

1) a) $\sin 75^\circ = \sin(45^\circ + 30^\circ) = \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6} + \sqrt{2}}{4}$

b) $\sin 15^\circ = \sin(45^\circ - 30^\circ) = \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6} - \sqrt{2}}{4}$

c) $\operatorname{tg} 135^\circ = \operatorname{tg}(180^\circ - 45^\circ) = \frac{\operatorname{tg} 180^\circ - \operatorname{tg} 45^\circ}{1 + \operatorname{tg} 180^\circ \cdot \operatorname{tg} 45^\circ} = \frac{0 - 1}{1 + 0 \cdot 1} = \frac{-1}{1} = -1$

d) $\operatorname{tg} 285^\circ = \operatorname{tg}(360^\circ - 75^\circ) = \frac{\operatorname{tg} 360^\circ - \operatorname{tg} 75^\circ}{1 + \operatorname{tg} 360^\circ \cdot \operatorname{tg} 75^\circ} = \frac{0 - \operatorname{tg} 75^\circ}{1 + 0 \cdot \operatorname{tg} 75^\circ} = -\operatorname{tg} 75^\circ = -\frac{\sin 75^\circ}{\cos 75^\circ} =$
 $= -\frac{\sin 75^\circ}{\sin 15^\circ} = -\frac{\frac{\sqrt{6} + \sqrt{2}}{4}}{\frac{\sqrt{6} - \sqrt{2}}{4}} = -\frac{\sqrt{6} + \sqrt{2}}{\sqrt{6} - \sqrt{2}} = -\frac{(\sqrt{6} + \sqrt{2})(\sqrt{6} + \sqrt{2})}{(\sqrt{6} - \sqrt{2})(\sqrt{6} + \sqrt{2})} = -\frac{(\sqrt{6})^2 + 2\sqrt{6}\sqrt{2} + (\sqrt{2})^2}{(\sqrt{6})^2 - (\sqrt{2})^2} =$
 $= -\frac{6 + 2\sqrt{12} + 2}{6 - 2} = -\frac{8 + 2\sqrt{12}}{4} = -\frac{8 + 4\sqrt{3}}{4} = -2 - \sqrt{3}$

2) a) $\cos(3x) = \cos(2x+x) = \cos 2x \cos x - \sin 2x \sin x = (\cos^2 x - \sin^2 x) \cos x - 2\sin x \cos x \cdot \sin x =$
 $= (\cos^2 x - 1 + \cos^2 x) \cos x - 2\sin^2 x \cos x = (2\cos^2 x - 1) \cos x - 2(1 - \cos^2 x) \cos x =$
 $= (2\cos^3 x - 1 - 2 + 2\cos^2 x) \cos x = \boxed{(4\cos^2 x - 3) \cos x} = \boxed{4\cos^3 x - 3\cos x}$

b) $\cos(4x) = \cos(2(2x)) = \cos^2(2x) - \sin^2(2x) = (\cos^2 x - \sin^2 x)^2 - (2\sin x \cos x)^2 =$
 $= (\cos^2 x - 1 + \cos^2 x)^2 - 4\sin^2 x \cos^2 x = (2\cos^2 x - 1)^2 - 4(1 - \cos^2 x) \cos^2 x =$
 $= 4\cos^4 x - 4\cos^2 x + 1 - 4\cos^2 x + 4\cos^4 x = \boxed{8\cos^4 x - 8\cos^2 x + 1}$

3)
$$\begin{aligned} \sin(\alpha + \beta) &= \sin \alpha \cos \beta + \cos \alpha \sin \beta \\ \sin(\alpha - \beta) &= \sin \alpha \cos \beta - \cos \alpha \sin \beta \\ \hline \sin(\alpha + \beta) + \sin(\alpha - \beta) &= 2\sin \alpha \cos \beta \end{aligned}$$

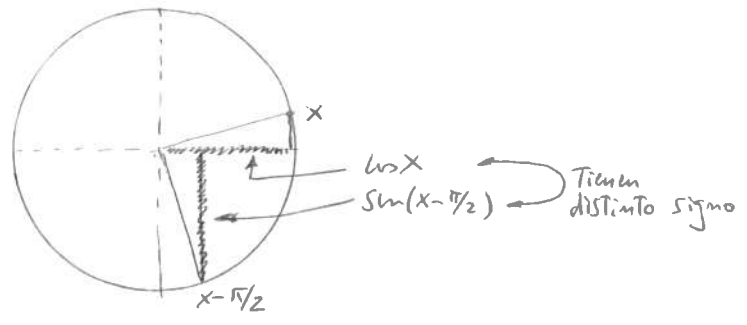
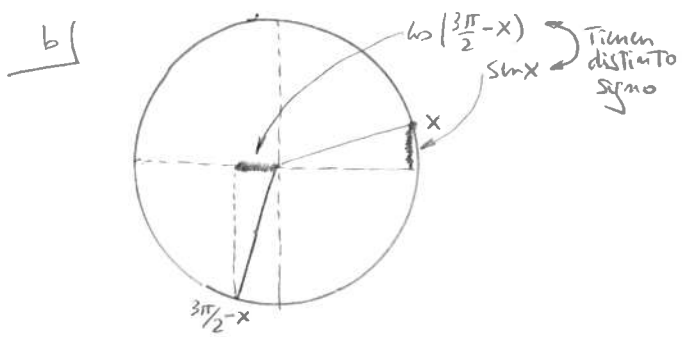
$$\left. \begin{aligned} -\cos(\alpha + \beta) &= -\cos \alpha \cos \beta + \sin \alpha \sin \beta \\ \cos(\alpha - \beta) &= \cos \alpha \cos \beta + \sin \alpha \sin \beta \\ \hline \cos(\alpha - \beta) - \cos(\alpha + \beta) &= 2\sin \alpha \sin \beta \end{aligned} \right\}$$

a) $\alpha = 3x, \beta = x$
 $\sin(3x+x) + \sin(3x-x) = 2\sin 3x \cos x \rightarrow \sin 4x + \sin 2x = 2\sin 3x \cos x$
 $\frac{\sin 4x + \sin 2x}{2} = \sin 3x \cos x$

b) $\alpha = 3x, \beta = x$
 $\cos(3x-x) - \cos(3x+x) = 2\sin 3x \sin x \rightarrow \cos 2x - \cos 4x = 2\sin 3x \sin x$
 $\frac{\cos 2x - \cos 4x}{2} = \sin 3x \sin x$

4) a) $\cos\left(\frac{3\pi}{2} - x\right) = \cos \frac{3\pi}{2} \cos x + \sin \frac{3\pi}{2} \sin x = 0 \cdot \cos x + (-1) \cdot \sin x = \boxed{-\sin x}$

$\sin\left(x - \frac{\pi}{2}\right) = \sin x \cos \frac{\pi}{2} - \cos x \sin \frac{\pi}{2} = \sin x \cdot 0 - \cos x \cdot 1 = \boxed{-\cos x}$



$$\textcircled{5} \text{ a) } \operatorname{Tgd} = -\frac{40}{9} \rightarrow 1 + \left(-\frac{40}{9}\right)^2 = \sec^2 d ; 1 + \frac{1600}{81} = \sec^2 d ; \sec^2 d = \frac{1681}{81} ;$$

$$\sec d = -\sqrt{\frac{1681}{81}} = -\frac{41}{9} \Rightarrow \cos d = -\frac{9}{41} \rightarrow \sin d = \operatorname{Tgd} \cdot \cos d = \frac{-40}{9} \cdot \frac{-9}{41} = \frac{40}{41}$$

$$\sin(2d) = 2\sin d \cos d = 2 \cdot \frac{40}{41} \cdot \frac{-9}{41} = \boxed{\frac{-720}{1681}}$$

$$\cos(2d) = \cos^2 d - \sin^2 d = \left(\frac{-9}{41}\right)^2 - \left(\frac{40}{41}\right)^2 = \frac{81}{1681} - \frac{1600}{1681} = \boxed{\frac{-1519}{1681}}$$

$$\text{b) } \operatorname{Tgd} = -\frac{40}{9} \rightarrow d = -77'32 + 180 = 102'68 \rightarrow 2d = 205'36 \rightarrow \begin{cases} \sin 205'36 = \boxed{-0'43} \approx \frac{-720}{1681} \\ \cos 205'36 = \boxed{-0'90} \approx \frac{-1519}{1681} \end{cases}$$

$$\textcircled{6} \text{ a) } \cos^2 \frac{x}{2} - \sin^2 \frac{x}{2} = \sin x ; \cos 2\frac{x}{2} = \sin x ; \cos x = \sin x ; 1 = \operatorname{Tg} x \Rightarrow x = \begin{cases} 45^\circ = \frac{\pi}{4} \\ 225^\circ = \frac{5\pi}{4} \end{cases}$$

$$\text{b) } \cos(2x) + \sin x = 4\sin^2 x$$

$$\cos^2 x - \sin^2 x + \sin x = 4\sin^2 x ; 1 - \sin^2 x - \sin^2 x + \sin x = 4\sin^2 x ; -6\sin^2 x + \sin x + 1 = 0 ;$$

$$\sin x = \frac{-1 \pm \sqrt{1+24}}{-12} = \frac{-1 \pm 5}{-12} \begin{cases} \frac{-4}{-12} = -\frac{1}{3} \Rightarrow x = \begin{cases} 340'53^\circ = \frac{189\pi}{180} \\ 159'47^\circ = \frac{111\pi}{180} \end{cases} \\ \frac{-6}{-12} = \frac{1}{2} \Rightarrow x = \begin{cases} 30^\circ = \frac{\pi}{6} \\ 150^\circ = \frac{5\pi}{6} \end{cases} \end{cases}$$

$$\text{c) } \sin 3x + \cos 3x = \sqrt{2}$$

$$(\sin 3x + \cos 3x)^2 = (\sqrt{2})^2 ; \sin^2 3x + 2\sin 3x \cos 3x + \cos^2 3x = 2 ; 1 + \sin 6x = 2 ; \sin 6x = 1 ;$$

$$\sin 6x = 1 \Rightarrow 6x = 90^\circ \pm N \cdot 360^\circ ; x = 15^\circ \pm N \cdot 60^\circ = \begin{cases} 15^\circ = \frac{\pi}{12} \\ 75^\circ = \frac{5\pi}{12} \\ 135^\circ = \frac{9\pi}{12} \\ 195^\circ = \frac{13\pi}{12} \\ 255^\circ = \frac{17\pi}{12} \\ 315^\circ = \frac{21\pi}{12} \end{cases} \leftarrow \text{No sense solution}$$

otra forma:

$$\sin 3x + \cos 3x = \sqrt{2}$$

$$\frac{1}{\sqrt{2}} \sin 3x + \frac{1}{\sqrt{2}} \cos 3x = 1 ; \frac{\sqrt{2}}{2} \sin 3x + \frac{\sqrt{2}}{2} \cos 3x = 1 ; \cos 45^\circ \sin 3x + \sin 45^\circ \cos 3x = 1 ;$$

$$\sin(3x + 45^\circ) = 1 \Rightarrow 3x + 45^\circ = 90^\circ \pm N \cdot 360^\circ ; 3x = 45^\circ \pm N \cdot 360^\circ ; x = 15^\circ \pm N \cdot 120^\circ = \begin{cases} 15^\circ \\ 135^\circ \\ 255^\circ \end{cases} \checkmark$$

$$\text{d) } \sin 2x \cdot \cos x = 3\sin^2 x$$

$$2\sin x \cos^2 x = 3\sin^2 x ; 2\sin x \cos^2 x - 3\sin^2 x = 0 ; 2\sin x (1 - \sin^2 x) - 3\sin^2 x = 0 ;$$

$$2\sin x - 2\sin^3 x - 3\sin^2 x = 0 ; \sin x \cdot (2 - 2\sin^2 x - 3\sin x) = 0 \begin{cases} \sin x = 0 \\ -2\sin^2 x - 3\sin x + 2 = 0 \end{cases}$$

$$\sin x = 0 \rightarrow \boxed{x = \begin{cases} 0 \\ \pi \end{cases}} = \begin{cases} 0^\circ \\ 180^\circ \end{cases}$$

$$-2\sin^2 x - 3\sin x + 2 = 0 ; \sin x = \frac{3 \pm \sqrt{9+16}}{-4} = \frac{3 \pm 5}{-4} \begin{cases} \frac{8}{-4} = -2 \\ \frac{-2}{-4} = \frac{1}{2} \end{cases} \rightarrow \boxed{x = \begin{cases} \frac{\pi}{6} \\ \frac{5\pi}{6} \end{cases}} = \begin{cases} 30^\circ \\ 150^\circ \end{cases}$$

$$e) \cos^2 x - \sin^2 x = \frac{1}{2}$$

$$\cos 2x = \frac{1}{2} \rightarrow 2x = \begin{cases} 60^\circ \pm N \cdot 360 \\ 300^\circ \pm N \cdot 360 \end{cases} \quad ; \quad x = \begin{cases} 30^\circ \pm N \cdot 180^\circ \\ 150^\circ \pm N \cdot 180^\circ \end{cases}$$

$$x = \begin{cases} 30^\circ = \frac{\pi}{6} \\ 210^\circ = \frac{7\pi}{6} \\ 150^\circ = \frac{5\pi}{6} \\ 330^\circ = \frac{11\pi}{6} \end{cases}$$

$$f) \operatorname{tg} x \cdot \operatorname{sec} x = \sqrt{2}$$

$$\frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} = \sqrt{2} \quad ; \quad \sin x = \sqrt{2} \cdot \cos^2 x \quad ; \quad \sin x = \sqrt{2} (1 - \sin^2 x) \quad ; \quad \sin x - \sqrt{2} + \sqrt{2} \sin^2 x = 0$$

$$\sqrt{2} \cdot \sin^2 x + \sin x - \sqrt{2} = 0$$

$$\sin x = \frac{-1 \pm \sqrt{1+8}}{2\sqrt{2}} = \frac{-1 \pm 3}{2\sqrt{2}} = \begin{cases} \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \rightarrow x = \begin{cases} 45^\circ = \frac{\pi}{4} \\ 135^\circ = \frac{3\pi}{4} \end{cases} \\ \frac{-4}{2\sqrt{2}} = \frac{-2}{\sqrt{2}} = -\sqrt{2} \end{cases}$$

$$g) 3 \cos x = 2 \sec x - 5$$

$$3 \cos x = \frac{2}{\cos x} - 5 \quad ; \quad 3 \cos^2 x = 2 - 5 \cos x \quad ; \quad 3 \cos^2 x + 5 \cos x - 2 = 0$$

$$\cos x = \frac{-5 \pm \sqrt{25+24}}{6} = \frac{-5 \pm 7}{6} \begin{cases} \frac{2}{6} = \frac{1}{3} \rightarrow x = \begin{cases} 71^\circ = 0,40\pi \\ 289^\circ = 1,61\pi \end{cases} \\ \frac{-12}{6} = -2 \end{cases}$$

$$h) \log_2 \cos x + 1 = \log_2 (\sec x)$$

$$\log_2 \cos x - \log_2 \sec x = -1 \quad ; \quad \log_2 \frac{\cos x}{\sec x} = -1 \quad ; \quad \log_2 \frac{\cos x}{1/\sin x} = -1 \quad ; \quad \log_2 \cos x \sin x = -1 \rightarrow$$

$$\rightarrow \cos x \sin x = 2^{-1} \quad ; \quad \cos x \sin x = \frac{1}{2} \quad ; \quad 2 \cos x \sin x = 1 \quad ; \quad \sin 2x = 1$$

$$\sin 2x = 1 \rightarrow 2x = 90^\circ \pm N \cdot 360^\circ \quad ; \quad x = 45^\circ \pm N \cdot 180^\circ \quad ; \quad x = \begin{cases} 45^\circ = \frac{\pi}{4} \\ 225^\circ = \frac{5\pi}{4} \end{cases}$$

$$i) \operatorname{tg}(2x) = -\operatorname{tg} x$$

$$\frac{2 \operatorname{tg} x}{1 - \operatorname{tg}^2 x} = -\operatorname{tg} x \quad ; \quad 2 \operatorname{tg} x = -\operatorname{tg} x + \operatorname{tg}^3 x \quad ; \quad 3 \operatorname{tg} x - \operatorname{tg}^3 x = 0 \quad ; \quad \operatorname{tg} x \cdot (3 - \operatorname{tg}^2 x) = 0$$

$$\operatorname{tg} x \cdot (3 - \operatorname{tg}^2 x) = 0 \begin{cases} \operatorname{tg} x = 0 \rightarrow x = \begin{cases} 0 \\ \pi \end{cases} \\ \operatorname{tg}^2 x = 3 \quad ; \quad \operatorname{tg} x = \begin{cases} \sqrt{3} \rightarrow x = \begin{cases} 60^\circ = \frac{\pi}{3} \\ 240^\circ = \frac{4\pi}{3} \end{cases} \\ -\sqrt{3} \rightarrow x = \begin{cases} 120^\circ = \frac{2\pi}{3} \\ 300^\circ = \frac{5\pi}{3} \end{cases} \end{cases} \end{cases}$$

$$7) a) \cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha = 1 - \sin^2 \alpha - \sin^2 \alpha = 1 - 2\sin^2 \alpha \Rightarrow 2\sin^2 \alpha = 1 - \cos 2\alpha \quad ; \quad \sin^2 \alpha = \frac{1 - \cos 2\alpha}{2}$$

$$\sin \alpha = \pm \sqrt{\frac{1 - \cos 2\alpha}{2}} \Rightarrow \boxed{\sin \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos \alpha}{2}}} \quad \checkmark$$

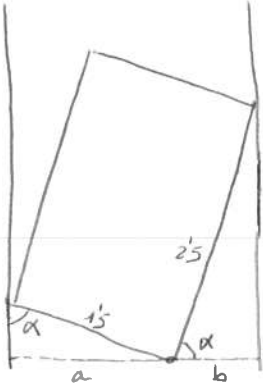
$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha = \cos^2 \alpha - (1 - \cos^2 \alpha) = 2\cos^2 \alpha - 1 \Rightarrow 2\cos^2 \alpha = 1 + \cos 2\alpha \quad ; \quad \cos^2 \alpha = \frac{1 + \cos 2\alpha}{2}$$

$$\cos \alpha = \pm \sqrt{\frac{1 + \cos 2\alpha}{2}} \Rightarrow \boxed{\cos \frac{\alpha}{2} = \pm \sqrt{\frac{1 + \cos \alpha}{2}}} \quad \checkmark$$

$$b) \cos 80^\circ = 0,17 \rightarrow \cos 40^\circ = + \sqrt{\frac{1+0,17}{2}} = \boxed{0,76} \rightarrow \sin 20^\circ = + \sqrt{\frac{1-0,76}{2}} = \boxed{0,34}$$

$$\begin{aligned} \frac{\sin 75^\circ + \sin 45^\circ}{\sin 75^\circ - \sin 45^\circ} &= \frac{\frac{\sqrt{6} + \sqrt{2}}{4} + \frac{\sqrt{2}}{2}}{\frac{\sqrt{6} + \sqrt{2}}{4} - \frac{\sqrt{2}}{2}} = \frac{\sqrt{6} + \sqrt{2} + 2\sqrt{2}}{\sqrt{6} + \sqrt{2} - 2\sqrt{2}} = \frac{\sqrt{6} + 3\sqrt{2}}{\sqrt{6} - \sqrt{2}} = \\ &= \frac{(\sqrt{6} + 3\sqrt{2})(\sqrt{6} + \sqrt{2})}{(\sqrt{6} - \sqrt{2})(\sqrt{6} + \sqrt{2})} = \frac{6 + \sqrt{12} + 3\sqrt{12} + 6}{6 - 2} = \frac{12 + 4\sqrt{12}}{4} = \boxed{3 + \sqrt{12}} = \boxed{3 + 2\sqrt{3}} \end{aligned}$$

9



$$\begin{aligned} \sin \alpha &= \frac{a}{1.5} \rightarrow a = 1.5 \sin \alpha \\ \cos \alpha &= \frac{b}{2.5} \rightarrow b = 2.5 \cos \alpha \\ 2 &= 1.5 \sin \alpha + 2.5 \cos \alpha \\ 4 &= 3 \sin \alpha + 5 \cos \alpha \rightarrow \cos \alpha = \frac{4 - 3 \sin \alpha}{5} \\ \sin^2 \alpha + \cos^2 \alpha &= 1 \\ \sin^2 \alpha + \left(\frac{4 - 3 \sin \alpha}{5} \right)^2 &= 1 \end{aligned}$$

$$\sin^2 \alpha + \frac{16 - 24 \sin \alpha + 9 \sin^2 \alpha}{25} = 1 ; \quad 25 \sin^2 \alpha + 16 - 24 \sin \alpha + 9 \sin^2 \alpha = 25 ;$$

$$34 \sin^2 \alpha - 24 \sin \alpha - 9 = 0 ; \quad \sin \alpha = \frac{24 \pm \sqrt{576 + 1224}}{68} = \begin{cases} 0.98 \Rightarrow \alpha = \boxed{77.65^\circ} = \boxed{77^\circ 38' 59.64''} \\ -0.27 \Rightarrow \alpha = \begin{cases} 102.35 \\ 344.28 \\ 164.28 \end{cases} \end{cases}$$

Los ángulos son: $\boxed{\alpha = 77.65^\circ}$ y $\boxed{90 - \alpha = 12.35^\circ}$

10

$$\sin a = \frac{3}{5} \rightarrow \cos a = -\sqrt{1 - \frac{9}{25}} = -\frac{4}{5} \rightarrow \operatorname{tga} = \frac{3/5}{-4/5} = -\frac{3}{4}$$

$$\cos b = \frac{5}{13} \rightarrow \sin b = -\sqrt{1 - \frac{25}{169}} = -\frac{12}{13} \rightarrow \operatorname{tg} b = \frac{-12/13}{5/13} = -\frac{12}{5}$$

$$\sin(a+b) = \sin a \cos b + \cos a \sin b = \frac{3}{5} \cdot \frac{5}{13} + \frac{-4}{5} \cdot \frac{-12}{13} = \boxed{\frac{63}{65}} = 0.9692$$

$$\operatorname{tg}(a-b) = \frac{\operatorname{tga} - \operatorname{tg} b}{1 + \operatorname{tga} \cdot \operatorname{tg} b} = \frac{-3/4 + 12/5}{1 + \frac{-3}{4} \cdot \frac{-12}{5}} = \boxed{\frac{33}{56}} = 0.5893$$

$$\begin{aligned} \sin(2a+b) &= \sin 2a \cos b + \cos 2a \sin b = 2 \sin a \cos a \cos b + (\cos^2 a - \sin^2 a) \sin b = \\ &= 2 \cdot \frac{3}{5} \cdot \frac{-4}{5} \cdot \frac{5}{13} + \left(\frac{16}{25} - \frac{9}{25} \right) \cdot \frac{-12}{13} = \boxed{-\frac{204}{325}} = -0.6277 \end{aligned}$$

b) $\sin a = \frac{3}{5} \mid \rightarrow a = 143^\circ$ $\cos b = \frac{5}{13} \mid \rightarrow b = 293^\circ$
 $90 < a < 180$ $270 < b < 360$

$$\sin(a+b) = \sin(143+293) = 0.9692 \quad \checkmark$$

$$\operatorname{tg}(a-b) = \operatorname{tg}(143-293) = 0.5893 \quad \checkmark$$

$$\sin(2a+b) = \sin(2 \cdot 143 + 293) = -0.6277 \quad \checkmark$$

11) $\sin B + \sin C = \cos B + \cos C$

$\sin B - \cos B = \cos C - \sin C$ Multiplicamos por $\frac{\sqrt{2}}{2}$

$\frac{\sqrt{2}}{2} \sin B - \frac{\sqrt{2}}{2} \cos B = \frac{\sqrt{2}}{2} \cos C - \frac{\sqrt{2}}{2} \sin C$ Ambos miembros con el desarrollo del seno de una resta

$\sin(B - 45^\circ) = \sin(45^\circ - C)$ los ángulos son: o iguales o suplementarios

$B - 45 = 45 - C$

$B + C = 90$

$A = 90^\circ$ ✓

$B - 45 = 180 - (45 - C)$

$B - 45 = 180 - 45 + C$

$B = 180 + C$ Es absurdo, ya que B no puede superar 180°.

Es absurdo, ya que B no puede superar 180°.

12)

a) $\cos^4 a - \sin^4 a = 2\cos^2 a - 1$

$\cos^4 a - \sin^4 a = (\cos^2 a + \sin^2 a) \cdot (\cos^2 a - \sin^2 a) = 1 \cdot (\cos^2 a - 1 + \cos^2 a) = 2\cos^2 a - 1$ ✓

b) $(\cos a + \sin a)^2 = \cos^2 a + 2\cos a \sin a + \sin^2 a = 1 + \sin 2a$ ✓

c) $(\operatorname{cosec} a + \operatorname{ctg} a) \cdot (\operatorname{cosec} a - \operatorname{ctg} a) = \operatorname{cosec}^2 a - \operatorname{ctg}^2 a = \frac{1}{\sin^2 a} - \frac{\cos^2 a}{\sin^2 a} = \frac{1 - \cos^2 a}{\sin^2 a} = \frac{\sin^2 a}{\sin^2 a} = 1$ ✓

13)

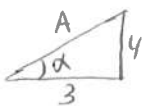
a) $\frac{\sec^2 a - \cos^2 a}{\operatorname{tg}^2 a} = \frac{\frac{1}{\cos^2 a} - \cos^2 a}{\frac{\sin^2 a}{\cos^2 a}} = \frac{(1 - \cos^4 a) / \cos^2 a}{\sin^2 a / \cos^2 a} = \frac{(1 + \cos^2 a)(1 - \cos^2 a)}{\sin^2 a} = \frac{(1 + \cos^2 a) \cdot \sin^2 a}{\sin^2 a} = 1 + \cos^2 a = 2 - \sin^2 a$

b) $\frac{\operatorname{cosec} a}{1 + \operatorname{ctg}^2 a} = \frac{\frac{1}{\sin a}}{1 + \frac{\cos^2 a}{\sin^2 a}} = \frac{\frac{1}{\sin a}}{\frac{\sin^2 a + \cos^2 a}{\sin^2 a}} = \frac{1}{\sin a} \cdot \frac{\sin^2 a}{\sin^2 a} = \sin a$

14)

b) $3\sin x + 4\cos x = A \sin(x + \alpha)$

$3\sin x + 4\cos x = A \sin x \cos \alpha + A \cos x \sin \alpha$



$3 = A \cos \alpha$
 $4 = A \sin \alpha$
 $A = \frac{3}{\cos \alpha}$
 $4 = \frac{3}{\cos \alpha} \cdot \sin \alpha$; $\frac{4}{3} = \operatorname{tg} \alpha \Rightarrow \alpha = 53'13''$

$9 = A^2 \cos^2 \alpha$
 $+ 16 = A^2 \sin^2 \alpha$
 $25 = A^2 \Rightarrow A = 5$

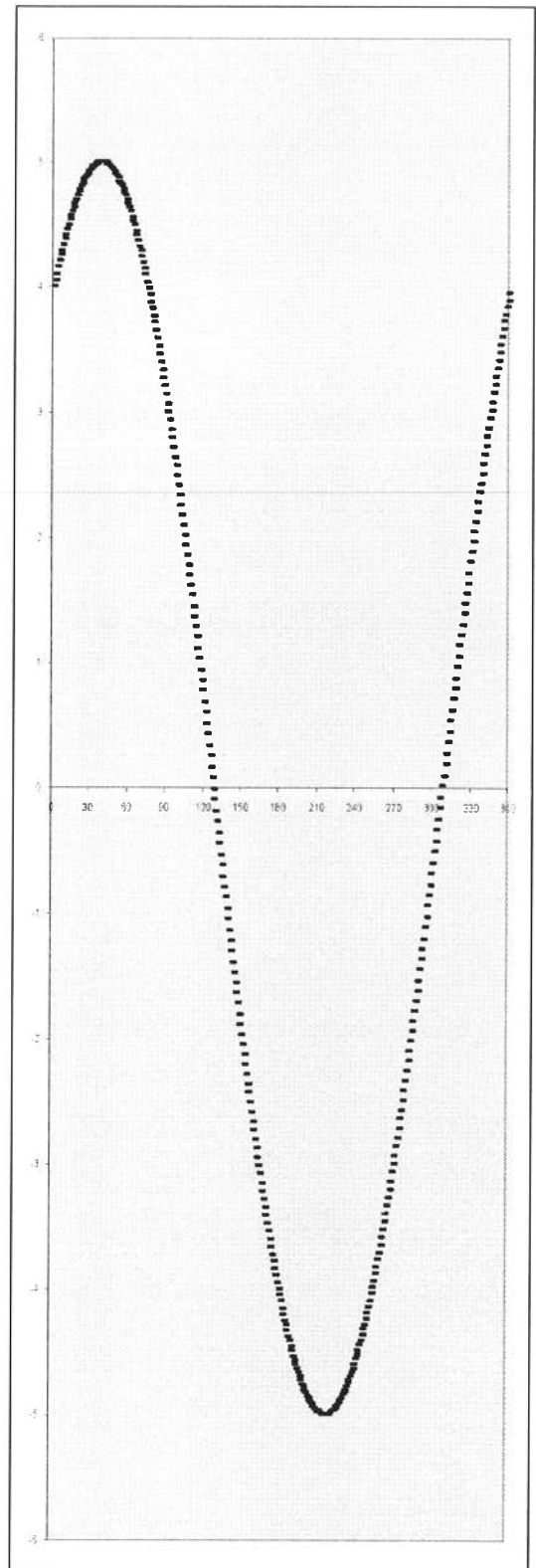
$3\sin x + 4\cos x = 5 \cdot \sin(x + 53'15'')$

El mayor valor de $3\sin x + 4\cos x$ será 5, para $x + 53'15'' = 90^\circ \Rightarrow x = 36'87''$

El menor valor de $3\sin x + 4\cos x$ será -5, para $x + 53'15'' = 270^\circ \Rightarrow x = 216'87''$

ángulo	3seno+4coseno	ángulo	3seno+4coseno	ángulo	3seno+4coseno	ángulo	3seno+4coseno
0	4	90	3	180	-4	270	-3
1	4,051748	91	2,92973346	181	-4,051748	271	-2,92973346
2	-4,102261798	92	2,858574494	182	-4,102261798	272	-2,858574494
3	4,151526008	93	2,786544779	183	-4,151526008	273	-2,786544779
4	4,199525622	94	2,713666256	184	-4,199525622	274	-2,713666256
5	-4,246246021	95	2,639961123	185	-4,246246021	275	-2,639961123
6	4,291672971	96	2,565451833	186	-4,291672971	276	-2,565451833
7	-4,335792637	97	2,490161081	187	-4,335792637	277	-2,490161081
8	4,378591578	98	2,414111802	188	-4,378591578	278	-2,414111802
9	-4,420056758	99	2,337327162	189	-4,420056758	279	-2,337327162
10	4,460175545	100	2,259830548	190	-4,460175545	280	-2,259830548
11	-4,49893572	101	2,181645569	191	-4,49893572	281	-2,181645569
12	4,536325475	102	2,102796039	192	-4,536325475	282	-2,102796039
13	-4,572333422	103	2,023305977	193	-4,572333422	283	-2,023305977
14	4,606948592	104	1,943199596	194	-4,606948592	284	-1,943199596
15	-4,64016044	105	1,862501298	195	-4,64016044	285	-1,862501298
16	4,671958851	106	1,781235665	196	-4,671958851	286	-1,781235665
17	-4,702334138	107	1,699427449	197	-4,702334138	287	-1,699427449
18	4,731277048	108	1,617101571	198	-4,731277048	288	-1,617101571
19	-4,758778766	109	1,534283109	199	-4,758778766	289	-1,534283109
20	4,784830913	110	1,450997289	200	-4,784830913	290	-1,450997289
21	-4,809425555	111	1,367269481	201	-4,809425555	291	-1,367269481
22	4,832555199	112	1,28312519	202	-4,832555199	292	-1,28312519
23	-4,854212799	113	1,198590046	203	-4,854212799	293	-1,198590046
24	4,87439176	114	1,113689801	204	-4,87439176	294	-1,113689801
25	-4,893085933	115	1,028450314	205	-4,893085933	295	-1,028450314
26	4,910289626	116	0,942897552	206	-4,910289626	296	-0,942897552
27	-4,925997596	117	0,857057574	207	-4,925997596	297	-0,857057574
28	4,94020506	118	0,770956527	208	-4,94020506	298	-0,770956527
29	-4,952907689	119	0,68462064	209	-4,952907689	299	-0,68462064
30	4,964101615	120	0,598076211	210	-4,964101615	300	-0,598076211
31	-4,973783428	121	0,511349602	211	-4,973783428	301	-0,511349602
32	4,981950177	122	0,424467232	212	-4,981950177	302	-0,424467232
33	-4,988599377	123	0,337455564	213	-4,988599377	303	-0,337455564
34	4,993729001	124	0,250341104	214	-4,993729001	304	-0,250341104
35	-4,997337486	125	0,163150387	215	-4,997337486	305	-0,163150387
36	4,999423734	126	0,075909974	216	-4,999423734	306	-0,075909974
37	-4,99998711	127	-0,011353562	217	-4,99998711	307	0,011353562
38	4,99902744	128	-0,09861364	218	4,99902744	308	0,09861364
39	-4,996545019	129	-0,18584368	219	-4,996545019	309	-0,18584368
40	4,992540602	130	-0,273017109	220	4,992540602	310	0,273017109
41	-4,987015408	131	-0,360107375	221	-4,987015408	311	-0,360107375
42	4,979971121	132	-0,447087949	222	4,979971121	312	0,447087949
43	-4,971409887	133	-0,533932335	223	-4,971409887	313	-0,533932335
44	4,961334313	134	-0,620614081	224	4,961334313	314	0,620614081
45	-4,949747468	135	-0,707106781	225	-4,949747468	315	-0,707106781
46	4,936652883	136	-0,79338409	226	4,936652883	316	0,79338409
47	-4,922054545	137	-0,879419726	227	-4,922054545	317	-0,879419726
48	4,905956902	138	-0,965187483	228	4,905956902	318	0,965187483
49	-4,888364857	139	-1,050661234	229	-4,888364857	319	-1,050661234
50	4,869283768	140	-1,135814943	230	4,869283768	320	1,135814943
51	-4,848719449	141	-1,220622673	231	-4,848719449	321	-1,220622673
52	4,826678162	142	-1,305058588	232	4,826678162	322	1,305058588
53	-4,803166623	143	-1,389096971	233	-4,803166623	323	-1,389096971
54	4,778191992	144	-1,472712221	234	4,778191992	324	1,472712221
55	-4,751761878	145	-1,555878868	235	-4,751761878	325	-1,555878868
56	4,723884332	146	-1,63857158	236	4,723884332	326	1,63857158
57	-4,694567844	147	-1,720765167	237	-4,694567844	327	-1,720765167
58	4,663821345	148	-1,802434592	238	4,663821345	328	1,802434592
59	-4,631654202	149	-1,883554978	239	-4,631654202	329	-1,883554978
60	4,598076211	150	-1,964101615	240	4,598076211	330	1,964101615
61	-4,563097602	151	-2,044049968	241	-4,563097602	331	-2,044049968
62	4,52672903	152	-2,123375683	242	4,52672903	332	2,123375683
63	-4,488981572	153	-2,202024598	243	-4,488981572	333	-2,202024598
64	4,449866726	154	-2,280062745	244	4,449866726	334	2,280062745
65	-4,409396408	155	-2,357376363	245	-4,409396408	335	-2,357376363
66	4,367582945	156	-2,433971901	246	4,367582945	336	2,433971901
67	-4,324439074	157	-2,509826028	247	-4,324439074	337	-2,509826028
68	4,279977937	158	-2,584915638	248	4,279977937	338	2,584915638
69	-4,234213078	159	-2,659217857	249	-4,234213078	339	-2,659217857
70	4,187158436	160	-2,732710053	250	4,187158436	340	2,732710053
71	-4,138828345	161	-2,805369839	251	-4,138828345	341	-2,805369839
72	4,089237526	162	-2,877175082	252	4,089237526	342	2,877175082
73	-4,038401087	163	-2,94810391	253	-4,038401087	343	-2,94810391
74	3,986334511	164	-3,018134716	254	3,986334511	344	3,018134716
75	-3,933053659	165	-3,08724617	255	-3,933053659	345	-3,08724617
76	3,878574761	166	-3,155417218	256	3,878574761	346	3,155417218
77	-3,822914412	167	-3,222627096	257	-3,822914412	347	-3,222627096
78	3,766089565	168	-3,28885533	258	3,766089565	348	3,28885533
79	-3,708117532	169	-3,354081748	259	-3,708117532	349	-3,354081748
80	3,64901597	170	-3,418286479	260	3,64901597	350	3,418286479
81	-3,588802882	171	-3,481449967	261	-3,588802882	351	-3,481449967
82	3,52749661	172	-3,543552972	262	3,52749661	352	3,543552972
83	-3,465115829	173	-3,604576576	263	-3,465115829	353	-3,604576576
84	3,401679539	174	-3,664502192	264	3,401679539	354	3,664502192
85	-3,337207065	175	-3,723311564	265	-3,337207065	355	-3,723311564
86	3,271718046	176	-3,78098678	266	3,271718046	356	3,78098678
87	-3,205232429	177	-3,83751027	267	-3,205232429	357	-3,83751027
88	3,137770468	178	-3,892864818	268	3,137770468	358	3,892864818
89	-3,069352711	179	-3,947033561	269	-3,069352711	359	-3,947033561

Maximo: 4,99998711 Minimo: -4,99998711



Como vemos en la tabla adjunta, realizada con una hoja de cálculo, el mayor valor y el menor valor de $3\text{sen}\alpha + 4\text{cos}\alpha$ son respectivamente 4,99998711 y -4,99998711 para, aproximadamente, 37° y 217° . Por lo estudiado analíticamente, estos valores son realmente 5 y -5. Mostramos también la gráfica que, con forma sinusoidal, oscila entre estos valores.