(1) $5x-7y-2=0 \rightarrow y=\frac{5x-2}{7}$ $\rightarrow$ pendiente $=\frac{5}{7} \Rightarrow \vec{u}=(7,5)$; $\rightarrow P(-1,-1)$ (por ejemplo)

$\boxed{\begin{cases} x=-1+7r \\ y=-1+5r \end{cases}}$ (Hay otras soluciones)

(2) $\alpha = 120° \rightarrow$ pendiente $= \operatorname{tg} 120° = -\sqrt{3}$ $\quad \boxed{y-1=-\sqrt{3}(x-2)}$

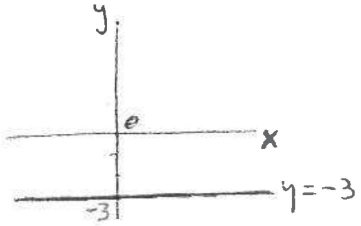
(3)

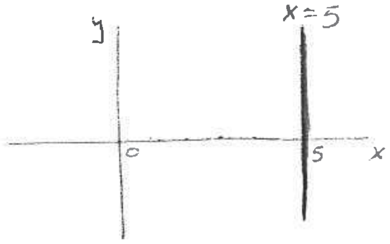
$\vec{AB}=(3,10)$

$C = A+\frac{1}{3}\vec{AB} = \boxed{\left(4,\frac{25}{3}\right)}$

$D = A+\frac{2}{3}\vec{AB} = \boxed{\left(5,\frac{35}{3}\right)}$

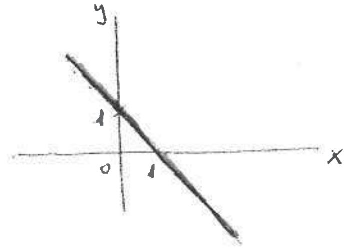
(4)

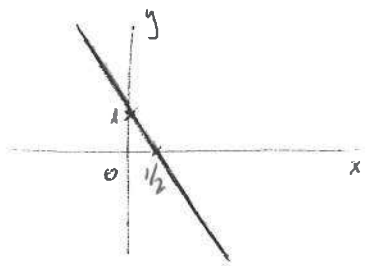




$\begin{cases} x=-t \\ y=1+t \end{cases}$

t	x	y
0	0	1
1	1	0





$\begin{cases} x=-3-r \\ y=7+2r \end{cases}$

$r=-3 \Rightarrow x=0,\ y=1$

$r=-\frac{7}{2} \Rightarrow x=\frac{1}{2},\ y=0$

r	x	y
-3	0	1
-7/2	1/2	0

(5) $A\left(-\frac{1}{2},2\right)$, $B(5,-1)$, $C\left(3,-\frac{1}{3}\right)$

$\vec{AB} = \left(5+\frac{1}{2},-1-2\right) = \left(\frac{11}{2},-3\right) \rightarrow$ pendiente $= \frac{-3}{11/2} = -\frac{6}{11}$

$\vec{AC} = \left(3+\frac{1}{2},-\frac{1}{3}-2\right) = \left(\frac{7}{2},-\frac{7}{3}\right) \rightarrow$ pendiente $= \frac{-7/3}{7/2} = -\frac{2}{3}$

$\boxed{\text{No están alineados}}$

(6) $-x+y+3=0 \rightarrow y=x-3 \rightarrow$ pendiente $=1$

$y+2=1(x-1)$ $\quad \boxed{y=x-3}$ $\quad \boxed{-x+y+3=0}$

(7) $y=\frac{3-5x}{6} \rightarrow \vec{v}=(6,-5)$

$\begin{cases} x=-1+2r \\ y=3-r \end{cases} \rightarrow \vec{w}=(2,-1)$

$\cos\alpha = \frac{|\vec{v}\cdot\vec{w}|}{|\vec{v}|\,|\vec{w}|} = \frac{|12+5|}{\sqrt{36+25}\sqrt{4+1}} = \frac{17}{\sqrt{61}\sqrt{5}} = \frac{17}{\sqrt{305}} \Rightarrow \boxed{\alpha = 13°}$

(8) $3x+2y-5=0 \rightarrow m=-\frac{3}{2}$
$3x+2y+7=0 \rightarrow m=-\frac{3}{2}$
$\rightarrow$ Paralelas (son distintas)

$x+3y-4=0 \rightarrow m=-\frac{1}{3}$
$x+2y-5=0 \rightarrow m=-\frac{1}{2}$
$\rightarrow$ secantes

$x+y-3=0 \rightarrow x+y-3=0$
$2x+2y-6=0 \rightarrow x+y-3=0$
$\rightarrow$ coincidentes.

(9) $\left.\begin{array}{l} 4x+y-14=0 \\ 7x+2y-28=0 \end{array}\right\} \rightarrow \begin{array}{l} 8x+2y-28=0 \\ 7x+2y-28=0 \\ \hline \boxed{x \qquad =0} \end{array} \rightarrow \boxed{y=14}$ $\quad P(0,14)$

$(0,14) \in 3x+my-7=0 \Rightarrow 0+14m-7=0 \Rightarrow \boxed{m=\frac{1}{2}}$

(10) $r_1 \equiv x=3$

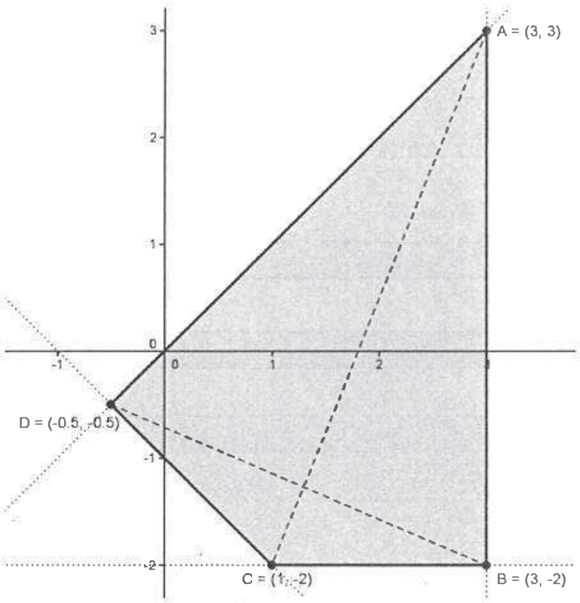
$r_2 \equiv y-x=0$

$r_3 \equiv y+x+1=0$

$r_4 \equiv y+2=0$

$\left.\begin{array}{l} r_1 \\ r_2 \end{array}\right\} \Rightarrow (3,3)$ $\quad \left.\begin{array}{l} r_1 \\ r_3 \end{array}\right\} \Rightarrow \cancel{(3,-4)}$ $\quad \left.\begin{array}{l} r_1 \\ r_4 \end{array}\right\} \Rightarrow (3,-2)$

$\left.\begin{array}{l} r_2 \\ r_3 \end{array}\right\} \Rightarrow \left(-\frac{1}{2},-\frac{1}{2}\right)$ $\quad \left.\begin{array}{l} r_2 \\ r_4 \end{array}\right\} \Rightarrow \cancel{(-2,-2)}$ $\quad \left.\begin{array}{l} r_3 \\ r_4 \end{array}\right\} \Rightarrow (1,-2)$



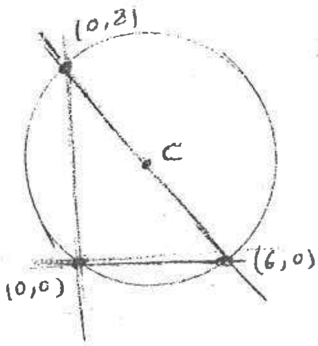
$\vec{AC} = (-2,-5)$ $\quad \frac{x-3}{2} = \frac{y-3}{5} \rightarrow \boxed{y=\frac{5x-9}{2}}$

$\vec{DB} = \left(3+\frac{1}{2}, -2+\frac{1}{2}\right) = \left(\frac{7}{2}, -\frac{3}{2}\right)$ $\quad \frac{x-3}{7} = \frac{y+2}{-3}$

$\boxed{y=\frac{-3x-5}{7}}$

Diagonales: $\boxed{\begin{array}{l} -5x+2y+9=0 \\ 3x+7y+5=0 \end{array}}$

(11)



$4x+3y-24=0$ $\quad x=0 \Rightarrow y=8 \quad (0,8)$

$\qquad\qquad\qquad\quad y=0 \Rightarrow x=6 \quad (6,0)$

Como es un triángulo, el punto medio es el centro de la circunferencia : $C(3,4)$

$R=\sqrt{3^2+4^2}=5$ $\quad \boxed{\text{Area} = 25\pi \text{ u.s.}}$

(12) $y=\frac{2-4x}{3} \rightarrow 3y=2-4x \rightarrow 4x+3y-2=0$

$4x+3y+C=0$

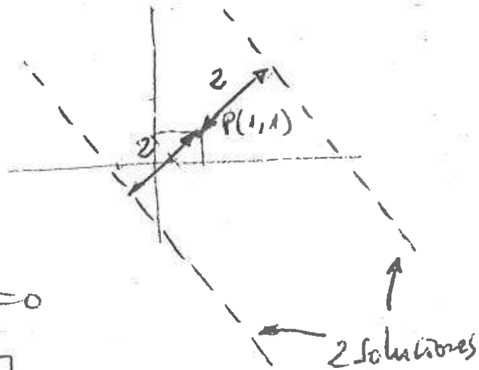
$2=\frac{|4+3+C|}{\sqrt{16+9}} \Rightarrow |7+C|=10$

$7+C=10 \qquad 7+C=-10$

$\boxed{C=3} \qquad \boxed{C=-17}$

$4x+3y+3=0 \qquad 4x+3y-17=0$

$\boxed{y=\frac{-4x-3}{3}} \qquad \boxed{y=\frac{17-4x}{3}}$



13) $\frac{|3x-y-5|}{\sqrt{9+1}} = \frac{|-6x+2y+8|}{\sqrt{36+4}} \longrightarrow \frac{|3x-y-5|}{\sqrt{10}} = \frac{|-6x+2y+8|}{\sqrt{40}} \longrightarrow$

$\rightarrow \frac{|3x-y-5|}{1} = \frac{|-6x+2y+8|}{2}$

Solución

Las rectas son paralelas, por lo que sólo hay una recta solución, que será también paralela a las rectas dadas.

$3x-y-5 = \frac{-6x+2y+8}{2}$

$3x-y-5 = -3x+y+4$

$\boxed{6x-2y-9=0}$

$3x-y-5 = -\frac{-6x+2y+8}{2}$

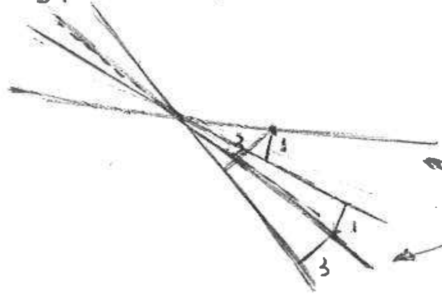
$3x-y-5 = 3x-y-4$

$-5=-4$

Sin solución

14) $\frac{|2x+y-3|}{\sqrt{4+1}} = 3\,\frac{|2x+4y-6|}{\sqrt{4+16}}$

$\frac{|2x+y-3|}{\cancel{\sqrt{5}}} = 3\,\frac{|2x+4y-6|}{2\cancel{\sqrt{5}}}$



2 soluciones

$2x+y-3 = \frac{3}{2}(2x+4y-6)$

$4x+2y-6 = 6x+12y-18$

$0 = 2x+10y-12$

$\boxed{0 = x+5y-6}$

$2x+y-3 = -\frac{3}{2}(2x+4y-6)$

$4x+2y-6 = -6x-12y+18$

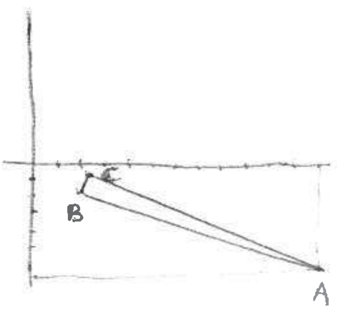
$10x+14y-24=0$

$\boxed{5x+7y-12=0}$

15) $\left.\begin{array}{l} x+2y+2=0 \\ 2x+3y-3=0 \end{array}\right\} \longrightarrow \begin{array}{l} 2x+4y+4=0 \\ 2x+3y-3=0 \\ \hline y+7=0 \end{array} \longrightarrow \boxed{y=-7}\ \boxed{x=12}$ $A(12,-7)$

$\left.\begin{array}{l} x+2y+2=0 \\ 3x-y-8=0 \end{array}\right\} \longrightarrow \begin{array}{l} x+2y+2=0 \\ 6x-2y-16=0 \\ \hline 7x \quad -14=0 \end{array} \longrightarrow \boxed{x=2}\ \boxed{y=-2}$ $B(2,-2)$

$\left.\begin{array}{l} 2x+3y-3=0 \\ 3x-y-8=0 \end{array}\right\} \longrightarrow \begin{array}{l} 2x+3y-3=0 \\ 9x-3y-24=0 \\ \hline 11x \quad -27=0 \end{array} \longrightarrow \boxed{x=\frac{27}{11}} \rightarrow \boxed{y=-\frac{7}{11}}$ $C\left(\frac{27}{11}, -\frac{7}{11}\right)$



Ecuación BC

$\vec{BC} = \left(\frac{27}{11}-2, -\frac{7}{11}+2\right) = \left(+\frac{5}{11}, \frac{15}{11}\right)$

Pendiente $= \frac{15/11}{+5/11} = +3$

$\left.\begin{array}{l} B(2,-2) \\ m=-3 \end{array}\right| \Rightarrow y+2 = +3(x-2)$

$\boxed{-3x+y+8=0}$

Altura h_a

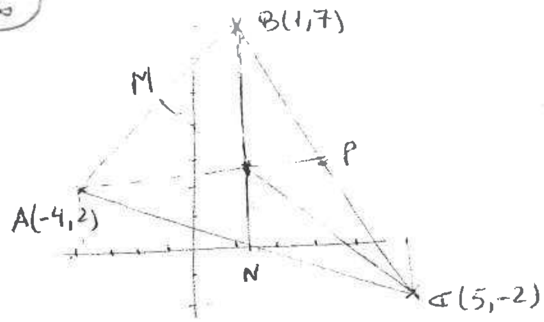
$h_a = d(A, \overline{BC}) = \frac{|-3\cdot 12-7+8|}{\sqrt{9+1}} = \frac{35}{\sqrt{10}}$

Base $\overline{BC}$

$\overline{BC} = |\vec{BC}| = \sqrt{\frac{25}{121}+\frac{225}{121}} = \sqrt{\frac{250}{121}} = \frac{5\sqrt{10}}{11}$

Área $= \frac{\frac{5\sqrt{10}}{11}\cdot\frac{35}{\sqrt{10}}}{2} = \boxed{\frac{175}{22}} = 7'955$ u.s.

(16)



$M = \left(-\frac{3}{2}, \frac{9}{2}\right)$

$\vec{MC} = \left(5+\frac{3}{2}, -2-\frac{9}{2}\right) = \left(\frac{13}{2}, \frac{-13}{2}\right) \rightarrow$ Pend $= -1$

Mediana vértice C : $y+2 = -1(x-5)$ $\boxed{y = 3-x}$

$N = (1/2, 0)$

$\vec{BN} = \left(\frac{1}{2}-1, 0-7\right) = \left(-\frac{1}{2}, -7\right) \rightarrow$ Pend $= 14$

Mediana vértice B : $y-7 = 14(x-1)$ $\boxed{y = 14x-7}$

$P = (3, 5/2)$

$\vec{AP} = \left(3+4, \frac{5}{2}-2\right) = \left(7, \frac{1}{2}\right) \rightarrow$ Pend $= \frac{1}{14}$

Mediana vértice A : $y-2 = \frac{1}{14}(x+4)$ $\boxed{y = \frac{x+32}{14}}$

$$\left.\begin{array}{l} y = 3-x \\ y = 14x-7 \end{array}\right\} \rightarrow 3-x = 14x-7$$

$-15x = -10$

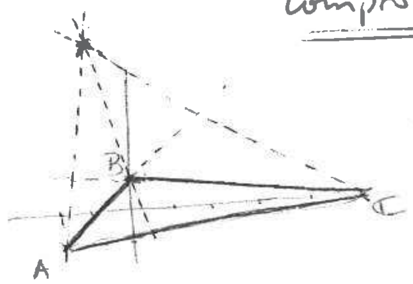
$x = \frac{10}{15} = \frac{2}{3}$

$y = 3 - \frac{2}{3} = \frac{7}{3}$

$\boxed{\text{Baricentro} = \left(\frac{2}{3}, \frac{7}{3}\right)}$

Comprobación : $x = \frac{2}{3} \rightarrow y = \frac{2/3+32}{14} = \frac{7}{3}$ ✓

(17)



A(-2,-1)
B(0,1)
C(5,0)

$\vec{AB} = (2, +2) \rightarrow$ Pend $= +1$

Altura vértice C : $y = -1(x-5)$ $\boxed{y = -x+5}$

$\vec{AC} = (7,1) \rightarrow$ pend $= \frac{1}{7}$

Altura vértice B : $y-1 = -7x$ $\boxed{y = 1-7x}$

$\vec{BC} = (5,-1) \rightarrow$ pend $= -\frac{1}{5}$

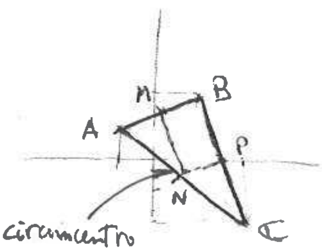
Altura vértice A : $y+1 = 5(x+2)$ $\boxed{y = 5x+9}$

$$\left.\begin{array}{l} y = -x+5 \\ y = 1-7x \end{array}\right\} \quad -x+5 = 1-7x \quad ; \quad 6x = -4 \quad x = -\frac{2}{3} \Rightarrow y = \frac{2}{3}+5 = \frac{17}{3}$$

$\boxed{\text{Ortocentro } \left(-\frac{2}{3}, \frac{17}{3}\right)}$

Comprobación : $x = -\frac{2}{3} \rightarrow y = 5 \cdot \frac{-2}{3} + 9 = \frac{17}{3}$ ✓

(18)



A(-1,1)
B(1,2)
C(2,-2)

Comprobación : $x = \frac{5}{6} \rightarrow y = \frac{2 \cdot \frac{5}{6} - 3}{8} = -\frac{1}{6}$ ✓

$M = \left(0, \frac{3}{2}\right)$

$\vec{AB} = (2,1) \rightarrow$ pend $= \frac{1}{2}$

Mediatriz lado $\vec{AB}$: $y - \frac{3}{2} = -2x$ $\boxed{y = \frac{3}{2} - 2x}$

$N = \left(\frac{1}{2}, -\frac{1}{2}\right)$

$\vec{AC} = (3,-3) \rightarrow$ pend $= -1$

Mediatriz lado $\vec{AC}$: $y + \frac{1}{2} = 1 \cdot \left(x - \frac{1}{2}\right)$ $\boxed{y = x-1}$

$P = \left(\frac{3}{2}, 0\right)$

$\vec{BC} = (1,-4) \rightarrow$ pend $= -4$

Mediatriz lado $\vec{BC}$: $y = \frac{1}{4}\left(x - \frac{3}{2}\right)$ $\boxed{y = \frac{2x-3}{8}}$

$$\left.\begin{array}{l} y = \frac{3}{2} - 2x \\ y = x-1 \end{array}\right\} \quad x-1 = \frac{3}{2} - 2x \quad ; \quad 3x = \frac{5}{2} \quad x = \frac{5}{6} \Rightarrow y = \frac{5}{6} - 1 = -\frac{1}{6}$$

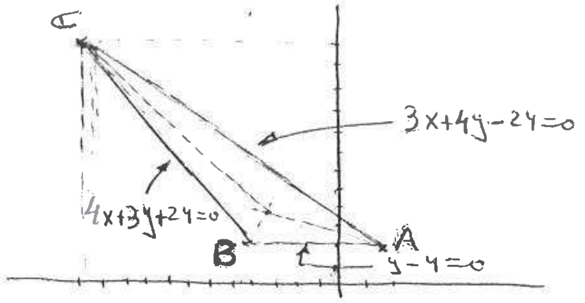
$\boxed{\text{Circuncentro } \left(\frac{5}{6}, -\frac{1}{6}\right)}$

(19)

$$\left.\begin{array}{l} y-4=0 \\ 3x+4y-24=0 \end{array}\right\} \rightarrow \boxed{\begin{array}{l} x=\frac{8}{3} \\ y=4 \end{array}} \quad A\left(\tfrac{8}{3},4\right)$$

$$\left.\begin{array}{l} y-4=0 \\ 4x+3y+24=0 \end{array}\right\} \rightarrow \boxed{\begin{array}{l} x=-9 \\ y=4 \end{array}} \quad B(-9,4)$$

$$\left.\begin{array}{l} 3x+4y-24=0 \\ 4x+3y+24=0 \end{array}\right\} \rightarrow \boxed{\begin{array}{l} x=-24 \\ y=24 \end{array}} \quad C(-24,24)$$



$$\frac{|y-4|}{1} = \frac{|3x+4y-24|}{5} \quad \begin{cases} y-4 = \frac{3x+4y-24}{5} \rightarrow 5y-20 = 3x+4y-24 \rightarrow \cancel{y=3x-4} \\ y-4 = -\frac{3x+4y-24}{5} \rightarrow 5y-20 = -3x-4y+24 \rightarrow \boxed{y=\frac{-3x+44}{9}} \end{cases}$$

$$\frac{|y-4|}{1} = \frac{|4x+3y+24|}{5} \quad \begin{cases} y-4 = \frac{4x+3y+24}{5} \rightarrow 5y-20 = 4x+3y+24 \rightarrow \boxed{y=2x+22} \\ y-4 = -\frac{4x+3y+24}{5} \rightarrow 5y-20 = -4x-3y-24 \rightarrow \cancel{y=\frac{-x-1}{2}} \end{cases}$$

$$\frac{|3x+4y-24|}{5} = \frac{|4x+3y+24|}{5} \quad \begin{cases} 3x+4y-24 = 4x+3y+24 \rightarrow \cancel{y=x+48} \\ 3x+4y-24 = -4x-3y-24 \rightarrow \boxed{y=-x} \end{cases}$$

$$\left.\begin{array}{l} y=2x+22 \\ y=-x \end{array}\right\} \rightarrow 2x+22=-x \rightarrow \boxed{\begin{array}{l} x=-\frac{22}{3} \\ y=\frac{22}{3} \end{array}} \qquad \boxed{\text{Incentro } \left(-\frac{22}{3}, \frac{22}{3}\right)}$$

Comprobación: $x=-22/3 \rightarrow y = \frac{-3\cdot\frac{22}{3}+44}{9} = \frac{22}{3}$ ✓

(20) $\vec{a}(3,-4)$, $\vec{b}(5,x)$

a) $\vec{a} \parallel \vec{b} \Longrightarrow \frac{5}{3} = \frac{x}{-4} \Longrightarrow \boxed{x=-\frac{20}{3}}$

b) $\vec{a} \perp \vec{b} \Longrightarrow (3,-4)\cdot(5,x)=0 \Longrightarrow 15-4x=0 \Longrightarrow \boxed{x=\frac{15}{4}}$

c) $\alpha=60^\circ \Longrightarrow \vec{a}\cdot\vec{b} = |\vec{a}|\,|\vec{b}| \cos 60^\circ$

$$15-4x = \sqrt{9+16}\,\sqrt{25+x^2}\cdot\frac{1}{2}$$

$$30-8x = 5\sqrt{25+x^2}$$

$$(30-8x)^2 = 25(25+x^2)$$

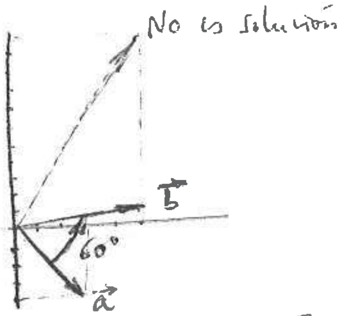
$$900-480x+64x^2 = 625+25x^2$$

$$39x^2-480x+275=0$$

$$x = \frac{480 \pm \sqrt{480^2-4\times39\times275}}{78} = \frac{480 \pm 250\sqrt{3}}{78} = \frac{240 \pm 125\sqrt{3}}{78}$$

$\cancel{\parallel}$

cos 60



Otro método: $(3-4i)\cdot 1_{60^\circ} = (3-4i)\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right) = \left(\frac{3}{2}+2\sqrt{3}\right) + i\left(\frac{3\sqrt{3}}{2}-2\right)$

El vector $(5,x)$ debe ser paralelo a $\left(\frac{3}{2}+2\sqrt{3}, \frac{3\sqrt{3}}{2}-2\right)$ luego:

$$\frac{5}{\frac{3}{2}+2\sqrt{3}} = \frac{x}{\frac{3\sqrt{3}}{2}-2} \Longrightarrow$$

$$x = \frac{\frac{15\sqrt{3}}{2}-10}{\frac{3}{2}+2\sqrt{3}} = \frac{15\sqrt{3}-20}{3+4\sqrt{3}} = \frac{(15\sqrt{3}-20)(3-4\sqrt{3})}{(3+4\sqrt{3})(3-4\sqrt{3})} = \frac{45\sqrt{3}-180-60+80\sqrt{3}}{9-48} = \frac{125\sqrt{3}-240}{-39} = \boxed{\frac{240-125\sqrt{3}}{39}}$$

21) $\vec{a}\cdot\vec{b}=16 \quad |\vec{a}|=2 \quad |\vec{b}|=12$

$$(3\vec{a}-\vec{b})^2 = 9\,\vec{a}\cdot\vec{a} - 6(\vec{a}\cdot\vec{b}) + \vec{b}\cdot\vec{b} = 9|\vec{a}|^2 - 6(\vec{a}\cdot\vec{b}) + |\vec{b}|^2 = 36 - 96 + 144 = \boxed{84}$$

$$(4\vec{a}+5\vec{b})(4\vec{a}-5\vec{b}) = 16\,\vec{a}\cdot\vec{a} \cancel{- 20\,\vec{a}\cdot\vec{b}} \cancel{+ 20\,\vec{b}\cdot\vec{a}} - 25\,\vec{b}\cdot\vec{b} =$$
$$= 16|\vec{a}|^2 - 25|\vec{b}|^2 = 16\cdot 4 - 25\cdot 144 = \boxed{-3536}$$

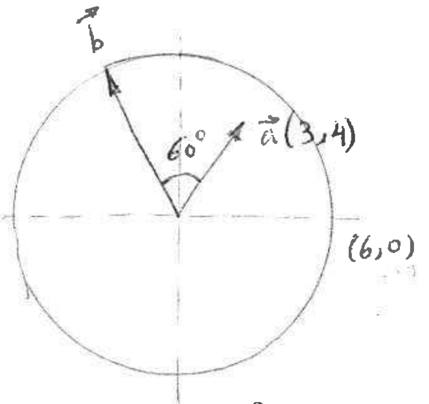
22) $|\vec{a}+\vec{b}|=8 \Rightarrow (\vec{a}+\vec{b})(\vec{a}+\vec{b})=64 \Rightarrow \vec{a}\cdot\vec{a}+2\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{b}=64$

$|\vec{a}-\vec{b}|=6 \Rightarrow (\vec{a}-\vec{b})(\vec{a}-\vec{b})=36 \Rightarrow \vec{a}\vec{a}-2\vec{a}\vec{b}+\vec{b}\vec{b}=36$

$$4\,\vec{a}\cdot\vec{b} = 28$$

$$\boxed{\vec{a}\cdot\vec{b}=7}$$

23)



$\vec{b}=(b_1,b_2)$

$\vec{a}\cdot\vec{b}=|\vec{a}|\,|\vec{b}|\cos 60^\circ$

$3b_1+4b_2=5\cdot 6\cdot \frac{1}{2}$

$3b_1+4b_2=15 \longrightarrow b_2=\frac{15-3b_1}{4}$

$b_1^2+b_2^2=36$

$$b_1^2+\frac{(15-3b_1)^2}{16}=36 \quad ; \quad 16b_1^2+225-90b_1+9b_1^2=576$$
$$25b_1^2-90b_1-351=0$$

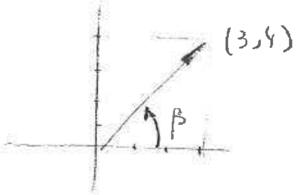
$$b_1=\frac{90\pm\sqrt{8100+35100}}{50}=\frac{90\pm 10\sqrt{432}}{50}=\frac{90\pm 120\sqrt{3}}{50}=\frac{9\pm 12\sqrt{3}}{5}$$

Como $\vec{b}$ pertenece al 2º cuadrante, escojo la solución negativa:

$$\boxed{b_1=\frac{9-12\sqrt{3}}{5}} \Rightarrow b_2=\frac{15-3\,\frac{9-12\sqrt{3}}{5}}{4}=\frac{75-27+36\sqrt{3}}{20}=$$
$$=\frac{48+36\sqrt{3}}{20}=\boxed{\frac{12+9\sqrt{3}}{5}}$$

$$\boxed{\vec{b}=\left(\frac{9-12\sqrt{3}}{5},\ \frac{12+9\sqrt{3}}{5}\right)}=\boxed{(-2'357,\ 5'518)}$$

Otra forma:



$$tg\,\beta=\frac{4}{3} \Rightarrow \beta=53,13^\circ$$

Hay que girar $6+0i$ un ángulo de $60+53,13^\circ=113,13^\circ$

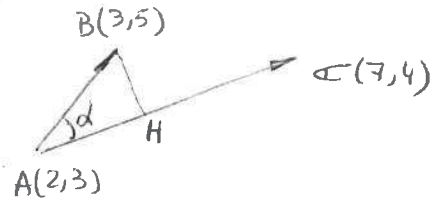
$$b_1+i\,b_2=(6+0i)\cdot 1_{113,13^\circ}=6\,(\cos 113,13^\circ+i\,\mathrm{sen}\,113,13^\circ)=$$
$$=-2'357+i\,5'518$$

$$\boxed{\vec{b}=(-2'357,\ 5'518)}$$

Otra forma: $\vec{b}=(3+4i)\cdot\left(\frac{6}{5}\right)_{60^\circ}=(3+4i)\cdot\frac{6}{5}(\cos 60^\circ+i\,\mathrm{sen}\,60^\circ)=\frac{6}{5}(3+4i)\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)=$

$=\frac{6}{5}\left(\frac{3}{2}+\frac{3\sqrt{3}}{2}i+\frac{4}{2}i-\frac{4\sqrt{3}}{2}\right)=\frac{3}{5}\left(3-4\sqrt{3}+i(3\sqrt{3}+4)\right)\Rightarrow \boxed{\vec{b}=\left(\frac{9-12\sqrt{3}}{5},\ \frac{9\sqrt{3}+12}{5}\right)}$

(24)



$\overrightarrow{AB} = (1,2)$
$\overrightarrow{AC} = (5,1)$

$\overrightarrow{AB}\cdot\overrightarrow{AC} = |\overrightarrow{AB}|\,|\overrightarrow{AC}|\cos\alpha$

$$\cos\alpha = \frac{(1,2)(5,1)}{\sqrt{1+4}\sqrt{25+1}} = \frac{7}{\sqrt{130}} \Rightarrow \boxed{\alpha = 52^{\circ}}$$

$$\overline{AH} = \overline{AB}\cdot\cos\alpha = \sqrt{1+4}\cdot\frac{7}{\sqrt{130}} = \boxed{\frac{7}{\sqrt{26}}} = 1'373$$

$$H = A + \frac{\overline{AH}}{\overline{AC}}\overrightarrow{AC} = (2,3) + \frac{7/\sqrt{26}}{\sqrt{26}}(5,1) = (2,3) + \frac{7}{26}(5,1) =$$

$$= \left(2+\frac{35}{26},\ 3+\frac{7}{26}\right) = \boxed{\left(\frac{87}{26},\ \frac{85}{26}\right)} = (3'346,\ 3'269)$$

Otro método:

$$\left.\begin{array}{l}\dfrac{x-2}{5} = \dfrac{y-3}{1}\\[2mm] \dfrac{x-3}{1} = \dfrac{y-5}{-5}\end{array}\right\} \rightarrow \left.\begin{array}{l}x-2 = 5y-15\\ -5x+15 = y-5\end{array}\right\} \rightarrow \left.\begin{array}{l}x-5y+13=0\\ -5x-y+20=0\end{array}\right\} \rightarrow \left.\begin{array}{l}5x-25y+65=0\\ -5x-y+20=0\end{array}\right\}$$

$$-26y+85=0$$

$$\boxed{y = \frac{85}{26}} \Rightarrow \boxed{x = \frac{87}{26}}$$

(25)

Recta $\overline{BF'}$

$\overrightarrow{BF'} = (3,-2) \rightarrow m = -\frac{2}{3}$

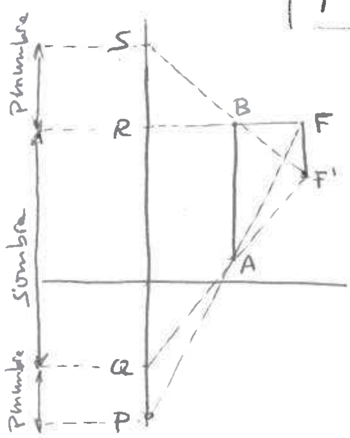
$y-6 = -\frac{2}{3}(x-4)$

$3y-18 = -2x+8$

$y = \frac{26-2x}{3}$

$x=0 \Rightarrow y = \frac{26}{3}$ $\boxed{S\left(0,\frac{26}{3}\right)}$

$\boxed{R(0,6)}$



Recta $\overline{AF}$

$\overrightarrow{AF} = (3,5) \rightarrow m = \frac{5}{3}$

$y-1 = \frac{5}{3}(x-4)$

$3y-3 = 5x-20$

$y = \frac{5x-17}{3}$

$x=0 \rightarrow y = \frac{-17}{3}$ $\boxed{P\left(0,-\frac{17}{3}\right)}$

Recta $\overline{AF'}$

$\overrightarrow{AF'} = (3,3) \rightarrow m=1$

$y-1 = 1\cdot(x-4)$

$y = x-3$

$x=0 \rightarrow y=-3$ $\boxed{Q(0,-3)}$

Sombra $= 6+3 =$ $\boxed{9 \text{ unidades}}$

$$\left.\begin{array}{l}\text{Penumbra}_1 = \dfrac{26}{3} - 6 = \dfrac{8}{3}\\[2mm] \text{Penumbra}_2 = \dfrac{17}{3} - 3 = \dfrac{8}{3}\end{array}\right\} \quad \text{Penumbra} = \frac{8}{3}+\frac{8}{3} = \boxed{\frac{16}{3}\text{ unidades}}$$

26

a) $\vec{CB} = \vec{OB} - \vec{OC} = \boxed{\vec{b} - \vec{c}}$

$\vec{AC} = \vec{OC} - \vec{OA} = \vec{OC} + \vec{OB} = \boxed{\vec{b} + \vec{c}}$

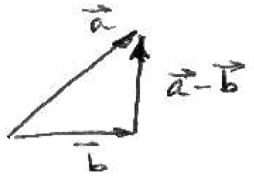
$\underbrace{-\vec{OA} = \vec{OB}}_{\text{Porque } \vec{AB} \text{ es un diámetro}}$

b) $\vec{AC} \cdot \vec{CB} = (\vec{b} + \vec{c}) \cdot (\vec{b} - \vec{c}) = \vec{b} \cdot \vec{b} - \cancel{\vec{b} \cdot \vec{c}} + \cancel{\vec{c} \cdot \vec{b}} - \vec{c} \cdot \vec{c} =$

$= |\vec{b}|^2 - |\vec{c}|^2 = R^2 - R^2 = 0 \Rightarrow \vec{AC} \perp \vec{CB} \Rightarrow \boxed{C = 90^\circ}$

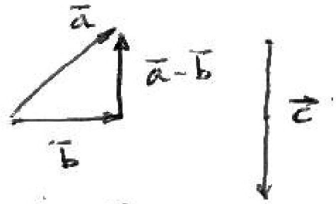
27

a)



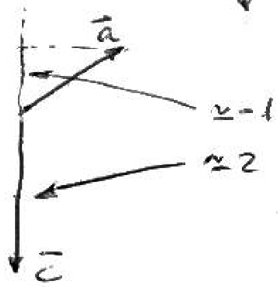
b) $|\vec{c}| \simeq \boxed{2}$

c)



$\Rightarrow \vec{c} = -2(\vec{a} - \vec{b}) = \boxed{2\vec{b} - 2\vec{a}}$

d)



$\Rightarrow \vec{a} \cdot \vec{c} \simeq \boxed{-2}$