INEQUALITIES

A) Linear inequalities (one unknown):

"Solving'' an inequality means finding all of its solutions. A "solution'' of an inequality is a number which when substituted for the variable makes the inequality a true statement.
Just like with equations, the solution to an inequality is a value that makes the inequality true. You can solve inequalities in the same way you can solve equations, by following these rules.

- You may add any positive or negative number to both sides of an inequality.
- You may multiply or divide both sides of an inequality by any positive number.
- **Watchout!** If you multiply or divide both sides of an inequality by a negative number, reverse the direction of the inequality sign!

Exercises:

1º) Pag. 472. (19c.1)
2º) solve: $3(2x+2) > 3(3x+4)$
3º) $\frac{5x-7}{3} < x+5$
4º) $1-2(x+5) \geq -3$
5º) $2x-5 < 2(x+1)+x$
6ª) $x+\frac{1-x}{6} < 2-\frac{2+x}{2}$
7ª) $3x-\frac{1-2x}{2} \leq 4+x$

How to solving graphical inequalities (two unknowns):

Example 1:

How to solving the graphical of the inequality x + y > 5.

Solution:

Draw the graph of x + y = 5.
x + y=5 ⇒ y =5 – x

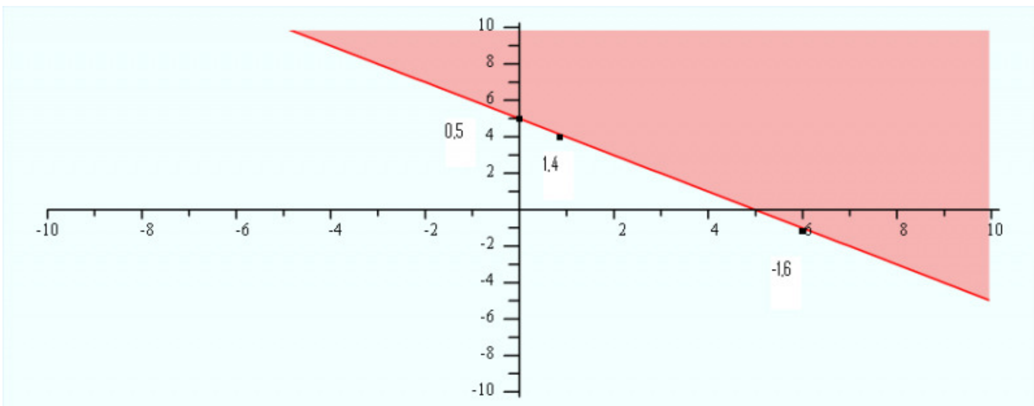
x	-1	0	1
y	6	5	4

Plot the points (-1,6), (0,5) and (1,4) in the graph and join them to get the straight line x + y = 5. Take the point (0,0) and substitute in the inequality

x + y > 5.

0 + 0 > 5 which is false.

So the region not containing the points (0,0) is the desired region and it is shaded linear graphical inequalities . The shaded region in the graph represents the inequality x + y > 5. All the points in the shaded region satisfy x + y > 5 and they are the solutions of x + y > 5.



example 2:

How to solving the graphing x + 4y < 10

Solution:

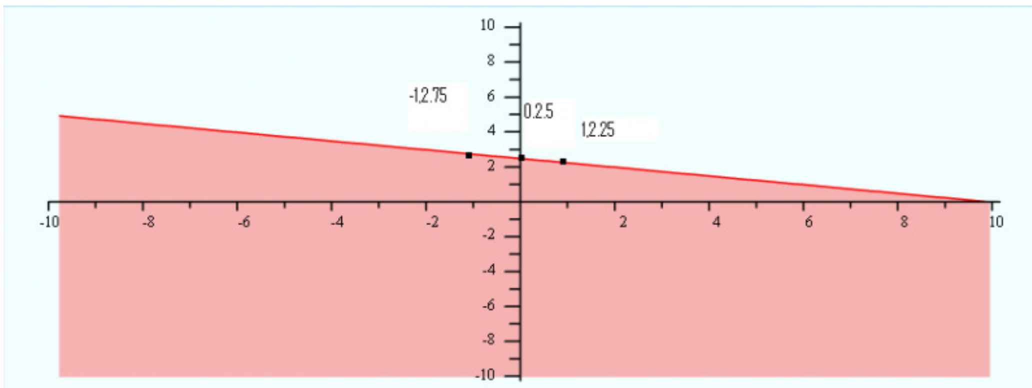
Draw the graph of x + 4y = 10 $\Rightarrow$ $y = \frac{10-x}{4}$

x	-1	0	1
y	2.75	2.5	2.25

Plot the points (-1, 2.75), (0,2.25) and (1, 2.25) in the graph and join them to get the dotted line x + 4y = 10. The given inequality x + 4y < 10 does not contain equality sign. So the points on the line will not satisfy x + 4y < 10 and they should be excluded. In order to show this the dotted line is drawn inequalities. Take the point (0,0) and substitute it in the inequality x + 4y < 10,

0 + 0 < 10   which is true.

Therefore the region containing the points (0,0) is the desired region and it is shaded linear graphical inequalities. The shaded region represents the inequality x + 4y < 10. All the points in the shaded region are the solutions of x + 4y < 10.



Solving System of Linear Inequalities Graphically (two unknowns)

E x a m p l e .

Solve the system of simultaneous inequalities:

$$\begin{cases} 5x - 7y > -11, \\ 2x + 3y < 10. \end{cases}$$

S o l u t i o n .

Firstly we build graphs of the linear functions: 5*x* – 7*y* = – 11 and 2*x* + 3*y* = 10. For each of them we find a half-plane, in which the given inequalities are valid. We know, that it is sufficient to check a validity of the inequalities at any one point; here the easiest point for check is an origin of coordinates *O* (0, 0). Substituting its coordinates in our inequalities instead of *x* and *y* , we receive: 5 · 0 – 7 · 0 = 0 > – 11, hence the lower half-plane (yellow) is the solution for the first inequality; 2 · 0 + 3 · 0 = 0 < 10, therefore the second inequality has as a solution also the lower half-plane (blue).

An intersection of these half-planes (an aqua area) is the solution of our system of simultaneous inequalities.

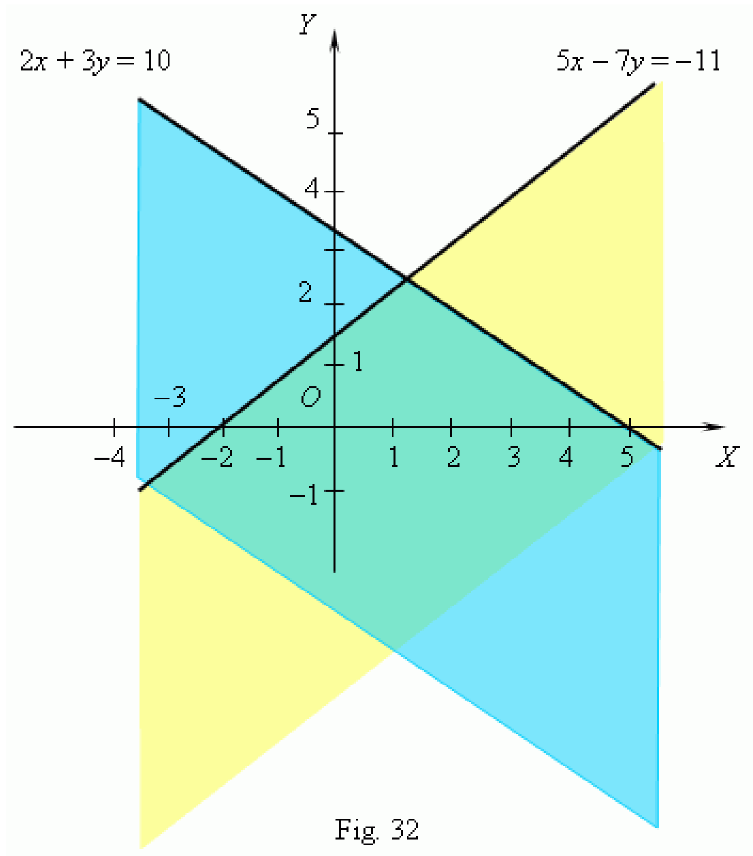


Fig. 32

Exercises:

1) Sketch the graph $3x + 4y < 12$
2) Sketch the graph $4x - 5y > 3$
3) Sketch the graph $3x < 6y$
4) Sketch the graph $2x > y$
5) Solve graphically,

a) $\begin{cases} x < 2 \\ x \geqslant 0 \end{cases}$ c) $\begin{cases} 3x - 1 \geqslant 7 - x \\ 1 - x < 1 - 2x \end{cases}$

b) $\begin{cases} 2x - 3 < 1 - x \\ 4 - 2x \geqslant 6 \end{cases}$ d) $\begin{cases} 5x + 2 \geqslant 3x + 4 \\ -3x + 3 \leqslant 2x - 7 \end{cases}$

6) Solve graphically,

a) $\begin{cases} x + y < 1 \\ x - y < 1 \end{cases}$ c) $\begin{cases} 2x + y < -2 \\ 4x + 2y < 1 \end{cases}$

b) $\begin{cases} x + 2y < 1 \\ 3x - y \leqslant 2 \end{cases}$ d) $\begin{cases} 3x \geqslant y - 2 \\ -2y \leqslant 4x - 2 \end{cases}$

B) Quadratic, polynomial and rational inequalities:

<u>Method</u>: factorising and making a sign diagram

Exercises:

1)
a) $x^2 - 2x - 3 > 0$
b) $x^2 - 8x + 12 \geqslant 0$
c) $4x^2 + 4x - 3 \leqslant 0$
d) $x^2 + 1 < 0$
e) $x(x + 1)(x - 2) > 0$
f) $x^3 - 9x \leqslant 0$

2)
a) $\dfrac{1}{x+2} < 0$
b) $\dfrac{x}{x-2} \geqslant 0$
c) $\dfrac{x+3}{x-5} > 0$
d) $\dfrac{x+4}{1-x} \leqslant 0$
e) $\dfrac{x^2-1}{x^2} < 0$
f) $\dfrac{x-4}{x^2-3x+2} \geqslant 0$