Other equations:

1.- Biquadratic Equations	Biquadratic equations are quartic equations with no odd-degree terms:	$\mathbf{ax^4 + bx^2 + c = 0}$
Solving Biquadratic Equations	To solve biquadratic equations, change $\mathbf{x^2 = t, x^4 = t^2}$; this generates a quadratic equation with the unknown, **t**: $\mathbf{at^2 + bt + c = 0}$ For every positive value of **t** there are **two values of x**, find: $x = \pm\sqrt{t}$	Solve: $x^4 - 13x^2 + 36 = 0$ $x^2 = t$ $t^2 - 13t + 36 = 0$ $t = \frac{13 \pm \sqrt{169-144}}{2} = \frac{13 \pm 5}{2} = \begin{cases} \nearrow t_1 = \frac{18}{2} = 9 \\ \searrow t_2 = \frac{8}{2} = 4 \end{cases}$ $x^2 = 9 \quad x = \pm\sqrt{9} = \begin{cases} \nearrow x_1 = 3 \\ \searrow x_2 = -3 \end{cases}$ $x^2 = 4 \quad x = \pm\sqrt{4} = \begin{cases} \nearrow x_3 = 2 \\ \searrow x_4 = -2 \end{cases}$
	The same procedure can be used to solve the equations of the type: ax6 + bx3 + c = 0 ax8 + bx4 + c = 0 ax10 + bx5 + c = 0	Solve: $x^6 - 7x^3 + 6 = 0$ $x^3 = t$ $t^2 - 7t + 6 = 0$ $t = \frac{7 \pm \sqrt{49-24}}{2} = \frac{7 \pm 5}{2} = \begin{cases} \nearrow t_1 = \frac{12}{2} = 6 \\ \searrow t_2 = \frac{2}{2} = 1 \end{cases}$ $x^3 = 6 \quad\quad x = \sqrt[3]{6}$ $x^3 = 1 \quad x = \sqrt[3]{1} \quad x = 1$
2.- Polinomial equations with a degree greater than 2.	The solutions of this equation are called the *roots* of the polynomial; they are the *zeroes* of the function *f* (corresponding to the points where the graph of *f* meets the *x*-axis). A number *a* is a root of *P* if and only if the polynomial *x* – *a* (of degree one in *x*) divides *P*.	$ax^n + bx^{n-1} + cx^{n-2} + \ldots + mx + n = 0$

Solving polynomial equations	*1º) Factorize the polynomial:* *a) Try to use common factors* *b)Try Ruffini's rule to find possible roots (for integer roots only)* *Once the polynomial is factorized use the <u>null factor law</u> (When the product of two or more numbers is zero, then at least one of them must be zero)*	*Solve* $2x^4 - 4x^3 - 6x^2 = 0$ *Factorising, using common factors:* $2x^2(x^2 - 2x - 3) = 0$ *Using the null factor, if the product is 0, one of the factors must be 0, then:* • *If* $2x^2 = 0$*, then <u>x=0</u> (it is the first solution, in this case is a double solution)* • *If* $(x^2 - 2x - 3) = 0$*, then, using the quadratic formula, <u>x = 3</u> and <u>x = -1</u>*
3.- Rational Equations	Rational polynomial equations are of the form $\frac{P(x)}{Q(x)} = 0$, where P(x) and Q(x) are polynomials.	$\frac{1}{x^2 - x} - \frac{1}{x - 1} = 0$
Solving Rational Equations	To solve rational equations multiply both sides of the equation by the least common multiple of the denominators.	$\frac{1}{x^2 - x} - \frac{1}{x - 1} = 0$ $x^2 - x = x(x - 1)$ $lcm(x^2 - x,\ x - 1) = x(x - 1)$ $1 - x = 0 \qquad x = 1$ Verify the solution: $\frac{1}{1 - 1} - \frac{1}{1 - 1} = 0 \qquad \frac{1}{0} - \frac{1}{0} = 0$ The equation has no solution because for x = 1, the denominators are annulled.
	Other example	$\frac{1}{x - 2} + \frac{1}{x + 2} = \frac{1}{x^2 - 4}$ $x^2 - 4 = (x - 2) \cdot (x + 2)$ $lcm(x - 2, x + 2,\ x^2 - 4) = (x - 2) \cdot (x + 2)$ $x + 2 + x - 2 = 1 \qquad 2x = 1 \qquad x = \frac{1}{2}$

<table>
<tr><td></td><td></td><td>

$$\frac{1}{\frac{1}{2}-2}+\frac{1}{\frac{1}{2}+2}=\frac{1}{\left(\frac{1}{2}\right)^2-4}$$

$$\frac{1}{-\frac{3}{2}}+\frac{1}{\frac{5}{2}}=\frac{1}{\frac{-15}{4}} \qquad -\frac{2}{3}+\frac{2}{5}=-\frac{4}{15}$$

The solution is: $x=\frac{1}{2}$</td></tr>
<tr><td>4.- Radical Equations</td><td>The irrational equations or radical equations, are those with the unknown value under the radical sign.</td><td>

$\sqrt{2x-3}-x=-1$</td></tr>
<tr><td>Solving Radical Equations</td><td>1. Isolate a radical in one of the two members and pass the other terms to the other member.
2. Square both members.
3. Solve the equation obtained.
4. Check if the solutions obtained verify the initial equation. Keep in mind that squaring an equation can only be done if another has the same solution as those that are given.
5. If the equation has several radicals, repeat the first two phases of the process to remove all of them.</td><td>

Solve: $\sqrt{2x-3}-x=-1$
1. Isolated the radical:
$\sqrt{2x-3}=-1+x$
2. Square both members:
$\left(\sqrt{2x-3}\right)^2=(-1+x)^2$
$2x-3=1-2x+x^2$
3. Solve the equation:
$x^2-4x+4=0$
$x=\frac{4\pm\sqrt{16-16}}{2}=\frac{4}{2}=2$
4.Verify:
$\sqrt{2\cdot 2-3}-2=-1 \qquad 1-2=-1$
The equation has the solution x = 2.</td></tr>
<tr><td></td><td>Other example:</td><td>

$\sqrt{x}+\sqrt{x-4}=2$
$\sqrt{x}=2-\sqrt{x-4}$
$\left(\sqrt{x}\right)^2=\left(2-\sqrt{x-4}\right)^2$
$x=4-4\sqrt{x-4}+x-4$
$4\sqrt{x-4}=0 \qquad \sqrt{x-4}=0$
$\left(\sqrt{x-4}\right)^2=0^2 \qquad x-4=0 \qquad x=4$
$\sqrt{4}+\sqrt{4-4}=2 \qquad 2+0=2$
The equation has the solution x = 4.</td></tr>
</table>

Exercises:

1) Solve:

a) $x^4 - 10x^2 + 9 = 0$
b) $x^4 - 61x^2 + 900 = 0$
c) $x^4 - 25x^2 + 144 = 0$
d) $x^4 - 16x^2 - 225 = 0$
e) $x^4 - 5x^2 + 4 = 0$
f) $x^6 - 10x^3 + 9 = 0$
g) $x^4 - 26x^2 = -25$
h) $x^6 - 64x^3 = 0$
i) $3x^3 - 12x^2 + 12x = 0$

2) Solve:

a) $x^3 + 2x^2 - x - 2 = 0$
b) $x^3 - 6x^2 + 3x + 10 = 0$
c) $x^3 - 3x + 2 = 0$
d) $x^3 + 3x^2 - 4x - 12 = 0$

3) Solve:

a) $\frac{3}{x} = 1 + \frac{x-13}{6}$

b) $\frac{2}{x} - \frac{2-x}{x+3} = 1$

c) $\frac{x}{x+1} + \frac{2}{x+2} = 3$

4) Solve:

a) $\sqrt{5x+4} - 1 = 2x$
b) $3\sqrt{x-1} + 11 = 2x$
c) $\sqrt{2x-1} + \sqrt{x+4} = 6$
d) $\sqrt{x^2 + 5x + 1} = x + 2$
e) $\sqrt{40 - x^2} + 4 = x$
f) $\sqrt{2x-1} + \sqrt{x+4} = 6$
g) $\sqrt{6+x} + 2x = -2$

Solutions:

1 e) $x = \pm 2 \quad o \quad x = \pm 1$
1 f) $x = \sqrt[3]{9} \quad o \quad x = 1$
1 g) $x = \pm 5 \quad o \quad x = \pm 1$
1 h) $x = 4$
1 i) *x = 0 o x= 2 (double solution)*

4 d) x=3
4 e) x= 6
4 f) x= 5
4 g) x= –2