HOJA 10: FUNCIONES REALES DE VARIABLE REAL

1. a) $Dom(f) = [0, +\infty)$; $Rec(f) = [0, +\infty)$

   b) $Dom(f) = \mathbb{R} - \{0\}$; $Rec(f) = (0, +\infty)$

   c) $Dom(f) = (-\infty, 4]$; $Rec(f) = [0, +\infty)$

   d) $Dom(f) = \mathbb{R} - \{0\}$; $Rec(f) = (-\infty, -2] \cup [2, +\infty)$

   e) $Dom(f) = \mathbb{R} - \{2\}$; $Rec(f) = \mathbb{R} - \{1\}$

   f) $Dom(f) = \mathbb{R} - \{0\}$; $Rec(f) = [2, +\infty)$

o

2. a) $f(x+4) = 2(x+4)^2 + 3(x+4) - 1 = 2x^2 + 19x + 43$

   b) $f(2-x) = 2(2-x)^2 + 3(2-x) - 1 = 2x^2 - 11x + 13$

   c) $f(-x) = 2(-x)^2 + 3(-x) - 1 = 2x^2 - 3x - 1$

   d) $f(x^2) = 2(x^2)^2 + 3x^2 - 1 = 2x^4 + 3x^2 - 1.$

o

3. a) $x - 4 = 0 \Rightarrow x = 4$; $Dom(f) = \mathbb{R} - \{4\}$

$y = \frac{2x-3}{x-4} \rightarrow xy - 4y = 2x - 3 \rightarrow xy - 2x = 4y - 3 \rightarrow x(y-2) = 4y - 3$

$\rightarrow x = \frac{4y-3}{y-2} \rightarrow y - 2 = 0 \rightarrow y = 2 \Rightarrow Rec(f) = \mathbb{R} - \{2\}.$

   b) $g(x) = -3 \Rightarrow \frac{2x-3}{x-4} = -3 \rightarrow 2x - 3 = -3x + 12 \rightarrow 5x = 15 \rightarrow \boxed{x = 3}$

o

4. $f(1) = 1 \rightarrow a + b = 1$
   $f(2) = 5 \rightarrow 2a + \frac{b}{2} = 5$

$$\left.\begin{array}{l} a + b = 1 \\ 4a + b = 10 \\ \hline 3a = 9 \end{array}\right\} \rightarrow \boxed{a = 3}$$

$b = 1 - a \rightarrow \boxed{b = -2}.$

o

⑤ a) $Dom(f) = [0, 10]$

b) $f(0) = 0$; $f(10) = \frac{10}{11}$

c) $\frac{x}{x+1} = 5 \rightarrow x = 5x + 5 \rightarrow -4x = 5 \rightarrow \boxed{x = \frac{-5}{4}}$

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⑥ $\frac{a+10}{a-8} = a \rightarrow a + 10 = a^2 - 8a \rightarrow a^2 - 9a - 10 = 0 \rightarrow a = \begin{cases} 10 \\ -1 \notin Dom(f) \end{cases}$

luego: $\boxed{a = 10}$

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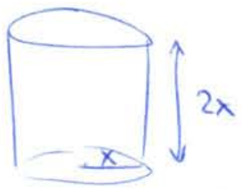
⑦ a)



$V = \pi r^2 h \rightarrow V(x) = \pi x^2 \cdot 2x \rightarrow V(x) = 2\pi x^3$

$Dom(V) = (0, +\infty)$.

b) $S = 2\pi r^2 + 2\pi r h \rightarrow S(x) = 2\pi x^2 + 2\pi x \cdot 2x = 6\pi x^2$

$Dom(S) = (0, +\infty)$

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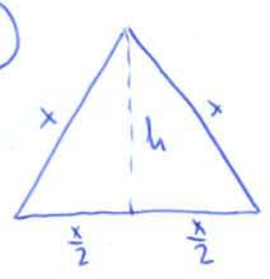
⑧



$h = \sqrt{x^2 - \frac{x^2}{4}} = \sqrt{\frac{3x^2}{4}} = \frac{x\sqrt{3}}{2}$

$A(x) = \frac{1}{2} \cdot x \cdot \frac{x\sqrt{3}}{2} = \frac{x^2\sqrt{3}}{4}$

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⑨ a) $Dom(f) = \mathbb{R}$.

b) $4x - 8 \geq 0 \rightarrow x \geq 2 \rightarrow Dom(f) = [2, +\infty)$

c) $x^2 - 4 > 0 \rightarrow (x-2)(x+2) > 0 \rightarrow Dom(f) = (-\infty, -2) \cup (2, +\infty)$

d) $x^3 + 1 = 0 \rightarrow x = \sqrt[3]{-1} = -1 \rightarrow Dom(f) = \mathbb{R} - \{-1\}$

e) $5x - x^2 = 0 \rightarrow x(5-x) = 0 \begin{cases} x = 0 \\ x = 5 \end{cases} \rightarrow Dom(f) = \mathbb{R} - \{0, 5\}$

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10 a) $f_1(x) = x+1$; $Dom(f_1) = (-\infty, 0)$
$f_2(x) = \sqrt{x}$; $Dom(f_2) = (0, 4]$
$f_3(x) = \frac{x^2-2}{x-5}$; $Dom(f_3) = (4,5) \cup (5, +\infty)$
$\Bigg\}$ $Dom(f) = \mathbb{R} - \{0, 5\}$

b) $g_1(x) = \frac{1}{x^2-1}$; $Dom(g_1) = \mathbb{R} - \{-1\}$
$g_2(x) = \frac{3x+5}{4}$; $Dom(g_2) = [1, 2]$
$g_3(x) = \sqrt{x^2-4}$; $Dom(g_3) = [2, +\infty)$
$\Bigg\}$ $Dom(g) = \mathbb{R} - \{-1\}$

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11 a) $Dom(f) = \mathbb{R} - \{-3, 3\}$; $Rec(f) = \mathbb{R}$.

b) $\nexists\, f(-3)$ ya que $-3 \notin Dom(f)$.

c) No es periódica ; d) Simetría impar.

e) A.V: $x = 3$; $x = -3$; A.H.: $y = 0$

f) Estrictamente creciente en todo su dominio; no tiene máx. ni mín relativos ni absolutos.

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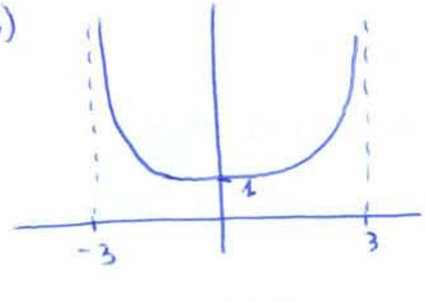
12 a) $Dom(f) = (-3, 3)$

c) $x = 3$; $x = -3$

b)



(con GDC)

d) $Rec(f) = [1, +\infty)$

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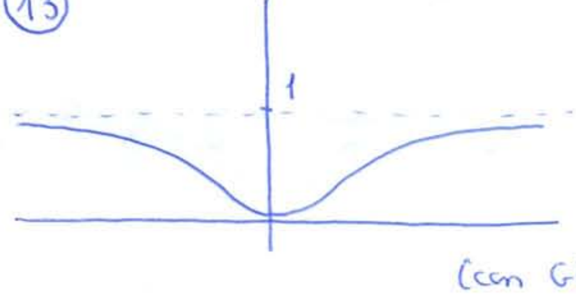

(con GDC)

a) $Dom(f) = \mathbb{R}$
$Rec(f) = [0, 1)$

b) Decreciente en $(-\infty, 0)$
Creciente en $(0, +\infty)$

c) Mín. absoluto y relativo en $(0,0)$. No hay máx. absolutos ni relativos.

d) Simetría par ; e) A.H: $y=1$.

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(14) a) $f(-x) = (-x)^2 - 6(-x) + 8 = x^2 + 6x + 8$ No tiene.

b) $f(-x) = (-x) \cdot |-x| = -x \cdot |x| = -f(x)$ Impar.

c) $f(-x) = |(-x)^3| = |-x^3| = |x^3| = f(x)$ Par.

d) $f(-x) = \dfrac{(-x)^3}{1-(-x)^2} = \dfrac{-x^3}{1-x^2} = -f(x)$ Impar.

e) $f(-x) = \dfrac{(-x)^2}{(-x)-2} = \dfrac{x^2}{-x-2}$ No tiene.

f) $f(-x) = \dfrac{(-x)^4}{1+(-x)^2} = \dfrac{x^4}{1+x^2} = f(x)$ Par.

g) $f(-x) = \sqrt[3]{(-x)^3+1} = \sqrt[3]{-x^3+1}$ No tiene.

h) $f(-x) = \sqrt[5]{(-x)^5+(-x)^3} = \sqrt[5]{-x^5-x^3} = \sqrt[5]{(-1)\cdot(x^5+x^3)} = -\sqrt[5]{x^5+x^3} =$

$= -f(x)$ Impar.

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(15) a) $f(g(x)) = f(x+2) = 3(x+2) = 3x+6$

b) $\left.\begin{array}{l} f^{-1}(18) = a \Rightarrow f(a) = 18 \Rightarrow 3a = 18 \Rightarrow a = 6 \\ g^{-1}(18) = b \Rightarrow g(b) = 18 \Rightarrow b+2 = 18 \Rightarrow b = 16 \end{array}\right\} a+b = 22$

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(16) a) $f(g(4)) = f\left(\dfrac{1}{4-3}\right) = f(1) = 2 \cdot 1 = 2$

b) $y = \dfrac{1}{x-3} \Rightarrow x = \dfrac{1}{y-3} \Rightarrow y-3 = \dfrac{1}{x} \Rightarrow y = \dfrac{1}{x} + 3 = g^{-1}(x)$

c) $\text{Dom}(g^{-1}) = \mathbb{R} - \{0\}$

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17 a) $y=\frac{2}{x-1} \to x=\frac{2}{y-1} \to y-1=\frac{2}{x} \to y=1+\frac{2}{x}=f^{-1}(x)$

b) $y=\frac{x+1}{2} \to x=\frac{y+1}{2} \to y+1=2x \to y=2x-1=f^{-1}(x)$

c) $y=\frac{2x-3}{x} \to x=\frac{2y-3}{y} \to xy=2y-3 \to xy-2y=-3 \to$

$\to 2y-xy=3 \to y(2-x)=3 \to y=\frac{3}{2-x}=f^{-1}(x)$

d) $y=\frac{2x^3-5}{x^3} \to x=\frac{2y^3-5}{y^3} \to xy^3=2y^3-5 \to 2y^3-xy^3=5$

$\to y^3(2-x)=5 \to y^3=\frac{5}{2-x} \to y=\sqrt[3]{\frac{5}{2-x}}=f^{-1}(x)$

e) $y=\frac{3x+4}{2x-3} \to x=\frac{3y+4}{2y-3} \to 2yx-3x=3y+4 \to$

$\to 2xy-3y=3x+4 \to y(2x-3)=3x+4 \to y=\frac{3x+4}{2x-3}=f^{-1}(x)$

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18 a) $y=\sqrt{3x-1}=\sqrt{g(x)}=f(g(x))=(f\circ g)(x)$

b) $y=3\sqrt{x}-1=3f(x)-1=g(f(x))=(g\circ f)(x)$

c) $y=3x^2-1=3h(x)-1=g(h(x))=(g\circ h)(x)$

d) $y=(3x-1)^2=[g(x)]^2=h(g(x))=(h\circ g)(x)$

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19 a) $x^2+6x+2=x^2+6x+9-9+2=(x+3)^2-7$

b) $(f\circ g)(x)=f(g(x))=x^2+6x+2=f(x+3) \to$

$f(x+3)=(x+3)^2-7 \to f(x)=x^2-7$

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(20) a) $f^{-1}(2)=a \Rightarrow f(a)=2 \Rightarrow 3a+5=2 \Rightarrow 3a=-3 \Rightarrow a=-1$

b) $(g\circ f)(-4)= g(f(-4))= g(-7)= 2(1+7)= 16.$

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(21) $f^{-1}(5)=a \Rightarrow f(a)=5 \Rightarrow \sqrt{3-2a}=5 \Rightarrow 3-2a=25 \Rightarrow 2a=-22$

$\Rightarrow a=-11$

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(22) a) $(g\circ f)(3)= g(f(3)) = g(2^3)= g(8)= \frac{8}{8-2}=\frac{8}{6}=\frac{4}{3}$

b) $g^{-1}(5)=a \Rightarrow g(a)=5 \Rightarrow \frac{a}{a-2}=5 \Rightarrow a=5a-10 \Rightarrow$

$\Rightarrow -4a=-10 \Rightarrow a=\frac{10}{4}=\frac{5}{2}$

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(23) a) $y=\frac{8}{x} \Rightarrow x=\frac{8}{y} \Rightarrow y=\frac{8}{x} \Rightarrow f^{-1}(x)=\frac{8}{x}$

b) i) $(f^{-1}\circ g)(x)= f^{-1}(g(x))= f^{-1}(x^2)= \frac{8}{x^2}$

ii) $\frac{8}{x^2}=x \Rightarrow 8=x^3 \Rightarrow x=\sqrt[3]{8}=2$

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(24) a) $y=4x-2 \Rightarrow x=4y-2 \Rightarrow 4y=x+2 \Rightarrow y=\frac{x+2}{4}=f^{-1}(x)$

b) $(f\circ g)(1)= f(g(1))= f(-2\cdot 1^2+8)= f(6)= 4\cdot 6-2=22$

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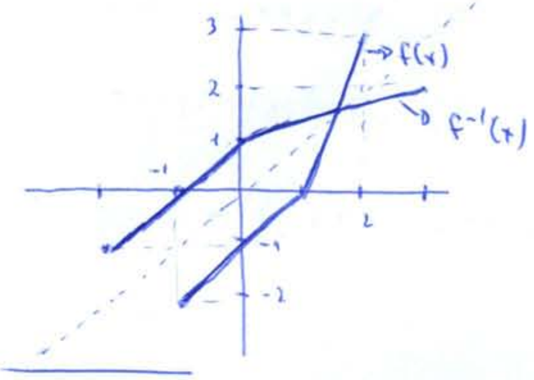
(25) a) $f(2)=3$;

$f^{-1}(-1)=0$;

b)



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26 $f(x)=g(x) \Rightarrow Kx^2+Kx = x-0'8 \rightarrow Kx^2+Kx-x+0'8=0$

$\rightarrow Kx^2+(K-1)x+0'8=0$ → la ec. cuadrática ha de tener dos soluciones distintas, luego $\Delta > 0 \Rightarrow b^2-4ac>0$:

$(K-1)^2-4\cdot K\cdot 0'8>0 \Rightarrow K^2-2K+1-3'2K>0 \Rightarrow$

$\Rightarrow K^2-5'2K+1>0 \Rightarrow (K-5)(K-0'2)>0 \Rightarrow K\in(-\infty,0'2)\cup(5,+\infty)$

27 a) $f(-3)=-1$; $f^{-1}(1)=0$

b) $Dom(f^{-1})=Rec(f)=[-3,3]$

c)