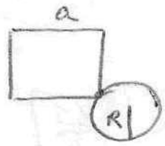
(1)



$$4a+2\pi R=240 \Rightarrow a = 60 - \frac{\pi R}{2}$$

$$S = a^2 + \pi R^2 = \left(60 - \frac{\pi R}{2}\right)^2 + \pi R^2 = 3600 - 60\pi R + \frac{\pi^2 R^2}{4} + \pi R^2 =$$

$$= \frac{\pi}{4}(\pi+4)R^2 - 60\pi R + 3600$$

$$S' = \frac{\pi}{2}\cdot(\pi+4)R - 60\pi$$

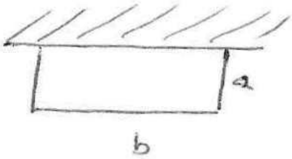
$$S' = 0 \Rightarrow R = \frac{60\pi}{\frac{\pi}{2}(\pi+4)} = \frac{120}{\pi+4}$$

$-$ $\quad 30/\pi+1 \quad$ $+$

signo S'

$\boxed{\text{Mínima área para } R = \frac{120}{\pi+4}}$

(2)



$$2a+b = 1000 \Rightarrow b = 1000 - 2a$$

$$A = ab = a(1000-2a) = 1000a - 2a^2$$

$$A' = 1000 - 4a$$

$$A' = 0 \Rightarrow \boxed{a = 250 \text{ m} \Rightarrow b = 500 \text{ m}}$$

$+$ $\quad 250 \quad$ $-$

signo A'

$\boxed{\text{Máxima Área para } a = 250 \text{ m}, \; b = 500 \text{ m}}$

(3)

$$S = x + \frac{25}{x}$$

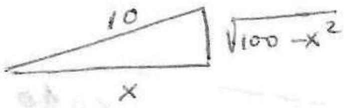
$$S' = 1 - \frac{25}{x^2} \qquad S' = 0 \Rightarrow x = \pm 5$$

$+$ $\quad -5 \quad$ $-$ $\quad$ $-$ $\quad 5 \quad$ $+$

signo S'

$\boxed{\text{Mínima suma para } x = 5}$

(4)



$$A = \frac{x\sqrt{100-x^2}}{2} = \frac{1}{2}\sqrt{100x^2 - x^4}$$

$$A' = \frac{1}{2}\cdot\frac{200x - 4x^3}{2\sqrt{100x^2-x^4}} = \frac{50x - x^3}{\sqrt{100x^2-x^4}} = \frac{x(50-x^2)}{x\sqrt{100-x^2}} = \frac{50-x^2}{\sqrt{100-x^2}}$$

$$A' = 0 \Rightarrow x = \pm\sqrt{50}$$

$0 \quad + \quad \sqrt{50} \quad -$

signo A'

$\boxed{\text{Máxima Área para } x = \sqrt{50}, \; \sqrt{100-x^2} = \sqrt{50}}$

⑤ $a+b^2=9 \Rightarrow a=9-b^2$

$S=a+b=9-b^2+b$

$S'=-2b+1 \qquad S'=0 \Rightarrow \boxed{b=\frac{1}{2}} \Rightarrow a=9-\frac{1}{4}=\boxed{\frac{35}{4}}$

$+ \quad \frac{1}{2} \quad -$

signo S'

$\boxed{\text{Máximo para } b=1/2,\ a=35/4}$

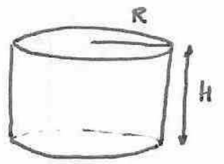
⑥ $P=x\cdot(20-x)=20x-x^2$

$P'=20-2x \qquad P'=0 \Rightarrow x=10$

$+ \quad 10 \quad -$

signo P'

$\boxed{\text{Máximo producto para } x=10,\ 20-x=10}$

⑦ 

$V=160\,l=160\ dm^3$

$\pi R^2 H=160 \longrightarrow H=\frac{160}{\pi R^2}$

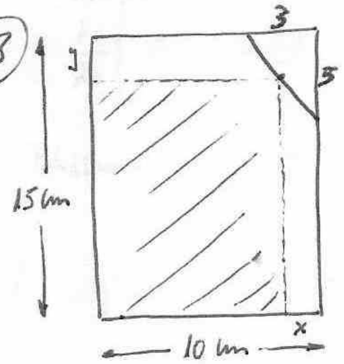
$S=2\cdot\pi R^2+2\pi R H=2\pi R^2+2\pi R\frac{160}{\pi R^2}=2\pi R^2+\frac{320}{R}$

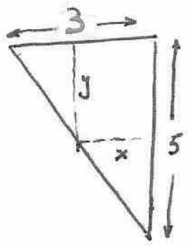
$\frac{dS}{dR}=4\pi R-\frac{320}{R^2}=\frac{4\pi R^3-320}{R^2}$

$\frac{dS}{dR}=0 \Rightarrow 4\pi R^3=320\ ;\ R=\sqrt[3]{\frac{320}{4\pi}}=\sqrt[3]{\frac{80}{\pi}}$

R	0		$\sqrt[3]{80/\pi}$		$+\infty$
S		$\searrow$		$\nearrow$	
dS/dR		$-$	0	$+$	

Mínima superficie para $\boxed{R=\sqrt[3]{\frac{80}{\pi}}\ dm} \longrightarrow H=\frac{160}{\pi\sqrt[3]{\left(\frac{80}{\pi}\right)^2}}=\frac{160\sqrt[3]{\frac{80}{\pi}}}{\pi\frac{80}{\pi}}=\boxed{2\sqrt[3]{\frac{80}{\pi}}\ dm}$

⑧ 



$\frac{3}{5}=\frac{3-x}{y} \Rightarrow y=\frac{5(3-x)}{3}$

$S=(15-y)\cdot(10-x)=\left(15-\frac{5(3-x)}{3}\right)\cdot(10-x)=\frac{45-15+5x}{3}\cdot(10-x)=\frac{30+5x}{3}\cdot(10-x)$

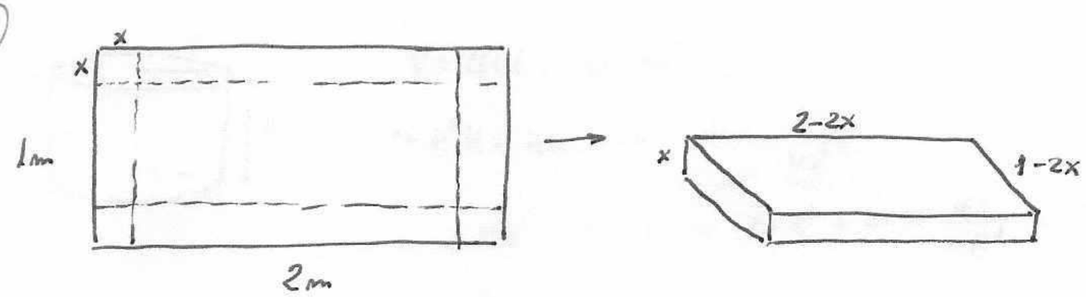
$\frac{dS}{dx}=\frac{5}{3}(10-x)+\frac{30+5x}{3}\cdot(-1)=\frac{50-5x-30-5x}{3}=\frac{20-10x}{3}$

$\frac{dS}{dx}=0 \Rightarrow 20=10x\ ;\ x=2$

x	0		2		10
S		$\searrow$		$\nearrow$	
$\frac{dS}{dx}$		$-$	0	$+$	

Mínima superficie para $\boxed{x=2\,cm} \longrightarrow \boxed{y=\frac{5}{3}\ cm}$

(9)



$$V = x\cdot(2-2x)(1-2x) = 2x(1-x)(1-2x) = 2(x-3x^2+2x^3)$$

$$\frac{dV}{dx} = 2(1-6x+6x^2)$$

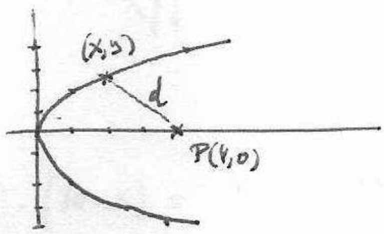
$$\frac{dV}{dx}=0 \Rightarrow 6x^2-6x+1=0 \; ; \quad x = \frac{6\pm\sqrt{36-24}}{12} = \frac{6\pm\sqrt{12}}{12} = \frac{6\pm 2\sqrt{3}}{12} = \frac{3\pm\sqrt{3}}{6} \begin{cases} \frac{3+\sqrt{3}}{6} \\ \frac{3-\sqrt{3}}{6} \end{cases}$$

x	0		$\frac{3-\sqrt{3}}{6}$		$\frac{3+\sqrt{3}}{6}$		1
V		$\nearrow$		$\searrow$		$\nearrow$	
$\frac{dV}{dx}$		$+$	0	$-$	0	$+$	

Máximo volumen para $\boxed{x = \frac{3-\sqrt{3}}{6} \text{ m}}$

(10)

$$y^2 = 4x \longrightarrow y = \pm\sqrt{4x}$$



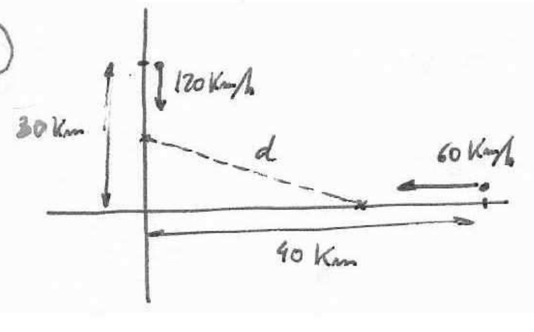
$$d = \sqrt{(x-4)^2+y^2} = \sqrt{(x-4)^2+4x} = \sqrt{x^2-4x+16}$$

$$d' = \frac{2x-4}{2\sqrt{x^2-4x+16}} = \frac{x-2}{\sqrt{x^2-4x+16}}$$

x	0		2		$+\infty$
d		$\searrow$		$\nearrow$	
d'		$-$	0	$+$	

Mínima distancia para $x=2 \Rightarrow$ Puntos $(2, \pm\sqrt{8})$

(11)



$t=0$ $\begin{cases} \text{locomotora 1 está en el punto } (40,0) \\ \text{locomotora 2 " " " " } (0,30) \end{cases}$

t horas $\begin{cases} \text{locomotora 1 " " " " } (40-60t,0) \\ \text{locomotora 2 " " " " } (0,30-120t) \end{cases}$

$$d=\sqrt{(40-60t)^2+(30-120t)^2}=\sqrt{1600-4800t+3600t^2+900-7200t+14400t^2}=$$

$$=\sqrt{2500-12000t+18000t^2}=10\sqrt{25-120t+180t^2}$$

$$d'=10\,\frac{-120+360t^2}{2\sqrt{25-120t+180t^2}}=5\,\frac{120(-1+3t^2)}{\sqrt{25-120t+180t^2}}=600\,\frac{3t^2-1}{\sqrt{25-120t+180t^2}}$$

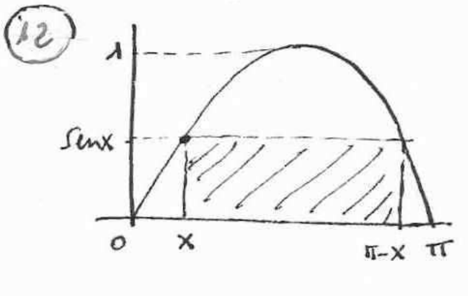
$$d'=0 \Rightarrow 3t^2=1 \quad ; \quad t=\pm\sqrt{1/3} \begin{cases} \sqrt{1/3} \\ \cancel{-\sqrt{1/3}} \end{cases}$$

t	0		$\sqrt{1/3}$		$+\infty$
d		$\searrow$		$\nearrow$	
d'		$-$	0	$+$	

Mínima distancia para $\boxed{t=\sqrt{1/3}\ \text{h}}$ $\longrightarrow$ $d_{min}=10\sqrt{25-\frac{120}{\sqrt{3}}+\frac{180}{3}}=$

$$=\boxed{10\sqrt{85-\frac{120}{\sqrt{3}}}\ \text{Km}}$$

(12)



$$\text{Area}=(\pi-2x)\,\text{sen}\,x \qquad x\in(0,\pi/2)$$

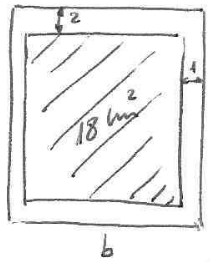
$$\frac{dA}{dx}=-2\,\text{sen}\,x+(\pi-2x)\cos x$$

$$\frac{dA}{dx}=0 \rightarrow -2\,\text{sen}\,x+(\pi-2x)\cos x=0 \rightarrow$$

$$\rightarrow 2\,\text{sen}\,x=(\pi-2x)\cos x \rightarrow 2\,\text{tg}\,x=\pi-2x \Longrightarrow$$ Resuelta con Calculadora gráfica $\boxed{x=0'710246}$

Area Máxima $=(\pi-2\cdot 0'71)\,\text{sen}\,0'71=\boxed{1'021}$

(13)



$$(b-2)\cdot(a-4)=18 \Rightarrow b=2+\frac{18}{a-4}$$

$$S=a\cdot b=a\cdot\left(2+\frac{18}{a-4}\right)=a\,\frac{2a-8+18}{a-4}=\frac{2a^2+10a}{a-4}$$

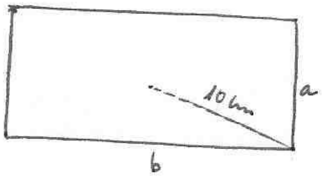
$$\frac{dS}{da}=\frac{(4a+10)(a-4)-(2a^2+10a)\cdot 1}{(a-4)^2}=\frac{4a^2-16a+\cancel{10a}-40-2a^2-\cancel{10a}}{(a-4)^2}=\frac{2a^2-16a-40}{(a-4)^2}$$

$$\frac{dS}{da}=0 \Rightarrow 2a^2-16a-40=0\ ;\quad a^2-8a-20=0\ ;\quad a=\frac{8\pm\sqrt{64+80}}{2}=\frac{8\pm 12}{2}\ \begin{cases}10\\ \cancel{-2}\end{cases}$$

a	0		10		$+\infty$
S		↘		↗	
$\frac{dS}{da}$		$-$	0	$+$	

Mínimo gasto en papel para $\boxed{a=10\text{ cm}} \longrightarrow b=2+\frac{18}{10-4}=\boxed{5\text{ cm}}$

(14)



$$\left(\frac{a}{2}\right)^2+\left(\frac{b}{2}\right)^2=10^2$$

$$\frac{a^2}{4}+\frac{b^2}{4}=100 \Rightarrow b=\sqrt{400-a^2}$$

$$\text{Área}=a\cdot b=a\sqrt{400-a^2}$$

$$\frac{dA}{da}=\sqrt{400-a^2}+a\cdot\frac{-\cancel{2}a}{\cancel{2}\sqrt{400-a^2}}=\frac{400-a^2-a^2}{\sqrt{400-a^2}}=\frac{400-2a^2}{\sqrt{400-a^2}}$$

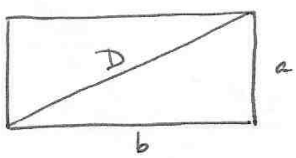
$$\frac{dA}{da}=0 \Rightarrow 400=2a^2\ ;\quad a=\sqrt{200}$$

a	0		$\sqrt{200}$		20
A		↗		↘	
$\frac{dA}{da}$		$+$	0	$-$	

Máximo área para $\boxed{a=\sqrt{200}\text{ cm}} \Rightarrow b=\sqrt{400-200}=\boxed{\sqrt{200}\text{ cm}}$

El rectángulo de máximo área es un cuadrado.

(15)



$$2a+2b=12 \Rightarrow b=6-a$$

$$D=\sqrt{a^2+b^2}=\sqrt{a^2+(6-a)^2}=\sqrt{36-12a+2a^2}$$

$$\frac{dD}{da}=\frac{-12+4a}{2\sqrt{36-12a+2a^2}}=\frac{2a-6}{\sqrt{36-12a+2a^2}}$$

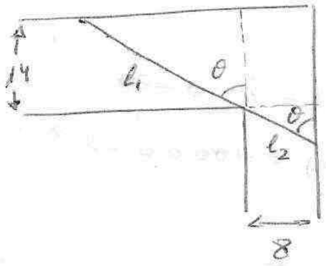
$$\frac{dD}{da}=0 \Rightarrow a=3\text{ cm}$$

a	0		3		6
D		↘		↗	
$\frac{dD}{da}$		$-$	0	$+$	

Mínima diagonal para $\boxed{a=3\text{ cm}} \longrightarrow \boxed{b=3\text{ cm}}$

El rectángulo de mínima diagonal es un cuadrado.

1



$\cos\theta = \dfrac{14}{l_1} \Rightarrow l_1 = 14\sec\theta$

$\operatorname{sen}\theta = \dfrac{8}{l_2} \Rightarrow l_2 = 8\operatorname{cosec}\theta$

$$L = 14\sec\theta + 8\operatorname{cosec}\theta = \frac{14}{\cos\theta} + \frac{8}{\operatorname{sen}\theta} \Rightarrow L' = \frac{+14\operatorname{sen}\theta}{\cos^2\theta} + \frac{-8\cos\theta}{\operatorname{sen}^2\theta}$$

$$L' = 0 \Rightarrow \frac{14\operatorname{sen}\theta}{\cos^2\theta} = \frac{8\cos\theta}{\operatorname{sen}^2\theta} \Rightarrow \frac{\operatorname{sen}^3\theta}{\cos^3\theta} = \frac{8}{14} \Rightarrow$$

$$\operatorname{tg}^3\theta = \frac{8}{14} \Rightarrow \operatorname{tg}\theta = \sqrt[3]{\frac{8}{14}} \Rightarrow \theta = 39{,}69^\circ$$

$$\Downarrow$$

$$\boxed{L = 30{,}72\ m}$$

0 — 39'69° + 90°

signo L'

2

$$f(x) = \frac{\log_{10} x}{x}$$

$$f'(x) = \frac{x \cdot \frac{1}{\ln 10} \cdot \frac{1}{x} - \log_{10} x \cdot 1}{x^2} = \frac{\frac{1}{\ln 10} - \log_{10} x}{x^2}$$

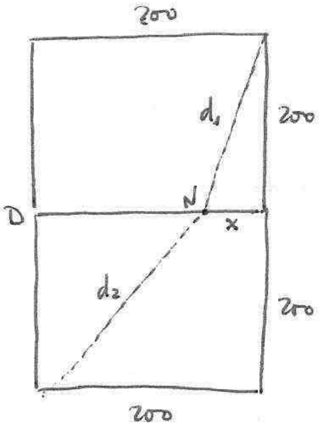
$$f'(x) = 0 \Rightarrow \frac{1}{\ln 10} = \log_{10} x \Rightarrow \frac{1}{\cancel{\ln 10}} = \frac{\ln x}{\cancel{\ln 10}} \Rightarrow x = e$$

Si $f(x)$ tiene un máximo para $x = e$, cumplirá que $f(e) \geq f(x)$

Concretamente $f(e) > f(\pi) \Rightarrow \dfrac{\log_{10} e}{e} > \dfrac{\log_{10}\pi}{\pi} \Rightarrow \pi\log_{10} e > e\log_{10}\pi \Rightarrow$

$$\Rightarrow \log_{10}(e^\pi) > \log_{10}(\pi^e) \Rightarrow \boxed{e^\pi > \pi^e}$$

3



$d_1 = \sqrt{200^2 + x^2}$ a 8 Km/h $\Rightarrow t_1 = \dfrac{\sqrt{200^2+x^2}}{8000}$

$d_2 = \sqrt{200^2 + (200-x)^2}$ a 10 Km/h $\Rightarrow t_2 = \dfrac{\sqrt{200^2+(200-x)^2}}{10000}$

$$T = t_1 + t_2 = \frac{\sqrt{200^2+x^2}}{8000} + \frac{\sqrt{200^2+(200-x)^2}}{10000}$$

Con calculadora gráfica se obtiene un tiempo mínimo de $x = 86{,}1745$

Luego $DN = 200 - 86{,}174579 = \boxed{113{,}825421\ m}$

El tiempo mínimo es $T = 0{,}050234125$ h $= \boxed{3 \text{ min } 0{,}84285 \text{ seg}}$