\vec{AB} = (-2,-2,2)$

$\vec{AC} = (-2,-1,2)$

$$\begin{vmatrix} x & y & z-2 \\ -2 & -2 & 2 \\ -2 & -1 & 2 \end{vmatrix} = 0 \quad ; \quad -2x - 2(z-2) = 0 \qquad \boxed{-x - z + 2 = 0}$$

b) Si la distancia del punto pedido P al plano π es $2\sqrt{2}$, sabemos que $2\sqrt{2}$ es la mínima distancia posible entre dicho punto P y cualquier punto del plano, por lo tanto, como M también dista $2\sqrt{2}$ del punto P, tiene que ser la proyección de P sobre el plano π.



$M = \frac{A+B}{2} = (1,1,1)$

$\vec{MP}$ es paralelo a $(-1,0,-1)$ y con módulo $2\sqrt{2}$:

$\vec{MP} = \lambda(-1,0,-1) = (-\lambda, 0, -\lambda)$

$|\vec{MP}| = 2\sqrt{2}$

$\sqrt{\lambda^2 + 0 + \lambda^2} = 2\sqrt{2}$; $2\lambda^2 = 8$; $\lambda^2 = 4$; $\lambda = \pm 2$

- $\lambda = 2 \rightarrow \vec{MP} = (-2,0,-2) \Rightarrow P_1 = M + \vec{MP} = (1,1,1) + (-2,0,-2) = \boxed{(-1,1,-1)}$
- $\lambda = -2 \rightarrow \vec{MP} = (2,0,2) \Rightarrow P_2 = M + \vec{MP} = (1,1,1) + (2,0,2) = \boxed{(3,1,3)}$

c) A(2,2,0), B(0,0,2), C(0,1,2), D(2,1,1)

$\vec{AB} = (-2,-2,2)$

$\vec{AC} = (-2,-1,2)$

$\vec{AD} = (0,-1,1)$

$$V.\text{tetraedro} = \left| \frac{1}{6} \cdot \begin{vmatrix} -2 & -2 & 2 \\ -2 & -1 & 2 \\ 0 & -1 & 1 \end{vmatrix} \right| = \left| \frac{1}{6}(2+4-4-4) \right| = \left| -\frac{1}{3} \right| = \boxed{1/3}$$

Otro procedimiento para (b)

Plano que pasa por P: $-x - z + D = 0$

$2\sqrt{2} = \frac{|-1-1+D|}{\sqrt{1+0+1}}$; $|D-2| = 4$; $D - 2 = \pm 4$; $D = 6$ o $D = -2$

$\boxed{-x - z + 6 = 0}$

$\boxed{-x - z - 2 = 0}$

Recta perpendicular al plano desde M:

$$\begin{cases} x = 1 - r \\ y = 1 \\ z = 1 - r \end{cases}$$

$$\begin{cases} -x - z + 6 = 0 \\ x = 1 - r \\ y = 1 \\ z = 1 - r \end{cases} \rightarrow -1 + r - 1 + r + 6 = 0 \; ; \; 2r = -4 \; ; \; r = -2 \rightarrow \boxed{P_2 (3,1,3)}$$

$$\begin{cases} -x - z - 2 = 0 \\ x = 1 - r \\ y = 1 \\ z = 1 - r \end{cases} \rightarrow -1 + r - 1 + r - 2 = 0 \; ; \; 2r = 4 \; ; \; r = 2 \rightarrow \boxed{P = (-1,1,-1)}$$

JUN 08

a) $\left.\begin{array}{l}3x+y=1\\ x-kz=2\end{array}\right\}$ $\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 3 & 1 & 0\\ 1 & 0 & -k\end{vmatrix} = -k\vec{i}-\vec{k}+3k\vec{j} = (-k,3k,-1)$ Vector Director 'r'

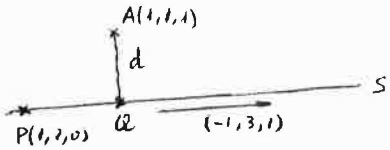
$r \parallel s \Rightarrow \frac{-k}{-1} = \frac{3k}{3} = \frac{-1}{1} \Rightarrow \boxed{k=-1}$

b) $r \perp s \Rightarrow (-k,3k,-1)\cdot(-1,3,1)=0$

$k+9k-1=0$

$10k=1 \quad \boxed{k=1/10}$

c) $Q=(1-t,\,2+3t,\,t)$



$\vec{QA} \perp (-1,3,1) \Rightarrow$

$\Rightarrow (+t,\,-1-3t,\,1-t)\cdot(-1,3,1)=0$

$-t-3-9t+1-t=0$

$-11t=2 \;;\; t=-\frac{2}{11} \longrightarrow \vec{QA}=\left(-\frac{2}{11},\,-1+\frac{6}{11},\,1+\frac{2}{11}\right)=\left(-\frac{2}{11},-\frac{5}{11},\frac{13}{11}\right)$

$d(A,s)=|\vec{QA}|=\sqrt{\frac{4}{121}+\frac{25}{121}+\frac{169}{121}}=\frac{\sqrt{198}}{11}=\boxed{\frac{3\sqrt{22}}{11}}$

SEPT 08

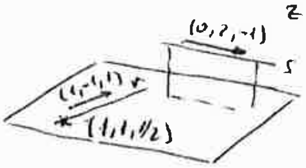
a) $r:\; x-1=1-y=z-\frac{1}{2} \Rightarrow \begin{cases}x=1+r\\ y=1-r\\ z=\frac{1}{2}+r\end{cases}$

$s:\; \vec{AB}=(0,2,-1) \Rightarrow \begin{cases}x=1\\ y=2s\\ z=1-s\end{cases}$

$\left.\begin{array}{l}x=1+r\\ y=1-r\\ z=\frac{1}{2}+r\\ x=1\\ y=2s\\ z=1-s\end{array}\right\} \longrightarrow \left.\begin{array}{l}1+r=1\\ 1-r=2s\\ \frac{1}{2}+r=1-s\end{array}\right\} \longrightarrow \begin{array}{l}r=0\\ -r-2s=-1\\ r+s=\frac{1}{2}\end{array}$ $\quad -2s=-1 \rightarrow s=\frac{1}{2}$ ✓ $\quad s=\frac{1}{2}$

Las rectas se cortan en 1 punto: $r=0 \rightarrow \begin{cases}x=1\\ y=1\\ z=1/2\end{cases}$ $\quad s=\frac{1}{2} \rightarrow \begin{cases}x=1\\ y=\frac{2}{2}=1\\ z=1-\frac{1}{2}=1/2\end{cases}$ $\quad \boxed{Q(1,1,1/2)}$

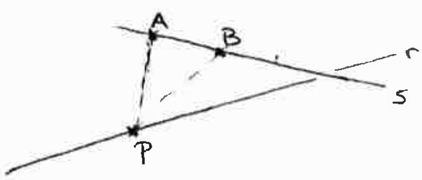
b) $\begin{vmatrix}x-1 & y-1 & z-\frac{1}{2}\\ 1 & -1 & 1\\ 0 & 2 & -1\end{vmatrix}=0$



$x-1+2z-1+y-1-2x+2=0$

$\boxed{-x+y+2z-1=0}$

c) $P(1+r,\,1-r,\,\frac{1}{2}+r)$



$\vec{PA}=(-r,\,-1+r,\,\frac{1}{2}-r)$

$\vec{PB}=(-r,\,1+r,\,-\frac{1}{2}-r)$

$$|\vec{PA}| = \sqrt{(-r)^2+(-1+r)^2+\left(\tfrac{1}{2}-r\right)^2} = \sqrt{r^2+1-2r+r^2+\tfrac{1}{4}-r+r^2} = \sqrt{3r^2-3r+\tfrac{5}{4}}$$

$$|\vec{PB}| = \sqrt{(-r)^2+(1+r)^2+\left(-\tfrac{1}{2}-r\right)^2} = \sqrt{r^2+1+2r+r^2+\tfrac{1}{4}+r+r^2} = \sqrt{3r^2+3r+\tfrac{5}{4}}$$

$$|\vec{PA}| = |\vec{PB}| \Rightarrow \sqrt{3r^2-3r+\tfrac{5}{4}} = \sqrt{3r^2+3r+\tfrac{5}{4}} \; ; \; 3r^2-3r+\tfrac{5}{4} = 3r^2+3r+\tfrac{5}{4}$$

$$-6r = 0$$

$$r = 0 \Rightarrow \boxed{P\left(1, 1, \tfrac{1}{2}\right)}$$

JUN 09

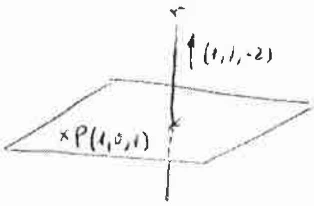
a)

$x + y - 2z + D = 0$

$1 + 0 - 2 + D = 0$

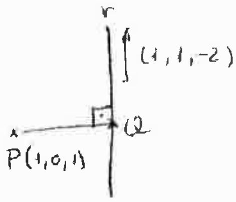
$D = 1$

$\boxed{x + y - 2z + 1 = 0}$



b)

$Q(6+r, 7+r, 4-2r)$



$\vec{PQ} = (5+r, 7+r, 3-2r)$

$\vec{PQ} \perp (1,1,-2) \Rightarrow (5+r, 7+r, 3-2r)\cdot(1,1,-2) = 0$

$5+r+7+r-6+4r = 0$

$6r = -6$

$r = -1 \longrightarrow \boxed{Q(5,6,6)}$

$\longrightarrow \vec{PQ} = (4,6,5)$

$d(P,r) = |\vec{PQ}| = \sqrt{16+36+25} = \boxed{\sqrt{77}}$

SEPT 09

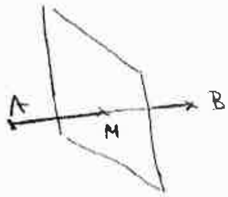
a)

$C = A + \frac{1}{3}\vec{AB} = (2,-1,1) + \frac{1}{3}(-4,4,0) = \boxed{\left(\frac{2}{3}, \frac{1}{3}, 1\right)}$

$D = A + \frac{2}{3}\vec{AB} = (2,-1,1) + \frac{2}{3}(-4,4,0) = \boxed{\left(-\frac{2}{3}, \frac{5}{3}, 1\right)}$

b)

$M = \frac{A+B}{2} = (0,1,1)$



$\vec{AB} = (-4,4,0)$

$-4x + 4y + D = 0$

$-4\cdot 0 + 4\cdot 1 + D = 0 \rightarrow D = -4 \; ; \; -4x + 4y - 4 = 0 \; ; \; \boxed{x - y + 1 = 0}$

JUN 10 fase Especif.

a) $P(1,2,3)$, $Q(1,-1,3)$ | $\vec{PQ}=(0,-3,0)$

$$\left.\begin{aligned} x&=1+0\cdot r\\ y&=2-3r\\ z&=3+0\cdot r\end{aligned}\right\} \to r\equiv \boxed{\begin{aligned} x&=1\\ y&=2-r\\ z&=3\end{aligned}}$$

$A(1,0,1)$, $B(2,-1,3)$, $C(4,1,0)$ | $\vec{AB}=(1,-1,2)$, $\vec{AC}=(3,1,-1)$

$$\begin{vmatrix} x-1 & y & z-1\\ 1 & -1 & 2\\ 3 & 1 & -1\end{vmatrix}=0$$

$$-(x-1)+7y+4(z-1)=0$$

$$\pi\equiv \boxed{-x+7y+4z-3=0}$$

b)
$$\left.\begin{aligned} x&=1\\ y&=2-r\\ z&=3\\ -x+7y&+4z-3=0\end{aligned}\right\} \quad -1+14-7r+12-3=0$$

$$-7r=-22 \to r=\frac{22}{7} \Rightarrow y=2-\frac{22}{7}=-\frac{8}{7}$$

La recta corta al plano en el punto $\boxed{(1,-\frac{8}{7},3)}$

JUN 10 fase especif.

a) $\frac{x-1}{2}=\frac{y-2}{3}=z \longrightarrow \left\{\begin{aligned} x&=1+2r\\ y&=2+3r\\ z&=r\end{aligned}\right.$

$\frac{x+1}{3}=\frac{y-1}{2}=z \longrightarrow \left\{\begin{aligned} x&=-1+3t\\ y&=1+2t\\ z&=t\end{aligned}\right.$

$$\left.\begin{aligned} 1+2r&=-1+3t\\ 2+3r&=1+2t\\ r&=t\end{aligned}\right\} \to \left.\begin{aligned} 2r-3t&=-2\\ 3r-2t&=-1\\ r-t&=0\end{aligned}\right\}$$

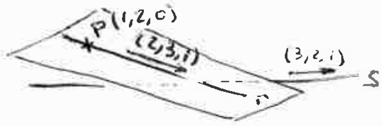
$$A=\begin{pmatrix} 2 & -3\\ 3 & -2\\ 1 & -1\end{pmatrix} \quad \begin{vmatrix} 2 & -3\\ 3 & -2\end{vmatrix}=5\neq 0 \Rightarrow r(A)=2$$

$$A^*=\left(\begin{array}{cc|c} 2 & -3 & -2\\ 3 & -2 & -1\\ 1 & -1 & 0\end{array}\right) \quad \begin{vmatrix} 2 & -3 & -2\\ 3 & -2 & -1\\ 1 & -1 & 0\end{vmatrix}=6+3-4-2=3\neq 0 \Rightarrow r(A^*)=3$$

$\to$ S. Incompatible

Como $r(A)=2$, las rectas no son paralelas, por lo tanto: $\boxed{\text{se cruzan}}$

b)
$$\begin{vmatrix} x-1 & y-2 & z\\ 2 & 3 & 1\\ 3 & 2 & 1\end{vmatrix}=0$$

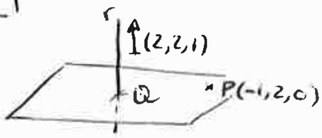


$$(x-1)+(y-2)-5z=0$$

$$\boxed{x+y-5z-3=0}$$

JUN 10 fase General

a)



$\pi\equiv 2x+2y+z+D=0$

$P(-1,2,0)\in\pi \Rightarrow -2+4+0+D=0$; $D=-2$ $\quad \boxed{\pi\equiv 2x+2y+z-2=0}$

b)
$$\left.\begin{aligned} x&=1+2r\\ y&=2r\\ z&=r\\ 2x+2y&+z-2=0\end{aligned}\right\} \quad 2(1+2r)+2\cdot 2r+r-2=0 \ ; \ \cancel{2}+4r+4r+r-\cancel{2}=0 \ ; \ r=0$$

$$\Downarrow$$

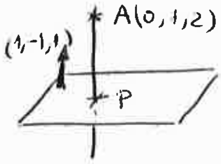
$$\boxed{Q(1,0,0)}$$

c) $d(P;r)=|\vec{PQ}|=|(2,-2,0)|=\boxed{\sqrt{8}}$

JUN 10 fase General

a)

$r \equiv \frac{x}{1} = \frac{y-1}{-1} = \frac{z-2}{1}$



b) $\pi \equiv x - y + z - 4 = 0$

$r \equiv \begin{cases} x = r \\ y = 1 - r \\ z = 2 + r \end{cases}$ $\rightarrow$ $r - (1-r) + (2+r) - 4 = 0$

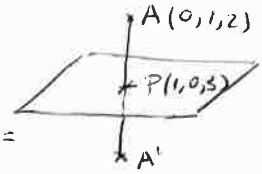
$r - 1 + r + 2 + r - 4 = 0$

$3r = 3$; $r = 1$ $\Rightarrow$ $P(1,0,3)$

c) $A' = A + 2\vec{AP} =$

$= (0,1,2) + 2 \cdot (1,-1,1) =$

$= (2,-1,4)$



SEPT 10 fase Específica

a) $A(1,0,0)$, $B(0,2,0)$, $C(0,0,-1)$ | $\vec{AB} = (-1,2,0)$, $\vec{AC} = (-1,0,-1)$

$\begin{vmatrix} x-1 & y & z \\ -1 & 2 & 0 \\ -1 & 0 & -1 \end{vmatrix} = 0$

$-2(x-1) - y + 2z = 0$

$\pi_1 \equiv -2x - y + 2z + 2 = 0$

$P(3,0,0)$, $Q(0,6,0)$, $R(0,0,-3)$ | $\vec{PQ} = (-3,6,0)$, $\vec{PR} = (-3,0,-3)$

$\begin{vmatrix} x-3 & y & z \\ -3 & 6 & 0 \\ -3 & 0 & -3 \end{vmatrix} = 0$

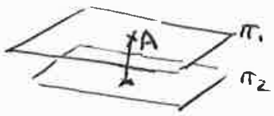
$-18(x-3) - 9y + 18z = 0$

$-2(x-3) - y + 2z = 0$

$\pi_2 \equiv -2x - y + 2z + 6 = 0$

b) $\begin{cases} -2x - y + 2z + 2 = 0 \\ -2x - y + 2z + 6 = 0 \end{cases}$ Obviamente, son paralelos

c) Elijo un punto cualquiera de π_1: por ejemplo $A(1,0,0)$:



$d(A(1,0,0)\ ;\ \pi_2 \equiv -2x - y + 2z + 6 = 0) =$

$= \frac{|-2-0+0+6|}{\sqrt{4+1+4}} = \frac{4}{3}$

SEPT 10 fase Específica

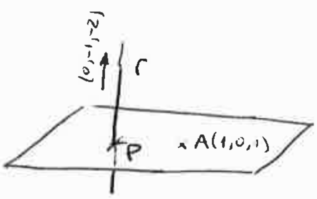
a) $B(0,1,1)$, $C(0,0,-1)$ | $\vec{BC} = (0,-1,-2)$

$r \equiv \begin{cases} x = 0 \\ y = -r \\ z = -1 - 2r \end{cases}$

b) $\pi \equiv 0 \cdot x - y - 2z + D = 0$

$A(1,0,1) \in \pi \rightarrow -2 + D = 0$; $D = 2$

$\pi \equiv -y - 2z + 2 = 0$



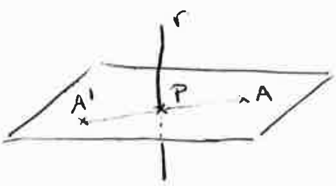
c) $\begin{cases} x = 0 \\ y = -r \\ z = -1 - 2r \\ -y - 2z + 2 = 0 \end{cases}$ $\rightarrow$ $+r + 2 + 4r + 2 = 0$; $5r = -4$; $r = -\frac{4}{5}$ $\rightarrow$ $y = 4/5$, $z = -1 + \frac{8}{5} = \frac{3}{5}$

$P\left(0, \frac{4}{5}, \frac{3}{5}\right)$

d) $A' = A + 2\vec{AP} =$

$= (1,0,1) + 2\left(-1, \frac{4}{5}, -\frac{2}{5}\right) =$

$= \left(-1, \frac{8}{5}, \frac{1}{5}\right)$



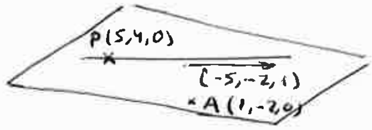
SEPT 10 fase General

$$r \equiv \begin{cases} x-2y+z+3=0 \\ y+2z-4=0 \end{cases} \longrightarrow \begin{cases} x-2y=-z-3 \\ y=4-2z \end{cases}$$

$$x = \frac{\begin{vmatrix} -z-3 & -2 \\ 4-2z & 1 \end{vmatrix}}{\begin{vmatrix} 1 & -2 \\ 0 & 1 \end{vmatrix}} = \frac{-z-3+8-4z}{1} = 5-5z$$

$$y = \frac{\begin{vmatrix} 1 & -z-3 \\ 0 & 4-2z \end{vmatrix}}{1} = 4-2z$$

$$r \equiv \begin{cases} x = 5-5r \\ y = 4-2r \\ z = r \end{cases}$$



$\vec{PA} = (-4,-6,0)$

$$\begin{vmatrix} x-1 & y+2 & z \\ -4 & -6 & 0 \\ -5 & -2 & 1 \end{vmatrix} = 0$$

$-6(x-1) + 4(y+2) - 22z = 0$

$$\boxed{-6x+4y-22z+14=0}$$

SEPT 10 fase General

a) $P(1,2,1)$, $\vec{u}(1,-1,1)$ $\longrightarrow$ $r \equiv \begin{cases} x = 1+r \\ y = 2-r \\ z = 1+r \end{cases}$

$A(2,3,2)$, $B(3,2,3)$ $\longrightarrow$ $\vec{AB} = (1,-1,1)$ $\Rightarrow$ $s \equiv \begin{cases} x = 2+t \\ y = 3-t \\ z = 2+t \end{cases}$

b) $\begin{cases} 1+r = 2+t \\ 2-r = 3-t \\ 1+r = 2+t \end{cases} \longrightarrow \begin{cases} r-t = 1 \\ -r+t = 1 \\ r-t = 1 \end{cases}$ $\quad A = \begin{pmatrix} 1 & -1 \\ -1 & 1 \\ 1 & -1 \end{pmatrix}$

$|1| \neq 0 \Rightarrow r(A) \geq 1$

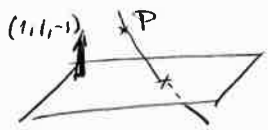
$\begin{vmatrix} 1 & -1 \\ -1 & 1 \end{vmatrix} = 0$, $\begin{vmatrix} 1 & -1 \\ 1 & -1 \end{vmatrix} = 0$ $\Rightarrow r(A) = 1$

$A^* = \left(\begin{array}{cc|c} 1 & -1 & 1 \\ -1 & 1 & 1 \\ 1 & -1 & 1 \end{array}\right)$ $\quad \begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} = 2 \neq 0 \Rightarrow r(A^*) = 2$ $\longrightarrow$ Sistema Incompatible

Al ser incompatible, las rectas pueden ser paralelas o cruzarse, como $r(A)=1$, las rectas son **paralelas**.

JUN 11 fase Específica

a) $P(-1,2,0)$, $\pi \equiv x+y-z+2=0$ $\quad -1+2-0+2=0$, $3 \neq 0$. Por lo tanto $P \notin \pi$



En consecuencia hay infinitas rectas que, pasando por P, cortan al plano π. Serviría cualquier vector salvo que sea paralelo al plano, por ejemplo $\vec{u}(3,2,1)$, ya que

$(3,2,1)\cdot(1,1,-1) = 3+2-1 = 4 \neq 0$.

$$\begin{cases} x = -1+3r \\ y = 2+2r \\ z = r \end{cases}$$

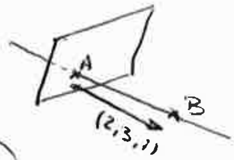
b) $$d(P(-1,2,0);\ \pi \equiv x+y-z+2=0) = \frac{|-1+2-0+2|}{\sqrt{1+1+1}} = \frac{3}{\sqrt{3}} = \boxed{\sqrt{3}}$$

JUN 11 fase específ

a) $A(0,-1,2)$, $B(2,2,3)$ | $\vec{AB} = (2,3,1)$

$$\left.\begin{array}{l} x = 2r \\ y = -1+3r \\ z = 2+r \end{array}\right\} \equiv r$$

b) $\pi \equiv 2x+3y+z+D=0$



$A(0,-1,2) \in \pi \Rightarrow$

$\Rightarrow 0-3+2+D=0$; $D=1$

$$\pi \equiv 2x+3y+z+1=0$$

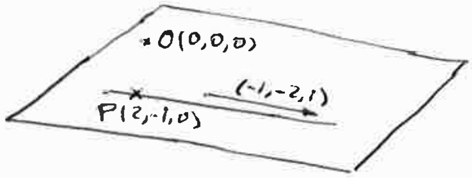
JUN 11 fase general

$r \equiv \left\{\begin{array}{l} 2x-y-5=0 \\ x+z-2=0 \end{array}\right. \longrightarrow \left.\begin{array}{l} 2x-y=5 \\ x = 2-z \end{array}\right\}$

$$y = \frac{\begin{vmatrix} 2 & 5 \\ 1 & 2-z \end{vmatrix}}{\begin{vmatrix} 2 & -1 \\ 1 & 0 \end{vmatrix}} = \frac{4-2z-5}{1} = -1-2z$$

$r \equiv \left\{\begin{array}{l} x = 2-r \\ y = -1-2r \\ z = r \end{array}\right.$

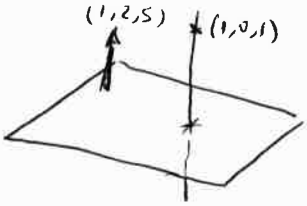
a) $\vec{OP} = (2,-1,0)$



$$\begin{vmatrix} x & y & z \\ 2 & -1 & 0 \\ -1 & -2 & 1 \end{vmatrix} = 0$$

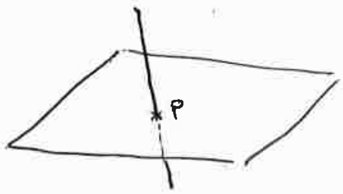
$-x-2y-5z=0$; $\pi \equiv x+2y+5z=0$

b) $\left.\begin{array}{l} x = 1+r \\ y = 2 \\ z = 1+5r \end{array}\right\}$



JUN 11 fase general

$r \equiv \frac{x+2}{1} = \frac{y-1}{4} = \frac{z}{3} \longrightarrow r \equiv \left\{\begin{array}{l} x = -2+r \\ y = 1+4r \\ z = 3r \end{array}\right.$



a) b)

$\left.\begin{array}{l} x=-2+r \\ y = 1+4r \\ z = 3r \\ x+5y-3z=15 \end{array}\right\} \rightarrow -2+r+5+20r-9r=15$

$12r = 12$

$r=1 \longrightarrow \begin{array}{l} x=-1 \\ y=5 \\ z=3 \end{array}$

La recta corta al plano en el punto $P(-1,5,3)$

JUL 11 fase específ

$r \equiv \frac{x}{2} = y = \frac{z}{2-a} \longrightarrow \vec{v_r} = (2,1,2-a)$

$s \equiv \left.\begin{array}{l} x-bz=0 \\ 2x-y-z+1=0 \end{array}\right\}$ $\vec{v_s} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & -b \\ 2 & -1 & -1 \end{vmatrix} = -b\vec{i} + (1-2b)\vec{j} - \vec{k} = (-b, 1-2b, -1)$

$\vec{v_r} \parallel \vec{v_s} \Rightarrow \frac{-b}{2} = \frac{1-2b}{1} = \frac{-1}{2-a}$

$-b = 2-4b$

$3b = 2$; $b = \frac{2}{3}$

$-b(2-a) = -2$

$-\frac{2}{3}(2-a) = -2$

$-4+2a=-6$

$2a=-2$

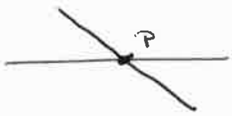
$a=-1$

Comprobación: $a=-1 \longrightarrow \vec{v_r} = (2,1,3)$

$b = 2/3 \longrightarrow \vec{v_s} = (-\frac{2}{3}, -\frac{1}{3}, -1)$ $\longrightarrow \vec{v_s} = -\frac{1}{3}\vec{v_r}$

JUL 11 fase específ.

$r \equiv \frac{x-1}{3} = \frac{y+2}{2} = z+1 \longrightarrow \begin{cases} x = 1+3r \\ y = -2+2r \\ z = -1+r \end{cases}$



a) $\left.\begin{array}{l} 1+3r = 1+t \\ -2+2r = m+3t \\ -1+r = -1+3t \end{array}\right\} ; \quad \left.\begin{array}{l} 3r-t=0 \\ 2r-3t = m+2 \\ r-3t=0 \end{array}\right\}$

$A = \begin{pmatrix} 3 & -1 \\ 2 & -3 \\ 1 & -3 \end{pmatrix} \qquad \begin{vmatrix} 3 & -1 \\ 2 & -3 \end{vmatrix} = -7 \neq 0 \Rightarrow r(A) = 2$

$A^* = \left(\begin{array}{cc|c} 3 & -1 & 0 \\ 2 & 3 & m+2 \\ 1 & -3 & 0 \end{array}\right) \qquad \begin{vmatrix} 3 & -1 & 0 \\ 2 & 3 & m+2 \\ 1 & -3 & 0 \end{vmatrix} = -m-2+9m+18 = 8m+16$

$8m+16=0 \longrightarrow m=-2$

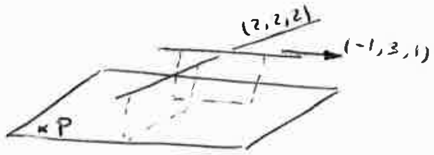
- Si $m \neq -2 \Rightarrow \left.\begin{array}{l} r(A^*)=3 \\ r(A)=2 \end{array}\right\} \longrightarrow$ Sistema Incompatible. Las rectas no se cortarán.
- Si $\boxed{m=-2} \Rightarrow \left.\begin{array}{l} r(A^*)=2 \\ r(A)=2 \end{array}\right\} \longrightarrow$ S. Compatible Determinado. Las rectas se cortan en un punto.

b) $\left.\begin{array}{l} 3r-t=0 \\ 2r-3t=0 \\ r-3t=0 \end{array}\right\} \longrightarrow \begin{array}{l} r=0 \\ t=0 \end{array} \Longrightarrow \begin{array}{l} x=1 \\ y=-2 \\ z=-1 \end{array}$ | Se cortan en el punto $P(1,-2,-1)$ |

JUL 11 fase general

$s' \equiv \left.\begin{array}{l} 2x-2y=4 \\ y-z=-3 \end{array}\right\} \longrightarrow \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -2 & 0 \\ 0 & 1 & -1 \end{vmatrix} = 2\vec{i}+2\vec{j}+2\vec{k} = (2,2,2)$ es el vector director de s'

$\begin{vmatrix} x-1 & y-1 & z-2 \\ -1 & 3 & 1 \\ 2 & 2 & 2 \end{vmatrix} = 0$



$4(x-1)+4(y-1)-8(z-2)=0$

$(x-1)+(y-1)-2(z-2)=0 \longrightarrow \boxed{x+y-2z+2=0}$

JUL 11 fase general

$r \equiv \frac{x-2}{1} = \frac{y+1}{3} = \frac{z-2}{4} \longrightarrow r \equiv \begin{cases} x = 2+r \\ y = -1+3r \\ z = 2+4r \end{cases}$

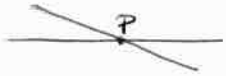
$s' \equiv \frac{x-1}{1} = \frac{y+2}{1} = \frac{z-3}{-1} \longrightarrow s' \equiv \begin{cases} x = 1+t \\ y = -2+t \\ z = 3-t \end{cases}$

$\left.\begin{array}{l} 2+r = 1+t \\ -1+3r = -2+t \\ 2+4r = 3-t \end{array}\right\} \longrightarrow \left.\begin{array}{l} r-t=-1 \\ 3r-t=-1 \\ 4r+t=1 \end{array}\right\}$

$A = \begin{pmatrix} 1 & -1 \\ 3 & -1 \\ 4 & 1 \end{pmatrix} \qquad \begin{vmatrix} 1 & -1 \\ 3 & -1 \end{vmatrix} = 2 \neq 0 \Rightarrow r(A)=2$ Es decir, las rectas no son paralelas ni coincidentes.

$A^* = \left(\begin{array}{cc|c} 1 & -1 & -1 \\ 3 & -1 & -1 \\ 4 & 1 & 1 \end{array}\right) \qquad \begin{vmatrix} 1 & -1 & -1 \\ 3 & -1 & -1 \\ 4 & 1 & 1 \end{vmatrix} = -1-3+4-4+3+1 = 0 \Longrightarrow r(A^*)=2$

$\left.\begin{array}{l} r(A)=2 \\ r(A^*)=2 \\ m=2 \end{array}\right\} \Rightarrow$ S. Compatible determinado. | Las rectas se cortan en un punto |



$\left.\begin{array}{l} r-t=-1 \\ 3r-t=-1 \\ \cancel{4r+t=1} \end{array}\right\}$

$r = \frac{\begin{vmatrix} -1 & -1 \\ -1 & -1 \end{vmatrix}}{\begin{vmatrix} 1 & -1 \\ 3 & -1 \end{vmatrix}} = \frac{0}{2} = 0 \longrightarrow \begin{array}{l} x=2 \\ y=-1 \\ z=2 \end{array}$

$s' = \frac{\begin{vmatrix} 1 & -1 \\ 3 & -1 \end{vmatrix}}{2} = \frac{-1+3}{2} = 1 \longrightarrow \begin{array}{l} x=2 \\ y=-1 \\ z=2 \end{array}$

| El punto de corte es $P(2,-1,2)$ |

JUN 12 fase específ.

[Es idéntico al de Septiembre 2010, fase específica]

JUN 12 fase específ.

$r \equiv \dfrac{x+1}{1} = \dfrac{y-1}{2} = \dfrac{z-2}{3} \longrightarrow r \equiv \begin{cases} x = -1+r \\ y = 1+2r \\ z = 2+3r \end{cases}$

a) b)

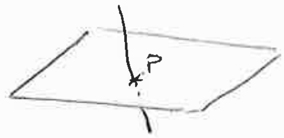
$r \equiv \begin{cases} x = -1+r \\ y = 1+2r \\ z = 2+3r \end{cases}$; $\pi \equiv 2x+4y-3z=15$

$2(-1+r)+4(1+2r)-3(2+3r)=15$

$-2+2r+4+8r-6-9r=15$

$r = 19 \longrightarrow \begin{cases} x = 18 \\ y = 39 \\ z = 59 \end{cases}$

La recta corta al plano en el punto $P(18,39,59)$

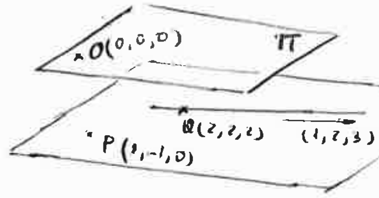


JUN 12 fase General

a) $\vec{PQ} = (1,3,2)$

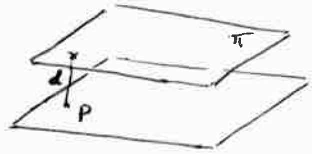
Los vectores $(1,2,3)$ y $(1,3,2)$ son paralelos al plano π, que, además, contiene a $(0,0,0)$:

$$\begin{vmatrix} x & y & z \\ 1 & 2 & 3 \\ 1 & 3 & 2 \end{vmatrix} = 0 ; \quad \pi \equiv \boxed{-5x+y+z=0}$$



b) $d = d(P(1,-1,0) ; \pi \equiv -5x+y+z=0) =$

$$= \frac{|-5-1+0|}{\sqrt{25+1+1}} = \frac{6}{\sqrt{27}} = \frac{6}{3\sqrt{3}} = \boxed{\frac{2}{\sqrt{3}}}$$



JUN 12 fase General

a) $r \equiv \begin{cases} x = 1+2t \\ y = -5-5t \\ z = -3+2t \end{cases}$; $\pi_1 \equiv x+2y+3z-1=0$

$1+2t-10-10t-9+6t-1=0$

$-2t = 19 \; ; \; t = -\frac{19}{2} \longrightarrow \begin{cases} x = -18 \\ y = 42'5 \\ z = -22 \end{cases}$

La recta r corta al plano π_1 en el punto $P(-18, 42'5, -22)$

$r \equiv \begin{cases} x = 1+2t \\ y = -5-5t \\ z = -3+2t \end{cases}$; $\pi_2 \equiv x+2y+4z-2=0$

$1+2t-10-10t-12+8t-2=0$

$0 \cdot t = 23$ Absurdo

La recta r es paralela al plano π_2

b) $\begin{cases} x+2y+3z-1=0 \\ x+2y+4z-2=0 \end{cases}$

$A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 4 \end{pmatrix}$ $\quad \begin{vmatrix} 2 & 3 \\ 2 & 4 \end{vmatrix} = 2 \neq 0 \Rightarrow \begin{matrix} r(A)=2 \\ r(A^*)=2 \\ n=3 \end{matrix}$

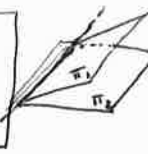
S. Compatible Indeterminado. Es decir, los planos no son paralelos ni coincidentes. Sino que se cortan sobre una recta de ecuación:

$\begin{cases} 2y+3z = 1-x \\ 2y+4z = 2-x \end{cases}$

$$y = \frac{\begin{vmatrix} 1-x & 3 \\ 2-x & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 3 \\ 2 & 4 \end{vmatrix}} = \frac{4-4x-6+3x}{2} = \frac{-2-x}{2} = -1-\frac{1}{2}x$$

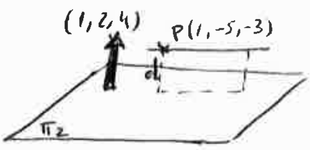
$$z = \frac{\begin{vmatrix} 2 & 1-x \\ 2 & 2-x \end{vmatrix}}{2} = \frac{4-2x-2+2x}{2} = 1$$

$\Rightarrow \begin{cases} x = r \\ y = -1-\frac{1}{2}r \\ z = 1 \end{cases}$



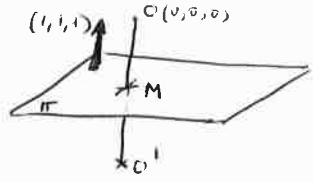
c) $d = d(P(1,-5,-3) ; \pi_2 \equiv x+2y+4z-2=0) =$

$$= \frac{|1-10-12-2|}{\sqrt{1+4+16}} = \boxed{\frac{23}{\sqrt{21}}}$$



(JUL 12 fase específ.)

La única opción es que O' sea simétrico de O respecto del plano π.



Recta $\overline{OO'}$

$$\left.\begin{array}{l} x=r \\ y=r \\ z=r \end{array}\right\}$$

Punto M, intersección de $\overline{OO'}$ con π

$$\left.\begin{array}{l} x=r \\ y=r \\ z=r \\ x+y+z=3 \end{array}\right\} \quad r+r+r=3 \; ; \; 3r=3 \; ; \; r=1 \rightarrow \boxed{M(1,1,1)}$$

$$O' = O + 2\cdot\vec{OM} = (0,0,0)+2\cdot(1,1,1) = \boxed{(2,2,2)}$$

Comprobación:

$$d(O(0,0,0)\,;\,\pi\equiv x+y+z-3=0) = \frac{|0+0+0-3|}{\sqrt{1+1+1}} = \frac{3}{\sqrt{3}}$$

$$d(O'(2,2,2)\,;\,\pi\equiv (x+y+z-3=0)) = \frac{|2+2+2-3|}{\sqrt{1+1+1}} = \frac{3}{\sqrt{3}} \quad ✓$$

(JUL 12 fase específ.)

a) A(1,0,0), B(0,2,0), C(0,3,0)

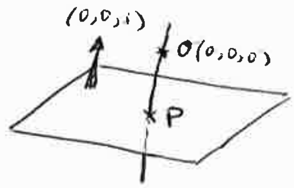
$\vec{AB} = (-1,2,0)$

$\vec{AC} = (-1,3,0)$

$$\begin{vmatrix} x-1 & y & z \\ -1 & 2 & 0 \\ -1 & 3 & 0 \end{vmatrix} = 0 \rightarrow -z=0 \; ; \; \boxed{\pi\equiv z=0}$$

b)

$$\left.\begin{array}{l} x=0+0\cdot r \\ y=0+0\cdot r \\ z=0+1\cdot r \end{array}\right\} \rightarrow \boxed{\begin{array}{l} x=0 \\ y=0 \\ z=r \end{array}}$$



$$\left.\begin{array}{l} x=0 \\ y=0 \\ z=r \\ z=0 \end{array}\right\} \quad \boxed{r=0} \rightarrow \boxed{P(0,0,0)}$$

En el gráfico anterior, O y P coinciden.

(JUL 12 fase general)

a)

$$\left.\begin{array}{r} 2x-y+z=0 \\ z=3 \end{array}\right\} \qquad A=\begin{pmatrix} 2 & -1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \qquad \begin{vmatrix} -1 & 1 \\ 0 & 1 \end{vmatrix} = -1 \neq 0 \Rightarrow \left.\begin{array}{l} r(A)=2 \\ r(A^*)=2 \\ n=3 \end{array}\right\} \text{S. Compat. Indeterm.}$$

$$\left.\begin{array}{r} -y+z=-2x \\ z=3 \end{array}\right\} \qquad y=\frac{\begin{vmatrix} -2x & 1 \\ 3 & 1 \end{vmatrix}}{\begin{vmatrix} -1 & 1 \\ 0 & 1 \end{vmatrix}} = \frac{-2x-3}{-1} = 3+2x$$

$z=3$

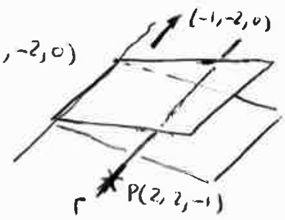
→ π_1 y π_2 se cortan sobre la recta:

$$\boxed{\begin{array}{l} x=r \\ y=3+2r \\ z=3 \end{array}}$$

b)

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -1 & 1 \\ 0 & 0 & 1 \end{vmatrix} = -\vec{i}-2\vec{j} = (-1,-2,0)$$

$$\boxed{r\equiv\left\{\begin{array}{l} x=2-t \\ y=2-2t \\ z=-1 \end{array}\right.}$$



JUL 12 fase General

a) A(0,2,1) B(1,-2,0) C(2,0,3)

$\vec{AB} = (1,-4,-1)$

$\vec{AC} = (2,-2,2)$

$$\begin{vmatrix} x & y-2 & z-1 \\ 1 & -4 & -1 \\ 2 & -2 & 2 \end{vmatrix} = 0$$

$-10x - 4(y-2) + 6(z-1) = 0$

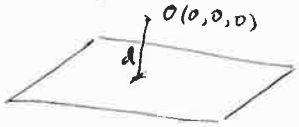
$-5x - 2(y-2) + 3(z-1) = 0$

$\pi \equiv \boxed{-5x - 2y + 3z + 1 = 0}$ Plano que contiene a A, B y C

$D(1,1,K) \in \pi \Rightarrow -5 - 2 + 3K + 1 = 0 \;;\; 3K = 6 \;;\; \boxed{K=2}$

b) $d(O(0,0,0) \;;\; \pi \equiv -5x - 2y + 3z + 1 = 0) =$

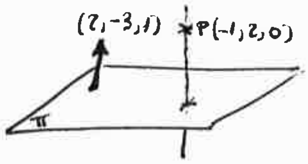
$= \dfrac{|-0-0+0+1|}{\sqrt{25+4+9}} = \boxed{\dfrac{1}{\sqrt{38}}}$



JUN 13 fase General

a)

$$\left.\begin{aligned} x &= -1 + 2r \\ y &= 2 - 3r \\ z &= r \end{aligned}\right\}$$

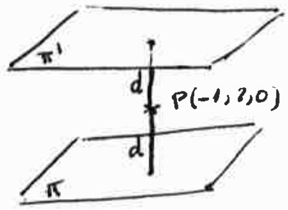


b) $d(P(-1,2,0) \;;\; \pi \equiv 2x - 3y + z - 8 = 0) = \dfrac{|2\cdot(-1) - 3\cdot 2 + 0 - 8|}{\sqrt{4+9+1}} = \boxed{\dfrac{16}{\sqrt{14}}}$

c) $\pi' \equiv 2x - 3y + z + D = 0$

$d(P(-1,2,0) \;;\; \pi' \equiv 2x - 3y + z + D = 0) = \dfrac{16}{\sqrt{14}}$

$\dfrac{|-2-6+D|}{\sqrt{4+9+1}} = \dfrac{16}{\sqrt{14}}$

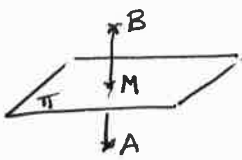


$|D-8| = 16$ → $D - 8 = 16 \Rightarrow D = 24 \rightarrow \boxed{\pi' \equiv 2x - 3y + z + 24 = 0}$

→ $D - 8 = -16 \Rightarrow D = -8 \rightarrow \pi \equiv 2x - 3y + z - 8 = 0$ ✓

JUN 13 fase General

a) $M = \dfrac{A+B}{2} = \boxed{\left(\dfrac{3}{2}, \dfrac{1}{2}, \dfrac{3}{2}\right)}$



b) $\vec{AB} = (1,3,1)$

$\pi \equiv x + 3y + z + D = 0$

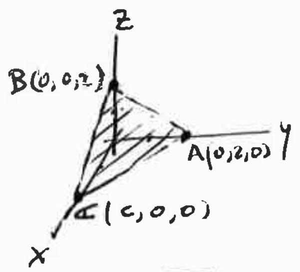
$M \in \pi \Rightarrow \dfrac{3}{2} + 3\cdot\dfrac{1}{2} + \dfrac{3}{2} + D = 0 \;;\; D = -\dfrac{9}{2} \Rightarrow \boxed{x + 3y + z - \dfrac{9}{2} = 0}$

JUN 13 fase Específica

$\vec{AB} = (0,-2,2)$

$\vec{AC} = (c,-2,0)$

$\vec{AB} \times \vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & -2 & 2 \\ c & -2 & 0 \end{vmatrix} = (4, 2c, 2c)$



$\text{Area} = \dfrac{1}{2}|\vec{AB} \times \vec{AC}| \Rightarrow 6 = \dfrac{1}{2}\sqrt{16 + 4c^2 + 4c^2} \;;\; 12 = \sqrt{16 + 8c^2} \;;\; 144 = 16 + 8c^2 \;;\; c^2 = 16 \;;\; c = \pm 4$

$c = 4 \rightarrow \vec{AB} \times \vec{AC} = (4,8,8) \;;\; 4x + 8y + 8z + D = 0 \;;\; 4\cdot 0 + 8\cdot 2 + 8\cdot 0 + D = 0 \;;\; D = -16 \;;\; \boxed{4x + 8y + 8z - 16 = 0}$

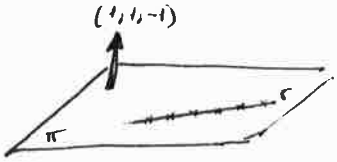
$c = -4 \rightarrow \vec{AB} \times \vec{AC} = (4,-8,-8) \;;\; 4x - 8y - 8z + D = 0 \;;\; 4\cdot 0 - 8\cdot 2 - 8\cdot 0 + D = 0 \;;\; D = 16 \;;\; \boxed{4x - 8y - 8z + 16 = 0}$

JUN 13 fase Específica

$r \equiv \begin{cases} x=0 \\ y-z=0 \end{cases} \Rightarrow r \equiv \begin{cases} x=0 \\ y=r \\ z=r \end{cases}$

a) $\pi \equiv x+y-z=0$, $r \equiv \begin{cases} x=0 \\ y=r \\ z=r \end{cases}$ $\Big\}$ $0+r-r=0$; $0=0 \Rightarrow$ **Recta contenida en el plano**

b) Cualquier recta que tenga $(1,1,-1)$ como vector director puede servir:



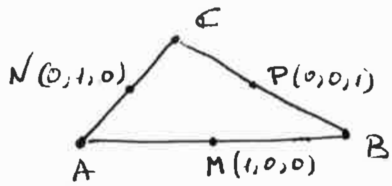
$$\boxed{\frac{x-x_0}{1} = \frac{y-y_0}{1} = \frac{z-z_0}{-1}}$$

Por ejemplo: $\boxed{\frac{x}{1} = \frac{y}{1} = \frac{z}{-1}}$

c) Al estar la recta contenida en el plano, dicha mínima distancia es $\boxed{0}$.

JUL 13 fase General

$A(a_1, a_2, a_3)$
$B(b_1, b_2, b_3)$
$C(c_1, c_2, c_3)$



a) $M = \frac{A+B}{2}$; $(1,0,0) = \left(\frac{a_1+b_1}{2}, \frac{a_2+b_2}{2}, \frac{a_3+b_3}{2}\right) \Rightarrow \begin{cases} b_1 = 2-a_1 \\ b_2 = -a_2 \\ b_3 = -a_3 \end{cases} \rightarrow B(2-a_1, -a_2, -a_3)$

$N = \frac{A+C}{2}$; $(0,1,0) = \left(\frac{a_1+c_1}{2}, \frac{a_2+c_2}{2}, \frac{a_3+c_3}{2}\right) \Rightarrow \begin{cases} c_1 = -a_1 \\ c_2 = 2-a_2 \\ c_3 = -a_3 \end{cases} \rightarrow C(-a_1, 2-a_2, -a_3)$

$P = \frac{B+C}{2}$; $(0,0,1) = \left(\frac{2-a_1-a_1}{2}, \frac{-a_2+2-a_2}{2}, \frac{-a_3-a_3}{2}\right) \Rightarrow \boxed{a_1=1}$ $\boxed{a_2=1}$ $\boxed{a_3=-1}$

$\boxed{\begin{array}{l} A(1,1,-1) \\ B(1,-1,1) \\ C(-1,1,1) \end{array}}$

b) $\vec{AB} \times \vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & -2 & 2 \\ -2 & 0 & 2 \end{vmatrix} = (-4,-4,-4)$

$\text{Area} = \frac{1}{2}|\vec{AB} \times \vec{AC}| = \frac{1}{2}\sqrt{16+16+16} = \frac{1}{2}\sqrt{48} = \boxed{2\sqrt{3}}$

JUL 13 fase General

$r \equiv \begin{cases} 3x+y=0 \\ 4x+z=0 \end{cases} \rightarrow \begin{cases} y=-3x \\ z=-4x \end{cases}$; $\boxed{\begin{array}{l} x=r \\ y=-3r \\ z=-4r \end{array}} \rightarrow \vec{v}_r(1,-3,-4)$

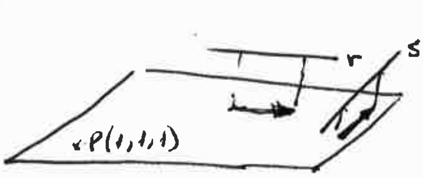
$s \equiv \begin{cases} x-y=2 \\ y-z=-3 \end{cases} \rightarrow \begin{cases} x=2+y \\ z=3+y \end{cases}$; $\boxed{\begin{array}{l} x=2+t \\ y=t \\ z=3+t \end{array}} \rightarrow \vec{v}_s(1,1,1)$

Vectores Directores

$\begin{vmatrix} x-1 & y-1 & z-1 \\ 1 & -3 & -4 \\ 1 & 1 & 1 \end{vmatrix} = 0$; $1\cdot(x-1) - 5(y-1) + 4(z-1) = 0$

$x-1-5y+5+4z-4=0$

$\boxed{x-5y+4z=0}$



También: $\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 1 & 0 \\ 4 & 0 & 1 \end{vmatrix} = (1,3,-4)$; $\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{vmatrix} = (1,1,1)$ Para hallar los vector directores.

JUL 13 fase Específica

a) $\vec{AB} = (-1,0,0)$

$$s \equiv \left.\begin{array}{l} x = 1 - r \\ y = 1 \\ z = 0 \end{array}\right\} \Rightarrow \boxed{s \equiv \left\{\begin{array}{l} y = 1 \\ z = 0 \end{array}\right.}$$

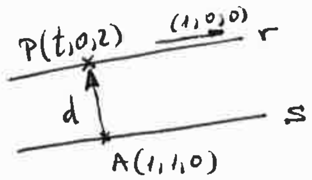
b) $$r \equiv \left\{\begin{array}{l} y = 0 \\ z = 2 \end{array}\right. \Rightarrow \boxed{r \equiv \left\{\begin{array}{l} x = t \\ y = 0 \\ z = 2 \end{array}\right.}$$

$$\left.\begin{array}{l} 1 - r = t \\ 1 = 0 \\ 0 = 2 \end{array}\right\}$$ Sistema Incompatible.

Las rectas r, s ó son paralelas, ó se cruzan.

La recta r lleva la dirección del vector $(1,0,0)$
La recta s " " " " " $(-1,0,0)$ $\Rightarrow$ Son paralelas

c) $\vec{AP} = (t-1,-1,2)$



$\vec{AP} \perp (1,0,0) \Rightarrow t-1+0+0=0$

$t=1 \rightarrow P(1,0,2)$

$\rightarrow \vec{AP} = (0,-1,2) \Rightarrow d = |\vec{AP}| = \sqrt{0+1+4} = \boxed{\sqrt{5}}$

JUL 13 fase Específ.

$(a,b,c) \longrightarrow (a+b, a+b+c, a+b)$

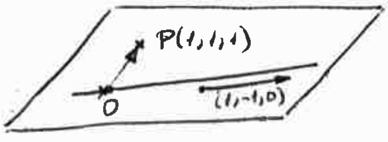
a) $$\left.\begin{array}{l} a+b=0 \\ a+b+c=0 \\ a+b=0 \end{array}\right\} ; \quad \begin{array}{l} a+b=0 \\ a+b+c=0 \\ \cancel{a+b=0} \end{array}$$

$a+b=0 \rightarrow \boxed{b=-a}$

$a+b+c=0 \rightarrow 0+c=0 ; \boxed{c=0}$

Los puntos que se mueven al origen son: $\boxed{(r,-r,0)}$ $(r \in \mathbb{R})$

b) Elegimos un punto cualquiera de la recta $(r,-r,0)$. Por ejemplo $O(0,0,0)$



$\vec{OP} = (1,1,1)$

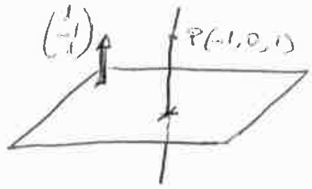
$$\begin{vmatrix} x-1 & y-1 & z-1 \\ 1 & 1 & 1 \\ 1 & -1 & 0 \end{vmatrix} = 0 ; \quad 1\cdot(x-1) + 1\cdot(y-1) - 2(z-1) = 0$$

$x-1+y-1-2z+2=0$

$\boxed{x+y-2z=0}$

JUN 14 fase General

a) $\left.\begin{array}{l} x=-1+r \\ y=-r \\ z=1+r \end{array}\right\}$

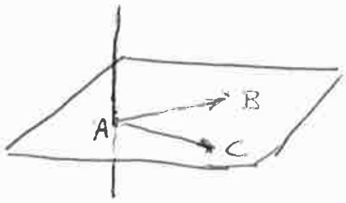


b) $d(P(-1,0,1),\ x-y+z+2=0) = \dfrac{|-1-0+1+2|}{\sqrt{1^2+(-1)^2+1^2}} = \boxed{\dfrac{2}{\sqrt{3}}}$

JUN 14 fase General

a) $\vec{AB} = \begin{pmatrix}2-0\\2+1\\3-2\end{pmatrix} = \begin{pmatrix}2\\3\\1\end{pmatrix}$

$\vec{AC} = \begin{pmatrix}0-0\\0+1\\3-2\end{pmatrix} = \begin{pmatrix}0\\1\\1\end{pmatrix}$



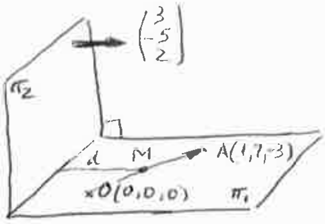
$\begin{vmatrix} x & y & z-3 \\ 2 & 3 & 1 \\ 0 & 1 & 1 \end{vmatrix} = 0$; $2x-2y+2(z-3)=0$; $2x-2y+2z-6=0$

$\boxed{\pi:\ x-y+z-3=0}$

b) $\left.\begin{array}{l} x=r \\ y=-1-r \\ z=2+r \end{array}\right\}$

JUN 14 fase Específica

a) $\vec{OA} = \begin{pmatrix}1\\2\\-3\end{pmatrix}$



$\begin{vmatrix} x & y & z \\ 1 & 2 & -3 \\ 3 & -5 & 2 \end{vmatrix} = 0$; $-11x-11y-11z=0$;

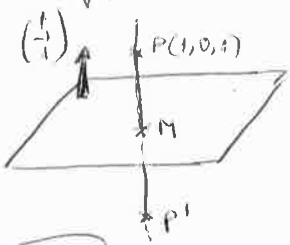
$\boxed{\pi_1:\ x+y+z=0}$

b) $M = \dfrac{O+A}{2} = \left(\dfrac{0+1}{2}, \dfrac{0+2}{2}, \dfrac{0-3}{2}\right) = \left(\dfrac{1}{2}, 1, -\dfrac{3}{2}\right)$

$d\left(M\left(\frac{1}{2},1,-\frac{3}{2}\right),\ 3x-5y+2z-11=0\right) = \dfrac{\left|3\cdot\frac{1}{2}-5\cdot 1+2\cdot\left(-\frac{3}{2}\right)-11\right|}{\sqrt{9+25+4}} = \dfrac{35/2}{\sqrt{38}} = \boxed{\dfrac{35}{2\sqrt{38}}}$

JUN 14 fase Específica

b) $\left.\begin{array}{l} x-y+z+1=0 \\ \left.\begin{array}{l} x=1+r \\ y=-r \\ z=1+r \end{array}\right\} \end{array}\right\} \rightarrow$



$\rightarrow 1+r-(-r)+1+r+1=0$; $3r=-3$; $r=-1$ $\Rightarrow$ $\begin{array}{l} x=1-1=0 \\ y=-(-1)=1 \\ z=1-1=0 \end{array}$ $\boxed{M(0,1,0)}$

a) $\vec{OP'} = \vec{OP} + 2\cdot\vec{PM} = \begin{pmatrix}1\\0\\1\end{pmatrix} + 2\cdot\begin{pmatrix}0-1\\1-0\\0-1\end{pmatrix} = \begin{pmatrix}1\\0\\1\end{pmatrix} + \begin{pmatrix}-2\\2\\-2\end{pmatrix} =$

$= \begin{pmatrix}-1\\2\\-1\end{pmatrix} \Rightarrow \boxed{P'(-1,2,-1)}$

Julio 14 fase General

$r_1:\ x=z=0 \longrightarrow r_1: \left.\begin{array}{l} x=0 \\ y=r \\ z=0 \end{array}\right\}$

$r_2: \left.\begin{array}{l} x+y+z=5 \\ 2x-y+3z=1 \end{array}\right\} \rightarrow \left.\begin{array}{l} x+y=5-z \\ 2x-y=1-3z \end{array}\right\}$ $(z\in\mathbb{R})$

$x = \dfrac{\begin{vmatrix} 5-z & 1 \\ 1-3z & -1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix}} = \dfrac{-5+z-1+3z}{-1-2} = \dfrac{4z-6}{-3} = 2-\dfrac{4}{3}z$

$y = \dfrac{\begin{vmatrix} 1 & 5-z \\ 2 & 1-3z \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix}} = \dfrac{1-3z-10+2z}{-3} = \dfrac{-z-9}{-3} = 3+\dfrac{1}{3}z$ $\Rightarrow$

$\Rightarrow r_2: \left.\begin{array}{l} x=2-4t \\ y=3+t \\ z=3t \end{array}\right\}$

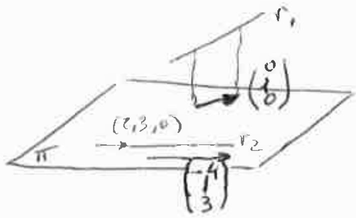
a)

$r_1: \begin{cases} x=0 \\ y=r \\ z=0 \end{cases} \quad r_2: \begin{cases} x=2-4t \\ y=3+t \\ z=3t \end{cases}$; $\begin{cases} 0=2-4t \\ r=3+t \\ 0=3t \end{cases}$; $\begin{cases} t=1/2 \\ r=3+t \\ t=0 \end{cases}$

Sistema incompatible.
Como los vectores $\begin{pmatrix}0\\1\\0\end{pmatrix}$ y $\begin{pmatrix}-4\\1\\3\end{pmatrix}$ no son paralelos, las **rectas se cruzan.**

b)

$$\begin{vmatrix} x-2 & y-3 & z \\ 0 & 1 & 0 \\ -4 & 1 & 3 \end{vmatrix} = 0$$

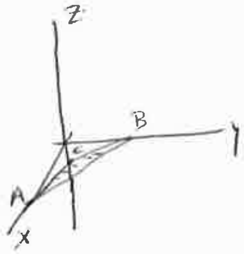


$3(x-2)+4z=0$

$\pi: \boxed{3x+4z-6=0}$

Julio 14 fase General

$\begin{cases} x+y-2z-1=0 \\ y=0 \\ z=0 \end{cases} \rightarrow x=1 \quad \boxed{A(1,0,0)}$



$\begin{cases} x+y-2z-1=0 \\ x=0 \\ z=0 \end{cases} \rightarrow y=1 \quad \boxed{B(0,1,0)}$

$\begin{cases} x+y-2z-1=0 \\ x=0 \\ y=0 \end{cases} \rightarrow z=-\frac{1}{2} \quad \boxed{C(0,0,-\frac{1}{2})}$

$\vec{AB} = \begin{pmatrix}-1\\1\\0\end{pmatrix}$; $\vec{AC} = \begin{pmatrix}-1\\0\\-1/2\end{pmatrix}$ $\quad \vec{AB} \times \vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 1 & 0 \\ -1 & 0 & -1/2 \end{vmatrix} = \begin{pmatrix}-1/2\\-1/2\\1\end{pmatrix}$

$\text{Área} = \frac{1}{2}|\vec{AB} \times \vec{AC}| = \frac{1}{2}\sqrt{\frac{1}{4}+\frac{1}{4}+1} = \frac{1}{2}\sqrt{\frac{3}{2}} = \frac{1}{2}\frac{\sqrt{6}}{2} = \boxed{\frac{\sqrt{6}}{4}\ u^2}$

Julio 14 fase Específica

$r_1: \begin{cases} x-z=2 \\ 2x-y=1 \end{cases} \rightarrow \begin{cases} z=x-2 \\ y=2x-1 \end{cases} \Rightarrow r_1: \begin{cases} x=r \\ y=-1+2r \\ z=-2+r \end{cases}$

$r_2: \begin{cases} x+y=1 \\ 2y-z=-1 \end{cases} \rightarrow \begin{cases} x=1-y \\ z=1+2y \end{cases} \Rightarrow r_2: \begin{cases} x=1-t \\ y=t \\ z=1+2t \end{cases}$

a)

$\begin{cases} x=r \\ y=-1+2r \\ z=-2+r \\ x=1-t \\ y=t \\ z=1+2t \end{cases}$ $\begin{cases} r=1-t \\ -1+2r=t \\ -2+r=1+2t \end{cases}$; $\begin{cases} r+t=1 \\ 2r-t=1 \\ r-2t=3 \end{cases}$

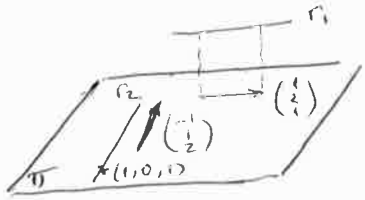
$A=\begin{pmatrix}1&1\\2&-1\\1&-2\end{pmatrix}$ $\quad \begin{vmatrix}1&1\\2&-1\end{vmatrix}=-3\neq 0 \Rightarrow r(A)=2$

$A^*=\begin{pmatrix}1&1&1\\2&-1&1\\1&-2&3\end{pmatrix}$ $\quad \begin{vmatrix}1&1&1\\2&-1&1\\1&-2&3\end{vmatrix} = -3-4+1+1-6+2=-9 \Downarrow r(A^*)=3$

El sistema es incompatible y como $r(A)=2$, es decir, que los vectores no son paralelos, las **rectas se cruzan.**

b)

$$\begin{vmatrix} x-1 & y & z-1 \\ 1 & 2 & 1 \\ -1 & 1 & 2 \end{vmatrix} = 0$$



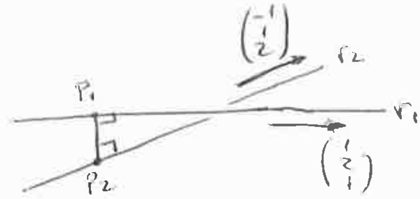
$3(x-1)-3y+3(z-1)=0$

$x-1-y+z-1=0$

$\boxed{\pi: x-y+z-2=0}$

c)

$P_1(r, -1+2r, -2+r)$
$P_2(1-t, t, 1+2t)$ $\Bigg\} \to \overrightarrow{P_1P_2} = \begin{pmatrix} 1-t-r \\ t+1-2r \\ 3+2t-r \end{pmatrix}$



$$\left.\begin{array}{l} \overrightarrow{P_1P_2}\cdot\begin{pmatrix}-1\\1\\2\end{pmatrix}=0 \\ \overrightarrow{P_1P_2}\cdot\begin{pmatrix}1\\2\\1\end{pmatrix}=0 \end{array}\right\} \left.\begin{array}{l} -1+t+r+t+1-2r+6+4t-2r=0 \\ 1-t-r+2t+2-4r+3+2t-r=0 \end{array}\right\} ; \left.\begin{array}{l} 6t-3r=-6 \\ 3t-6r=-6 \end{array}\right\} ;$$

$$; \left.\begin{array}{l} 2t-r=-2 \\ t-2r=-2 \end{array}\right\} \qquad t=\frac{\begin{vmatrix}-2 & -1\\ -2 & -2\end{vmatrix}}{\begin{vmatrix}2 & -1\\ 1 & -2\end{vmatrix}} = \frac{2}{-3} = \frac{-2}{3}$$

$$r=\frac{\begin{vmatrix}2 & -2\\ 1 & -2\end{vmatrix}}{\begin{vmatrix}2 & -1\\ 1 & -2\end{vmatrix}} = \frac{-2}{-3} = \frac{2}{3}$$

$$\Rightarrow \overrightarrow{P_1P_2} = \begin{pmatrix} 1+\frac{2}{3}-\frac{2}{3} \\ -\frac{2}{3}+1-\frac{4}{3} \\ 3-\frac{4}{3}-\frac{2}{3} \end{pmatrix} = \begin{pmatrix}1\\-1\\1\end{pmatrix}$$

$$d(r_1, r_2) = |\overrightarrow{P_1P_2}| = \sqrt{1^2+(-1)^2+1^2} = \boxed{\sqrt{3}}$$

También:

$d(r_1, r_2) = d(Q, \pi)$

Siendo Q cualquier punto de r_1, por ejemplo $Q(0,-1,-2)$

$$d(r_1, r_2) = d\big(Q(0,-1,-2),\ \pi: x-y+z-2=0\big) = \frac{|0-(-1)+(-2)-2|}{\sqrt{1+1+1}} = \frac{3}{\sqrt{3}} = \boxed{\sqrt{3}} \ ✓$$

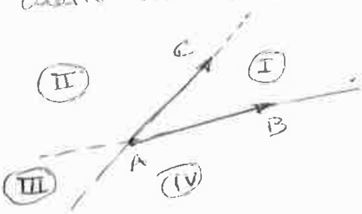
(Julio 14 fase Específica)

$$\overrightarrow{AB} = \begin{pmatrix}-2\\-4\\-4\end{pmatrix} \qquad \overrightarrow{AC} = \begin{pmatrix}-5\\-9\\-7\end{pmatrix} \qquad \overrightarrow{AD} = \begin{pmatrix}-3\\-5\\-3\end{pmatrix}$$

Veamos que los cuatro puntos son coplanarios:

$$[\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}] = \begin{vmatrix}-2 & -4 & -4\\ -5 & -9 & -7\\ -3 & -5 & -3\end{vmatrix} = -54-100-84+108+60+70 = -238+238=0 \ ✓$$

Veamos cómo están situados los cuatro vértices:



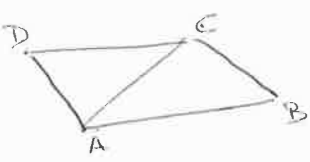
(I) $r>0$, $t>0$
(II) $r<0$, $t>0$
(III) $r<0$, $t<0$
(IV) $r>0$, $t>0$

$\overrightarrow{AD} = r\overrightarrow{AB} + t\overrightarrow{AC}$

$$\left.\begin{array}{l} -2r-5t=-3 \\ -4r-9t=-5 \\ -4r-7t=-3 \end{array}\right\} ; \left.\begin{array}{l} 2r+5t=3 \\ 4r+9t=5 \\ \cancel{4r+7t=3} \end{array}\right\}.$$

$$r=\frac{\begin{vmatrix}3 & 5\\ 5 & 9\end{vmatrix}}{\begin{vmatrix}2 & 5\\ 4 & 9\end{vmatrix}} = \frac{2}{-2} = -1 \qquad t=\frac{\begin{vmatrix}2 & 3\\ 4 & 5\end{vmatrix}}{\begin{vmatrix}2 & 5\\ 4 & 9\end{vmatrix}} = \frac{-2}{-2} = 1$$

En consecuencia, como $\left.\begin{array}{l} r<0 \\ t>0 \end{array}\right\} \Rightarrow$ D pertenece a II



$$\text{Área } \widehat{ABC} = \frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{AC}|$$

$$\text{Área } \widehat{ACD} = \frac{1}{2}|\overrightarrow{AC}\times\overrightarrow{AD}|$$

$$\vec{AB}\times\vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & -4 & -4 \\ -5 & -9 & -7 \end{vmatrix} = \begin{pmatrix} -8 \\ 6 \\ -2 \end{pmatrix} \Rightarrow \text{Área } \widehat{ABC} = \frac{\sqrt{64+36+4}}{2} = \frac{\sqrt{104}}{2}$$

$$\vec{AC}\times\vec{AD} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -5 & -9 & -7 \\ -3 & -5 & -3 \end{vmatrix} = \begin{pmatrix} -8 \\ 6 \\ -2 \end{pmatrix} \Rightarrow \text{Área } \widehat{ACD} = \frac{\sqrt{64+36+4}}{2} = \frac{\sqrt{104}}{2}$$

$$\text{Área Polígono } ABCD = \frac{\sqrt{104}}{2} + \frac{\sqrt{104}}{2} = \boxed{\sqrt{104}\ u^2}$$

Junio 15 fase General

$$r_1: \begin{cases} 2x-y=1 \\ x-z=2 \end{cases} ; \quad \begin{cases} y=-1+2x \\ z=-2+x \end{cases} \quad r_1: \begin{cases} x=r \\ y=-1+2r \\ z=-2+r \end{cases}$$

$$r_2: \begin{cases} 2x-y=2 \\ y-2z=-2 \end{cases} ; \quad \begin{cases} x=1+\frac{1}{2}y \\ z=1+\frac{1}{2}y \end{cases} \quad r_2: \begin{cases} x=1+t \\ y=2t \\ z=-1+t \end{cases}$$

a)

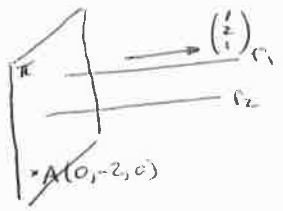
$$\left.\begin{matrix} x=r \\ y=-1+2r \\ z=-2+r \\ x=1+t \\ y=2t \\ z=-1+t \end{matrix}\right\} \quad \begin{cases} r=1+t \\ -1+2r=2t \\ -2+r=-1+t \end{cases} ; \quad \begin{cases} r-t=1 \\ 2r-2t=1 \\ r-t=1 \end{cases}$$

$$A = \begin{pmatrix} 1 & -1 \\ 2 & -2 \\ 1 & -1 \end{pmatrix} \quad r(A)=1 \text{ porque las dos columnas son iguales.}$$

$$A^* = \begin{pmatrix} 1 & -1 & 1 \\ 2 & -2 & 1 \\ 1 & -1 & 1 \end{pmatrix} \quad \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = -1 \Rightarrow r(A^*)=2$$

El sistema es incompatible, como $r(A)=1$, es decir, que las rectas son paralelas, no son la misma recta. r_1 y r_2 son paralelas no coincidentes

b)

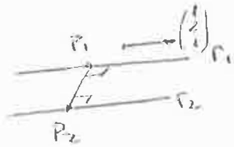


$$x+2y+z+D=0$$

$$0-4+0+D=0 ; \quad D=4$$

$$\boxed{\pi: x+2y+z+4=0}$$

c)



$P_1(0,-1,-2)$ ← (Valdría cualquiera de r_1)

$P_2(1+t, 2t, -1+t)$

$$\vec{P_1P_2} = \begin{pmatrix} 1+t \\ 1+2t \\ 1+t \end{pmatrix}$$

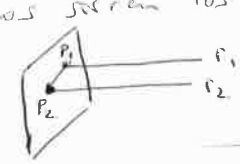
$$\vec{P_1P_2}\cdot\begin{pmatrix}1\\2\\1\end{pmatrix}=0 \Rightarrow 1+t+2+4t+1+t=0 ; \quad 6t=-4 ; \quad t=-\frac{4}{6}=-\frac{2}{3} \Rightarrow$$

$$\Rightarrow \vec{P_1P_2} = \begin{pmatrix} 1-\frac{2}{3} \\ 1-\frac{4}{3} \\ 1-\frac{2}{3} \end{pmatrix} = \begin{pmatrix} 1/3 \\ -1/3 \\ 1/3 \end{pmatrix}$$

$$d(r_1, r_2) = |\vec{P_1P_2}| = \sqrt{\frac{1}{9}+\frac{1}{9}+\frac{1}{9}} = \sqrt{\frac{3}{9}} = \boxed{\frac{\sqrt{3}}{3}}$$

También

Al ser el plano π perpendicular a las rectas, nos sirven los puntos intersección de las rectas con el plano:



$$\left.\begin{matrix} r_1: \begin{cases} x=r \\ y=-1+2r \\ z=-2+r \end{cases} \\ \pi: x+2y+z+4=0 \end{matrix}\right\} \Rightarrow r-2+4r-2+r+4=0 ; \quad 6r=0 ; \quad r=0 \rightarrow P_1(0,-1,-2)$$

$r_2: \begin{cases} x = 1+t \\ y = 2t \\ z = -1+t \end{cases}$

$\pi: x+2y+z+4=0$

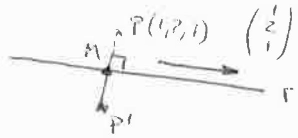
$1+t+4t-1+t+4=0 \; ; \; 6t=-4 \; ; \; t=\frac{-2}{3} \rightarrow P_2\left(1-\frac{2}{3}, \frac{-4}{3}, -1-\frac{2}{3}\right) = \left(\frac{1}{3}, \frac{-4}{3}, \frac{-5}{3}\right)$

$$\overrightarrow{P_1P_2} = \begin{pmatrix} \frac{1}{3}-0 \\ \frac{-4}{3}+1 \\ \frac{-5}{3}+2 \end{pmatrix} = \begin{pmatrix} 1/3 \\ -1/3 \\ -5/3 \end{pmatrix}$$

$$d(r_1, r_2) = |\overrightarrow{P_1P_2}| = \sqrt{\frac{1}{9}+\frac{1}{9}+\frac{1}{9}} = \sqrt{\frac{3}{9}} = \boxed{\frac{\sqrt{3}}{3}} \checkmark$$

Junio 15 fase General

$$r: \frac{x-1}{1} = \frac{y-1}{2} = \frac{z+1}{1} \longrightarrow r: \begin{cases} x = 1+r \\ y = 1+2r \\ z = -1+r \end{cases}$$



a) $M(1+r, 1+2r, -1+r)$

$$\overrightarrow{PM} = \begin{pmatrix} 1+r-1 \\ 1+2r-2 \\ -1+r-1 \end{pmatrix} = \begin{pmatrix} r \\ 2r-1 \\ r-2 \end{pmatrix}$$

$$\overrightarrow{PM} \cdot \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = 0 \Rightarrow r+4r-2+r-2=0 \; ; \; 6r=4 \; ; \; r=\frac{2}{3} \Rightarrow \overrightarrow{PM} = \begin{pmatrix} 2/3 \\ 4/3-1 \\ 2/3-2 \end{pmatrix} = \begin{pmatrix} 2/3 \\ 1/3 \\ -4/3 \end{pmatrix}$$

$$\overrightarrow{OP'} = \overrightarrow{OP} + 2\,\overrightarrow{PM} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + 2\begin{pmatrix} 2/3 \\ 1/3 \\ -4/3 \end{pmatrix} = \begin{pmatrix} 1+4/3 \\ 2+2/3 \\ 1-8/3 \end{pmatrix} = \begin{pmatrix} 7/3 \\ 8/3 \\ -5/3 \end{pmatrix} \quad \boxed{P'\left(\frac{7}{3}, \frac{8}{3}, -\frac{5}{3}\right)}$$

b) $d(P, r) = |\overrightarrow{PM}| = \sqrt{\frac{4}{9}+\frac{1}{9}+\frac{16}{9}} = \boxed{\frac{\sqrt{21}}{3}}$

c) $d(P, P') = \frac{1}{2}|\overrightarrow{PM}| = \boxed{\frac{\sqrt{21}}{6}}$

Junio 15 fase específica

$$\begin{cases} 2x+2y+az=1 \\ 2x+ay+2z=-2 \\ ax+2y+2z=1 \end{cases}$$

Que se corten sobre una recta significa que es un sistema compatible indeterminado con rangos: $r(A) = r(A^*) = 2$

$$A = \begin{pmatrix} 2 & 2 & a \\ 2 & a & 2 \\ a & 2 & 2 \end{pmatrix} \qquad A^* = \begin{pmatrix} 2 & 2 & a & 1 \\ 2 & a & 2 & -2 \\ a & 2 & 2 & 1 \end{pmatrix} \qquad \begin{matrix} r(A) \leq 3 \\ r(A^*) \leq 3 \end{matrix}$$

$$|A| = \begin{vmatrix} 2 & 2 & a \\ 2 & a & 2 \\ a & 2 & 2 \end{vmatrix} = 4a+4a+4a-a^3-8-8 = -a^3+12a-16$$

$$|A| = 0 \rightarrow -a^3+12a-8=0 \; ;$$

	-1	0	12	-16
2		-2	-4	16
	-1	-2	+8	0
2		-2	-8	
	-1	-4	0	
-4		4		
	-1	0		

- Si $\begin{matrix} a \neq 2 \\ a \neq -4 \end{matrix} \Rightarrow \begin{matrix} r(A)=3 \\ r(A^*)=3 \\ n=3 \end{matrix} \rightarrow$ Solución única. Los 3 planos se cortarían en 1 punto.
- Si $a=2$: $A = \begin{pmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{pmatrix}$ $r(A)=1$

$$A^* = \begin{pmatrix} 2 & 2 & 2 & 1 \\ 2 & 2 & 2 & -2 \\ 2 & 2 & 2 & 1 \end{pmatrix} \qquad \begin{vmatrix} 2 & 1 \\ 2 & -2 \end{vmatrix} = -6 \Rightarrow r(A^*)=2$$

Sistema Incompatible. Los tres planos no tendrían ningún punto común. (Son dos planos idénticos y uno paralelo)

• Si $a=-4$: $A=\begin{pmatrix} 2 & 2 & -4 \\ 2 & -4 & 2 \\ -4 & 2 & 2 \end{pmatrix}$ $\begin{vmatrix} 2 & 2 \\ 2 & -4 \end{vmatrix} = -12 \neq 0 \Rightarrow r(A)=2$

$A^{*}=\left(\begin{array}{ccc|c} 2 & 2 & -4 & 1 \\ 2 & -4 & 2 & -2 \\ -4 & 2 & 2 & 1 \end{array}\right)$ $\begin{vmatrix} 2 & 2 & 1 \\ 2 & -4 & -2 \\ -4 & 2 & 1 \end{vmatrix} = -8+4+16-16-4+8=0 \Rightarrow r(A^{*})=2$; $n=3$

$\Rightarrow$ Sist. Compatible Indeterminado.

Los 3 planos tienen una recta común $\Rightarrow$ $\boxed{a=-4}$

b| c|

$\left.\begin{array}{l} 2x+2y-4z=1 \\ 2x-4y+2z=-2 \\ -4x+2y+2z=1 \end{array}\right\}$ $\left.\begin{array}{l} 2x+2y=1+4z \\ 2x-4y=-2-2z \end{array}\right\}$ $(z\in\mathbb{R})$

$x=\dfrac{\begin{vmatrix} 1+4z & 2 \\ -2-2z & -4 \end{vmatrix}}{\begin{vmatrix} 2 & 2 \\ 2 & -4 \end{vmatrix}} = \dfrac{-4-16z+4+4z}{-12} = z$

$y=\dfrac{\begin{vmatrix} 2 & 1+4z \\ 2 & -2-2z \end{vmatrix}}{\begin{vmatrix} 2 & 2 \\ 2 & -4 \end{vmatrix}} = \dfrac{-4-4z-2-8z}{-12} = \dfrac{-6-12z}{-12} = \dfrac{1}{2}+z$

$\boxed{\left.\begin{array}{l} x=r \\ y=\frac{1}{2}+r \\ z=r \end{array}\right\}}$ $\boxed{\text{Vector director: } \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}}$

También: $\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 2 & -4 \\ 2 & -4 & 2 \end{vmatrix} = \begin{pmatrix} -12 \\ -12 \\ -12 \end{pmatrix} \rightarrow$ vector director $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ ✓

Junio 15 fase Específica

$r: \left.\begin{array}{l} x-y+z=2 \\ 2x-2y+z=2 \end{array}\right\}$ $\left.\begin{array}{l} -y+z=2-x \\ -2y+z=2-2x \end{array}\right\}$ $(x\in\mathbb{R})$

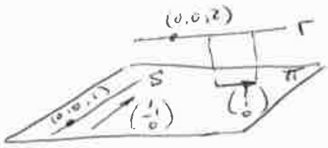
$y=\dfrac{\begin{vmatrix} 2-x & 1 \\ 2-2x & 1 \end{vmatrix}}{\begin{vmatrix} -1 & 1 \\ -2 & 1 \end{vmatrix}} = \dfrac{2-x-2+2x}{1} = x$

$z=\dfrac{\begin{vmatrix} -1 & 2-x \\ -2 & 2-2x \end{vmatrix}}{\begin{vmatrix} -1 & 1 \\ -2 & 1 \end{vmatrix}} = \dfrac{-2+2x+4-2x}{1} = 2$

$r: \left.\begin{array}{l} x=r \\ y=r \\ z=2 \end{array}\right\}$

$s: \left.\begin{array}{l} x+y=0 \\ z=1 \end{array}\right\} \rightarrow s: \left.\begin{array}{l} x=t \\ y=-t \\ z=1 \end{array}\right\}$

a| $\left.\begin{array}{l} r: \begin{cases} x=r \\ y=r \\ z=2 \end{cases} \\ s: \begin{cases} x=t \\ y=-t \\ z=1 \end{cases} \end{array}\right\}$ $\begin{array}{l} r=t \\ r=-t \\ 2=1 \end{array}$ ← Absurdo. Sistema Incompatible. Las rectas no tienen puntos comunes y, como no son paralelas, $\boxed{\text{se cruzan.}}$



b| $\begin{vmatrix} x & y & z-1 \\ 1 & 1 & 0 \\ 1 & -1 & 0 \end{vmatrix} = 0$

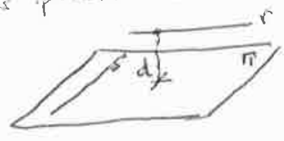
$-2(z-1)=0$

$\boxed{\pi: z-1=0}$

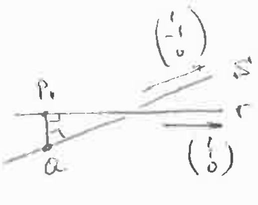
Valdría cualquier punto de r.

c| $d(r,s)=d(r,\pi)=d\left(P(0,0,2), \pi: z-1=0\right)=$

$=\dfrac{|2-1|}{\sqrt{0^2+0^2+1^2}}=\boxed{1}$



También: Buscando los puntos P y Q:

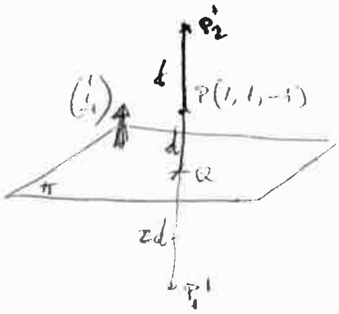


$P(r, r, 2)$, $Q(t, -t, 1)$ $\rightarrow$ $\vec{PQ} = \begin{pmatrix} t-r \\ -t-r \\ -1 \end{pmatrix}$

$\vec{PQ} \cdot \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = 0;\ t-r-t-r = 0 \ ;\ -2r = 0 \ ;\ r = 0$

$\vec{PQ} \cdot \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} = 0;\ t-r+t-r = 0 \ ;\ 2t = 0 \ ;\ t = 0$ $\rightarrow$ $\vec{PQ} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$

$d(r, s) = |\vec{PQ}| = \sqrt{0+0+1} = \boxed{1}$ ✓



Julio 15 fase General

a) $r: \begin{cases} x = 1+r \\ y = 1+r \\ z = -1-r \end{cases}$ $\pi: x+y-z=0$

$1+r+1+r+1+r=0$

$3r=-3$

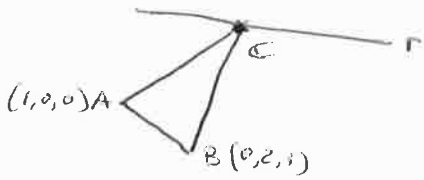
$r=-1$ $\rightarrow$ $x = 1-1 = 0$, $y = 1-1 = 0$, $z = -1+1 = 0$ $\boxed{Q(0,0,0)}$

b) $\vec{OP_1'} = \vec{OP} + 3\vec{PQ} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} + 3 \cdot \begin{pmatrix} 0-1 \\ 0-1 \\ 0+1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} + 3\begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ -2 \\ 2 \end{pmatrix} \Rightarrow \boxed{P_1'(-2,-2,2)}$

$\vec{OP_2'} = \vec{OP} + \vec{QP} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ -2 \end{pmatrix} \Rightarrow \boxed{P_2'(2,2,-2)}$

Julio 15 fase General

a) $r: \begin{cases} y=0 \\ z=10 \end{cases} \rightarrow r: \begin{cases} x=t \\ y=0 \\ z=10 \end{cases}$



$C \in r \Rightarrow C = (t, 0, 10)$

$\vec{AC} = \begin{pmatrix} t-1 \\ 0 \\ 10 \end{pmatrix}$, $\vec{BC} = \begin{pmatrix} t \\ -2 \\ 9 \end{pmatrix}$ $\quad |\vec{AC}| = |\vec{BC}| \Rightarrow \sqrt{(t-1)^2+0^2+10^2} = \sqrt{t^2+(-2)^2+9^2}$

$t^2-2t+1+100 = t^2+4+81$

$-2t = -16$, $t = 8$ $\Rightarrow$ $\boxed{C = (8,0,10)}$

b) $\vec{AC} = \begin{pmatrix} 7 \\ 0 \\ 10 \end{pmatrix}$, $\vec{BC} = \begin{pmatrix} 8 \\ -2 \\ 9 \end{pmatrix}$ $\quad \vec{AC} \times \vec{BC} = \begin{vmatrix} i & j & k \\ 7 & 0 & 10 \\ 8 & -2 & 9 \end{vmatrix} = \begin{pmatrix} 20 \\ 17 \\ -14 \end{pmatrix}$

$\text{Area} = \frac{1}{2}|\vec{AC} \times \vec{BC}| = \frac{1}{2}\sqrt{20^2+17^2+(-14)^2} = \boxed{\frac{\sqrt{885}}{2}\ u^2}$

Julio 15 fase Específica

$r: \begin{cases} x-2=0 \\ y+z+1=0 \end{cases} \longrightarrow r: \begin{cases} x=2 \\ y=r \\ z=-1-r \end{cases}$

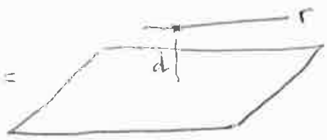
a) $r: \begin{cases} x=2 \\ y=r \\ z=-1-r \end{cases}$ $\pi: x-y-z+2=0$

$2-r-(-1-r)+2=0 \ ;\ 2-r+1+r+2=0 \ ;\ 5=0$ Absurdo.

Por lo tanto la $\boxed{\text{recta es paralela al plano.}}$

b) Valdría cualquier punto de r

$d(r, \pi) = d(P(2,0,-1), \pi: x-y-z+2=0) =$



$= \frac{|2-0+1+2|}{\sqrt{1+1+1}} = \boxed{\frac{5}{\sqrt{3}}}$

Julio 15 fase Específica

$$r: \begin{cases} x-2=0 \\ x+y+z-2=0 \end{cases} \rightarrow \begin{cases} x=2 \\ 2+y+z-2=0 \end{cases} \rightarrow r: \begin{cases} x=2 \\ y=r \\ z=-r \end{cases}$$

$$s: \begin{cases} x+y-z-4=0 \\ y+z=0 \end{cases} \rightarrow \begin{cases} x+y-z-4=0 \\ z=-y \end{cases} \rightarrow \begin{cases} x+y+y-4=0 \\ z=-y \end{cases} \rightarrow \begin{cases} x=4-2y \\ z=-y \end{cases} \rightarrow$$

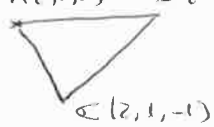
$$\rightarrow s': \begin{cases} x=4-2t \\ y=t \\ z=-t \end{cases}$$

Veamos los tres vértices del Triángulo:

$$\left.\begin{array}{l} r: \begin{cases} x=2 \\ y=r \\ z=-r \end{cases} \\ \text{Eje } X: \begin{cases} y=0 \\ z=0 \end{cases} \end{array}\right\} \rightarrow r=0 \rightarrow A(2,0,0)$$

$$\left.\begin{array}{l} s': \begin{cases} x=4-2t \\ y=t \\ z=-t \end{cases} \\ \text{Eje } X: \begin{cases} y=0 \\ z=0 \end{cases} \end{array}\right\} \rightarrow t=0 \rightarrow B(4,0,0)$$

$$\left.\begin{array}{l} r: \begin{cases} x=2 \\ y=r \\ z=-r \end{cases} \\ s': \begin{cases} x=4-2t \\ y=t \\ z=-t \end{cases} \end{array}\right\} \rightarrow \begin{cases} 2=4-2t \\ r=t \\ -r=-t \end{cases} ; \begin{cases} 2t=2 \\ r=t \\ r=t \end{cases} ; \begin{array}{l} \boxed{t=1} \rightarrow \boxed{C(2,1,-1)} \checkmark \\ \boxed{r=1} \rightarrow C(2,1,-1) \end{array}$$



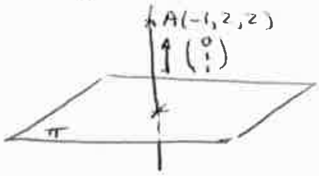
$$\left.\begin{array}{l} \vec{AB} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} \\ \vec{AC} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \end{array}\right\} \vec{AB} \times \vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 0 & 0 \\ 0 & 1 & -1 \end{vmatrix} = \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix}$$

$$\text{Area} = \frac{1}{2}\left|\vec{AB} \times \vec{AC}\right| = \frac{1}{2}\sqrt{0+4+4} = \frac{\sqrt{8}}{2} = \boxed{\sqrt{2}\ u^2}$$

Junio 16 fase General

$$r: \begin{cases} x-y-z+1=0 \\ y-z=0 \end{cases} \qquad \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & -1 \\ 0 & 1 & -1 \end{vmatrix} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \text{ es el vector director de la recta}$$

a)



$0 \cdot x + 1 \cdot y + 1 \cdot z + D = 0;$

$y + z + D = 0$

$2 + 2 + D = 0 \; ; \; D = -4$

$\boxed{\pi: y + z - 4 = 0}$

b)

$$\left.\begin{array}{l} \left.\begin{array}{l} x=-1 \\ y=3+r \\ z=3+r \end{array}\right| \\ y+z-4=0 \end{array}\right\} \quad 3+r+3+r-4=0$$

$2r = -2$

$\boxed{r=-1} \rightarrow \boxed{Q(-1,2,2)}$ Que coincide con A.

Junio 16 fase General

a) $\vec{AB} = \begin{pmatrix} 2 \\ m+5 \\ 0 \end{pmatrix}$

$\vec{AC} = \begin{pmatrix} 3 \\ 3 \\ 0 \end{pmatrix}$

A, B, C alineados $\Rightarrow \vec{AB} \parallel \vec{AC} \Rightarrow \frac{2}{3} = \frac{m+5}{3} = \frac{0}{0} \Rightarrow \boxed{m = -3}$

<u>También</u>

Recta AC : $\left.\begin{array}{l} x = 2+3r \\ y = -5+3r \\ z = 2 \end{array}\right\}$

$B \in AC \Rightarrow \left.\begin{array}{l} 2+3r = 4 \\ -5+3r = m \\ 2 = 2 \end{array}\right\} \rightarrow r = 2/3 \rightarrow m = -5+3\cdot\frac{2}{3} = -3$ ✓

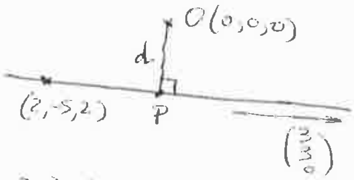
b) $\left.\begin{array}{l} x = 2+3r \\ y = -5+3r \\ z = 2 \end{array}\right\} \rightarrow \boxed{\frac{x-2}{3} = \frac{y+5}{3}} = \frac{z-2}{0}$

$x-2 = y+5$

$x - y - 7 = 0$

Recta : $\boxed{\left.\begin{array}{l} x - y - 7 = 0 \\ z = 2 \end{array}\right\}}$

c) $P(2+3r, -5+3r, 2)$



$\vec{OP} = (2+3r, -5+3r, 2)$

$\vec{OP} \perp \begin{pmatrix} 3 \\ 3 \\ 0 \end{pmatrix} \Rightarrow (2+3r)\cdot 3 + (-5+3r)\cdot 3 + 2\cdot 0 = 0$

$6 + 9r - 15 + 9r = 0$

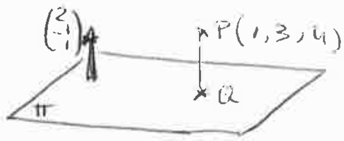
$18r = 9$

$r = 1/2 \longrightarrow P\left(2+\frac{3}{2}, -5+\frac{3}{2}, 2\right)$

$\boxed{P(7/2, -7/2, 2)}$

Junio 16 fase Específica

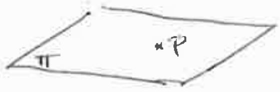
a) $r: \left.\begin{array}{l} x = 1+2r \\ y = 3-r \\ z = 4+r \end{array}\right\}$
$\pi: 2x - y + z - 3 = 0$ $\Bigg\} \longrightarrow$



$\rightarrow 2(1+2r) - (3-r) + (4+r) - 3 = 0$; $2 + 4r - 3 + r + 4 + r - 3 = 0$; $6r = 0$; $r = 0$

$r = 0 \rightarrow \boxed{Q(1,3,4)}$ ¡que coincide con P!

b) Obviamente, el simétrico es de nuevo $\boxed{P(1,3,4)}$



Junio 16 fase Específica

recta $\parallel \pi: 4x+y+z+2=0 \implies$ Vector Director $\perp \begin{pmatrix}4\\1\\1\end{pmatrix}$

recta $\perp$ $x = \frac{y}{-2} = z-5 \implies$ Vector Director $\perp \begin{pmatrix}1\\-2\\1\end{pmatrix}$ $\Bigg\} \implies$

$$\implies \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & 1 & 1 \\ 1 & -2 & 1 \end{vmatrix} = \begin{pmatrix}3\\-3\\-9\end{pmatrix}$$

lo vector director de la recta, podemos usar $\begin{pmatrix}1\\-1\\-3\end{pmatrix}$

$$r: \begin{cases} x = 2 + r \\ y = -1 - r \\ z = 1 - 3r \end{cases} \rightarrow \boxed{x-2 = \frac{y+1}{-1} = \frac{z-1}{-3}}$$

$-x+2 = y+1$; $0 = x+y-1$

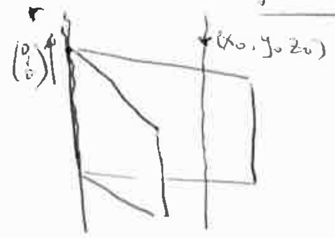
$-3y-3 = -z+1$; $-3y+z-4=0$

$$\boxed{r: \begin{cases} x+y-1=0 \\ -3y+z-4=0 \end{cases}}$$

Julio 16 fase General

a) $\begin{cases} \pi_1: x+z=0 \\ \pi_2: z-3=0 \end{cases} \rightarrow \begin{cases} x=-z \\ z=3 \end{cases} \rightarrow \begin{cases} x=-3 \\ z=3 \end{cases} \rightarrow \boxed{r: \begin{cases} x=-3 \\ y=r \\ z=3 \end{cases}}$ Se corta sobre esta recta.

b) Cualquier recta paralela al vector $\begin{pmatrix}0\\1\\0\end{pmatrix}$ será paralela a π_1 y a π_2:



$$\boxed{\frac{x-x_0}{0} = \frac{y-y_0}{1} = \frac{z-z_0}{0}}$$

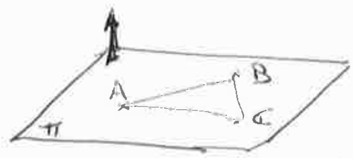
$x - x_0 = 0$; $0 = z - z_0$

Serviría: $\boxed{\begin{cases} x-x_0=0 \\ z-z_0=0 \end{cases}}$ $\begin{pmatrix} x_0 \in \mathbb{R} \\ z_0 \in \mathbb{R} \end{pmatrix}$

Julio 16 fase General

a) $\vec{AB} = \begin{pmatrix}1\\0\\-1\end{pmatrix}$

$\vec{AC} = \begin{pmatrix}2\\-2\\-4\end{pmatrix}$



$$\vec{AB} \times \vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & -1 \\ 2 & -2 & -4 \end{vmatrix} = \begin{pmatrix}-2\\2\\-2\end{pmatrix}$$

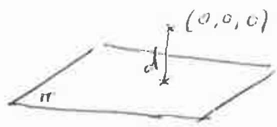
nos puede servir $\begin{pmatrix}-1\\1\\-1\end{pmatrix}$ como vector normal.

$\pi: -x+y-z+D=0$

$A \in \pi \rightarrow -0+2-1+D=0$, $D=-1$

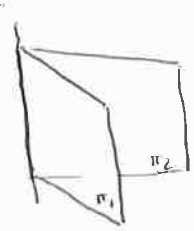
$$\boxed{\pi: -x+y-z-1=0}$$

b) $d(O(0,0,0), \pi: -x+y-z-1=0) = \frac{|-0+0-0-1|}{\sqrt{1+1+1}} = \boxed{\frac{1}{\sqrt{3}}}$



Julio 16 fase Específica

$\pi_1: x+y+z-6=0$
$\pi_2: 2x+3y+z+5=0$ $\longrightarrow$ $\begin{cases} x+y=6-z \\ 2x+3y=-5-z \end{cases}$

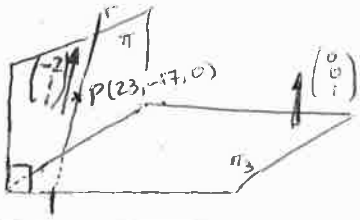


$$x=\frac{\begin{vmatrix} 6-z & 1 \\ -5-z & 3 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix}}=\frac{18-3z+5+z}{3-2}=23-2z$$

$$y=\frac{\begin{vmatrix} 1 & 6-z \\ 2 & -5-z \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix}}=\frac{-5-z-12+2z}{1}=-17+z$$

Los planos π_1 y π_2 se cortan sobre la recta : $r: \begin{cases} x=23-2r \\ y=-17+r \\ z=r \end{cases}$

$$\begin{vmatrix} x-23 & y+17 & z \\ -2 & 1 & 1 \\ 0 & 0 & 1 \end{vmatrix}=0$$

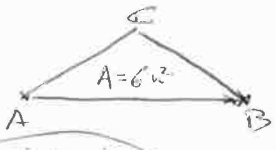


$1\cdot(x-23)+2(y+17)+0z=0$

$x-23+2y+34=0$; $\pi: \boxed{x+2y+11=0}$

Julio 16 fase Específica

$\vec{AB}=\begin{pmatrix}3\\2\\0\end{pmatrix}$, $\vec{AC}=\begin{pmatrix}-6\\a+3\\0\end{pmatrix}$ $\Big\}$ $\vec{AB}\times\vec{AC}=\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 2 & 0 \\ -6 & a+3 & 0 \end{vmatrix}=$



$=\begin{pmatrix}0\\0\\3a+9+12\end{pmatrix}=\begin{pmatrix}0\\0\\3a+21\end{pmatrix}$

$\text{Area}=\frac{1}{2}|\vec{AB}\times\vec{AC}|=\frac{1}{2}\sqrt{0^2+0^2+(3a+21)^2}=\frac{|3a+21|}{2}$

$\text{Area}=6 \Rightarrow \frac{|3a+21|}{2}=6$; $|3a+21|=12$

$3a+21=12$, $3a=-9$, $\boxed{a=-3}$; $3a+21=-12$, $3a=-32$, $\boxed{a=-32/3}$

Modelo 17

Ver Junio 11 fase General.

Modelo 17

$r: \frac{x+1}{1}=\frac{y-1}{2}=\frac{z-21}{3} \longrightarrow r: \begin{cases} x=-1+r \\ y=1+2r \\ z=21+3r \end{cases}$

a) $r: \begin{cases} x=-1+r \\ y=1+2r \\ z=21+3r \end{cases}$, $\pi: 2x+4y-3z=15$

$2(-1+r)+4(1+2r)-3(21+3r)=15$;
$-2+2r+4+8r-63-9r=15$;
$r=76 \longrightarrow x=75$, $y=153$, $z=249$

La recta corta al plano en el punto P(75, 153, 249)