① $A = \begin{pmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & a \end{pmatrix}$ $\begin{vmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & a \end{vmatrix} = a^3 - 3a + 2 = (a-1)^2(a+2)$

* Si $\left.\begin{matrix} a \neq 1 \\ a \neq -2 \end{matrix}\right| \Rightarrow \left.\begin{matrix} r=3 \\ r^+=3 \\ n=3 \end{matrix}\right\}$ S.C.D.

$$x = \frac{\begin{vmatrix} 1 & 1 & 1 \\ 1 & a & 1 \\ -1 & 1 & a \end{vmatrix}}{a^3-3a+2} = \frac{a^2-1}{a^3-3a+2} = \frac{a+1}{a^2+a-2}$$

$$y = \frac{\begin{vmatrix} a & 1 & 1 \\ 1 & 1 & 1 \\ 1 & -1 & a \end{vmatrix}}{a^3-3a+2} = \frac{a^2-1}{a^3-3a+2} = \frac{a+1}{a^2+a-2}$$

$$z = \frac{\begin{vmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & -1 \end{vmatrix}}{a^3-3a+2} = \frac{-(a+3)(a-1)}{a^3-3a+2} = \frac{-(a+3)}{a^2+a-2}$$

* Si $a=1 \Rightarrow \left.\begin{matrix} x+y+z=1 \\ x+y+z=1 \\ x+y+z=-1 \end{matrix}\right\}$ S. Incompatible

* Si $a=-2 \Rightarrow \left.\begin{matrix} -2x+y+z=1 \\ x-2y+z=1 \\ x+y-2z=-1 \end{matrix}\right\}$ $A = \begin{pmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{pmatrix}$ $\begin{vmatrix} -2 & 1 \\ 1 & -2 \end{vmatrix} = 4-1 = 3 \Rightarrow \boxed{r=2}$

$A^+ = \begin{pmatrix} -2 & 1 & 1 & 1 \\ 1 & -2 & 1 & 1 \\ 1 & 1 & -2 & -1 \end{pmatrix}$ $\begin{vmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -1 \end{vmatrix} = 3 \Rightarrow \boxed{r^+=3}$ $\Rightarrow$ S. Incomp.

② $A = \begin{pmatrix} 1 & 1 & -1 \\ 2 & 3 & a \\ 1 & a & 3 \end{pmatrix}$ $\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = 1 \neq 0 \Rightarrow r \geq 2$ $\begin{vmatrix} 1 & 1 & -1 \\ 2 & 3 & a \\ 1 & a & 3 \end{vmatrix} = -a^2 - a + 6$

$-a^2 - a + 6 = 0 \Rightarrow \begin{matrix} a=-3 \\ a=2 \end{matrix}$

• Si $\left.\begin{matrix} a \neq -3 \\ a \neq 2 \end{matrix}\right\} \Rightarrow \boxed{r=3}$ $\boxed{\begin{matrix} r^+=3 \\ n=3 \end{matrix}} \Rightarrow$ S.C.D. $\boxed{x=1 \quad y=\frac{1}{a+3} \quad z=\frac{1}{a+3}}$

• Si $a=-3 \Rightarrow \boxed{r=2}$ $A^+ = \begin{pmatrix} 1 & 1 & -1 & 1 \\ 2 & 3 & -3 & 3 \\ 1 & -3 & 3 & 2 \end{pmatrix}$ $\begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & -3 & 2 \end{vmatrix} = 5 \Rightarrow \boxed{r^+=3}$ Incompat

• Si $a=2 \Rightarrow \boxed{r=2}$ $A^+ = \begin{pmatrix} 1 & 1 & -1 & 1 \\ 2 & 3 & 2 & 3 \\ 1 & 2 & 3 & 2 \end{pmatrix}$ $\begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & 2 & 2 \end{vmatrix} = 0 \Rightarrow \boxed{r^+=2}$ $\boxed{n=3}$ S.C. Indet

$\left.\begin{matrix} x+y=1+z \\ 2x+3y=3-2z \end{matrix}\right\}$ $\begin{matrix} x=5z \\ y=1-4z \\ z \in \mathbb{R} \end{matrix}$

③ $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1+a & 1 \\ 1 & 1 & 1+a \end{pmatrix}$ $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+a & 1 \\ 1 & 1 & 1+a \end{vmatrix} = (1+a)^2 + 1 + 1 - (1+a) - (1+a) - 1 =$

$= 1 + 2a + a^2 + 1 - 1 - a - 1 - a = a^2$

* Si $a \neq 0 \Rightarrow \left.\begin{matrix} r=3 \\ r^+=3 \\ n=3 \end{matrix}\right\}$ S.C.D

$$x = \frac{\begin{vmatrix} a & 1 & 1 \\ 2a & 1+a & 1 \\ 0 & 1 & 1+a \end{vmatrix}}{a^2} = \frac{a(1+2a+a^2) + 2a - 2a(1+a) - a}{a^2} =$$

$$= \frac{a + 2a^2 + a^3 - 2a^2 - a}{a^2} = \frac{a^3}{a^2} = \boxed{a}$$

$$y = \frac{\begin{vmatrix} 1 & a & 1 \\ 1 & 2a & 1 \\ 1 & 0 & 1+a \end{vmatrix}}{a^2} = \frac{2a(1+a) + a - 2a - a(1+a)}{a^2} =$$

$$= \frac{2a + 2a^2 + \cancel{a} - 2a - \cancel{a} - a^2}{a^2} = \boxed{1}$$

$$z = \frac{\begin{vmatrix} 1 & 1 & a \\ 1 & 1+a & 2a \\ 1 & 1 & 0 \end{vmatrix}}{a^2} = \frac{a + 2a - a(1+a) - 2a}{a^2} = \boxed{-1}$$

* Si $a=0 \Rightarrow$ $\left.\begin{array}{l} x+y+z=0 \\ x+y+z=0 \\ x+y+z=0 \end{array}\right\}$ S.C.I.

$$\boxed{\begin{array}{l} x = \text{cualquier nº real} \\ y = \text{cualquier nº real} \\ z = -x-y \end{array}}$$

(4) $A = \begin{pmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & a \end{pmatrix}$ $\qquad \begin{vmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & a \end{vmatrix} = a^3 - 3a + 2 = (a-1)^2(a+2)$

* Si $\left.\begin{array}{l} a \neq 1 \\ a \neq -2 \end{array}\right| \Rightarrow \left.\begin{array}{l} r=3 \\ r^*=3 \\ n=3 \end{array}\right\}$ S.C.D

$$x = \frac{\begin{vmatrix} 1 & 1 & 1 \\ a & a & 1 \\ a^2 & 1 & a \end{vmatrix}}{a^3-3a+2} = \frac{a^2+a+a^2-a^3-a^2-1}{a^3-3a+2} = \frac{-a^3+a^2+a-1}{a^3-3a+2} = \boxed{\frac{-(a+1)}{a+2}}$$

$$y = \frac{\begin{vmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & a^2 & a \end{vmatrix}}{a^3-3a+2} = \frac{a^3+a^2+1-a-a-a^3}{a^3-3a+2} = \frac{a^2-2a+1}{a^3-3a+2} = \boxed{\frac{1}{a+2}}$$

$$z = \frac{\begin{vmatrix} a & 1 & 1 \\ 1 & a & a \\ 1 & 1 & a^2 \end{vmatrix}}{a^3-3a+2} = \frac{a^4+1+\cancel{a}-\cancel{a}-a^2-a^2}{a^3-3a+2} = \frac{a^4-2a^2+1}{a^3-3a+2} = \boxed{\frac{(a+1)^2}{a+2}}$$

* Si $a=1 \Rightarrow$ $\left.\begin{array}{l} x+y+z=1 \\ x+y+z=1 \\ x+y+z=1 \end{array}\right\}$ S.C.I.

$$\boxed{\begin{array}{l} x \in \mathbb{R} \\ y \in \mathbb{R} \\ z = 1-x-y \end{array}}$$

* Si $a=-2 \Rightarrow$ $\left.\begin{array}{l} -2x+y+z=1 \\ x-2y+z=-2 \\ x+y-2z=4 \end{array}\right\}$

$A = \begin{pmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{pmatrix}$ $\qquad \begin{vmatrix} -2 & 1 \\ 1 & -2 \end{vmatrix} = 4-1 = 3 \Rightarrow \boxed{r=2}$

$A^* = \begin{pmatrix} -2 & 1 & 1 & 1 \\ 1 & -2 & 1 & -2 \\ 1 & 1 & -2 & 4 \end{pmatrix}$ $\qquad \begin{vmatrix} -2 & 1 & 1 \\ 1 & -2 & -2 \\ 1 & 1 & 4 \end{vmatrix} = 16+1-2+2-4-4 = 9 \Downarrow \boxed{r^*=3}$

S. Incompatible.

(5) $A = \begin{pmatrix} 4-a & 2 & 1 \\ 2 & 4-a & 2 \\ 2 & 4 & 8-a \end{pmatrix}$

$$\begin{vmatrix} 4-a & 2 & 1 \\ 2 & 4-a & 2 \\ 2 & 4 & 8-a \end{vmatrix} = (4-a)^2(8-a) + 8 + 8 - 2(4-a) - 4(8-a) - 8(4-a) = (16-2a+a^2)(8-a) + 16 - 8 + 2a - 32 + 4a - 32 + 8a = 128 - 16a + 8a^2 - 16a + 2a^2 - a^3 - 56 + 14a =$$

$$= -a^3 + 16a^2 - 66a + 72 = -(a-4)(a-(6+3\sqrt{2}))(a-(6-3\sqrt{2}))$$

* Si $\left.\begin{array}{l} a \neq 4 \\ a \neq 6+3\sqrt{2} \\ a \neq 6-3\sqrt{2} \end{array}\right\} \Rightarrow \begin{array}{l} r=3 \\ r^*=3 \\ n=3 \end{array}$ S.C.D

$$\boxed{\begin{array}{l} x=0 \\ y=0 \\ z=0 \end{array}}$$

* Si $a=4 \Rightarrow A = \begin{pmatrix} 0 & 2 & 1 \\ 2 & 0 & 2 \\ 2 & 4 & 4 \end{pmatrix}$ $\qquad \begin{vmatrix} 0 & 2 \\ 2 & 0 \end{vmatrix} = -4 \Rightarrow \begin{array}{l} r=2 \\ r^*=2 \\ n=3 \end{array}$ S.C.I

$$\left.\begin{array}{l} 2y = -z \\ 2x = -2z \end{array}\right\} \qquad \boxed{\begin{array}{l} x = -z \\ y = -\frac{z}{2} \\ z \in \mathbb{R} \end{array}}$$

* Si $a = 6+3\sqrt{2} \Longrightarrow A = \begin{pmatrix} -2-3\sqrt{2} & 2 & 1 \\ 2 & -2-3\sqrt{2} & 2 \\ 2 & 4 & 2-3\sqrt{2} \end{pmatrix}$

$\begin{vmatrix} 2 & -2-3\sqrt{2} \\ 2 & 4 \end{vmatrix} = 8+4+6\sqrt{2} = 12+6\sqrt{2} \Longrightarrow \boxed{\begin{matrix} r=2 \\ n=3 \end{matrix}}$ S.C.I.

$$\left.\begin{matrix} 2x - (2+3\sqrt{2})y = -2z \\ 2x + 4y = (-2+3\sqrt{2})z \end{matrix}\right\}$$

$$x = \frac{\begin{vmatrix} -2z & -2-3\sqrt{2} \\ (-2+3\sqrt{2})z & 4 \end{vmatrix}}{12+6\sqrt{2}} = \frac{-8-4+6\sqrt{2}-6\sqrt{2}+18}{12+6\sqrt{2}}z = \frac{6z}{12+6\sqrt{2}} = \boxed{\frac{z}{2+\sqrt{2}}}$$

$$y = \frac{\begin{vmatrix} 2 & -2z \\ 2 & (-2+3\sqrt{2})z \end{vmatrix}}{12+6\sqrt{2}} = \frac{-4+6\sqrt{2}+4}{12+6\sqrt{2}}z = \boxed{\frac{\sqrt{2}\,z}{2+\sqrt{2}}}$$

$z \in \mathbb{R}$

Si $a = 6-3\sqrt{2} \Longrightarrow A = \begin{pmatrix} -2+3\sqrt{2} & 2 & 1 \\ 2 & -2+3\sqrt{2} & 2 \\ 2 & 4 & 2+3\sqrt{2} \end{pmatrix}$

$\begin{vmatrix} 2 & -2+3\sqrt{2} \\ 2 & 4 \end{vmatrix} = 8+4-6\sqrt{2} = 12-6\sqrt{2} \Longrightarrow \boxed{\begin{matrix} r=2 \\ n=3 \end{matrix}}$ S.C.I.

$$\left.\begin{matrix} 2x - (2-3\sqrt{2})y = -2z \\ 2x + 4y = -(2+3\sqrt{2})z \end{matrix}\right\}$$

$$x = \frac{\begin{vmatrix} -2z & -(2-3\sqrt{2}) \\ (-2-3\sqrt{2})z & 4 \end{vmatrix}}{12-6\sqrt{2}} = \boxed{\frac{z}{2-\sqrt{2}}}$$

$$y = \frac{\begin{vmatrix} 2 & -2z \\ 2 & -(2+3\sqrt{2})z \end{vmatrix}}{12-6\sqrt{2}} = \boxed{\frac{-\sqrt{2}\,z}{2-\sqrt{2}}}$$

$z \in \mathbb{R}$

(6) $A = \begin{pmatrix} 2 & a & 4 \\ 1 & 1 & 7 \\ a & -1 & 13 \end{pmatrix}$

$\begin{vmatrix} 1 & 7 \\ -1 & 13 \end{vmatrix} = 13+7 = 20 \Longrightarrow r \geq 2$

$\begin{vmatrix} 2 & a & 4 \\ 1 & 1 & 7 \\ a & -1 & 13 \end{vmatrix} = 26 - 4 + 7a^2 - 4a + 14 - 13a = 7a^2 - 17a + 36 \neq 0$

$\boxed{\begin{matrix} r=3 \\ n=3 \end{matrix}} \Longrightarrow$ S.C.D. $\boxed{\begin{matrix} x=0 \\ y=0 \\ z=0 \end{matrix}}$

(7) $A = \begin{pmatrix} 1 & a^2 \\ a & 1 \\ a^2 & a \end{pmatrix}$ $\quad |1| = 1 \Longrightarrow r \geq 1$

$A^* = \begin{pmatrix} 1 & a^2 & -a \\ a & 1 & -a^2 \\ a^2 & a & -1 \end{pmatrix}$

$\begin{vmatrix} 1 & a^2 & -a \\ a & 1 & -a^2 \\ a^2 & a & -1 \end{vmatrix} = -1 - \cancel{a^3} - a^6 + \cancel{a^3} + a^3 + a^3 = -a^6 + 2a^3 - 1 =$

$= -(a-1)^2(a^2+a+1)^2$

* Si $a \neq 1 \Rightarrow \begin{matrix} r^+ = 3 \\ r < 3 \end{matrix}$ S. Incompatible.

* Si $a = 1 \Rightarrow \left.\begin{matrix} x+y=-1 \\ x+y=-1 \\ \cancel{x+y=-1} \end{matrix}\right\}$ S.C.I $\boxed{\begin{matrix} x \in \mathbb{R} \\ y = -1-x \end{matrix}}$

(8) $A = \begin{pmatrix} 1 & \lambda & -1 \\ 2 & -1 & \lambda \\ 1 & 10 & -6 \end{pmatrix}$

$$\begin{vmatrix} 2 & -1 \\ 1 & 10 \end{vmatrix} = 20+1 = 21 \Rightarrow r \geq 2$$

$$\begin{vmatrix} 1 & \lambda & -1 \\ 2 & -1 & \lambda \\ 1 & 10 & -6 \end{vmatrix} = 6-20+\lambda^2-1+12\lambda-10\lambda = \lambda^2+2\lambda-15 = (\lambda-3)(\lambda+5)$$

* Si $\left.\begin{matrix} \lambda \neq 3 \\ \lambda \neq -5 \end{matrix}\right| \Rightarrow \begin{matrix} r=3 \\ r^+=3 \\ n=3 \end{matrix}$ S.C.D.

$$x = \frac{\begin{vmatrix} 2 & \lambda & -1 \\ 5 & -1 & \lambda \\ 1 & 10 & -6 \end{vmatrix}}{(\lambda-3)(\lambda+5)} = \frac{12-50+\lambda^2-1+30\lambda-20\lambda}{(\lambda-3)(\lambda+5)} = \boxed{\frac{\lambda^2+10\lambda+11}{\lambda^2+2\lambda-15}}$$

$$y = \frac{\begin{vmatrix} 1 & 2 & -1 \\ 2 & 5 & \lambda \\ 1 & 1 & -6 \end{vmatrix}}{(\lambda-3)(\lambda+5)} = \frac{-30-2+2\lambda+5+24-\lambda}{(\lambda-3)(\lambda+5)} = \frac{\lambda-3}{(\lambda-3)(\lambda+5)} = \boxed{\frac{1}{\lambda+5}}$$

$$z = \frac{\begin{vmatrix} 1 & \lambda & 2 \\ 2 & -1 & 5 \\ 1 & 10 & 1 \end{vmatrix}}{(\lambda-3)(\lambda+5)} = \frac{-1+40+5\lambda+2-2\lambda-50}{(\lambda-3)(\lambda+5)} = \frac{3\lambda-9}{(\lambda-3)(\lambda+5)} = \boxed{\frac{3}{\lambda+5}}$$

* Si $\lambda = 3 \Rightarrow \left\{\begin{matrix} \boxed{r=2} \\ A^+ = \begin{pmatrix} 1 & 3 & -1 & 2 \\ 2 & -1 & 3 & 5 \\ 1 & 10 & -6 & 1 \end{pmatrix} \end{matrix}\right.$ $\begin{vmatrix} 1 & 3 & 2 \\ 2 & -1 & 5 \\ 1 & 10 & 1 \end{vmatrix} = -1+40+15+2-6-50 = 0 \Rightarrow$

$\Rightarrow \begin{matrix} r=2 \\ r^+=2 \\ n=3 \end{matrix}$ S.C.I

$$\left.\begin{matrix} 2x-y = 5-3z \\ x+10y = 1+6z \end{matrix}\right\}$$

$$x = \frac{\begin{vmatrix} 5+3z & -1 \\ 1+6z & 10 \end{vmatrix}}{21} = \frac{50+30z+1+6z}{21} = \boxed{\frac{36z+51}{21}}$$

$$y = \frac{\begin{vmatrix} 2 & 5+3z \\ 1 & 1+6z \end{vmatrix}}{21} = \frac{2+12z-5-3z}{21} = \boxed{\frac{9z-3}{21}}$$

$z \in \mathbb{R}$

Si $\lambda = -5 \Rightarrow \left\{\begin{matrix} \boxed{r=2} \\ A^+ = \begin{pmatrix} 1 & -5 & -1 & 2 \\ 2 & -1 & -5 & 5 \\ 1 & 10 & -6 & 1 \end{pmatrix} \end{matrix}\right.$ $\begin{vmatrix} 1 & -5 & 2 \\ 2 & -1 & 5 \\ 1 & 10 & 1 \end{vmatrix} = -1+40-25+2+10-50 = -23$

$\Downarrow$

$\boxed{r^+=3}$

$\boxed{\begin{matrix} r=2 \\ r^+=3 \end{matrix}}$ S. Incompatible

(9)

$$\begin{pmatrix} 3 & -3k-10 & 6-2k & -18-6k \\ 1 & 5-k & 4+k & -10+3k \\ 1 & -k & 2 & -6 \end{pmatrix} \to \begin{pmatrix} 0 & -10 & -2k & -6k \\ 0 & 5 & 2+k & -4+3k \\ 1 & -k & 2 & -6 \end{pmatrix} \to$$

$$\to \begin{pmatrix} 0 & 0 & 4 & -8 \\ 0 & 5 & 2+k & -4+3k \\ 1 & -k & 2 & -6 \end{pmatrix} \to \left.\begin{array}{l} 4z=-8 \\ 5y+(2+k)z=-4+3k \\ x-ky+2z=-6 \end{array}\right\} \Rightarrow$$

$$\Rightarrow \boxed{z=-2} \Rightarrow 5y-2(2+k)=-4+3k \;;\; 5y=5k \Rightarrow \boxed{y=k}$$

$$x-k^2-4=-6 \Rightarrow \boxed{x=k^2-2}$$

S.C.D

Otra forma

$$A=\begin{pmatrix} 3 & -3k-10 & 6-2k \\ 1 & 5-k & 4+k \\ 1 & -k & 2 \end{pmatrix} \;;\; |A|=\begin{vmatrix} 3 & -3k-10 & 6-2k \\ 1 & 5-k & 4+k \\ 1 & -k & 2 \end{vmatrix} \underset{\substack{F2=F2-F3 \\ F1=F1-3F3}}{=} \begin{vmatrix} 0 & -10 & -2k \\ 0 & 5 & 2+k \\ 1 & -k & 2 \end{vmatrix} = -20-10k+10k=-20\neq 0 \Rightarrow \text{rango } A=3$$

$$\Downarrow$$

S.C.D

$$x=\frac{\begin{vmatrix} -18-6k & -3k-10 & 6-2k \\ -10+3k & 5-k & 4+k \\ -6 & -k & 2 \end{vmatrix}}{-20} \underset{\substack{F2=F2-F3 \\ F1=F1-3F3}}{=} \frac{\begin{vmatrix} -6k & -10 & -2k \\ -4+3k & 5 & 2+k \\ -6 & -k & 2 \end{vmatrix}}{-20} \underset{C1=C1-3\cdot C3}{=} \frac{\begin{vmatrix} 0 & -10 & -2k \\ -10 & 5 & 2+k \\ -12 & -k & 2 \end{vmatrix}}{-20} = \frac{-20k^2+240+\cancel{120k}-\cancel{120k}-200}{-20} = \boxed{k^2-2}$$

$$y=\frac{\begin{vmatrix} 3 & -18-6k & 6-2k \\ 1 & -10+3k & 4+k \\ 1 & -6 & 2 \end{vmatrix}}{-20} \underset{C2=C2-3\cdot C3}{=} \frac{\begin{vmatrix} 3 & -36 & 6-2k \\ 1 & -22 & 4+k \\ 1 & -12 & 2 \end{vmatrix}}{-20} \underset{\substack{F1=F1-3F3 \\ F2=F2-F1}}{=} \frac{\begin{vmatrix} 0 & 0 & -2k \\ 0 & -10 & 2+k \\ 1 & -12 & 2 \end{vmatrix}}{-20} = \frac{-20k}{-20} = \boxed{k}$$

$$z=\frac{\begin{vmatrix} 3 & -3k-10 & -18-6k \\ 1 & 5-k & -10+3k \\ 1 & -k & -6 \end{vmatrix}}{-20} \underset{\substack{F1=F1-3F3 \\ F2=F2-F1}}{=} \frac{\begin{vmatrix} 0 & -10 & -6k \\ 0 & 5 & -4+3k \\ 1 & -k & -6 \end{vmatrix}}{-20} = \frac{40-\cancel{30k}+\cancel{30k}}{-20} = \boxed{-2}$$

(10)

$$A=\begin{pmatrix} 2 & -k & 4 \\ k & -1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

$$\begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix} = -1-1=-2 \Rightarrow r\geq 2$$

$$\begin{vmatrix} 2 & -k & 4 \\ k & -1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = -2+4k-k+\cancel{4}+k^2-\cancel{2} = k^2+3k = k(k+3)$$

* Si $\left.\begin{matrix} K\neq 0 \\ K\neq -3 \end{matrix}\right| \Rightarrow \boxed{\begin{matrix} r=3 \\ r^+=3 \\ n=3 \end{matrix}}$ S.C.D

$$x = \frac{\begin{vmatrix} 2 & -K & 4 \\ -3 & -1 & 1 \\ 1 & 1 & 1 \end{vmatrix}}{K(K+3)} = \boxed{\frac{-4}{K}} \quad y = \frac{\begin{vmatrix} 2 & 2 & 4 \\ K & -3 & 1 \\ 1 & 1 & 1 \end{vmatrix}}{K(K+3)} = \boxed{\frac{2}{K}} \quad z = \frac{\begin{vmatrix} 2 & -K & 2 \\ K & -1 & -3 \\ 1 & 1 & 1 \end{vmatrix}}{K(K+3)} = \boxed{\frac{K+2}{K}}$$

* Si $K=0 \Rightarrow \left\{ \begin{matrix} \boxed{r=2} \\ A^+ = \begin{pmatrix} 2 & 0 & 4 & 2 \\ 0 & -1 & 1 & -3 \\ 1 & 1 & 1 & 1 \end{pmatrix} \quad \begin{vmatrix} 0 & 4 & 2 \\ -1 & 1 & -3 \\ 1 & 1 & 1 \end{vmatrix} = -2-12-2+4 = -12 \Rightarrow \boxed{r^+=3} \end{matrix}\right.$

$\Downarrow$

Sistema Incompatible

* Si $K=-3 \Rightarrow \left\{ \begin{matrix} \boxed{r=2} \\ A^+ = \begin{pmatrix} 2 & 3 & 4 & 2 \\ -3 & -1 & 1 & -3 \\ 1 & 1 & 1 & 1 \end{pmatrix} \quad \begin{vmatrix} 3 & 4 & 2 \\ -1 & 1 & -3 \\ 1 & 1 & 1 \end{vmatrix} = 3-2-12-2+4+9 = 0 \Rightarrow \boxed{r^+=2} \end{matrix}\right.$

$\left.\begin{matrix} r=2 \\ r^+=2 \\ n=3 \end{matrix}\right\} \Rightarrow$ S.C.I. $\quad \left.\begin{matrix} -y+z=-3+3x \\ y+z=1-x \end{matrix}\right\}$

$\boxed{x\in\mathbb{R}}$

$$y = \frac{\begin{vmatrix} -3+3x & 1 \\ 1-x & 1 \end{vmatrix}}{\begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix}} = \frac{-3+3x-1+x}{-2} = \frac{4x-4}{-2} = \boxed{2-2x}$$

$$z = \frac{\begin{vmatrix} -1 & -3+3x \\ 1 & 1-x \end{vmatrix}}{-2} = \frac{-1+x+3-3x}{-2} = \frac{2-2x}{-2} = \boxed{x-1}$$

(11) $A = \begin{pmatrix} 1 & 2 \\ 3 & 1 \\ m & 1 \end{pmatrix} \quad \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = 1-6=-5 \Rightarrow \boxed{\begin{matrix} r=2 \\ r^+=2 \\ n=2 \end{matrix}}$ S.C.D $\quad \boxed{\begin{matrix} y=0 \\ z=0 \end{matrix}}$

(12) $A = \begin{pmatrix} m & -1 \\ 1 & -m \end{pmatrix} \quad \begin{vmatrix} m & -1 \\ 1 & -m \end{vmatrix} = -m^2+1 = -(m+1)(m-1)$

* Si $\left.\begin{matrix} m\neq 1 \\ m\neq -1 \end{matrix}\right| \Rightarrow \boxed{\begin{matrix} r=2 \\ r^+=2 \\ n=2 \end{matrix}}$ S.C.D.

$$x = \frac{\begin{vmatrix} 1 & -1 \\ 2m-1 & -m \end{vmatrix}}{-(m+1)(m-1)} = \frac{-m+2m-1}{-(m+1)(m-1)} = \boxed{\frac{-1}{m+1}}$$

$$y = \frac{\begin{vmatrix} m & 1 \\ 1 & 2m-1 \end{vmatrix}}{-(m+1)(m-1)} = \frac{2m^2-m-1}{-(m+1)(m-1)} = \boxed{\frac{-(2m+1)}{m+1}}$$

* Si $m=1 \Rightarrow \left.\begin{matrix} x-y=1 \\ \cancel{x-y=1} \end{matrix}\right\}$ S.C.I. $\quad \boxed{\begin{matrix} x\in\mathbb{R} \\ y=x-1 \end{matrix}}$

* Si $m=-1 \Rightarrow \left.\begin{matrix} -x-y=1 \\ x+y=-3 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x+y=-1 \\ x+y=-3 \end{matrix}\right\}$ S. Incompatible

(13) $A=\begin{pmatrix}1&1&-1\\2&3&a\\1&1&3\end{pmatrix}$

$\begin{vmatrix}1&1\\2&3\end{vmatrix}=3-2=1\neq 0 \Rightarrow r\geq 2$

$\begin{vmatrix}1&1&-1\\2&3&a\\1&1&3\end{vmatrix}=9-2+a+3-6-a=4\neq 0 \Rightarrow \boxed{\begin{matrix}r=3\\r^+=3\\m=3\end{matrix}}$ S.C.D. (No depende de a)

$x=\dfrac{\begin{vmatrix}1&1&-1\\3&3&a\\2&1&3\end{vmatrix}}{4}=\dfrac{9-3+2a+6-9-a}{4}=\boxed{\dfrac{a+3}{4}}$

$y=\dfrac{\begin{vmatrix}1&1&-1\\2&3&a\\1&2&3\end{vmatrix}}{4}=\dfrac{9-4+a+3-6-2a}{4}=\boxed{\dfrac{2-a}{4}}$

$z=\dfrac{\begin{vmatrix}1&1&1\\2&3&3\\1&1&2\end{vmatrix}}{4}=\dfrac{6+2+3-3-4-3}{4}=\boxed{\dfrac{1}{4}}$

(14) $\left.\begin{matrix}x+2y+3z=1\\x+Ky+3z=3\\y=z\\3y-z=2\end{matrix}\right\}\equiv\left.\begin{matrix}x+5y=1\\x+(k+3)y=3\\2y=2\end{matrix}\right\}\equiv\left.\begin{matrix}x+5=1\\x+k+3=3\end{matrix}\right\}\equiv$

$\boxed{z=y}$ $\boxed{\begin{matrix}y=1\\z=y=1\end{matrix}}$

$\equiv\left.\begin{matrix}x=-4\\x=-k\end{matrix}\right\}\Rightarrow$ • Si $k=4$ S.C.D. $\boxed{\begin{matrix}x=-4\\y=1\\z=1\end{matrix}}$

$\boxed{\begin{matrix}y=1\\z=1\end{matrix}}$ • Si $k\neq 4$ S. Incompatible

(También se puede discutir por rangos de matrices)

(15) $A=\begin{pmatrix}1&1&1\\1&2&-3\\K&-1&-1\\1&-1&1\end{pmatrix}$ $\begin{vmatrix}1&1\\1&2\end{vmatrix}=1\Rightarrow r\geq 2$

$\begin{vmatrix}1&1&1\\1&2&-3\\1&-1&1\end{vmatrix}=\begin{vmatrix}0&2&0\\1&2&-3\\1&-1&1\end{vmatrix}=-2\begin{vmatrix}1&-3\\1&1\end{vmatrix}=-8\Rightarrow\boxed{r=3}$

$A^+=\begin{pmatrix}1&1&1&2\\1&2&-3&8\\K&-1&-1&1\\1&-1&1&-2\end{pmatrix}$ $\begin{vmatrix}1&1&1&2\\1&2&-3&8\\K&-1&-1&1\\1&-1&1&-2\end{vmatrix}=16-8K$

$16-8K=0\rightarrow (K=2)$

• Si $K\neq 2\Rightarrow\boxed{\begin{matrix}r^+=4\\r=3\end{matrix}}$ S. Incompatible

• Si $K=2\Rightarrow\boxed{\begin{matrix}r^+=3\\r=3\\m=3\end{matrix}}$ S.C.D. $\left.\begin{matrix}x+y+z=2\\x+2y-3z=8\\x-y+z=-2\end{matrix}\right\}$ $\boxed{\begin{matrix}x=1\\y=2\\z=-1\end{matrix}}$

(16)
$$\begin{pmatrix} 1 & 0 & -3 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & -3 & 2 & 0 \\ -4 & 0 & 0 & \lambda \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad \begin{vmatrix} 1 & 0 & -3 \\ 0 & 1 & 0 \\ 0 & -3 & 2 \end{vmatrix} = 2 \Rightarrow \text{rango } A \geq 3$$

$$\begin{vmatrix} 1 & 0 & -3 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & -3 & 2 & 0 \\ -4 & 0 & 0 & \lambda \end{vmatrix} \underset{C3 \to C3 + 3\cdot C1}{=} \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & -3 & 2 & 0 \\ -4 & 0 & -12 & \lambda \end{vmatrix} = 1 \cdot \begin{vmatrix} 1 & 0 & -1 \\ -3 & 2 & 0 \\ 0 & -12 & \lambda \end{vmatrix} = 2\lambda - 36$$

$$2\lambda - 36 = 0 \Rightarrow \boxed{\lambda = 18}$$

- Si $\lambda \neq 18 \Rightarrow \left.\begin{array}{l} n = 4 \\ \text{rango } A = 4 \end{array}\right| \Rightarrow$ S.C.D $\Rightarrow \boxed{\begin{array}{l} x=0 \\ y=0 \\ z=0 \\ t=0 \end{array}}$

- Si $\lambda = 18 \Rightarrow \left.\begin{array}{l} n = 4 \\ \text{rango } A < 4 \end{array}\right| \Rightarrow$ S.C.I

$$\left.\begin{array}{l} x - 3z = 0 \\ y = t \\ -3y + 2z = 0 \end{array}\right\}$$

$$x = \frac{\begin{vmatrix} 0 & 0 & -3 \\ t & 1 & 0 \\ 0 & -3 & 2 \end{vmatrix}}{2} = \boxed{\frac{9t}{2}} \qquad \boxed{y = t} \qquad z = \frac{\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & t \\ 0 & -3 & 0 \end{vmatrix}}{2} = \boxed{\frac{3t}{2}} \qquad \boxed{t \in \mathbb{R}}$$

Otra forma:

$$\left.\begin{array}{l} x - 3z = 0 \\ y - t = 0 \\ -3y + 2z = 0 \\ -4x + \lambda t = 0 \end{array}\right\} \to \boxed{y = t} \qquad \left.\begin{array}{l} x - 3z = 0 \\ -3t + 2z = 0 \\ -4x + \lambda t = 0 \end{array}\right\} \to x = 3z \qquad \left.\begin{array}{l} -3t + 2z = 0 \\ -12z + \lambda t = 0 \end{array}\right\}$$

$$\left.\begin{array}{l} 2z - 3t = 0 \\ -12z + \lambda t = 0 \end{array}\right\} \qquad \begin{pmatrix} 2 & -3 \\ -12 & \lambda \end{pmatrix} \begin{pmatrix} z \\ t \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{vmatrix} 2 & -3 \\ -12 & \lambda \end{vmatrix} = 2\lambda - 36 \quad ; \quad 2\lambda - 36 = 0 \quad ; \quad \lambda = 18$$

- Si $\lambda \neq 18 \Rightarrow$ S.C.D. $\Rightarrow \boxed{\begin{array}{l} z=0 \\ t=0 \end{array}} \to \boxed{\begin{array}{l} x=0 \\ y=0 \end{array}}$

- Si $\lambda = 18 \Rightarrow$ S.C.I $\Rightarrow 2z - 3t = 0 \to \boxed{\begin{array}{l} z = \frac{3t}{2} \\ t \in \mathbb{R} \end{array}} \to \boxed{\begin{array}{l} x = \frac{9t}{2} \\ y = t \end{array}}$