JUN 94

i) $A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & a \\ 1 & a & 1 \end{pmatrix}$ $\quad \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = -1 \Rightarrow$ rango $A \geq 2$

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & a \\ 1 & a & 1 \end{vmatrix} = 1+2a+a-1-2-a^2 = -a^2+3a-2$$

$$-a^2+3a-2=0 \rightarrow a = \begin{cases} 2 \\ 1 \end{cases}$$

- Si $\left.\begin{matrix} a\neq 2 \\ a \neq 1 \end{matrix}\right| \Rightarrow$ rango $A = 3 \Rightarrow$ S.C.D.

- Si $a=2$: $\boxed{\text{rango } A = 2}$

$$\left.\begin{matrix} x+y+z=1 \\ 2x+y+2z=2 \\ x+2y+z=1 \end{matrix}\right\} \quad A^* = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 1 & 2 & 2 \\ 1 & 2 & 1 & 1 \end{pmatrix} \Rightarrow \boxed{\text{rango } A^* = 2} \Rightarrow \text{S.C.I.}$$

$$\left.\begin{matrix} x+y=1-z \\ 2x+y=2-2z \end{matrix}\right\} \quad \boxed{\begin{matrix} x=1-z \\ y=0 \\ z\in\mathbb{R} \end{matrix}}$$

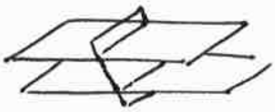
- Si $a=1$: $\boxed{\text{rango } A = 2}$

$$\left.\begin{matrix} x+y+z=0 \\ 2x+y+z=1 \\ x+y+z=1 \end{matrix}\right\} \Rightarrow \boxed{\text{S. Incomp.}}$$

ii)
- Si $\left.\begin{matrix} a\neq 2 \\ a \neq 1 \end{matrix}\right| \rightarrow$ Los 3 planos tienen un punto común.

- Si $a=2 \rightarrow$ Los 3 planos tienen una recta común

- Si $a=1 \rightarrow$ El 1^{er} y 3^{er} plano son paralelos, el 2° es secante.



SEPT 94

i) $A \cdot X = B$

B es una matriz columna con todos sus elementos nulos.

S sistema homogéneo $\Leftrightarrow b_i = 0 \quad \forall i$

ii) $A \cdot X = 0$

Sea $X = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$ la solución trivial.

Obviamente: $\quad a_{i1}\cdot 0 + a_{i2}\cdot 0 + \dots + a_{in}\cdot 0 = 0 \quad \forall i$

iii) Si rango $A <$ n° incógnitas $\Rightarrow$ Sistema Compatible Indeterm.

JUN 95

$$A^* = \begin{pmatrix} 1 & 1 & -1 & 2 \\ m & 1 & 1 & 1 \\ 1 & -1 & 3 & 1 \\ 4 & 2 & 0 & m \end{pmatrix}$$

$$|A^*| = \begin{vmatrix} 1 & 1 & -1 & 2 \\ m & 1 & 1 & 1 \\ 1 & -1 & 3 & 1 \\ 4 & 2 & 0 & m \end{vmatrix} = \begin{vmatrix} 1 & 1 & -1 & 2 \\ m+1 & 2 & 0 & 3 \\ 4 & 2 & 0 & 7 \\ 4 & 2 & 0 & m \end{vmatrix} = -(-1)\begin{vmatrix} m+1 & 2 & 3 \\ 4 & 2 & 7 \\ 4 & 2 & m \end{vmatrix} =$$

$F2 \rightarrow F2+F1$
$F3 \rightarrow F3+3\cdot F1$

$F3 \rightarrow F3-F2$

$$= +1 \begin{vmatrix} m+1 & 2 & 3 \\ 4 & 2 & 7 \\ 0 & 0 & m-7 \end{vmatrix} = 2(m+1)(m-7) - 8(m-7) = (2m+2-8)(m-7) =$$

$$= (2m-6)(m-7)$$

- Si $\left.\begin{matrix} m \neq 3 \\ m \neq 7 \end{matrix}\right| \Rightarrow$ rango $A^+ = 4 \Rightarrow$ S. Incompatible

- Si $m = 7 \longrightarrow$ $\boxed{\text{rango } A^+ < 4}$

$$\left.\begin{aligned} x+y-z &= 2 \\ 7x+y+z &= 1 \\ x-y+3z &= 1 \\ 4x+2y \quad &= 7 \end{aligned}\right\} \qquad A = \begin{pmatrix} 1 & 1 & -1 \\ 7 & 1 & 1 \\ 1 & -1 & 3 \\ 4 & 2 & 0 \end{pmatrix}$$

$\begin{vmatrix} 1 & 1 \\ 7 & 1 \end{vmatrix} = -6 \neq 0 \Rightarrow$ rango $A \geq 2$

$\begin{vmatrix} 1 & 1 & -1 \\ 7 & 1 & 1 \\ 1 & -1 & 3 \end{vmatrix} = -8 \Rightarrow \left.\begin{matrix} \text{rango } A = 3 \\ \text{rango } A^+ = 3 \end{matrix}\right| \Rightarrow$ S.C.D.

$$\left.\begin{aligned} x+y-z &= 2 \\ 7x+y+z &= 1 \\ x-y+3z &= 1 \end{aligned}\right\} \longrightarrow \boxed{\begin{aligned} x &= -1 \\ y &= 11/2 \\ z &= 5/2 \end{aligned}}$$

- Si $m = 3 \longrightarrow$ $\boxed{\text{rango } A^+ < 4}$

$$\left.\begin{aligned} x+y-z &= 2 \\ 3x+y+z &= 1 \\ x-y+3z &= 1 \\ 4x+2y \quad &= 3 \end{aligned}\right\} \qquad A = \begin{pmatrix} 1 & 1 & -1 \\ 3 & 1 & 1 \\ 1 & -1 & 3 \\ 4 & 2 & 0 \end{pmatrix}$$

$\begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} = -2 \neq 0 \Rightarrow$ rango $A \geq 2$

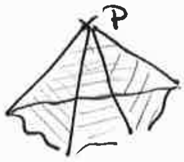
$\left.\begin{matrix} \begin{vmatrix} 1 & 1 & -1 \\ 3 & 1 & 1 \\ 1 & -1 & 3 \end{vmatrix} = 0 \\ \begin{vmatrix} 1 & 1 & -1 \\ 3 & 1 & 1 \\ 4 & 2 & 0 \end{vmatrix} = 0 \end{matrix}\right| \Rightarrow \boxed{\text{rango } A = 2}$

$$A^+ = \begin{pmatrix} 1 & 1 & -1 & 2 \\ 3 & 1 & 1 & 1 \\ 1 & -1 & 3 & 1 \\ 4 & 2 & 0 & 3 \end{pmatrix}$$

$\begin{vmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 1 & -1 & 1 \end{vmatrix} = -8 \Rightarrow \boxed{\text{rango } A^+ = 3}$

Sistema Incomp.

$\boxed{ii}$ Para $m=7$ los cuatro planos tienen un punto común. En cualquier otro caso, no tienen ningún punto común.



OTRO PROCEDIMIENTO :

$$A = \begin{pmatrix} 1 & 1 & -1 \\ m & 1 & 1 \\ 1 & -1 & 3 \\ 4 & 2 & 0 \end{pmatrix}$$

$\begin{vmatrix} -1 & 3 \\ 2 & 0 \end{vmatrix} = -6 \Rightarrow$ rango $A \geq 0 \quad \forall m$

$\begin{vmatrix} 1 & 1 & -1 \\ 1 & -1 & 3 \\ 4 & 2 & 0 \end{vmatrix} = 0$

$\begin{vmatrix} m & 1 & 1 \\ 1 & -1 & 3 \\ 4 & 2 & 0 \end{vmatrix} = 2+12+4-6m = 18-6m$

$18-6m=0 \Rightarrow m=3$

- Si: $m=3 \Rightarrow \boxed{\text{rango } A=2}$ y se estudiaría como caso particular, ya hecho en el otro procedimiento.
- Si $m \neq 3 \Rightarrow \boxed{\text{rango } A=3}$

Como $|A^+| = (2m-6)(m-7)$, según lo hecho en el otro procedimiento, tendríamos que para $m=3 \Rightarrow \boxed{\text{rango } A^+ \neq 3}$ Luego es S.C.D.

SEPT 95

$$\left.\begin{array}{l} x=1 \\ y=1 \\ z=1 \end{array}\right| \rightarrow \begin{array}{l} a+1+a^2=3 \rightarrow a^2+a-2=0 \rightarrow a = 1,\ -2 \\ -1-7+8=0 \rightarrow 0=0 \ \checkmark \\ 1+a^3+a^2=-3 \end{array}$$

$a=1$: $1+1^3+1^2=-3$ falso.

$a=-2$: $1-8+4=-3$ ✓

Luego: $\boxed{a=-2}$

$$\left.\begin{array}{l} -2x+y+4z=3 \\ -x-7y+8z=0 \\ x-8y+4z=-3 \end{array}\right\}$$

$$A=\begin{pmatrix} -2 & 1 & 4 \\ -1 & -7 & 8 \\ 1 & -8 & 4 \end{pmatrix} \qquad \begin{vmatrix} -2 & 1 \\ -1 & -7 \end{vmatrix} = 15 \Rightarrow \text{rango } A \geq 2$$

$$\begin{vmatrix} -2 & 1 & 4 \\ -1 & -7 & 8 \\ 1 & -8 & 4 \end{vmatrix} = 0 \Rightarrow \boxed{\text{rango } A=2}$$

S.C.I.

$$A^+=\left(\begin{array}{ccc|c} -2 & 1 & 4 & 3 \\ -1 & -7 & 8 & 0 \\ 1 & -8 & 4 & -3 \end{array}\right) \qquad \begin{vmatrix} -2 & 1 & 3 \\ -1 & -7 & 0 \\ 1 & -8 & -3 \end{vmatrix} = 0 \Rightarrow \boxed{\text{rango } A^+=2}$$

$$\left.\begin{array}{l} -2x+y=3-4z \\ -x-7y=-8z \end{array}\right\}$$

$$x=\frac{\begin{vmatrix} 3-4z & 1 \\ -8z & -7 \end{vmatrix}}{15} = \frac{-21+28z+8z}{15} = \frac{36z-21}{15} = \boxed{\frac{12z-7}{5}}$$

$$y=\frac{\begin{vmatrix} -2 & 3-4z \\ -1 & -8z \end{vmatrix}}{15} = \frac{16z+3-4z}{15} = \frac{12z+3}{15} = \boxed{\frac{4z+1}{5}}$$

$z \in \mathbb{R}$

Para $\boxed{z=1 \rightarrow \begin{array}{l} x=1 \\ y=1 \end{array}}$

JUN96 | i|

Rango A = Rango A^+ = nº incógnitas $\Rightarrow$ S.C.D.

Rango A = Rango A^+ < nº incógnitas $\Rightarrow$ S.C.I.

Rango A $\neq$ Rango A^+ $\Rightarrow$ S. Incompatible

ii|

$S_1 \equiv AX = B_1$

$S_2 \equiv AX = B_2$

S_1 infinitas soluciones $\Rightarrow$ Rango A < nº incóg $\Rightarrow$ S_2 no es S.C.D.

iii|

$$\left.\begin{array}{l} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \\ a_{31}x + a_{32}y = b_3 \\ \dots \\ a_{m1}x + a_{m2}y = b_m \end{array}\right\}$$

S.C.I $\Rightarrow$ serán rectas coincidentes.

Al cambiar los términos independientes se obtienen rectas paralelas ó coincidentes pero no secantes.

SEPT 97| i|

$$\left.\begin{array}{l} Ax + By + Cz = D \\ A'x + B'y + C'z = D' \end{array}\right\} \equiv S$$

i| • Sean (x_1, y_1, z_1) $(x_2\ y_2\ z_2)$ dos soluciones del sistema.

Para que $(x_1+x_2, y_1+y_2, z_1+z_2)$ sea también solución del sistema debe suceder:

$(x_1\ y_1\ z_1)$ $(x_2\ y_2\ z_2)$ soluciones de S. $(x_1+x_2, y_1+y_2, z_1+z_2)$ solución de S $\Rightarrow$ $A(x_1+x_2) + B(y_1+y_2) + C(z_1+z_2) = D \Rightarrow$

$Ax_1 + Ax_2 + By_1 + By_2 + Cz_1 + Cz_2 = D \Rightarrow$

$\Rightarrow Ax_1 + Ax_2 + By_1 + By_2 + Cz_1 + Cz_2 = D \Rightarrow D + D = D \Rightarrow D = 0 \Rightarrow$

$\Rightarrow Ax_1 + By_1 + Cz_1 + Ax_2 + By_2 + Cz_2 = D$

$\Rightarrow$ S es un sistema homogéneo.

Al revés:

S sistema homogéneo $\Rightarrow$ $A(x_1+x_2) + B(y_1+y_2) + C(z_1+z_2) =$
$= Ax_1 + Ax_2 + By_1 + By_2 + Cz_1 + Cz_2 = Ax_1 + By_1 + Cz_1 + Ax_2 + By_2 + Cz_2 =$
$= 0 + 0 = 0 \Rightarrow$ $(x_1+x_2, y_1+y_2, z_1+z_2)$ es solución de S.

Por lo tanto:

$(x_1+x_2, y_1+y_2, z_1+z_2)$ solución de S $\Leftrightarrow$ S homogéneo

• Sea (x, y, z) una solución de S. (cx, cy, cz) solución de S, $c \neq 1$ $\Rightarrow$ $Acx + Bcy + Ccz = D \Rightarrow$

$\Rightarrow c \cdot (Ax + By + Cz) = D \Rightarrow c \cdot D = D \Rightarrow (c-1)D = 0 \Rightarrow \boxed{D = 0}$ ya que $c \neq 1$

Al revés :

S sistema homogéneo $\Rightarrow A cx + Bcy + Ccz = c(Ax + By) + Cz) = c \cdot 0 = 0 \Rightarrow$

$\Rightarrow (cx, cy, cz)$ es solución de S

Por lo tanto :

(cx, cy, cz) solución de S $\forall c \neq 1$ $\Leftrightarrow$ S homogéneo

ii) S_1 equivalente a S_2 $\Leftrightarrow$ Toda solución de S_1 lo es de S_2 y viceversa.

- Multiplicando una ecuación por una constante $\neq 0$ se convierte un sistema en otro equivalente
- Sustituyendo una ecuación por la suma con otras ecuaciones convierte un sistema en otro equivalente.

$$\left.\begin{array}{l} x+y+z=6 \\ x-y-z=0 \end{array}\right\} \equiv \left.\begin{array}{l} 2x+2y+2z=12 \\ 2x \qquad\qquad = 6 \end{array}\right\}$$

JUN 98

i) $\left.\begin{array}{l} x+2y-z=0 \\ x-y \qquad = 8 \\ x \qquad -z=6 \\ 2x \qquad -2z=12 \end{array}\right\}$ $A = \begin{pmatrix} 1 & 2 & -1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \\ 2 & 0 & -2 \end{pmatrix}$

$$\begin{vmatrix} 1 & 2 & -1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{vmatrix} = 2 \Rightarrow \text{rango } A = 3$$

$$|A^+| = \begin{vmatrix} 1 & 2 & -1 & 0 \\ 1 & -1 & 0 & 8 \\ 1 & 0 & -1 & 6 \\ 2 & 0 & -2 & 12 \end{vmatrix} = 0 \Rightarrow \text{rango } A^+ = 3$$

$\Rightarrow$ S.C.D

$$\left.\begin{array}{l} x+2y-z=0 \\ x-y \qquad =8 \\ x \qquad -z=6 \end{array}\right\} \rightarrow \boxed{\begin{array}{l} x=5 \\ y=-3 \\ z=-1 \end{array}}$$

ii) La intersección de dos planos nunca resulta un único punto, podrá ser una recta ó no tener puntos comunes si fuesen paralelos ó infinitos puntos si son planos coincidentes. Por lo tanto, S_1 o S_2 no pueden ser ambos planos. De hecho, S_1 es una recta y S_2 un plano.

SEPT 98

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad a,b,c,d \neq 0$$

Rango $A = \begin{cases} \cancel{0} & \text{no puede ser, ya que } a,b,c,d \neq 0 \\ \boxed{1} & \\ \cancel{2} & \text{no puede ser, ya que el sistema tendría una única solución y ya conocemos dos: } (1,2) \ (0,0) \end{cases}$

Por lo tanto : $\boxed{\text{Rango } A = 1}$

ii) $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \left.\begin{array}{l} a+2b=0 \\ c+2d=0 \end{array}\right| \Rightarrow \begin{array}{l} a=-2b \\ c=-2d \end{array} \Rightarrow$

$\Rightarrow A = \begin{pmatrix} -2b & b \\ -2d & d \end{pmatrix}$ $(-2b, b)$ es perpendicular a $(1,2)$

Por ejemplo :

$(-2,1)$

$(-6,3)$ etc.

iii) No es posible, todos serán : $(-2b, b)$

JUN 2000) $\begin{pmatrix} 1 & \alpha \\ \beta & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1-\beta \\ \alpha \end{pmatrix}$

i) $\left.\begin{array}{l} x+\alpha y = 1-\beta \\ \beta x + y = \alpha \end{array}\right\}$

$P(2,-1)$ solución 1ª ecuación : $\left.\begin{array}{l} 2-\alpha = 1-\beta \\ 2\beta = \alpha \end{array}\right\} \Rightarrow \boxed{\begin{array}{l} \alpha = 2 \\ \beta = 1 \end{array}}$

$Q(2,0)$ " 2ª ecuación :

ii) $\left.\begin{array}{l} x+2y=0 \\ x+y=2 \end{array}\right\}$

$\begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} = -1 \Rightarrow \boxed{S.C.D.}$

SEPT 2000) i) $\left.\begin{array}{l} 2x+y-2z=1 \\ x-y+z=3 \end{array}\right\} \equiv \left\{\begin{array}{l} 2x+y-2z=1 \\ x-y+z=3 \\ \alpha(2x+y-2z)+\beta(x-y+z) = \alpha+3\beta \end{array}\right.$

$\alpha\cdot(-8+1-0)+\beta(-4-1+0) = \alpha+3\beta$

$-7\alpha-5\beta = \alpha+3\beta$

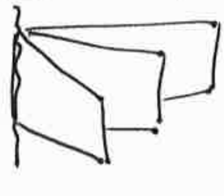
$-8\alpha = 8\beta \longrightarrow \boxed{\beta=-\alpha}$

Por ejemplo, con $\begin{array}{l} \alpha=1 \\ \beta=-1 \end{array}$ (equivale a restar las ecuaciones) :

$(2x+y-2z)-(x-y+z) = 1-3$

$\boxed{x+2y-3z=-2}$

ii) S son dos planos que se cortan sobre una recta, el nuevo sistema tendrá un Tercer plano que también pasa por dicha recta, luego los 3 planos pertenecen a un mismo haz.



JUN 2001

a) $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & -1 & 1 \end{pmatrix}$ $\quad \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = 1 \Rightarrow$ rango $A \geq 2$

$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & -1 & 1 \end{vmatrix} = 2 \Rightarrow \boxed{\text{rango } A = 3} \Rightarrow \boxed{\text{S.C.D. } \forall \lambda \in \mathbb{R}}$

b) Para $\lambda = 7$:

$$x = \frac{\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & 2 \\ 7 & -1 & 1 \end{vmatrix}}{2} = \frac{6}{2} = 3$$

$$y = \frac{\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 7 & 1 \end{vmatrix}}{2} = \frac{-6}{2} = -3$$

$$z = \frac{\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & -1 & 7 \end{vmatrix}}{2} = \frac{2}{2} = 1$$

SEPT 2001

$$A = \begin{pmatrix} 1+a & 1 & 1 \\ 1 & 1+a & 1 \\ 1 & 1 & 1+a \end{pmatrix}$$

a) $|A| = (1+a)^3 + 1 + 1 - (1+a) - (1+a) - (1+a) = \cancel{1} + \cancel{3a} + 3a^2 + a^3 + \cancel{2} - \cancel{3} - \cancel{3a} =$

$= a^3 + 3a^2 = a^2(a+3)$

$|A| = 0 \rightarrow a = 0,\ -3$

- Si $\begin{matrix} a \neq 0 \\ a \neq -3 \end{matrix} \Rightarrow$ S.C.D.

- Si $a = 0 \rightarrow \left.\begin{matrix} x+y+z=1 \\ x+y+z=1 \\ x+y+z=1 \end{matrix}\right\}$ S.C.I.

- Si $a = -3 \rightarrow \left.\begin{matrix} -2x+y+z=1 \\ x-2y+z=-2 \\ x+y-2z=10 \end{matrix}\right\}$

$A = \begin{pmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{pmatrix}$ $\quad \begin{vmatrix} -2 & 1 \\ 1 & -2 \end{vmatrix} = 3 \Rightarrow \boxed{\text{rango } A = 2}$

$A^+ = \begin{pmatrix} -2 & 1 & 1 & 1 \\ 1 & -2 & 1 & -2 \\ 1 & 1 & -2 & 10 \end{pmatrix}$ $\quad \begin{vmatrix} -2 & 1 & 1 \\ 1 & -2 & -2 \\ 1 & 1 & 10 \end{vmatrix} = 27 \Rightarrow \boxed{\text{rango } A^+ = 3}$

Sist. Incomp.

b) • Si $a = -3 \rightarrow \boxed{\begin{matrix} x = 1-y-z \\ y \in \mathbb{R} \\ z \in \mathbb{R} \end{matrix}}$

• Si $\begin{matrix} a \neq 0 \\ a \neq -3 \end{matrix} \rightarrow$

$$x = \frac{\begin{vmatrix} 1 & 1 & 1 \\ 1+a & 1+a & 1 \\ 1+a^2 & 1 & 1+a \end{vmatrix}}{a^2(a+3)} = \frac{\begin{vmatrix} 1 & 1 & 1 \\ a & a & 0 \\ a^2 & 0 & a \end{vmatrix}}{a^2(a+3)} = \frac{\cancel{a^2}\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ a & 0 & 1 \end{vmatrix}}{\cancel{a^2}(a+3)} =$$

$$= \frac{1-a-1}{a+3} = \boxed{\frac{-a}{a+3}}$$

F2 → F2 − F1
F3 → F3 − F1

$$y = \frac{\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+a & 1 \\ 1 & 1+a^2 & 1+a \end{vmatrix}}{a^2(a+3)} = \frac{\begin{vmatrix} 1+a & 1 & 1 \\ -a & a & 0 \\ -a & a^2 & a \end{vmatrix}}{a^2(a+3)} = \frac{\cancel{a^2}\begin{vmatrix} 1+a & 1 & 1 \\ -1 & 1 & 0 \\ -1 & a & 1 \end{vmatrix}}{\cancel{a^2}(a+3)} =$$

$$= \frac{1+a-a+1+1}{a+3} = \boxed{\frac{3}{a+3}}$$

$$z = \frac{\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+a & 1+a \\ 1 & 1 & 1+a^2 \end{vmatrix}}{a^2(a+3)} = \frac{\begin{vmatrix} 1+a & 1 & 1 \\ -a & a & a \\ -a & 0 & a^2 \end{vmatrix}}{a^2(a+3)} = \frac{\cancel{a}\begin{vmatrix} 1+a & 1 & 1 \\ -1 & 1 & 1 \\ -1 & 0 & a \end{vmatrix}}{\cancel{a^2}(a+3)} =$$

$$= \frac{a+a^2-1+1+a}{a+3} = \boxed{\frac{a^2+2a}{a+3}}$$

JUN 2002

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & -2 & 1 \\ 5 & \lambda & 3 \end{pmatrix}$$

$$\begin{vmatrix} 1 & 1 \\ 2 & -2 \end{vmatrix} = -4 \Rightarrow \text{rango } A \geq 2$$

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & -2 & 1 \\ 5 & \lambda & 3 \end{vmatrix} = -6+2\lambda+5+10-6-\lambda = \lambda+3$$

- Si $\lambda \neq -3 \Rightarrow$ S.C.D.
- Si $\lambda = -3 \Rightarrow \boxed{\text{rango } A = 2}$ → S.C.I.

$$A^+ = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & -2 & 1 & 2 \\ 5 & -3 & 3 & 5 \end{pmatrix} \quad \begin{vmatrix} 1 & 1 & 1 \\ 2 & -2 & 2 \\ 5 & -3 & 5 \end{vmatrix} = 0 \Rightarrow \boxed{\text{rango } A^+ = 2} \rightarrow \text{S.C.I.}$$

b)

$$\left.\begin{aligned} x+y &= 1-z \\ 2x-2y &= 2-z \end{aligned}\right\}$$

$$x = \frac{\begin{vmatrix} 1-z & 1 \\ 2-z & -2 \end{vmatrix}}{-4} = \frac{-2+2z-2+z}{-4} = \frac{3z-4}{-4} = \boxed{\frac{4-3z}{4}}$$

$$y = \frac{\begin{vmatrix} 1 & 1-z \\ 2 & 2-z \end{vmatrix}}{-4} = \frac{2-z-2+2z}{-4} = \boxed{-\frac{z}{4}}$$

$$\boxed{z \in \mathbb{R}}$$

JUN 2003

$$A = \begin{pmatrix} 1 & 0 & -3 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & -3 & 2 & 0 \\ -4 & 0 & 0 & \lambda \end{pmatrix}$$

$$\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 \Rightarrow \text{rango } A \geq 2$$

$$\begin{vmatrix} 1 & 0 & -3 \\ 0 & 1 & 0 \\ 0 & -3 & 2 \end{vmatrix} = 2 \Rightarrow \text{rango } A \geq 3$$

$$|A| = \begin{vmatrix} 1 & 0 & -3 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & -3 & 2 & 0 \\ -4 & 0 & 0 & \lambda \end{vmatrix} \underset{F4 \to F4 + 4 \cdot F1}{=} \begin{vmatrix} 1 & 0 & -3 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & -3 & 2 & 0 \\ 0 & 0 & -12 & \lambda \end{vmatrix} = 1 \cdot \begin{vmatrix} 1 & 0 & -1 \\ -3 & 2 & 0 \\ 0 & -12 & \lambda \end{vmatrix} =$$

$$= 2\lambda - 36$$

$$|A| = 0 \Rightarrow \lambda = 18$$

- Si $\lambda \neq 18 \Rightarrow \boxed{\text{rango } A = 4} \Rightarrow$ S.C.D.
- Si $\lambda = 18 \Rightarrow$ rango $A = 3$

$$A^+ = \left(\boxed{\begin{matrix} 1 & 0 & -3 \\ 0 & 1 & 0 \\ 0 & -3 & 2 \end{matrix}} \begin{matrix} 0 & -1 \\ -1 & 2 \\ 0 & 0 \end{matrix} \atop \begin{matrix} -4 & 0 & 0 & 18 & -5 \end{matrix}\right) \qquad \begin{vmatrix} 1 & 0 & -3 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & -3 & 2 & 0 \\ -4 & 0 & 0 & -5 \end{vmatrix} = 54 \Rightarrow \boxed{\text{rango } A^+ = 4} \Rightarrow \text{Sist. Incomp.}$$

b) $\lambda = 7$:

$$\left.\begin{array}{r} x - 3z = -1 \\ y - t = 2 \\ -3y + 2z = 0 \\ -4x + 7t = -5 \end{array}\right\} \longrightarrow \boxed{x = -\frac{67}{22} \quad y = -\frac{5}{11} \quad z = -\frac{15}{22} \quad t = -\frac{27}{11}}$$

Para no tener que hacer determinantes de 4° orden, mejor se eliminan incógnitas:

$$\begin{array}{l} x = 3z - 1 \\ y = t + 2 \end{array} \longrightarrow \left.\begin{array}{r} -3(t+2) + 2z = 0 \\ -4(3z-1) + 7t = -5 \end{array}\right\} \qquad \left.\begin{array}{r} -3t + 2z = 6 \\ 7t - 12z = -9 \end{array}\right\}$$

$$t = \frac{\begin{vmatrix} 6 & 2 \\ -9 & -12 \end{vmatrix}}{\begin{vmatrix} -3 & 2 \\ 7 & -12 \end{vmatrix}} = \frac{-72+18}{36-14} = \frac{-54}{22} = \boxed{\frac{-27}{11}}$$

$$z = \frac{\begin{vmatrix} -3 & 6 \\ 7 & -9 \end{vmatrix}}{22} = \frac{27-42}{22} = \boxed{\frac{-15}{22}}$$

$$x = 3 \cdot \frac{-15}{22} - 1 = \frac{-45-22}{+22} = \boxed{\frac{-67}{22}}$$

$$y = \frac{-27}{11} + 2 = \boxed{-\frac{5}{11}}$$

SEPT 03

$$A = \begin{pmatrix} 5 & 4 & 2 \\ 2 & 3 & 1 \\ 4 & -1 & m^2 \end{pmatrix}$$

$$\begin{vmatrix} 5 & 4 \\ 2 & 3 \end{vmatrix} = 7 \Rightarrow \text{rango } A \geq 2 \quad \forall m \in \mathbb{R}$$

$$|A| = \begin{vmatrix} 5 & 4 & 2 \\ 2 & 3 & 1 \\ 4 & -1 & m^2 \end{vmatrix} = 15m^2 - 4 + 16 - 24 - 8m^2 + 5 = 7m^2 - 7$$

$$|A| = 0 \Rightarrow m = \pm 1 \qquad \left\{\begin{array}{l} \text{Si } m = 1 \text{ o } m = -1 \rightarrow \text{rango } A = 2 \\ \text{Si } \left.\begin{array}{l} m \neq 1 \\ m \neq -1 \end{array}\right| \rightarrow \text{rango } A = 3 \end{array}\right.$$

• Si $m = 1$: $\left.\begin{array}{r} 5x + 4y + 2z = 0 \\ 2x + 3y + z = 0 \\ 4x - y + z = 0 \end{array}\right\}$ Al ser homogéneo, es compatible.

$$\left.\begin{array}{l} \text{rango } A = 2 \\ \text{n° incóg} = 3 \end{array}\right| \Rightarrow S.C.I. \qquad \left.\begin{array}{r} 5x + 4y = -2z \\ 2x + 3y = -z \end{array}\right\} \Rightarrow x = \frac{\begin{vmatrix} -2z & 4 \\ -z & 3 \end{vmatrix}}{\begin{vmatrix} 5 & 4 \\ 2 & 3 \end{vmatrix}} = \boxed{\frac{-2z}{7}}$$

$$y = \frac{\begin{vmatrix} 5 & -2z \\ 2 & -z \end{vmatrix}}{7} = \boxed{\frac{3z}{7}}$$

$$z \in \mathbb{R}$$

• Si $m = -1$: $\left.\begin{array}{r} 5x + 4y + 2z = 0 \\ 2x + 3y + z = 0 \\ 4x - y + z = -2 \end{array}\right\}$

$$A^+ = \begin{pmatrix} \boxed{\begin{matrix} 5 & 4 \\ 2 & 3 \end{matrix}} & \begin{matrix} 0 \\ 0 \end{matrix} \\ \begin{matrix} 4 & -1 \end{matrix} & -2 \end{pmatrix} \qquad |A^+| = \begin{vmatrix} 5 & 4 & 0 \\ 2 & 3 & 0 \\ 4 & -1 & -2 \end{vmatrix} = -14 \neq 0$$

SEPT 05

x = Nº billetes de 10 €
y = " " " 20 €
z = " " " 50 €

$$\left.\begin{array}{l} x+y+z=130 \\ 10x+20y+50z=3000 \end{array}\right\} \quad x=\alpha z \qquad \left.\begin{array}{l} x+y+z=130 \\ x+2y+5z=300 \\ x-\alpha z=0 \end{array}\right\}$$

a) $\boxed{\alpha=2}$

$$\left.\begin{array}{l} x+y+z=130 \\ x+2y+5z=300 \\ x-2z=0 \end{array}\right\}$$

$$x=\frac{\begin{vmatrix}130&1&1\\300&2&5\\0&0&-2\end{vmatrix}}{\begin{vmatrix}1&1&1\\1&2&5\\1&0&-2\end{vmatrix}}=\frac{80}{1}=80$$

$$y=\frac{\begin{vmatrix}1&130&1\\1&300&5\\1&0&-2\end{vmatrix}}{1}=\frac{10}{1}=10$$

$$z=\frac{\begin{vmatrix}1&1&130\\1&2&300\\1&0&0\end{vmatrix}}{1}=\frac{40}{1}=40$$

b) $\boxed{\alpha=3}$

$$\left.\begin{array}{l} x+y+z=130 \\ x+2y+5z=300 \\ x-3z=0 \end{array}\right\} \qquad \begin{vmatrix}1&1&1\\1&2&5\\1&0&-3\end{vmatrix}=0$$

$$M=\begin{pmatrix}1&1&1\\1&2&5\\1&0&-3\end{pmatrix} \quad \begin{vmatrix}1&1\\1&2\end{vmatrix}=1 \Rightarrow \text{rango } M=2$$

$$M^+=\begin{pmatrix}1&1&1&130\\1&2&5&300\\1&0&-3&0\end{pmatrix} \quad \begin{vmatrix}1&1&130\\1&2&300\\1&0&0\end{vmatrix}=40 \Rightarrow \text{rango } M^+=3$$

$\Rightarrow$ $\boxed{\text{Sistema Incompatible}}$ No tiene solución

c)

$$M^+=\begin{pmatrix}1&1&1&100\\1&2&5&d\\1&0&-3&0\end{pmatrix} \quad \begin{vmatrix}1&1&100\\1&2&d\\1&0&0\end{vmatrix}=d-200$$

Si $d\neq 200 \Rightarrow \left.\begin{array}{l}\text{rango } M^+=3\\ \text{rango } M=2\end{array}\right\} \Rightarrow$ S. Incompatible

Si $d=200 \Rightarrow \left.\begin{array}{l}\text{rango } M^+=2\\ \text{rango } M=2\end{array}\right\} \Rightarrow$ $\boxed{\text{S. Compatible Indeterminado}}$ Habrá infinitas soluciones

$n=3$

$\boxed{d=200\text{ €}}$

$$\left.\begin{array}{l} x+y=100-z \\ x+2y=200-5z \end{array}\right\} \qquad \boxed{\begin{array}{l} x=3z \\ y=100-4z \\ z\in\mathbb{R} \end{array}}$$

P. Ej.: $x=30 \quad y=60 \quad z=10$

$x=60 \quad y=20 \quad z=20$

JUN 05

$$\left.\begin{array}{l} x+y=1 \\ ay+z=0 \\ x+(1+a)y+az=a+1 \end{array}\right\}$$

a) $M=\begin{pmatrix}1&1&0\\0&a&1\\1&1+a&a\end{pmatrix}$ $\quad |M|=\begin{vmatrix}1&1&0\\0&a&1\\1&1+a&a\end{vmatrix}=a^2+1-(1+a)=a^2-a$

$|M|=0 \rightarrow a=\begin{cases}0\\1\end{cases}$

• Si $\left.\begin{array}{l}a\neq 0\\a\neq 1\end{array}\right| \Rightarrow$ rango $M=3 \Rightarrow$ $\boxed{\text{S. Compatible Determinado}}$

• Si $a=0$: $M=\begin{pmatrix}1&1&0\\0&0&1\\1&1&0\end{pmatrix}$ $\quad \begin{vmatrix}1&0\\0&1\end{vmatrix}=1 \Rightarrow$ rango $M=2$

$M^+=\begin{pmatrix}1&1&0&1\\0&0&1&0\\1&1&0&1\end{pmatrix}$ $\quad \begin{vmatrix}1&0&1\\0&1&0\\1&0&1\end{vmatrix}=0 \Rightarrow$ rango $M^+=2$

$n=3$

$\Rightarrow$ $\boxed{\text{S. Compat. Indetermi}}$

$\left.\begin{array}{l}\text{rango } A^{*}=3 \\ \text{rango } A=2\end{array}\right| \Longrightarrow$ Sistema Incompatible

• Si $\left.\begin{array}{l} m\neq 1 \\ m\neq -1\end{array}\right| \Longrightarrow \text{rango} A=3 \Longrightarrow$ S.C.D.

$$x=\frac{\begin{vmatrix}0&4&2\\0&3&1\\m-1&-1&m^2\end{vmatrix}}{\begin{vmatrix}5&4&2\\2&3&1\\4&-1&m^2\end{vmatrix}}=\frac{4(m-1)-6(m-1)}{7m^2-7}=\frac{2-2m}{7m^2-7}=\frac{-2(\cancel{m-1})}{7(m+1)(\cancel{m-1})}=\boxed{\frac{-2}{7(m+1)}}$$

$$y=\frac{\begin{vmatrix}5&0&2\\2&0&1\\4&m-1&m^2\end{vmatrix}}{7m^2-7}=\frac{4(m-1)-5(m-1)}{7m^2-7}=\frac{-m+1}{7m^2-7}=\frac{-(\cancel{m-1})}{7(m+1)(\cancel{m-1})}=\boxed{\frac{-1}{7(m+1)}}$$

$$z=\frac{\begin{vmatrix}5&4&0\\2&3&0\\4&-1&m-1\end{vmatrix}}{7m^2-7}=\frac{15(m-1)-8(m-1)}{7m^2-7}=\frac{\cancel{7}(\cancel{m-1})}{\cancel{7}(m+1)(\cancel{m-1})}=\boxed{\frac{1}{m+1}}$$

JUN 04

x = Edad del niño
y = " " padre
z = " " abuelo

$$\left.\begin{array}{l} y=\alpha x \\ 2z+x+y=182 \\ 2x+z=100\end{array}\right\} \qquad \left.\begin{array}{l} -\alpha x+y=0 \\ x+y+2z=182 \\ 2x+z=100\end{array}\right\}$$

a) $\boxed{\alpha=2}$

$$\left.\begin{array}{l} -2x+y=0 \\ x+y+2z=182 \\ 2x+z=100\end{array}\right\}$$

$$x=\frac{\begin{vmatrix}0&1&0\\182&1&2\\100&0&1\end{vmatrix}}{\begin{vmatrix}-2&1&0\\1&1&2\\2&0&1\end{vmatrix}}=\frac{18}{1}=18$$

$$y=\frac{\begin{vmatrix}-2&0&0\\1&182&2\\2&100&1\end{vmatrix}}{1}=36$$

$$z=\frac{\begin{vmatrix}-2&1&0\\1&1&182\\2&0&100\end{vmatrix}}{1}=64$$

b) $\boxed{\alpha=3}$

$$\left.\begin{array}{l} -3x+y=0 \\ x+y+2z=182 \\ 2x+z=100\end{array}\right\} \qquad \begin{vmatrix}-3&1&0\\1&1&2\\2&0&1\end{vmatrix}=0$$

$M=\begin{pmatrix}-3&1&0\\1&1&2\\2&0&1\end{pmatrix}$ $\begin{vmatrix}-3&1\\1&1\end{vmatrix}=-3-1=-4 \Rightarrow$ rango $M=2$

$M^{*}=\begin{pmatrix}-3&1&0&0\\1&1&2&182\\2&0&1&100\end{pmatrix}$ $\begin{vmatrix}-3&1&0\\1&0&182\\2&0&100\end{vmatrix}=-36 \Rightarrow$ rango $M^{*}=3$

$\Rightarrow$ $\boxed{\text{Sistema Incompatible}}$
No tiene solución

c) $M^{*}=\begin{pmatrix}-3&1&0&0\\1&1&2&200\\2&0&1&100\end{pmatrix}$ $\begin{vmatrix}-3&1&0\\1&1&200\\2&0&100\end{vmatrix}=0 \Rightarrow$ rango $M^{*}=2$
rango $M=2$
$n=3$
$\Rightarrow$ $\boxed{\text{S. Compatible Indeterminado}}$
Tiene infinitas soluciones

$$\left.\begin{array}{l} -3x+y=0 \\ x+y=182-2z\end{array}\right\} \Rightarrow \boxed{\begin{array}{l} x=\frac{81-z}{2} \\ y=\frac{243-3z}{2} \\ z\in\mathbb{R}\end{array}}$$

P.Ej: $z=61$ $y=20$ $x=10$
$z=51$ $y=30$ $x=15$

JUN 06

$$A=\begin{pmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & 0 \end{pmatrix} \qquad A^{+}=\begin{pmatrix} a & 1 & 1 & 1 \\ 1 & a & 1 & 1 \\ 1 & 1 & 0 & 1 \end{pmatrix}$$

$$\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} = -1 \neq 0 \Rightarrow r(A) \geq 2$$

$$|A| = \begin{vmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & 0 \end{vmatrix} = 1+1-a-a = 2-2a \quad ; \quad 2-2a=0 \Rightarrow a=1$$

- Si $a \neq 1 \Rightarrow \left.\begin{array}{l} r(A)=3 \\ r(A^{+})=3 \\ n=3 \end{array}\right| \Rightarrow$ S.C.D.

- Si $a=1 \Rightarrow r(A)=2$

$$A^{+}=\begin{pmatrix} 1 & \boxed{1 \quad 1} & 1 \\ 1 & 1 \quad 1 & 1 \\ 1 & \boxed{1 \quad 0} & 1 \end{pmatrix} \qquad \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} = 0 \Rightarrow \left.\begin{array}{l} r(A^{+})=2 \\ n=3 \end{array}\right\} \Rightarrow \text{S.C.I.}$$

Soluciones:

- Si $a \neq 1 \rightarrow$

$$x = \frac{\begin{vmatrix} 1 & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & 0 \end{vmatrix}}{2-2a} = \frac{1+1-a-1}{2-2a} = \frac{1-a}{2-2a} = \boxed{\frac{1}{2}}$$

$$y = \frac{\begin{vmatrix} a & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix}}{2-2a} = \frac{1+1-1-a}{2-2a} = \frac{1-a}{2-2a} = \boxed{\frac{1}{2}}$$

$$z = \frac{\begin{vmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & 1 \end{vmatrix}}{2-2a} = \frac{a^2+1+1-a-1-a}{2-2a} = \frac{a^2-2a+1}{2-2a} = \frac{(a-1)^2}{2-2a} =$$

$$= \frac{(a-1)^2}{-2(a-1)} = \frac{a-1}{-2} = \boxed{\frac{1-a}{2}}$$

- Si $a=1 \rightarrow$

$$\left.\begin{array}{l} y+z = 1-x \\ \boxed{y} = 1-x \end{array}\right\} \longrightarrow z = 1-x-y = 1-x-1+x = 0 \qquad \boxed{z=0}$$

$$\boxed{x \in \mathbb{R}}$$

SEPT 06

$$A=\begin{pmatrix} a+2 & a-1 & -1 \\ a & -1 & 1 \\ 1 & a & -1 \end{pmatrix} \qquad A^{+}=\begin{pmatrix} a+2 & a-1 & -1 & 3 \\ a & -1 & 1 & 3 \\ 1 & a & -1 & 1 \end{pmatrix}$$

$$|A| = \begin{vmatrix} a+2 & a-1 & -1 \\ a & -1 & 1 \\ 1 & a & -1 \end{vmatrix} = +\cancel{a}+\cancel{2}-\cancel{a^2}+\cancel{a}-\cancel{1}-\cancel{1}+\cancel{a}-a-a^2-2\cancel{a} = -a^2-a$$

$$-a^2-a=0;\ -a(a+1)=0;\ a = \begin{cases} -1 \\ 0 \end{cases}$$

- Si $\left.\begin{array}{l} a \neq -1 \\ a \neq 0 \end{array}\right| \Rightarrow \left.\begin{array}{l} r(A)=3 \\ r(A^{+})=3 \\ n=3 \end{array}\right| \Rightarrow$ S.C.D.

- Si $a=-1 \longrightarrow A=\begin{pmatrix} 1 & -2 & -1 \\ -1 & -1 & 1 \\ 1 & -1 & -1 \end{pmatrix} \qquad A^{+}=\begin{pmatrix} 1 & -2 & -1 & 3 \\ -1 & -1 & 1 & 3 \\ 1 & -1 & -1 & 1 \end{pmatrix}$

$$\begin{vmatrix} 1 & -2 \\ -1 & -1 \end{vmatrix} = -1-2 = -3 \neq 0 \Rightarrow r(A)=2$$

$$\begin{vmatrix} 1 & -2 & 3 \\ -1 & -1 & 3 \\ 1 & -1 & 1 \end{vmatrix} = -1+\cancel{3}-\cancel{6}+\cancel{3}-2+3 = 0 \Rightarrow r(A^{+})=2$$

S. Compatible Determinado

• $a=1$:

$$M = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 1 \end{pmatrix} \qquad \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} = 1 \Rightarrow \text{rango } M = 2$$

$$M^+ = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 2 & 1 & 2 \end{pmatrix} \qquad \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 2 & 2 \end{vmatrix} = 1 \Rightarrow \text{rango } M^+ = 3$$

$\Rightarrow \boxed{\text{S. Incompatible}}$

b) $\boxed{a=2}$

$$x = \frac{\begin{vmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 3 & 3 & 2 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 3 & 2 \end{vmatrix}} = \frac{4}{2} = \boxed{+2}$$

$$y = \frac{\begin{vmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 3 & 2 \end{vmatrix}}{-2} = \frac{-2}{+2} = \boxed{-1}$$

$$z = \frac{\begin{vmatrix} 1 & 1 & 1 \\ 0 & 2 & 0 \\ 1 & 3 & 3 \end{vmatrix}}{-2} = \frac{4}{+2} = \boxed{+2}$$

SEPT04

$$M = \begin{pmatrix} 2 & a & 1 \\ 1 & a & 0 \\ 0 & -1 & a \end{pmatrix}$$

$$|M| = \begin{vmatrix} 2 & a & 1 \\ 1 & a & 0 \\ 0 & -1 & a \end{vmatrix} = 2a^2 - 1 - a^2 = a^2 - 1$$

$$|M| = 0 \Rightarrow a = \begin{cases} -1 \\ 1 \end{cases}$$

a)

• Si $\left.\begin{matrix} a \neq 1 \\ a \neq -1 \end{matrix}\right\} \Rightarrow$ rango $M = 3 \Rightarrow \boxed{\text{S. Comp. Determinado}}$

• Si $a=1$:

$$M = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix} \qquad \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = 1 \Rightarrow \text{rango } M = 2$$

$$M^+ = \begin{pmatrix} 2 & 1 & 1 & 2 \\ 1 & 1 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{pmatrix} \qquad \begin{vmatrix} 2 & 1 & 2 \\ 1 & 1 & 1 \\ 0 & -1 & 0 \end{vmatrix} = 0 \Rightarrow \text{rango } M^+ = 2$$

$n = 3$

$\boxed{\text{S. Comp. Indet}}$

• Si $a=-1$:

$$M = \begin{pmatrix} 2 & -1 & 1 \\ 1 & -1 & 0 \\ 0 & -1 & -1 \end{pmatrix} \qquad \begin{vmatrix} 2 & -1 \\ 1 & -1 \end{vmatrix} = -1 \Rightarrow \text{rango } M = 2$$

$$M^+ = \begin{pmatrix} 2 & -1 & 1 & 2 \\ 1 & -1 & 0 & 1 \\ 0 & -1 & -1 & 0 \end{pmatrix} \qquad \begin{vmatrix} 2 & -1 & 2 \\ 1 & -1 & 1 \\ 0 & -1 & 0 \end{vmatrix} = 0 \Rightarrow \text{rango } M^+ = 2$$

$n = 3$

$\boxed{\text{S. Comp. Indet}}$

b) • Si $a=1$:

$$\left.\begin{matrix} 2x + y = 2 - z \\ x + y = 1 \end{matrix}\right\} \qquad \boxed{\begin{matrix} x = 1 - z \\ y = z \\ z \in \mathbb{R} \end{matrix}}$$

• Si $a=-1$:

$$\left.\begin{matrix} 2x - y = 2 - z \\ x - y = 1 \end{matrix}\right\} \qquad \boxed{\begin{matrix} x = 1 - z \\ y = -z \\ z \in \mathbb{R} \end{matrix}}$$

• Si $a=0 \longrightarrow A=\begin{pmatrix} 2 & -1 & -1 \\ 0 & -1 & 1 \\ 1 & 0 & -1 \end{pmatrix}$ $\quad A^+=\begin{pmatrix} 2 & -1 & -1 & 3 \\ 0 & -1 & 1 & 3 \\ 1 & 0 & -1 & 1 \end{pmatrix}$

$\begin{vmatrix} 2 & -1 \\ 0 & -1 \end{vmatrix} = -2 \neq 0 \Rightarrow r(A)=2$

$\begin{vmatrix} 2 & -1 & 3 \\ 0 & -1 & 3 \\ 1 & 0 & 1 \end{vmatrix} = -2-3+3 = -2 \neq 0 \Rightarrow r(A^+)=3$

S. Incompatible

Para $a=-1$:

$\left.\begin{aligned} x-2y &= z+3 \\ -x-y &= 3-z \end{aligned}\right\}$

$x = \dfrac{\begin{vmatrix} z+3 & -2 \\ 3-z & -1 \end{vmatrix}}{\begin{vmatrix} 1 & -2 \\ -1 & -1 \end{vmatrix}} = \dfrac{-z-3+6-2z}{-3} = \dfrac{3-3z}{-3} = \boxed{z-1}$

$y = \dfrac{\begin{vmatrix} 1 & z+3 \\ -1 & 3-z \end{vmatrix}}{-3} = \dfrac{3-z+z+3}{-3} = \boxed{-2}$

$\boxed{z \in \mathbb{R}}$

JUN 07

$x =$ nº vehículos marca A
$y =$ " " " B
$z =$ " " " C

$\left.\begin{aligned} x+y+z &= 21000 \\ 10000x+15000y+20000z &= 322000000 \\ \alpha x + y \quad &= 21000 \end{aligned}\right\}$

$\left.\begin{aligned} x+y+z &= 21000 \\ 2x+3y+4z &= 64400 \\ \alpha x + y \quad &= 21000 \end{aligned}\right\}$

b) $\alpha = 3$:

$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 3 & 1 & 0 \end{vmatrix} = 2+12-9-4 = 1$

$x = \dfrac{\begin{vmatrix} 21000 & 1 & 1 \\ 64400 & 3 & 4 \\ 21000 & 1 & 0 \end{vmatrix}}{1} =$ 1400 vehículos marca A

$y = \dfrac{\begin{vmatrix} 1 & 21000 & 1 \\ 2 & 64400 & 4 \\ 3 & 21000 & 0 \end{vmatrix}}{1} =$ 16 800 vehículos marca B

$z = \dfrac{\begin{vmatrix} 1 & 1 & 21000 \\ 2 & 3 & 64400 \\ 3 & 1 & 21000 \end{vmatrix}}{1} =$ 2 800 vehículos marca C

c) $A=\begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ \alpha & 1 & 0 \end{pmatrix}$ $\quad A^+=\begin{pmatrix} 1 & 1 & 1 & 21000 \\ 2 & 3 & 4 & 64400 \\ \alpha & 1 & 0 & 21000 \end{pmatrix}$

$\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = 1 \neq 0 \Rightarrow r(A) \geq 2$

$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ \alpha & 1 & 0 \end{vmatrix} = 2+4\alpha-3\alpha-4 = \alpha-2$

• Si $\alpha \neq 2 \Rightarrow \left.\begin{aligned} r(A)&=3 \\ r(A^+)&=3 \\ n&=3 \end{aligned}\right| \Rightarrow$ S.C.D.

• Si $\alpha = 2 \longrightarrow \boxed{r(A)=2}$

$$A^+ = \begin{pmatrix} \boxed{\begin{matrix} 1 & 1 \\ 2 & 3 \end{matrix}} & 1 & 21000 \\ & 4 & 64400 \\ 2 \quad 1 & 0 & 21000 \end{pmatrix}$$

$$\begin{vmatrix} 1 & 1 & 21000 \\ 2 & 3 & 64400 \\ 2 & 1 & 21000 \end{vmatrix} = 1400 \neq 0 \Rightarrow r(A^+) = 3 \longrightarrow \boxed{\text{S. Incompatible}}$$

$\boxed{\text{No tendrá solución real para } \alpha = 2}$

Sept 07

$$A = \begin{pmatrix} 2 & 1 \\ 1-a & -1 \\ a & 1 \end{pmatrix} \qquad A^+ = \begin{pmatrix} 2 & 1 & a \\ 1-a & -1 & 1 \\ a & 1 & a \end{pmatrix}$$

$$|A^+| = \begin{vmatrix} 2 & 1 & a \\ 1-a & -1 & 1 \\ a & 1 & a \end{vmatrix} = \cancel{-2a} + \cancel{a} - \cancel{a^2} + \cancel{a} + \cancel{a^2} - a + a^2 - 2 = a^2 - a - 2$$

$$a^2 - a - 2 = 0 \qquad a = \begin{cases} 2 \\ -1 \end{cases}$$

- Si $\left.\begin{matrix} a \neq 2 \\ a \neq -1 \end{matrix}\right| \Rightarrow r(A^+) = 3$ $\boxed{\text{S. Incompatible}}$

- Si $a = 2$: $A = \begin{pmatrix} 2 & 1 \\ -1 & -1 \\ 2 & 1 \end{pmatrix}$ $A^+ = \begin{pmatrix} 2 & 1 & 2 \\ -1 & -1 & 1 \\ 2 & 1 & 2 \end{pmatrix}$

$$\begin{vmatrix} 2 & 1 \\ -1 & -1 \end{vmatrix} = -2 + 1 = -1 \Rightarrow \left.\begin{matrix} r(A) = 2 \\ r(A^+) = 2 \\ n = 2 \end{matrix}\right\} \Rightarrow \text{S. C. D.}$$

- Si $a = -1$: $A = \begin{pmatrix} 2 & 1 \\ 2 & -1 \\ -1 & 1 \end{pmatrix}$

$$\begin{vmatrix} 2 & 1 \\ 2 & -1 \end{vmatrix} = -4 \Rightarrow \left.\begin{matrix} r(A) = 2 \\ r(A^+) = 2 \\ n = 2 \end{matrix}\right| \Rightarrow \text{S.C.D.}$$

Solución :

$a = 2$: $\left.\begin{matrix} 2x + y = 2 \\ -x - y = 1 \end{matrix}\right\}$ $\boxed{\begin{matrix} x = 3 \\ y = -4 \end{matrix}}$

$a = -1$ $\left.\begin{matrix} 2x + y = -1 \\ 2x - y = 1 \end{matrix}\right\}$ $\boxed{\begin{matrix} x = 0 \\ y = -1 \end{matrix}}$

SEPT 08

a)

$$A^+ = \left(\begin{array}{ccc|c} 2 & a & 6 & 0 \\ a & 2 & 4 & 2 \\ a & 2 & 6 & a-2 \end{array}\right)$$

$$\begin{vmatrix} 2 & 4 \\ 2 & 6 \end{vmatrix} = 4 \neq 0 \Rightarrow r \geq 2$$

$$\begin{vmatrix} 2 & a & 6 \\ a & 2 & 4 \\ a & 2 & 6 \end{vmatrix} = 24 + \cancel{12a} + 4a^2 - \cancel{12a} - 6a^2 - 16 = -2a^2 + 8$$

$$-2a^2 + 8 = 0 \ ; \quad a^2 = 4 \ ; \quad a = \pm 2$$

- Si $\left.\begin{array}{l} a \neq 2 \\ a \neq -2 \end{array}\right| \Rightarrow \boxed{r=3}$ $\quad \left.\begin{array}{l} r=3 \\ r^+=3 \\ n=3 \end{array}\right\} \Rightarrow \boxed{S.C.D.}$

- Si $a=2 \Rightarrow \boxed{r=2}$ $\quad A^+ = \left(\begin{array}{ccc|c} 2 & 2 & 6 & 0 \\ 2 & 2 & 4 & 2 \\ 2 & 2 & 6 & 0 \end{array}\right)$

$$\begin{vmatrix} 2 & 6 & 0 \\ 2 & 4 & 2 \\ 2 & 6 & 0 \end{vmatrix} = 0 \Rightarrow r^+ = 2$$

$\left.\begin{array}{l} r=2 \\ r^+=2 \\ n=3 \end{array}\right\} \Rightarrow \boxed{S.C.I}$

- Si $a=-2 \Rightarrow \boxed{r=2}$ $\quad A^+ = \left(\begin{array}{ccc|c} 2 & -2 & 6 & 0 \\ -2 & 2 & 4 & 2 \\ -2 & 2 & 6 & -4 \end{array}\right)$

$$\begin{vmatrix} -2 & 6 & 0 \\ 2 & 4 & 2 \\ 2 & 6 & -4 \end{vmatrix} = 32 + 24 + 48 + 24 = 128 \neq 0 \Rightarrow r^+ = 3$$

$\left.\begin{array}{l} r=2 \\ r^+=3 \end{array}\right\} \Rightarrow \boxed{S.\ Incompat.}$

b) $a=1$ Al ser $\left.\begin{array}{l} a \neq 2 \\ a \neq -2 \end{array}\right|$ es un sistema compatible Determinado.

$$\left.\begin{array}{l} 2x + y + 6z = 0 \\ x + 2y + 4z = 2 \\ x + 2y + 6z = -1 \end{array}\right\}$$

$$x = \frac{\begin{vmatrix} 0 & 1 & 6 \\ 2 & 2 & 4 \\ -1 & 2 & 6 \end{vmatrix}}{\begin{vmatrix} 2 & 1 & 6 \\ 1 & 2 & 4 \\ 1 & 2 & 6 \end{vmatrix}} = \frac{24 - 4 + \cancel{12} - \cancel{12}}{24 + \cancel{12} + 4 - \cancel{12} - 6 - 16} = \frac{20}{6} = \frac{10}{3}$$

$$y = \frac{\begin{vmatrix} 2 & 0 & 6 \\ 1 & 2 & 4 \\ 1 & -1 & 6 \end{vmatrix}}{6} = \frac{24 - 6 - 12 + 8}{6} = \frac{14}{6} = \frac{7}{3}$$

$$z = \frac{\begin{vmatrix} 2 & 1 & 0 \\ 1 & 2 & 2 \\ 1 & 2 & -1 \end{vmatrix}}{6} = \frac{-4 + 2 + 1 - 8}{6} = -\frac{9}{6} = -\frac{3}{2}$$

JUN 08

a) $A = \begin{pmatrix} x & y & x \\ y & 0 & y \\ 1 & z & z \end{pmatrix}$

$$|A| = \begin{vmatrix} x & y & x \\ y & 0 & y \\ 1 & z & z \end{vmatrix} = \cancel{xyz} + y^2 - y^2 z - \cancel{xyz} = y^2(1-z)$$

$$|A| = 0 \Rightarrow y^2(1-z) = 0 \nearrow \searrow \begin{matrix} y=0 \\ z=1 \end{matrix}$$

- Si $y=0$ ó $z=1$ no existe A^{-1}
- Si $y \neq 0$ y $z \neq 1$ existe A^{-1}

b) $$B \cdot A = (a \quad 2 \quad 3) \begin{pmatrix} x & y & x \\ y & 0 & y \\ 1 & z & z \end{pmatrix} = (ax+2y+3 \quad ay+3z \quad ax+2y+3z)$$

$$B \cdot A = C \longrightarrow (ax+2y+3 \quad ay+3z \quad ax+2y+3z) = (4 \quad 0 \quad 2)$$

$$\left.\begin{array}{l} ax+2y+3=4 \\ ay+3z=0 \\ ax+2y+3z=2 \end{array}\right\} \longrightarrow \left.\begin{array}{l} ax+2y=1 \\ ay+3z=0 \\ ax+2y+3z=2 \end{array}\right\}$$

$$M^+ = \left(\begin{array}{ccc|c} a & 2 & 0 & 1 \\ 0 & a & 3 & 0 \\ a & 2 & 3 & 2 \end{array}\right)$$

$$\begin{vmatrix} 2 & 0 \\ 2 & 3 \end{vmatrix} = 6 \neq 0 \Rightarrow r \geq 2$$

$$\begin{vmatrix} a & 2 & 0 \\ 0 & a & 3 \\ a & 2 & 3 \end{vmatrix} = 3a^2 + \cancel{6a} - \cancel{6a} = 3a^2$$

$$3a^2 = 0 \Rightarrow a = 0$$

- Si $a \neq 0 \Rightarrow \boxed{r=3}$; $\left.\begin{array}{l} r=3 \\ r^+=3 \\ n=3 \end{array}\right\} \Rightarrow$ Solución Única . $\boxed{\text{S.C.D.}}$

- Si $a=0$: $M^+ = \left(\begin{array}{ccc|c} 0 & 2 & 0 & 1 \\ 0 & 0 & 3 & 0 \\ 0 & 2 & 3 & 2 \end{array}\right)$ $\left.\begin{array}{l} r=2 \\ \\ \begin{vmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ 2 & 3 & 2 \end{vmatrix} = 12 \neq 0 \Rightarrow r^+=3 \end{array}\right\} \Rightarrow$ Sin Solución $\boxed{\text{S. Incomp.}}$

c) $a \neq 0$:

$$x = \frac{\begin{vmatrix} 1 & 2 & 0 \\ 0 & a & 3 \\ 2 & 2 & 3 \end{vmatrix}}{3a^2} = \frac{3a+12-6}{3a^2} = \frac{3a+6}{3a^2} = \boxed{\frac{a+2}{a^2}}$$

$$y = \frac{\begin{vmatrix} a & 1 & 0 \\ 0 & 0 & 3 \\ a & 2 & 3 \end{vmatrix}}{3a^2} = \frac{3a-6a}{3a^2} = \frac{-3a}{3a^2} = \boxed{-\frac{1}{a}}$$

$$z = \frac{\begin{vmatrix} a & 2 & 1 \\ 0 & a & 0 \\ a & 2 & 2 \end{vmatrix}}{3a^2} = \frac{2a^2-a^2}{3a^2} = \frac{a^2}{3a^2} = \boxed{\frac{1}{3}}$$

• Si $a=0$: $A^+ = \left(\begin{array}{ccc|c} 0 & 2 & 0 & 0 \\ -2 & 1 & 3 & 0 \\ 0 & -1 & 0 & 1 \end{array}\right)$

$\begin{vmatrix} 0 & 2 \\ -2 & 1 \end{vmatrix} = 4 \neq 0 \Rightarrow \boxed{r=2}$

$\begin{vmatrix} 0 & 2 & 0 \\ -2 & 1 & 0 \\ 0 & -1 & 1 \end{vmatrix} = 4 \neq 0 \Rightarrow \boxed{r^+=3}$ $\Big\} \Rightarrow \boxed{\text{S. Incompatible}}$

• Si $a=1$: $A^+ = \left(\begin{array}{ccc|c} 0 & 2 & 1 & 1 \\ -1 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$

$\begin{vmatrix} 0 & 2 \\ -1 & 1 \end{vmatrix} = 2 \neq 0 \Rightarrow \boxed{r=2}$

$\begin{vmatrix} 0 & 2 & 1 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{vmatrix} = 0 \Rightarrow \boxed{r^+=2}$ $\Big\} \Rightarrow \boxed{\text{S. C. I.}}$

$n=3$

• Si $a=2$: $A^+ = \left(\begin{array}{ccc|c} 0 & 2 & 2 & 2 \\ 0 & 1 & 3 & 0 \\ 0 & 1 & 0 & -1 \end{array}\right)$

$\begin{vmatrix} 2 & 3 \\ 1 & 3 \end{vmatrix} = 3 \neq 0 \Rightarrow \boxed{r=2}$

$\begin{vmatrix} 2 & 2 & 2 \\ 1 & 3 & 0 \\ 1 & 0 & -1 \end{vmatrix} = -6-6+2 = -10 \neq 0 \Rightarrow \boxed{r^+=3}$ $\Big\} \Rightarrow \boxed{\text{S. Incompatible}}$

b) $a=1$:

$$\left.\begin{array}{l} 2y + z = 1 \\ -x + y + 3z = 0 \\ 0 = 0 \end{array}\right\} \longrightarrow \left.\begin{array}{l} 2y = 1-z \\ -x + y = -3z \end{array}\right\} \qquad \boxed{\begin{array}{l} y = \frac{1-z}{2} \\ x = \frac{5z+1}{2} \\ z \in \mathbb{R} \end{array}}$$

Otro procedimiento:

$$\left.\begin{array}{l} 2y + az = a \\ (a-2)x + y + 3z = 0 \\ (a-1)y = 1-a \end{array}\right\}$$

→ • Si $a \neq 1 \Rightarrow y = \frac{1-a}{a-1} = -1 \longrightarrow \left.\begin{array}{l} -2 + az = a \\ (a-2)x - 1 + 3z = 0 \end{array}\right\} \Rightarrow$

→ • Si $a=1 \longrightarrow \left.\begin{array}{l} 2y + z = 1 \\ -x + y + 3z = 0 \\ 0 = 0 \end{array}\right\}$ (Que ya estudiamos antes)

$\Rightarrow \begin{array}{l} az = a+2 \\ (a-2)x + 3z = 1 \end{array}$

→ • Si $\left.\begin{array}{l} a \neq 0 \\ (a \neq 1) \end{array}\right\} \Rightarrow z = \frac{a+2}{a} \longrightarrow (a-2)x + 3\frac{a+2}{a} = 1 \Rightarrow$

→ • Si $a=0 \longrightarrow \left.\begin{array}{l} 0 = 2 \\ -2x + 3z = 1 \end{array}\right\} \longleftarrow \boxed{\text{Sin solución}}$

$\Rightarrow (a-2)x = 1 - \frac{3a+6}{a}$; $(a-2)x = \frac{a-3a-6}{a}$; $(a-2)x = \frac{-2a-6}{a} \Rightarrow$

$\Rightarrow$ → • Si $\left.\begin{array}{l} a \neq 2 \\ (a \neq 0) \\ (a \neq 1) \end{array}\right\} \Rightarrow x = \frac{-2a-6}{a(a-2)}$

→ • Si $a=2 \longrightarrow 0 \cdot x = \frac{-10}{2}$; $0 = -5 \longleftarrow \boxed{\text{Sin solución}}$

El resumen sería:

• Si $\left.\begin{array}{l} a \neq 2 \\ a \neq 0 \\ a \neq 1 \end{array}\right\} \Rightarrow \boxed{\text{S.C.D.}}$ $\boxed{x = \frac{-2a-6}{a(a-2)} \; ; \; y = -1 \; ; \; z = \frac{a+2}{a}}$

• Si $a=2 \Rightarrow \boxed{\text{S. Incomp}}$

• Si $a=1 \Rightarrow \boxed{\text{S. Incomp}}$

• Si $a=0 \Rightarrow \boxed{\text{S.C.I.}}$

JUN 09

$$A^+ = \left(\begin{array}{ccc|c} 1 & -a & 1 & a \\ a & -1 & 1 & 1 \\ -a & -1 & 1 & a \end{array}\right)$$

a)

$$\begin{vmatrix} 1 & -a & 1 \\ a & -1 & 1 \\ -a & -1 & 1 \end{vmatrix} = -1 - a + a^2 - a + a^2 + 1 = 2a^2 - 2a = 2a(a-1)$$

$$2a(a-1) = 0 \Rightarrow a = \begin{cases} 0 \\ 1 \end{cases}$$

• Si $\left.\begin{array}{l} a \neq 0 \\ a \neq 1 \end{array}\right| \Rightarrow \boxed{r=3}$ $\quad \left.\begin{array}{l} r=3 \\ r^+=3 \\ n=3 \end{array}\right\} \Rightarrow \boxed{\text{S.C.D.}}$

• Si $a=0 \Rightarrow A^+ = \left(\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 \\ 0 & -1 & 1 & 0 \end{array}\right)$

$$\left.\begin{array}{l} \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix} = -1 \neq 0 \Rightarrow \boxed{r=2} \\ \begin{vmatrix} 1 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & -1 & 0 \end{vmatrix} = 1 \neq 0 \Rightarrow \boxed{r^+=3} \end{array}\right\} \Rightarrow \boxed{\text{S. Incompatible}}$$

• Si $a=1 \Rightarrow A^+ = \left(\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 \end{array}\right)$

$$\left.\begin{array}{l} \begin{vmatrix} 1 & -1 \\ -1 & -1 \end{vmatrix} = -1-1 = -2 \neq 0 \Rightarrow \boxed{r=2} \\ \begin{vmatrix} 1 & -1 & 1 \\ 1 & -1 & 1 \\ -1 & -1 & 1 \end{vmatrix} = 0 \Rightarrow \boxed{r^+=2} \\ n=3 \end{array}\right\} \Rightarrow \boxed{\text{S. C. Indeterminado}}$$

b) $a=2$: Al ser $\left.\begin{array}{l} a\neq 0 \\ a \neq 1 \end{array}\right\}$ es un sistema compatible determinado.

$$x = \frac{\begin{vmatrix} 2 & -2 & 1 \\ 1 & -1 & 1 \\ 2 & -1 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & -2 & 1 \\ 2 & -1 & 1 \\ -2 & -1 & 1 \end{vmatrix}} = \frac{-2-1-4+2+2+2}{-1-2+4-2+4+1} = \boxed{\frac{-1}{4}}$$

$$y = \frac{\begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ -2 & 2 & 1 \end{vmatrix}}{4} = \frac{1+4-4+2-4-2}{4} = \boxed{\frac{-3}{4}}$$

$$z = \frac{\begin{vmatrix} 1 & -2 & 2 \\ 2 & -1 & 1 \\ -2 & -1 & 2 \end{vmatrix}}{4} = \frac{-2-4+4-4+8+1}{4} = \boxed{\frac{3}{4}}$$

SEPT 09

$$A^+ = \left(\begin{array}{ccc|c} 0 & 2 & a & a \\ a-2 & 1 & 3 & 0 \\ 0 & a-1 & 0 & 1-a \end{array}\right)$$

$$\begin{vmatrix} 0 & 2 & a \\ a-2 & 1 & 3 \\ 0 & a-1 & 0 \end{vmatrix} = a(a-2)(a-1)$$

$$a(a-2)(a-1) = 0 \Rightarrow a = \begin{cases} 0 \\ 1 \\ 2 \end{cases}$$

• Si $\left.\begin{array}{l} a \neq 0 \\ a \neq 1 \\ a \neq 2 \end{array}\right\} \Rightarrow \left.\begin{array}{l} r=3 \\ r^+=3 \\ n=3 \end{array}\right\} \Rightarrow \boxed{\text{S. C. D.}}$

JUN 10 fase General

$$\left.\begin{array}{l} 2x-y+z=2 \\ ax-y+z=1 \\ x+ay+z=1 \end{array}\right\}$$

a)

$$A=\begin{pmatrix} 2 & -1 & 1 \\ a & -1 & 1 \\ 1 & a & 1 \end{pmatrix} \qquad \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = 1 \neq 0 \Rightarrow r(A) \geq 2 \text{ para cualquier } a \in \mathbb{R}$$

$$|A| = \begin{vmatrix} 2 & -1 & 1 \\ a & -1 & 1 \\ 1 & a & 1 \end{vmatrix} = -2 + a^2 - \cancel{1} + \cancel{1} + a - 2a = a^2 - a - 2$$

$$a^2 - a - 2 = 0 \Rightarrow a = \begin{cases} 2 \\ -1 \end{cases}$$ Por lo tanto, si $a=2$ ó $a=-1$, $r(A)=2$.

- Si $\left.\begin{array}{l} a \neq 2 \\ a \neq -1 \end{array}\right\} \Rightarrow \left.\begin{array}{l} r(A)=3 \\ r(A^*)=3 \\ n=3 \end{array}\right\} \Rightarrow$ S. Compatible Determinado.

- Si $a=2$: $A^* = \left(\begin{array}{ccc|c} 2 & -1 & 1 & 2 \\ 2 & -1 & 1 & 1 \\ 1 & 2 & 1 & 1 \end{array}\right)$ $\begin{vmatrix} 2 & 1 & 2 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{vmatrix} = 2+8+1-2-2-2 = 5 \neq 0 \Rightarrow r(A^*)=3$

  $\left.\begin{array}{l} r(A)=2 \\ r(A^*)=3 \end{array}\right\} \Rightarrow$ S. Incompatible.

- Si $a=-1$: $A^* = \left(\begin{array}{ccc|c} 2 & -1 & 1 & 2 \\ -1 & -1 & 1 & 1 \\ 1 & -1 & 1 & 1 \end{array}\right)$ $\begin{vmatrix} 2 & 1 & 2 \\ -1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 2-2+1-2+1-2 = -2 \neq 0 \Rightarrow r(A^*)=3$

  $\left.\begin{array}{l} r(A)=2 \\ r(A^*)=3 \end{array}\right\} \Rightarrow$ S. Incompatible.

b) $\underline{\underline{a=0}}$

$$\left.\begin{array}{r} 2x-y+z=2 \\ -y+z=1 \\ +z=1 \end{array}\right\}$$

$$x = \frac{\begin{vmatrix} 2 & -1 & 1 \\ 1 & -1 & 1 \\ 1 & 0 & 1 \end{vmatrix}}{\begin{vmatrix} 2 & -1 & 1 \\ 0 & -1 & 1 \\ 1 & 0 & 1 \end{vmatrix}} = \frac{-2-1+1+1}{-2-1+1} = \frac{-1}{-2} = \boxed{\frac{1}{2}}$$

$$y = \frac{\begin{vmatrix} 2 & 2 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}}{-2} = \frac{2+2-1-2}{-2} = \boxed{-\frac{1}{2}}$$

$$z = \frac{\begin{vmatrix} 2 & -1 & 2 \\ 0 & -1 & 1 \\ 1 & 0 & 1 \end{vmatrix}}{-2} = \frac{-2-1+2}{-2} = \boxed{\frac{1}{2}}$$

JUN 10 fase Específica

$$\left.\begin{array}{r} ax+y+z=a \\ x+y+z=a \\ y+az=2 \end{array}\right\}$$

a) $A = \begin{pmatrix} a & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & a \end{pmatrix}$ $\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} = 1 \neq 0 \Rightarrow r(A) \geq 2$ para cualquier $a \in \mathbb{R}$

$$|A| = \begin{vmatrix} a & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & a \end{vmatrix} = a^2 + 1 - a - a = a^2 - 2a + 1$$

$a^2 - 2a + 1 = 0 \rightarrow a = 1$ Por lo Tanto, si $a=1$, $r(A)=2$

- Si $a \neq 1 \Rightarrow \left.\begin{array}{l} r(A)=3 \\ r(A^*)=3 \\ n=3 \end{array}\right\} \rightarrow$ S. Compatible Determinado.

- Si $a=1 \Rightarrow A^* = \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 2 \end{array}\right)$

$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 2 \end{vmatrix} = 2+1-2-1=0 \Rightarrow \left.\begin{array}{l} r(A)=2 \\ r(A^*)=2 \\ n=3 \end{array}\right\} \rightarrow$ S. Compatible Indeterminado

b) $\underline{a=1}$: $\left.\begin{array}{l} \cancel{x+y+z=1} \\ x+y+z=1 \\ y+z=2 \end{array}\right\} \rightarrow \left.\begin{array}{l} x+y=1-z \\ y=\boxed{2-z} \\ z\in\mathbb{R} \end{array}\right\} \rightarrow x+2-z=1-z \Rightarrow \boxed{x=-1}$

$$\boxed{\text{Soluciones}: \begin{array}{l} x=-1 \\ y=2-z \\ z\in\mathbb{R} \end{array}}$$

SEPT 10 fase General

$\left.\begin{array}{l} x+y+z=1 \\ 2x+y+mz=1 \\ 4x+y+m^2z=m \end{array}\right\}$

a) $A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & m \\ 4 & 1 & m^2 \end{pmatrix}$ $\quad \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = -1 \neq 0 \Rightarrow r(A) \geq 2$ para cualquier $m\in\mathbb{R}$

$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & m \\ 4 & 1 & m^2 \end{vmatrix} = m^2+2+4m-4-2m^2-m = -m^2+3m-2$

$-m^2+3m-2=0 \Rightarrow m = \begin{cases} 1 \\ 2 \end{cases}$ Por lo tanto, si $m=1$ ó $m=2$, $r(A)=2$

- Si $\left.\begin{array}{l} m\neq 1 \\ m\neq 2 \end{array}\right\} \Rightarrow \left.\begin{array}{l} r(A)=3 \\ r(A^*)=3 \\ n=3 \end{array}\right\} \Rightarrow$ S. Compatible Determinado

- Si $m=1$: $A^* = \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 2 & 1 & 1 & 1 \\ 4 & 1 & 1 & 1 \end{array}\right)$

$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 4 & 1 & 1 \end{vmatrix} = 0 \Rightarrow \left.\begin{array}{l} r(A)=2 \\ r(A^*)=2 \\ n=3 \end{array}\right\} \rightarrow$ S. Compatible Indeterminado

- Si $m=2$: $A^* = \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 2 & 1 & 2 & 1 \\ 4 & 1 & 4 & 2 \end{array}\right)$

$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 4 & 1 & 2 \end{vmatrix} = 2+2+4-4-4-1 = -1 \neq 0 \Rightarrow \left.\begin{array}{l} r(A)=2 \\ r(A^*)=3 \end{array}\right\} \Rightarrow$ S. Incompatible

b) $\underline{m=1}$: $\left.\begin{array}{l} x+y+z=1 \\ 2x+y+z=1 \\ \cancel{4x+y+z=1} \end{array}\right\} \rightarrow \left.\begin{array}{l} x+y=1-z \\ 2x+y=1-z \\ z\in\mathbb{R} \end{array}\right\}$

$x = \dfrac{\begin{vmatrix} 1-z & 1 \\ 1-z & 1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix}} = \dfrac{1-z-1+z}{-1} = \boxed{0}$ $\quad y = \dfrac{\begin{vmatrix} 1 & 1-z \\ 2 & 1-z \end{vmatrix}}{-1} = \dfrac{1-z-2+2z}{-1} = \boxed{1-z}$

$$\boxed{\text{Soluciones}: \begin{array}{l} x=0 \\ y=1-z \\ z\in\mathbb{R} \end{array}}$$

SEPT 10
fase
Específica

$$\left.\begin{aligned} 2x-y+z&=1\\ 3x+ay+2z&=3\\ x+2y+az&=2 \end{aligned}\right\}$$

a)

$$A=\begin{pmatrix} 2 & -1 & 1\\ 3 & a & 2\\ 1 & 2 & a \end{pmatrix} \qquad \begin{vmatrix} 2 & 1\\ 3 & 2 \end{vmatrix}=1\neq 0 \Rightarrow r(A)\geq 2 \text{ para cualquier } a\in\mathbb{R}$$

$$|A|=\begin{vmatrix} 2 & -1 & 1\\ 3 & a & 2\\ 1 & 2 & a \end{vmatrix} = 2a^2+6-2-a+3a-8 = 2a^2+2a-4$$

$$2a^2+2a-4=0 \Rightarrow a=\begin{cases}1\\-2\end{cases}$$ Por lo Tanto, si $a=1$ ó $a=-2$, $r(A)=2$

- Si $\left.\begin{matrix} a\neq 1\\ a\neq -2\end{matrix}\right\} \Rightarrow \left.\begin{matrix} r(A)=3\\ r(A^*)=3\\ n=3\end{matrix}\right\} \Rightarrow$ S. Compatible Determinado

- Si $a=1$: $A^*=\left(\begin{array}{ccc|c} 2 & -1 & 1 & 1\\ 3 & 1 & 2 & 3\\ 1 & 2 & 1 & 2 \end{array}\right)$

$$\begin{vmatrix} 2 & 1 & 1\\ 3 & 2 & 3\\ 1 & 1 & 2 \end{vmatrix} = 8+3+3-2-6-6=0 \Rightarrow \left.\begin{matrix} r(A)=2\\ r(A^*)=2\\ n=3\end{matrix}\right\} \rightarrow \text{S. Compatible IndeTerminado}$$

- Si $a=-2$: $A^*=\left(\begin{array}{ccc|c} 2 & -1 & 1 & 1\\ 3 & -2 & 2 & 3\\ 1 & 2 & -2 & 2 \end{array}\right)$

$$\begin{vmatrix} 2 & 1 & 1\\ 3 & 2 & 3\\ 1 & -2 & 2 \end{vmatrix} = 8-6+3-2-6+12 = 23-14=9\neq 0 \Rightarrow \left.\begin{matrix} r(A)=2\\ r(A^*)=3\end{matrix}\right\} \rightarrow \text{S. Incompatible}$$

b) $\underline{\underline{a=0}}$:

$$\left.\begin{aligned} 2x-y+z&=1\\ 3x+2z&=3\\ x+2y&=2 \end{aligned}\right\}$$

$$x=\frac{\begin{vmatrix} 1 & -1 & 1\\ 3 & 0 & 2\\ 2 & 2 & 0 \end{vmatrix}}{\begin{vmatrix} 2 & -1 & 1\\ 3 & 0 & 2\\ 1 & 2 & 0 \end{vmatrix}} = \frac{6-4-4}{6-2-8} = \frac{-2}{-4} = \boxed{\frac{1}{2}}$$

$$y=\frac{\begin{vmatrix} 2 & 1 & 1\\ 3 & 3 & 2\\ 1 & 2 & 0 \end{vmatrix}}{-4} = \frac{6+2-3-8}{-4} = \boxed{\frac{3}{4}}$$

$$z=\frac{\begin{vmatrix} 2 & -1 & 1\\ 3 & 0 & 3\\ 1 & 2 & 2 \end{vmatrix}}{-4} = \frac{6-3+6-12}{-4} = \boxed{\frac{3}{4}}$$

JUN 11
fase
General

$$\left.\begin{aligned} x+ay-z&=1+a\\ x+y-az&=2\\ x-y-z&=a \end{aligned}\right\}$$

a)

$$A=\begin{pmatrix} 1 & a & -1\\ 1 & 1 & -a\\ 1 & -1 & -1 \end{pmatrix} \qquad \begin{vmatrix} 1 & 1\\ 1 & -1 \end{vmatrix}=-2\neq 0 \Rightarrow r(A)\geq 2 \text{ para cualquier } a\in\mathbb{R}$$

$$|A|=\begin{vmatrix} 1 & a & -1\\ 1 & 1 & -a\\ 1 & -1 & -1 \end{vmatrix} = -1+1-a^2+1+a-a = 1-a^2$$

$$1-a^2=0 \Rightarrow a=\begin{cases}1\\-1\end{cases}$$ Por lo Tanto, si $a=1$ ó $a=-1$, $r(A)=2$

• Si $\left.\begin{array}{l} a\neq 1 \\ a\neq -1 \end{array}\right| \rightarrow \left.\begin{array}{l} r(A)=3 \\ r(A^*)=3 \\ n=3 \end{array}\right| \Rightarrow$ S. Compatible Determinado

• Si $a=1$: $A^* = \left(\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 1 & 1 & -1 & 2 \\ 1 & -1 & -1 & 1 \end{array}\right)$

$\left|\begin{array}{ccc} 1 & 1 & 2 \\ 1 & 1 & 2 \\ 1 & -1 & 1 \end{array}\right| = 0 \Rightarrow \left.\begin{array}{l} r(A)=2 \\ r(A^*)=2 \\ n=3 \end{array}\right| \Rightarrow$ S. Compatible Indeterminado

• Si $a=-1$: $A^* = \left(\begin{array}{ccc|c} 1 & -1 & -1 & 0 \\ 1 & 1 & 1 & 2 \\ 1 & -1 & -1 & -1 \end{array}\right)$

$\left|\begin{array}{ccc} 1 & -1 & 0 \\ 1 & 1 & 2 \\ 1 & -1 & -1 \end{array}\right| = -1-2-1+2 = -2 \neq 0 \Rightarrow \left.\begin{array}{l} r(A)=2 \\ r(A^*)=3 \end{array}\right| \Rightarrow$ S. Incompatible

b) $\underline{\underline{a=1}}$:

$\left.\begin{array}{l} \cancel{x+y-z=2} \\ x+y-z=2 \\ x-y-z=1 \end{array}\right\} \longrightarrow \left.\begin{array}{l} x+y=2+z \\ x-y=1+z \end{array}\right\}$

$z\in\mathbb{R}$

$$x = \frac{\left|\begin{array}{cc} 2+z & 1 \\ 1+z & -1 \end{array}\right|}{\left|\begin{array}{cc} 1 & 1 \\ 1 & -1 \end{array}\right|} = \frac{-2-z-1-z}{-1-1} = \frac{-3-2z}{-2} = \boxed{\frac{3+2z}{2}}$$

$$y = \frac{\left|\begin{array}{cc} 1 & 2+z \\ 1 & 1+z \end{array}\right|}{-2} = \frac{1+z-2-z}{-2} = \boxed{\frac{1}{2}}$$

Soluciones: $x = \dfrac{3+2z}{2}$

$y = \dfrac{1}{2}$

$z\in\mathbb{R}$

JUN 11 fase Específica

$\left.\begin{array}{l} x+y+z=a \\ x+ay+z=a \\ x+y+az=1 \end{array}\right\}$

a) $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & a \end{pmatrix}$

$|A| = \left|\begin{array}{ccc} 1 & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & a \end{array}\right| = a^2+1+1-a-a-1 = a^2-2a+1$

$a^2-2a+1=0 \rightarrow a=1$

• Si $a\neq 1 \Rightarrow \left.\begin{array}{l} r(A)=3 \\ r(A^*)=3 \\ n=3 \end{array}\right| \Rightarrow$ S. Compatible Determinado

• Si $a=1$: $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \rightarrow$ Obviamente $r(A)=1$

$A^* = \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{array}\right) \rightarrow$ Obviamente $r(A^*)=1$

$n=3$ $\Rightarrow$ S. Compatible Indeterminado

b) $\underline{a=1}$:

$\left.\begin{array}{l} x+y+z=1 \\ \cancel{x+y+z=1} \\ \cancel{x+y+z=1} \end{array}\right\} \rightarrow \boxed{\begin{array}{l} x=1-y-z \\ y\in\mathbb{R} \\ z\in\mathbb{R} \end{array}}$

JUL 11 fase General

$$y = ax^3 + bx^2 + cx + 4$$

$$\begin{array}{l} (1,10) \to 10 = a+b+c+4 \\ (-1,2) \to 2 = -a+b-c+4 \\ (2,26) \to 26 = 8a+4b+2c+4 \end{array} \Bigg\} \to \begin{array}{l} a+b+c=6 \\ -a+b-c=-2 \\ 8a+4b+2c=22 \end{array} \Bigg\} \to \begin{array}{l} a+b+c=6 \\ -a+b-c=-2 \\ 4a+2b+c=11 \end{array} \Bigg\}$$

$$M = \begin{pmatrix} 1 & 1 & 1 \\ -1 & 1 & -1 \\ 4 & 2 & 1 \end{pmatrix}$$

$$|M| = \begin{vmatrix} 1 & 1 & 1 \\ -1 & 1 & -1 \\ 4 & 2 & 1 \end{vmatrix} = 1 - \cancel{2} - 4 - 4 + 1 + \cancel{2} = -6 \neq 0 \Rightarrow \begin{array}{l} r(M)=3 \\ r(M^*)=3 \\ n=3 \end{array} \Bigg\} \Rightarrow \text{S. Compatible Determinado} \Rightarrow \boxed{\text{Solución única}}$$

$$a = \frac{\begin{vmatrix} 6 & 1 & 1 \\ -2 & 1 & -1 \\ 11 & 2 & 1 \end{vmatrix}}{-6} = \frac{6-4-11-11+2+12}{-6} = \frac{-6}{-6} = \boxed{1}$$

$$b = \frac{\begin{vmatrix} 1 & 6 & 1 \\ -1 & -2 & -1 \\ 4 & 11 & 1 \end{vmatrix}}{-6} = \frac{-2-\cancel{11}-24+8+6+\cancel{11}}{-6} = \frac{-12}{-6} = \boxed{2}$$

$$c = \frac{\begin{vmatrix} 1 & 1 & 6 \\ -1 & 1 & -2 \\ 4 & 2 & 11 \end{vmatrix}}{-6} = \frac{11-12-8-24+11+4}{-6} = \frac{-18}{-6} = \boxed{3}$$

$$\Rightarrow \boxed{y = x^3 + 2x^2 + 3x + 4}$$

JUL 11 fase General

a)

$$A = \begin{pmatrix} a & 2 & 1 \\ 0 & -1 & -a \\ 1 & -2 & a \end{pmatrix} \qquad \begin{vmatrix} 0 & -1 \\ 1 & -2 \end{vmatrix} = 1 \neq 0 \Rightarrow r(A) \geq 2 \text{ para cualquier } a \in \mathbb{R}$$

$$|A| = \begin{vmatrix} a & 2 & 1 \\ 0 & -1 & -a \\ 1 & -2 & a \end{vmatrix} = -a^2 - 2a + 1 - 2a^2 = -3a^2 - 2a + 1$$

$$-3a^2 - 2a + 1 = 0 \Rightarrow a = \begin{cases} -1 \\ 1/3 \end{cases}$$

- Si $\left.\begin{array}{l} a \neq -1 \\ a \neq 1/3 \end{array}\right| \Rightarrow r(A) = 3$
- Si $a = -1$ ó $a = 1/3 \Rightarrow r(A) = 2$

b) $a = -1$:

$$\begin{array}{r} \cancel{-x + 2y + z = 0} \\ -y + z = 0 \\ x - 2y - z = 0 \end{array} \Bigg\} \to \begin{array}{l} -y = -z \to \boxed{y = z} \\ x - 2y = z \to x = z + 2y = \boxed{3z} \end{array} \quad \boxed{z \in \mathbb{R}}$$

$$\boxed{\text{Soluciones: } \begin{array}{l} x = 3z \\ y = z \\ z \in \mathbb{R} \end{array}}$$

JUL 11 fase Específica

$$\begin{array}{l} -ax + y = 1 \\ -x + ay - z = 0 \\ x + y + az = 1 \end{array} \Bigg\}$$

a)

$$A = \begin{pmatrix} -a & 1 & 0 \\ -1 & a & -1 \\ 1 & 1 & a \end{pmatrix}$$

$$|A| = \begin{vmatrix} -a & 1 & 0 \\ -1 & a & -1 \\ 1 & 1 & a \end{vmatrix} = -a^3 - 1 + \cancel{a} - \cancel{a} = -a^3 - 1$$

$-a^3 - 1 = 0$; $a^3 = -1$; $a = \sqrt[3]{-1}$; $a = -1$ Por lo tanto, si $a \neq -1$, $r(A) = 3$

- Si $a \neq -1 \Rightarrow \left.\begin{array}{l} r(A)=3 \\ r(A^*)=3 \\ n=3 \end{array}\right| \Rightarrow$ S. Compatible Determinado

- Si $a=-1$: $A = \begin{pmatrix} 1 & 1 & 0 \\ -1 & -1 & -1 \\ 1 & 1 & -1 \end{pmatrix}$

$\begin{vmatrix} 1 & 0 \\ -1 & -1 \end{vmatrix} = 1 \neq 0 \Rightarrow \boxed{r(A)=2}$

$A^* = \left(\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ -1 & -1 & -1 & 0 \\ 1 & 1 & 1 & 1 \end{array}\right)$

$\begin{vmatrix} 1 & 0 & 1 \\ -1 & -1 & 0 \\ 1 & 1 & 1 \end{vmatrix} = -1-1+1 = -1 \neq 0 \Rightarrow \boxed{r(A^*)=3} \Longrightarrow$ S. Incompatible

b) $a=0$: $\left.\begin{array}{l} \boxed{y=1} \\ -x-z=0 \\ x+y=1 \end{array}\right\}$ $\rightarrow \boxed{x=0}$ $\rightarrow \boxed{z=0}$

$\boxed{\text{Solución: } \begin{array}{l} x=0 \\ y=1 \\ z=0 \end{array}}$

JUN 12 fase General

$\left.\begin{array}{l} x+y=1 \\ ay+z=0 \\ x+(1+a)y+az=a+1 \end{array}\right\}$

a) $A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & a & 1 \\ 1 & 1+a & a \end{pmatrix}$ $\quad \begin{vmatrix} 1 & 0 \\ a & 1 \end{vmatrix} = 1 \neq 0 \Rightarrow r(A) \geq 2$ para cualquier $a \in \mathbb{R}$

$|A| = \begin{vmatrix} 1 & 1 & 0 \\ 0 & a & 1 \\ 1 & 1+a & a \end{vmatrix} = a^2+1-(1+a) = a^2-a = a(a-1)$

$a(a-1)=0 \Rightarrow a = \begin{cases} 0 \\ 1 \end{cases}$ $\quad$ Si $a=0$ ó $a=1 \rightarrow r(A)=2$

- Si $\left.\begin{array}{l} a\neq 0 \\ a \neq 1 \end{array}\right| \Rightarrow \left.\begin{array}{l} r(A)=3 \\ r(A^*)=3 \\ n=3 \end{array}\right\} \Rightarrow$ S. Compatible Determinado

- Si $a=0$: $A^* = \left(\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{array}\right)$

$\begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix} = 1-1=0 \Rightarrow \left.\begin{array}{l} r(A)=2 \\ r(A^*)=2 \\ n=3 \end{array}\right\} \Rightarrow$ S. Compatible Indeterminado.

- Si $a=1$: $A^* = \left(\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 2 & 1 & 2 \end{array}\right)$

$\begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{vmatrix} = 2+1-2 = 1 \neq 0 \Rightarrow \left.\begin{array}{l} r(A)=2 \\ r(A^*)=3 \end{array}\right\} \Rightarrow$ S. Incompatible

b) $a=0$: $\left.\begin{array}{l} x+\boxed{y}=1 \\ \quad z=0 \\ \cancel{x}+\cancel{y}=\cancel{1} \end{array}\right\} \rightarrow \boxed{\begin{array}{l} y=1-x \\ z=0 \\ x\in\mathbb{R} \end{array}}$

$\boxed{\text{Soluciones: } \begin{array}{l} x\in\mathbb{R} \\ y=1-x \\ z=0 \end{array}}$

JUN 12 fase General

$$|A-xI| = \begin{vmatrix} 3-x & -1 & 0 \\ -1 & 3-x & 0 \\ 0 & 0 & 2-x \end{vmatrix} = (3-x)^2(2-x) - (2-x) = (2-x)\left((3-x)^2-1\right)$$

a) $|A-xI| = 0 \Rightarrow (2-x)\left((3-x)^2-1\right) = 0$

$\rightarrow 2-x=0 \Rightarrow x = \boxed{2}$

$\rightarrow (3-x)^2 = 1 \;;\; 3-x = \pm 1 \;;\; x = 3 \mp 1 = \begin{cases} \boxed{2} \\ \boxed{4} \end{cases}$

b) $M = \begin{pmatrix} 3-x & -1 & 0 \\ -1 & 3-x & 0 \\ 0 & 0 & 2-x \end{pmatrix}$

- Si $\left.\begin{array}{l} x \neq 2 \\ x \neq 4 \end{array}\right| \Rightarrow \left.\begin{array}{l} r(M)=3 \\ m=3 \end{array}\right\} \Rightarrow$ S.C. Determinado

- Si $x=2$: $M = \begin{pmatrix} \boxed{1} & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$|1| = 1 \neq 0 \Rightarrow r(M) \geq 1$

$\left.\begin{array}{l} \begin{vmatrix} 1 & -1 \\ -1 & 1 \end{vmatrix} = 0 \\ \begin{vmatrix} 1 & 0 \\ -1 & 0 \end{vmatrix} = 0 \end{array}\right| \Rightarrow \left.\begin{array}{l} r(M)=1 \\ m=3 \end{array}\right\} \Rightarrow$ S. Compatible Indeterminado

- Si $x=4$: $M = \begin{pmatrix} -1 & -1 & 0 \\ -1 & \boxed{-1} & \boxed{0} \\ 0 & \boxed{0} & \boxed{-2} \end{pmatrix}$

$\begin{vmatrix} -1 & 0 \\ 0 & -2 \end{vmatrix} = 2 \neq 0 \Rightarrow \left.\begin{array}{l} r(M)=2 \\ m=3 \end{array}\right| \Rightarrow$ S. Compatible Indeterminado

c) Al ser homogéneo, si es compatible Determinado, la solución es la Trivial : $\boxed{\begin{array}{l} x_1=0 \\ x_2=0 \\ x_3=0 \end{array}}$

JUN 12 fase Específica

$$\left.\begin{array}{r} (a-1)x+2y+(a-1)z = 1+a \\ (a+1)y-(a+1)z = 2 \\ x + y + az = a \end{array}\right\}$$

a) $A = \begin{pmatrix} a-1 & 2 & a-1 \\ 0 & a+1 & -(a+1) \\ 1 & 1 & a \end{pmatrix}$

$$|A| = \begin{vmatrix} a-1 & 2 & a-1 \\ 0 & a+1 & -(a+1) \\ 1 & 1 & a \end{vmatrix} = (a-1)(a+1)a - 2(a+1) - \cancel{(a+1)(a-1)} + \cancel{(a-1)(a+1)} =$$

$$= (a+1)(a^2-a-2)$$

$(a+1)(a^2-a-2) = 0$ $\rightarrow a+1=0 \;;\; \boxed{a=-1}$

$\rightarrow a^2-a-2=0 \;;\; \boxed{a = \begin{cases} -1 \\ 2 \end{cases}}$

Por lo tanto, si $a=-1$ ó $a=2$, $r(A) \neq 3$

- Si $\left.\begin{array}{l} a \neq -1 \\ a \neq 2 \end{array}\right| \Rightarrow \left.\begin{array}{l} r(A)=3 \\ r(A^*)=3 \\ m=3 \end{array}\right| \Rightarrow$ S. Compatible Determinado

- Si $a=-1 \Rightarrow A = \begin{pmatrix} -2 & 2 & -2 \\ 0 & 0 & 0 \\ 1 & 1 & -1 \end{pmatrix}$

$\begin{vmatrix} -2 & 2 \\ 1 & 1 \end{vmatrix} = -4 \neq 0 \Rightarrow \boxed{r(A)=2}$

$A^* = \left(\begin{array}{ccc|c} -2 & 2 & -2 & 0 \\ 0 & 0 & 0 & 2 \\ 1 & 1 & -1 & -1 \end{array}\right)$

$\begin{vmatrix} -2 & 2 & 0 \\ 0 & 0 & 2 \\ 1 & 1 & -1 \end{vmatrix} = 4+4 = 8 \Rightarrow \boxed{r(A^*)=3}$ $\Rightarrow$ S. Incompatible

• Si $a=2 \Rightarrow A=\begin{pmatrix}1&2&1\\0&3&-3\\1&1&2\end{pmatrix}$

$\begin{vmatrix}1&2\\0&3\end{vmatrix}=3\neq0 \Rightarrow r(A)=2$

$A^*=\left(\begin{array}{ccc|c}1&2&1&3\\0&3&-3&2\\1&1&2&2\end{array}\right)$

$\begin{vmatrix}1&2&3\\0&3&2\\1&1&2\end{vmatrix}=6+4-9-2=-1\neq0 \Rightarrow r(A^*)=3 \Longrightarrow$ S. Incompatible

b) $\underline{\underline{a=0}}$:

$$\left.\begin{array}{r}-x+2y-z=1\\ y-z=2\\ x+y=0\end{array}\right\}$$

$$x=\frac{\begin{vmatrix}1&2&-1\\2&1&-1\\0&1&0\end{vmatrix}}{\begin{vmatrix}-1&2&-1\\0&1&-1\\1&1&0\end{vmatrix}}=\frac{-2+1}{-2+1-1}=\frac{-1}{-2}=\boxed{\frac{1}{2}}$$

$$y=\frac{\begin{vmatrix}-1&1&-1\\0&2&-1\\1&0&0\end{vmatrix}}{-2}=\frac{-1+2}{-2}=\boxed{-\frac{1}{2}}$$

$$z=\frac{\begin{vmatrix}-1&2&1\\0&1&2\\1&1&0\end{vmatrix}}{-2}=\frac{4-1+2}{-2}=\boxed{-\frac{5}{2}}$$

JUL 12 fase General

$$\left.\begin{array}{r}x+y+z=2\\ ax+y=1\\ x+y+2z=3\end{array}\right\}$$

a) $A=\begin{pmatrix}1&1&1\\a&1&0\\1&1&2\end{pmatrix}$

$\begin{vmatrix}1&1\\1&0\end{vmatrix}=-1\neq0 \Rightarrow r(A)\geq2$ para cualquier $a\in\mathbb{R}$

$|A|=\begin{vmatrix}1&1&1\\a&1&0\\1&1&2\end{vmatrix}=2+a-1-2a=1-a$

$1-a=0 \Rightarrow a=1$. Por lo tanto, si $a=1$, $r(A)=2$

• Si $a\neq1 \Rightarrow \left.\begin{array}{l}r(A)=3\\ r(A^*)=3\\ n=3\end{array}\right| \Rightarrow$ S. Compatible Determinado

• Si $a=1$: $A^*=\left(\begin{array}{ccc|c}1&1&1&2\\1&1&0&1\\1&1&2&3\end{array}\right)$

$\begin{vmatrix}1&1&2\\1&0&1\\1&2&3\end{vmatrix}=4+1-3-2=0 \Rightarrow \left.\begin{array}{l}r(A)=2\\ r(A^*)=2\\ n=3\end{array}\right| \Rightarrow$ S. Compatible Indeterminado

b) $\underline{\underline{a=1}}$:

$$\left.\begin{array}{r}x+\boxed{y+z}=2\\ x+\boxed{y}=1\\ \cancel{x+y+2z=3}\end{array}\right\} \rightarrow \left.\begin{array}{l}y+z=2-x\\ \boxed{y=1-x}\\ \boxed{x\in\mathbb{R}}\end{array}\right\} \rightarrow 1-x+z=2-x \;;\; \boxed{z=1}$$

$$\boxed{\text{Soluciones: } \begin{array}{l}x\in\mathbb{R}\\ y=1-x\\ z=1\end{array}}$$

b] $\underline{a=-4}$: $\left.\begin{array}{l} \boxed{-4x+2y}+2z=-4 \\ \boxed{2x-4y}+2z=4 \\ \cancel{2x+2y-4z=0} \end{array}\right\} \longrightarrow \left.\begin{array}{l} -4x+2y=-4-2z \\ 2x-4y=4-2z \end{array}\right\}$ $\boxed{z\in\mathbb{R}}$

$$x=\frac{\begin{vmatrix} -4-2z & 2 \\ 4-2z & -4 \end{vmatrix}}{\begin{vmatrix} -4 & 2 \\ 2 & -4 \end{vmatrix}}=\frac{16+8z-8+4z}{12}=\frac{8+12z}{12}=\boxed{\frac{2+3z}{3}}$$

$$y=\frac{\begin{vmatrix} -4 & -4-2z \\ 2 & 4-2z \end{vmatrix}}{12}=\frac{-16+8z+8+4z}{12}=\frac{-8+12z}{12}=\boxed{\frac{-2+3z}{3}}$$

JUL 13 fase General

a] $|A|=\begin{vmatrix} -a & 2 & 0 \\ 1 & -1-a & 0 \\ 0 & 0 & 1-a \end{vmatrix}=-a(-1-a)(1-a)-2(1-a)=(1-a)(a+a^2-2)$

$|A|=0 \Rightarrow (1-a)(a^2+a-2)=0 \begin{cases} \to 1-a=0 \ ; \ a=1 \\ \to a^2+a-2=0 \ ; \ a=\begin{cases}1\\-2\end{cases}\end{cases}$

b] • Si $\left.\begin{array}{l} a\neq 1 \\ a\neq -2 \end{array}\right| \Rightarrow \left.\begin{array}{l} r(A)=3 \\ n=3 \end{array}\right| \to$ S. Compatible Determinado $\to$ Solución Trivial

• Si $a=1 \to \left.\begin{array}{l} r(A)<3 \\ n=3 \end{array}\right| \to$ S. Compatible Indeterminado

• Si $a=-2 \to \left.\begin{array}{l} r(A)<3 \\ n=3 \end{array}\right| \to$ S. Compatible Indeterminado

c] $\underline{a=1}$: $\left.\begin{array}{l} -x+2y=0 \\ x-2y=0 \\ 0\cdot z=0 \end{array}\right\}$

$A=\begin{pmatrix} -1 & 2 & 0 \\ 1 & -2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ $|-1|=-1\neq 0 \Rightarrow \boxed{r(A)=1}$

No alcanza el rango 2 porque las dos primeras filas son proporcionales entre sí y la 3ª fila es nula.

$\left.\begin{array}{l} \boxed{-x}+2y=0 \\ \cancel{x-2y=0} \\ \cancel{0\cdot z=0} \end{array}\right\}$ $-x=-2y$; $\boxed{x=2y}$ $\boxed{\begin{array}{l} y\in\mathbb{R} \\ z\in\mathbb{R} \end{array}}$

JUL 13 fase General

a] x = nº alumnos matriculados en la 1ª titulación
y = " " " " " 2ª "
z = " " " " " 3ª "

$\left.\begin{array}{l} x+y+z=352 \\ z=\dfrac{x}{3} \\ x-y=2z-2 \end{array}\right\}$; $\left.\begin{array}{l} x+y+z=352 \\ -x+3z=0 \\ x-y-2z=-2 \end{array}\right\} \to x=3z$

$\begin{array}{l} y+4z=352 \\ -y+z=-2 \\ \hline 5z=350 \end{array} \Rightarrow \boxed{z=70} \begin{cases} \to \boxed{y=72} \\ \to \boxed{x=210} \end{cases}$

Hay 210 matriculados en la 1ª titulación, 72 " " " 2ª " " y 70 " " " 3ª " "

JUN 13 fase General

$$A = \begin{pmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & a \end{pmatrix}$$

a) $|A| = a^3 + 1 + 1 - a - a - a = a^3 - 3a + 2$

$|A| = 0 \longrightarrow a^3 - 3a + 2 = 0$;

$$\begin{array}{r|rrrr} & 1 & 0 & -3 & 2 \\ 1 & & 1 & 1 & -2 \\ \hline & 1 & 1 & -2 & \boxed{0} \\ 1 & & 1 & 2 & \\ \hline & 1 & 2 & \boxed{0} & \\ -2 & & -2 & & \\ \hline & 1 & \boxed{0} & & \end{array}$$

$a = \begin{cases} 1 \\ -2 \end{cases}$

- Si $\left.\begin{array}{l} a \neq 1 \\ a \neq -2 \end{array}\right| \Rightarrow \left.\begin{array}{l} r(A) = 3 \\ r(A^*) = 3 \\ n = 3 \end{array}\right| \Rightarrow$ S.C.D.

- Si $a = 1$: $A = \begin{pmatrix} \boxed{1} & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \rightarrow \boxed{r(A) = 1}$

  $A^* = \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{array}\right)$ $\quad \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} = 1 \neq 0 \Rightarrow \boxed{r(A^*) = 2}$ $\Rightarrow$ Sistema Incompatible

- Si $a = -2$: $A = \begin{pmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{pmatrix}$ $\quad \begin{vmatrix} -2 & 1 \\ 1 & -2 \end{vmatrix} = 3 \neq 0 \Rightarrow \boxed{r(A) = 2}$

  $A^* = \left(\begin{array}{ccc|c} -2 & 1 & 1 & 0 \\ 1 & -2 & 1 & 0 \\ 1 & 1 & -2 & 1 \end{array}\right)$ $\quad \begin{vmatrix} -2 & 1 & 0 \\ 1 & -2 & 0 \\ 1 & 1 & 1 \end{vmatrix} = 4 - 1 = 3 \neq 0 \Rightarrow r(A^*) = 3$ $\Rightarrow$ Sistema Incompatible

b) $\underline{a = 0}$: $\left.\begin{array}{rl} y + z &= 0 \\ x \quad + z &= 0 \\ x + y \quad &= 1 \end{array}\right\}$ El sistema tendrá solución única.

$$x = \frac{\begin{vmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix}}{\begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix}} = \frac{1}{1+1} = \boxed{\frac{1}{2}} \; ; \; y = \frac{\begin{vmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix}}{2} = \boxed{\frac{1}{2}} \; ; \; z = \frac{\begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{vmatrix}}{2} = \boxed{-\frac{1}{2}}$$

JUN 13 fase Específica

$$A = \begin{pmatrix} a & 2 & 2 \\ 2 & a & 2 \\ 2 & 2 & a \end{pmatrix}$$

a) $|A| = a^3 + 8 + 8 - 4a - 4a - 4a = a^3 - 12a + 16$

$|A| = 0 \longrightarrow a^3 - 12a + 16 = 0$;

$$\begin{array}{r|rrrr} & 1 & 0 & -12 & 16 \\ 2 & & 2 & 4 & -16 \\ \hline & 1 & 2 & -8 & \boxed{0} \\ 2 & & 2 & 8 & \\ \hline & 1 & 4 & \boxed{0} & \\ -4 & & -4 & & \\ \hline & 1 & \boxed{0} & & \end{array}$$

$a = \begin{cases} 2 \\ -4 \end{cases}$

- Si $\left.\begin{array}{l} a \neq 2 \\ a \neq -4 \end{array}\right| \Rightarrow \left.\begin{array}{l} r(A) = 3 \\ r(A^*) = 3 \\ n = 3 \end{array}\right| \Rightarrow$ S.C.D.

- Si $a = 2$: $A = \begin{pmatrix} \boxed{2} & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{pmatrix} \rightarrow \boxed{r(A) = 1}$

  $A^* = \left(\begin{array}{ccc|c} 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & -2 \\ 2 & 2 & 2 & 0 \end{array}\right)$ $\quad \begin{vmatrix} 2 & 2 \\ 2 & -2 \end{vmatrix} = -8 \neq 0 \Rightarrow \boxed{r(A^*) = 2}$ $\Rightarrow$ Sistema Incompatible

- Si $a = -4$: $A = \begin{pmatrix} -4 & 2 & 2 \\ 2 & -4 & 2 \\ 2 & 2 & -4 \end{pmatrix}$ $\quad \begin{vmatrix} -4 & 2 \\ 2 & -4 \end{vmatrix} = 12 \neq 0 \Rightarrow \boxed{r(A) = 2}$

  $A^* = \left(\begin{array}{ccc|c} -4 & 2 & 2 & -4 \\ 2 & -4 & 2 & 4 \\ 2 & 2 & -4 & 0 \end{array}\right)$ $\quad \begin{vmatrix} -4 & 2 & -4 \\ 2 & -4 & 4 \\ 2 & 2 & 0 \end{vmatrix} = -16 + 16 - 32 + 32 = 0 \Rightarrow \boxed{r(A^*) = 2}$ $\quad \boxed{n = 3}$

$\Rightarrow$ S. Compatible Indeterminado

b) $\begin{vmatrix} 1 & 1 & 1 \\ -1 & 0 & 3 \\ 1 & -1 & -2 \end{vmatrix} = 1+3-2+3 = \boxed{5}$

JUL 13 fase Específica

$A = \begin{pmatrix} 1 & -1 & a \\ a & 3 & 1 \\ 2 & a & 2a \end{pmatrix}$

a) $|A| = \cancel{6a} + a^3 - 2 - \cancel{6a} + 2a^2 - a = a^3 + 2a^2 - a - 2$

$|A| = 0 \rightarrow a^3 + 2a^2 - a - 2 = 0$;

$$\begin{array}{r|rrrr} & 1 & 2 & -1 & -2 \\ 1 & & 1 & 3 & 2 \\ \hline & 1 & 3 & 2 & \boxed{0} \\ -1 & & -1 & -2 & \\ \hline & 1 & 2 & \boxed{0} & \\ -2 & & -2 & & \\ \hline & 1 & \boxed{0} & & \end{array}$$

$a = \begin{cases} 1 \\ -1 \\ -2 \end{cases}$

• Si $\left.\begin{array}{l} a \neq 1 \\ a \neq -1 \\ a \neq -2 \end{array}\right| \Rightarrow \left.\begin{array}{l} r(A)=3 \\ r(A^*)=3 \\ n=3 \end{array}\right| \Rightarrow$ S. C. D.

• Si $a=1$: $A = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 3 & 1 \\ 2 & 1 & 2 \end{pmatrix}$ $\begin{vmatrix} 1 & -1 \\ 1 & 3 \end{vmatrix} = 4 \neq 0 \Rightarrow \boxed{r(A)=2}$

$A^* = \left(\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 1 & 3 & 1 & 0 \\ 2 & 1 & 2 & 2 \end{array}\right)$ $\begin{vmatrix} 1 & -1 & 1 \\ 1 & 3 & 0 \\ 2 & 1 & 2 \end{vmatrix} = 6+1-6+2 = 3 \neq 0 \Rightarrow \boxed{r(A^*)=3}$ → Sistema Incompatible

• Si $a=-1$: $A = \begin{pmatrix} 1 & -1 & -1 \\ -1 & 3 & 1 \\ 2 & -1 & -2 \end{pmatrix}$ $\begin{vmatrix} 1 & -1 \\ -1 & 3 \end{vmatrix} = 2 \neq 0 \Rightarrow \boxed{r(A)=2}$

$A^* = \left(\begin{array}{ccc|c} 1 & -1 & -1 & 1 \\ -1 & 3 & 1 & 0 \\ 2 & -1 & -2 & 2 \end{array}\right)$ $\begin{vmatrix} 1 & -1 & 1 \\ -1 & 3 & 0 \\ 2 & -1 & 2 \end{vmatrix} = \cancel{6}+1-\cancel{6}-2 = -1 \neq 0 \Rightarrow \boxed{r(A^*)=3}$ → Sistema Incompatible

• Si $a=-2$: $A = \begin{pmatrix} 1 & -1 & -2 \\ -2 & 3 & 0 \\ 2 & -2 & -4 \end{pmatrix}$ $\begin{vmatrix} 1 & -1 \\ -2 & 3 \end{vmatrix} = 3-2 = 1 \neq 0 \Rightarrow \boxed{r(A)=2}$

$A^* = \left(\begin{array}{ccc|c} 1 & -1 & -2 & 1 \\ -2 & 3 & 0 & 0 \\ 2 & 2 & -4 & 2 \end{array}\right)$ $\begin{vmatrix} 1 & -1 & 1 \\ -2 & 3 & 0 \\ 2 & 2 & 2 \end{vmatrix} = \cancel{6}-\cancel{4}-\cancel{6}+\cancel{4} = 0 \Rightarrow \boxed{r(A^*)=2}$, $\boxed{n=3}$ → Sistema Compat. Indet.

b) $a=-2$

$\left.\begin{array}{l} x - y - 2z = 1 \\ -2x + 3y + z = 0 \\ \cancel{2x - 2y - 4z = 2} \end{array}\right\}$ $\quad \left.\begin{array}{l} x - y = 1 + 2z \\ -2x + 3y = -z \end{array}\right\}$ $\boxed{z \in \mathbb{R}}$

JUL 13 fase Específica

a) $p(x) = |A - x I_3| = \left| \begin{pmatrix} 0 & 2 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - x \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right| = \begin{vmatrix} -x & 2 & 0 \\ 1 & -1-x & 0 \\ 0 & 0 & 1-x \end{vmatrix} = -x(-1-x)(1-x) - 2(1-x) =$

$= (1-x)(x + x^2 - 2) = (1-x)(x-1)(x+2) = \boxed{-(x-1)^2(x+2)}$

b) $p(x) = 0 \Rightarrow \boxed{x = \begin{cases} 1 \\ -2 \end{cases}}$ Llamo: $M = A - x I_3$

c) • Si $\left.\begin{array}{l} x \neq 1 \\ x \neq -2 \end{array}\right| \Rightarrow \left.\begin{array}{l} r(M)=3 \\ n=3 \end{array}\right| \Rightarrow$ S. Compatible Determinado

• Si $x=1 \Rightarrow M = \begin{pmatrix} -1 & 2 & 0 \\ 1 & -2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ $\left.\begin{array}{l} |-1| \neq 0 \\ \begin{vmatrix} -1 & 2 \\ 1 & -2 \end{vmatrix} = 0 \end{array}\right\} \Rightarrow \left.\begin{array}{l} r(M)=1 \\ n=3 \end{array}\right| \Rightarrow$ Sistema Compatible Indeterminado

$\left.\begin{array}{l} -x_1 + 2x_2 = 0 \\ \cancel{x_1 - 2x_2 = 0} \\ \cancel{0x_3 = 0} \end{array}\right\}$ $\quad -x_1 = -2x_2 \quad \boxed{x_1 = 2x_2}$ $\quad \boxed{\begin{array}{l} x_2 \in \mathbb{R} \\ x_3 \in \mathbb{R} \end{array}}$

• Si $x=-2 \Rightarrow M=\begin{pmatrix} 2 & 2 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

$|2|=2\neq 0 \Rightarrow r(M)\geq 1$

$\begin{vmatrix} 2 & 2 \\ 1 & 1 \end{vmatrix}=0$; $\begin{vmatrix} 2 & 0 \\ 1 & 0 \end{vmatrix}=0$; $\begin{vmatrix} 2 & 0 \\ 0 & 3 \end{vmatrix}=6\neq 0 \Rightarrow r(M)=2$

$n=3$

$\Rightarrow$ S. Compatible Indeterminado.

$$\left.\begin{aligned} 2x_1+2x_2 &= 0 \\ \cancel{x_1+x_2} &= \cancel{0} \\ 3x_3 &= 0 \end{aligned}\right\} \to \boxed{\begin{aligned} x_1 &= -x_2 \\ x_2 &\in \mathbb{R} \\ x_3 &= 0 \end{aligned}}$$

JUN 14 fase General

$$\left.\begin{aligned} -ax+2y &= a \\ x-(1+a)y &= a \\ (1-a)z &= 1 \end{aligned}\right\}$$

a)

$A=\begin{pmatrix} -a & 2 & 0 \\ 1 & -(1+a) & 0 \\ 0 & 0 & 1-a \end{pmatrix}$ $\quad A^*=\left(\begin{array}{ccc|c} -a & 2 & 0 & a \\ 1 & -(1+a) & 0 & a \\ 0 & 0 & 1-a & 1 \end{array}\right)$ máx rango $A^*=3$

$|A| = a(1+a)(1-a)-2(1-a) = (1-a)(a+a^2-2)$

$|A|=0 \to 1-a=0$; $a=1$

$\to a^2+a-2=0$; $a=\frac{-1\pm\sqrt{1+8}}{2}=\frac{-1\pm 3}{2} \to 1,\ -2$

• Si $\left.\begin{aligned} a&\neq 1 \\ a&\neq -2 \end{aligned}\right| \Rightarrow \left.\begin{aligned} r(A)&=3 \\ r(A^*)&=3 \\ n&=2 \end{aligned}\right| \Rightarrow$ S. Compatible Determinado. Solución única.

• Si $a=1 \to A=\begin{pmatrix} -1 & 2 & 0 \\ 1 & -2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$|-1|=-1\neq 0 \Rightarrow r(A)\geq 1$

$\begin{vmatrix} -1 & 2 \\ 1 & -2 \end{vmatrix}=0 \Rightarrow r(A)=1$

$A^*=\begin{pmatrix} \boxed{-1} & 2 & 0 & 1 \\ 1 & -2 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ $\quad \begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix}=-2\neq 0 \Rightarrow r(A^*)=2$

$\Rightarrow$ S. Incompat. Sin Solución

• Si $a=-2 \to A=\begin{pmatrix} 2 & 2 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

$|2|=2\neq 0 \Rightarrow r(A)\geq 1$

$\begin{vmatrix} 2 & 0 \\ 1 & 0 \end{vmatrix}=0$

$\begin{vmatrix} 2 & 0 \\ 0 & 3 \end{vmatrix}=6\neq 0 \Rightarrow r(A)=2$

$A^*=\begin{pmatrix} \boxed{2} & 2 & \boxed{0} & -2 \\ 1 & 1 & 0 & -2 \\ \boxed{0} & 0 & \boxed{3} & 1 \end{pmatrix}$ $\quad \begin{vmatrix} 2 & 0 & -2 \\ 1 & 0 & -2 \\ 0 & 3 & 1 \end{vmatrix}=-6+12=6 \Rightarrow r(A^*)=3$

$\Rightarrow$ S. Incompatible. Sin Solución.

b) $a=-1$

$$\left.\begin{aligned} x+2y &= -1 \\ x &= -1 \\ 2z &= 1 \end{aligned}\right\} \to x=-1,\ z=1/2$$

$-1+2y=-1 \to y=0$

Solución

$x=-1$
$y=0$
$z=1/2$

JUN 14 fase General

a) $A=\begin{pmatrix} 1 & a & a+1 \\ 1 & 1 & 0 \\ 1 & a & a-1 \end{pmatrix}$

$|A|=\begin{vmatrix} 1 & a & a+1 \\ 1 & 1 & 0 \\ 1 & a & a-1 \end{vmatrix} = a-1+\cancel{a^2}+a-\cancel{a}-1-\cancel{a^2}+\cancel{a} = 2a-2$

Si $a\neq 1$, A tiene inversa

b) Si A tiene inversa el sistema de 3 ecuaciones y 3 incógnitas tendrá solución única.

Por otro lado, un sistema homogéneo nunca es incompatible

Por lo tanto, el sistema será compatible indeterminado para $a=1$

$\underline{a=1}$:

$$\left.\begin{array}{l} x+y+2z=0 \\ x+y \quad\quad = 0 \\ \cancel{x+y \quad\quad =0} \end{array}\right\} \longrightarrow \left.\begin{array}{l} x+2z=-y \\ x \quad\quad =-y \end{array}\right\} \quad y\in\mathbb{R}$$

$$x=\frac{\begin{vmatrix}-y & 2\\ -y & 0\end{vmatrix}}{\begin{vmatrix}1 & 2\\ 1 & 0\end{vmatrix}}=\frac{2y}{-2}=-y$$

$$z=\frac{\begin{vmatrix}1 & -y\\ 1 & -y\end{vmatrix}}{-2}=\frac{0}{-2}=0$$

Soluciones: $\boxed{\begin{array}{l} x=-y \\ y\in\mathbb{R} \\ z=0 \end{array}}$

Junio 15 fase Específica

$$\left.\begin{array}{l} x+y+z=2 \\ ax+2y+3z=0 \\ a^2x+4y+9z=-12 \end{array}\right\}$$

a) $A=\begin{pmatrix}1 & 1 & 1\\ a & 2 & 3\\ a^2 & 4 & 9\end{pmatrix}$ $\quad A^*=\begin{pmatrix}1 & 1 & 1 & 2\\ a & 2 & 3 & 0\\ a^2 & 4 & 9 & -12\end{pmatrix}$ máximo rango de $A^*=3$

$$|A|=\begin{vmatrix}1 & 1 & 1\\ a & 2 & 3\\ a^2 & 4 & 9\end{vmatrix}=18+4a+3a^2-2a^2-9a-12=a^2-5a+6$$

$|A|=0 \Rightarrow a^2-5a+6=0$; $a=\dfrac{5\pm\sqrt{25-24}}{2}=\dfrac{5\pm 1}{2}$ $\nearrow 3$, $\searrow 2$

- Si $\left.\begin{array}{l} a\neq 3 \\ a\neq 2\end{array}\right| \Rightarrow \left.\begin{array}{l} r(A)=3 \\ r(A^*)=3 \\ n=3\end{array}\right| \Rightarrow$ S. Compatible Determinado. Solución única.

- Si $a=3$: $A=\begin{pmatrix}1 & 1 & 1\\ 3 & 2 & 3\\ 9 & 4 & 9\end{pmatrix}$ $\quad \begin{vmatrix}1 & 1\\ 3 & 2\end{vmatrix}=-1\neq 0 \Rightarrow r(A)=2$

$A^*=\begin{pmatrix}1 & 1 & 1 & 2\\ 3 & 2 & 3 & 0\\ 9 & 4 & 9 & -12\end{pmatrix}$ $\quad \begin{vmatrix}1 & 1 & 2\\ 3 & 2 & 0\\ 9 & 4 & -12\end{vmatrix}=-24+24-36+36=0 \Rightarrow r(A^*)=2$

$n=3$

$\Rightarrow$ S. Compatible Indeterminado. Infinitas soluciones

- Si $a=2$: $A=\begin{pmatrix}1 & 1 & 1\\ 2 & 2 & 3\\ 4 & 4 & 9\end{pmatrix}$ $\quad \begin{vmatrix}1 & 1\\ 2 & 3\end{vmatrix}=1\neq 0 \Rightarrow r(A)=2$

$A^*=\begin{pmatrix}1 & 1 & 1 & 2\\ 2 & 2 & 3 & 0\\ 4 & 4 & 9 & -12\end{pmatrix}$ $\quad \begin{vmatrix}1 & 1 & 2\\ 2 & 3 & 0\\ 4 & 9 & -12\end{vmatrix}=-36+36+12=12\neq 0 \Rightarrow r(A^*)=3$

$\Rightarrow$ S. Incompatible. Sin solución

b) $\underline{a=3}$

$$\left.\begin{array}{l} x+y+z=2 \\ 3x+2y+3z=0 \\ \cancel{9x+4y+9z=-12} \end{array}\right\} \longrightarrow \left.\begin{array}{l} x+y=2-z \\ 3x+2y=-3z \end{array}\right\} \quad z\in\mathbb{R}$$

$$x=\frac{\begin{vmatrix}2-z & 1\\ -3z & 2\end{vmatrix}}{\begin{vmatrix}1 & 1\\ 3 & 2\end{vmatrix}}=\frac{4-2z+3z}{2-3}=\frac{4-z}{-1}=z-4$$

$$y=\frac{\begin{vmatrix}1 & 2-z\\ 3 & -3z\end{vmatrix}}{\begin{vmatrix}1 & 1\\ 3 & 2\end{vmatrix}}=\frac{-3z-6+3z}{-1}=\frac{-6}{-1}=6$$

Soluciones: $\boxed{\begin{array}{l} x=z-4 \\ y=6 \\ z\in\mathbb{R} \end{array}}$

Julio 14 fase General

$x =$ nº de canastas de 3 puntos
$y =$ " " " " 2 "
$z =$ " " " " 1 "

$$\left.\begin{array}{l} 3x+2y+z=64 \\ z=2+5x \\ y=2+z \end{array}\right\} \rightarrow \left.\begin{array}{l} 3x+2y+z=64 \\ -5x+z=2 \\ y-z=2 \end{array}\right\} \quad A^* = \left(\begin{array}{ccc|c} 3 & 2 & 1 & 64 \\ -5 & 0 & 1 & 2 \\ 0 & 1 & -1 & 2 \end{array}\right)$$

$$x = \frac{\begin{vmatrix} 64 & 2 & 1 \\ 2 & 0 & 1 \\ 2 & 1 & -1 \end{vmatrix}}{\begin{vmatrix} 3 & 2 & 1 \\ -5 & 0 & 1 \\ 0 & 1 & -1 \end{vmatrix}} = \frac{2+4+4-64}{-5-10-3} = \frac{-54}{-18} = 3$$

$$y = \frac{\begin{vmatrix} 3 & 64 & 1 \\ -5 & 2 & 1 \\ 0 & 2 & -1 \end{vmatrix}}{-18} = \frac{-6-10-320-6}{-18} = \frac{-342}{-18} = 19$$

$$z = \frac{\begin{vmatrix} 3 & 2 & 64 \\ -5 & 0 & 2 \\ 0 & 1 & 2 \end{vmatrix}}{-18} = \frac{-320+20-6}{-18} = \frac{-306}{-18} = 17$$

El equipo marcó 3 canastas de 3 puntos, 19 canastas de 2 puntos y 17 Tiros libres.

JUNIO 14 fase Específica

a) $A = \begin{pmatrix} 3a+5 & 7 & 12 \\ 2a+3 & 3 & 6 \\ 3a+4 & 2 & 6 \end{pmatrix}$ $r(A^*) = r(A)$ por ser un sist. Homogéneo

Para que un sistema homogéneo 3x3 tenga más de una solución, el rango de A debe ser menor que 3.

$$|A| = \begin{vmatrix} 3a+5 & 7 & 12 \\ 2a+3 & 3 & 6 \\ 3a+4 & 2 & 6 \end{vmatrix} \underset{F1=F1-F3}{=} \begin{vmatrix} 1 & 5 & 6 \\ 2a+3 & 3 & 6 \\ 3a+4 & 2 & 6 \end{vmatrix} \underset{F3=F3-F2}{=} \begin{vmatrix} 1 & 5 & 6 \\ 2a+3 & 3 & 6 \\ a+1 & -1 & 0 \end{vmatrix} =$$

$$= -6(2a+3)+30(a+1)-18(a+1)+6 =$$
$$= -12a-18+30a+30-18a-18+6 = 0 \Rightarrow r(A) \neq 3$$

Por lo tanto, para ningún valor de a la única solución es la Trivial.

b) $\underline{a=-1}$: $A = \begin{pmatrix} 2 & 7 & 12 \\ 1 & 3 & 6 \\ 1 & 2 & 6 \end{pmatrix}$ $\begin{vmatrix} 2 & 7 \\ 1 & 3 \end{vmatrix} = 6-7=-1 \Rightarrow r(A) \geq 2$

$\begin{vmatrix} 2 & 7 & 12 \\ 1 & 3 & 6 \\ 1 & 2 & 6 \end{vmatrix} = 36+24+42-36-42-24=0 \Rightarrow$

$\Rightarrow r(A)=2$

$$\left.\begin{array}{l} 2x+7y+12z=0 \\ x+3y+6z=0 \\ \cancel{x+2y+6z=0} \end{array}\right\} \rightarrow \left.\begin{array}{l} 2x+7y=-12z \\ x+3y=-6z \\ z\in\mathbb{R} \end{array}\right\}$$

$$x = \frac{\begin{vmatrix} -12z & 7 \\ -6z & 3 \end{vmatrix}}{\begin{vmatrix} 2 & 7 \\ 1 & 3 \end{vmatrix}} = \frac{-36z+42z}{6-7} = \frac{6z}{-1} = -6z$$

$$y = \frac{\begin{vmatrix} 2 & -12z \\ 1 & -6z \end{vmatrix}}{-1} = \frac{-12z+12z}{-1} = 0$$

Soluciones: $\boxed{\begin{array}{l} x=-6z \\ y=0 \\ z\in\mathbb{R} \end{array}}$

Junio 15 fase General

$$\left.\begin{array}{l} ax-ay+3z=a \\ -2x+3y-2z=-1 \\ 2x-y+z=a \end{array}\right\}$$

a) $A=\begin{pmatrix} a & -a & 3 \\ -2 & 3 & -2 \\ 2 & -1 & 1 \end{pmatrix}$ $\quad A^*=\begin{pmatrix} a & -a & 3 & a \\ -2 & 3 & -2 & -1 \\ 2 & -1 & 1 & a \end{pmatrix}$ máx. rango $A^*=3$

$$|A|=\begin{vmatrix} a & -a & 3 \\ -2 & 3 & -2 \\ 2 & -1 & 1 \end{vmatrix} = 3a+6+\cancel{4a}-18-\cancel{2a}-\cancel{2a}=3a-12$$

$|A|=0 \rightarrow a=4$

- Si $a\neq 4 \Rightarrow \left.\begin{array}{l} r(A)=3 \\ r(A^*)=3 \\ n=3 \end{array}\right| \Rightarrow$ S. Compatible Determinado. Solución única.

- Si $a=4$: $A=\begin{pmatrix} 4 & -4 & 3 \\ -2 & 3 & -2 \\ 2 & -1 & 1 \end{pmatrix}$ $\quad \begin{vmatrix} 4 & -4 \\ -2 & 3 \end{vmatrix} = 12-8=4\neq 0 \Rightarrow r(A)=2$

$A^*=\begin{pmatrix} 4 & -4 & 3 & 4 \\ -2 & 3 & -2 & -1 \\ 2 & -1 & 1 & 4 \end{pmatrix}$ $\quad \begin{vmatrix} 4 & -4 & 4 \\ -2 & 3 & -1 \\ 2 & -1 & 4 \end{vmatrix} = 48+4+\cancel{4}-24-32-\cancel{4}=4\neq 0 \Rightarrow$

$\Rightarrow r(A^*)=3$ Sistema Incompatible. Sin solución.

b) $a=1$: $A=\begin{pmatrix} 1 & -1 & 3 \\ -2 & 3 & -2 \\ 2 & -1 & 1 \end{pmatrix}$ $\quad A^*=\begin{pmatrix} 1 & -1 & 3 & 1 \\ -2 & 3 & -2 & -1 \\ 2 & -1 & 1 & 1 \end{pmatrix}$

$$x=\frac{\begin{vmatrix} 1 & -1 & 3 \\ -1 & 3 & -2 \\ 1 & -1 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & -1 & 3 \\ -2 & 3 & -2 \\ 2 & -1 & 1 \end{vmatrix}} = \frac{3+3+2-9-1-2}{3+6+4-18-2-2} = \frac{-4}{-9} = \boxed{\frac{4}{9}}$$

$$y=\frac{\begin{vmatrix} 1 & 1 & 3 \\ -2 & -1 & -2 \\ 2 & 1 & 1 \end{vmatrix}}{-9} = \frac{-1-6-4+6+2+2}{-9} = \boxed{\frac{1}{9}}$$

$$z=\frac{\begin{vmatrix} 1 & -1 & 1 \\ -2 & 3 & -1 \\ 2 & -1 & 1 \end{vmatrix}}{-9} = \frac{3+2+2-6-2-1}{-9} = \boxed{\frac{2}{9}}$$

Julio 14 fase Específica

$$\left.\begin{array}{l} ax-ay+z=2 \\ 3x+2y-2z=a \\ -ax+3y-z=2 \end{array}\right\}$$

a) $A=\begin{pmatrix} a & -a & 1 \\ 3 & 2 & -2 \\ -a & 3 & -1 \end{pmatrix}$ $\quad A^*=\begin{pmatrix} a & -a & 1 & 2 \\ 3 & 2 & -2 & a \\ -a & 3 & -1 & 2 \end{pmatrix}$ máximo rango $A^*=3$

$$|A|=\begin{vmatrix} a & -a & 1 \\ 3 & 2 & -2 \\ -a & 3 & -1 \end{vmatrix} = -\cancel{2a}+9-2a^2+\cancel{2a}-3a+6a = -2a^2+3a+9$$

$$|A|=0 \Rightarrow -2a^2+3a+9=0 \ ; \ a=\frac{-3\pm\sqrt{9+72}}{-4} = \frac{-3\pm 9}{-4} \begin{array}{l} -\frac{3}{2} \\ 3 \end{array}$$

- Si $\left.\begin{array}{l} a\neq -3/2 \\ a\neq 3 \end{array}\right| \Rightarrow \left.\begin{array}{l} r(A)=3 \\ r(A^*)=3 \\ n=2 \end{array}\right| \Rightarrow$ S. Compatible Determinado. Solución única

- Si $a=-3/2 \rightarrow$ $A=\begin{pmatrix} -3/2 & 3/2 & 1 \\ 3 & 2 & -2 \\ 3/2 & 3 & -1 \end{pmatrix}$ $\quad \begin{vmatrix} -3/2 & 3/2 \\ 3 & 2 \end{vmatrix} = -\frac{6}{2}-\frac{9}{2} = -\frac{15}{2}\neq 0 \Rightarrow r(A)=2$

$A^*=\begin{pmatrix} -3/2 & 3/2 & 1 & 2 \\ 3 & 2 & -2 & -3/2 \\ 3/2 & 3 & -1 & 2 \end{pmatrix}$ $\quad \begin{vmatrix} -3/2 & 3/2 & 2 \\ 3 & 2 & -3/2 \\ 3/2 & 3 & 2 \end{vmatrix} = -6+18-\frac{27}{8}-6-9-\frac{27}{4}$

$= -\frac{105}{8}\neq 0 \Rightarrow r(A^*)=3$

$\Rightarrow$ S. Incompatible. Sin solución

• Si $a=3 \longrightarrow A=\begin{pmatrix} 3 & -3 & 1 \\ 3 & 2 & -2 \\ -3 & 3 & -1 \end{pmatrix}$ $\quad \begin{vmatrix} 3 & -3 \\ 3 & 2 \end{vmatrix} = 6+9=15 \Rightarrow r(A)=2$

$A^*=\begin{pmatrix} \boxed{\begin{matrix} 3 & -3 \\ 3 & 2 \end{matrix}} & 1 & 2 \\ \begin{matrix} -3 & 3 \end{matrix} & -2 & 3 \\ & -1 & 2 \end{pmatrix}$ $\quad \begin{vmatrix} 3 & -3 & 2 \\ 3 & 2 & 3 \\ -3 & 3 & 2 \end{vmatrix} = 12+18+\cancel{27}+12+18-\cancel{27}=60 \Rightarrow r(A^*)=$

$\Rightarrow$ S. Incompatible . Sin solución

b] $a=1$:
$$\left.\begin{array}{l} x-y+z=2 \\ 3x+2y-2z=1 \\ -x+3y-z=2 \end{array}\right\}$$

$$x=\frac{\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -2 \\ 2 & 3 & -1 \end{vmatrix}}{\begin{vmatrix} 1 & -1 & 1 \\ 3 & 2 & -2 \\ -1 & 3 & -1 \end{vmatrix}} = \frac{-4+3+\cancel{4}-\cancel{4}-1+12}{-2+9-\cancel{2}+\cancel{2}-3+6} = \frac{10}{10} = \boxed{1}$$

$$y=\frac{\begin{vmatrix} 1 & 2 & 1 \\ 3 & 1 & -2 \\ -1 & 2 & -1 \end{vmatrix}}{10} = \frac{-1+6+4+1+6+4}{10} = \frac{20}{10} = \boxed{2}$$

$$z=\frac{\begin{vmatrix} 1 & -1 & 2 \\ 3 & 2 & 1 \\ -1 & 3 & 2 \end{vmatrix}}{10} = \frac{4+18+1+4+6-3}{10} = \frac{30}{10} = \boxed{3}$$

Julio 15 fase General

$$\left.\begin{array}{l} x+(a-1)y+z=1 \\ ax+(2a-2)y+2z=0 \\ (a+1)x+(3a-3)y+(a+3)z=0 \end{array}\right\}$$

$A=\begin{pmatrix} 1 & a-1 & 1 \\ a & 2a-2 & 2 \\ a+1 & 3a-3 & a+3 \end{pmatrix}$ $\quad A^*=\begin{pmatrix} 1 & a-1 & 1 & 1 \\ a & 2a-2 & 2 & 0 \\ a+1 & 3a-3 & a+3 & 0 \end{pmatrix}$ máx. rango $A^*=3$

$$|A|=\begin{vmatrix} 1 & a-1 & 1 \\ a & 2a-2 & 2 \\ a+1 & 3a-3 & a+3 \end{vmatrix} \underset{\uparrow}{=} \begin{vmatrix} 1 & a-1 & 1 \\ a-2 & 0 & 0 \\ a-2 & 0 & a \end{vmatrix} = -a(a-1)(a-2)$$

$F2 = F2-2F1$
$F3 = F3-3F1$

• Si $\left.\begin{array}{l} a\neq 0 \\ a\neq 1 \\ a\neq 2 \end{array}\right| \Rightarrow \left.\begin{array}{l} r(A)=3 \\ r(A^*)=3 \\ m=3 \end{array}\right\} \Rightarrow$ S. Compatible Determinado . Solución única

• Si $a=0 \longrightarrow A=\begin{pmatrix} 1 & -1 & 1 \\ 0 & -2 & 2 \\ 1 & -3 & 3 \end{pmatrix}$ $\quad \begin{vmatrix} 1 & -1 \\ 0 & -2 \end{vmatrix} = -2\neq 0 \Rightarrow r(A)=2$

$A^*=\begin{pmatrix} \boxed{\begin{matrix} 1 & -1 \\ 0 & -2 \end{matrix}} & 1 & 1 \\ & 2 & 0 \\ 1 \quad -3 & 3 & 0 \end{pmatrix}$ $\quad \begin{vmatrix} 1 & -1 & 1 \\ 0 & -2 & 0 \\ 1 & -3 & 0 \end{vmatrix} = 2\neq 0 \Rightarrow r(A^*)=3$ $\Bigg\} \Rightarrow$

$\Rightarrow$ S. Incompatible . Sin solución.

• Si $a=1 \longrightarrow A=\begin{pmatrix} 1 & 0 & 1 \\ 1 & 0 & 2 \\ 2 & 0 & 3 \end{pmatrix}$ $\quad \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = 1\neq 0 \Rightarrow r(A)=2$

$A^*=\begin{pmatrix} \boxed{\begin{matrix} 1 \\ 1 \end{matrix}} & 0 & \boxed{\begin{matrix} 1 \\ 2 \end{matrix}} & 1 \\ & 0 & & 0 \\ 2 & 0 & 4 & 0 \end{pmatrix}$ $\quad \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 0 \\ 2 & 4 & 0 \end{vmatrix} = 4-4=0 \Rightarrow r(A^*)=2$, $m=3$ $\Bigg\} \Rightarrow$

$\Rightarrow$ Sist. Compatible Indeterminado

Infinitas Soluciones

• Si $a=2 \rightarrow A=\begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 5 \end{pmatrix}$ $\begin{vmatrix} 1 & 1 \\ 3 & 5 \end{vmatrix} = 2 \neq 0 \Rightarrow r(A)=2$

$A^*=\begin{pmatrix} \boxed{1} & 1 & \boxed{1} & 1 \\ 2 & 2 & 2 & 0 \\ \boxed{3} & 3 & \boxed{5} & 0 \end{pmatrix}$ $\begin{vmatrix} 1 & 1 & 1 \\ 2 & 2 & 0 \\ 3 & 5 & 0 \end{vmatrix} = 10-6=4 \Rightarrow r(A^*)=3$ $\Big\} \Rightarrow$

$\Rightarrow$ S. Incompatible. Sin solución.

b) $\underline{a=1}$:

$$\left.\begin{array}{l} x + z = 1 \\ x + 2z = 0 \\ \cancel{2x + 4z = 0} \end{array}\right\}$$

$$x = \frac{\begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix}} = \frac{2}{1} = 2$$

$$z = \frac{\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix}}{1} = \frac{-1}{1} = -1$$

Soluciones: $\boxed{\begin{array}{l} x=2 \\ y\in\mathbb{R} \\ z=-1 \end{array}}$

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$$\left.\begin{array}{l} (a+1)x + y + z = a+1 \\ x + (a+1)y + z = a+3 \\ x + y + (a+1)z = -2a-4 \end{array}\right\}$$

$A=\begin{pmatrix} a+1 & 1 & 1 \\ 1 & a+1 & 1 \\ 1 & 1 & a+1 \end{pmatrix}$ $A^*=\begin{pmatrix} a+1 & 1 & 1 & a+1 \\ 1 & a+1 & 1 & a+3 \\ 1 & 1 & a+1 & -2a-4 \end{pmatrix}$ máx. rango $A^*=3$

$$|A| = \begin{vmatrix} a+1 & 1 & 1 \\ 1 & a+1 & 1 \\ 1 & 1 & a+1 \end{vmatrix} = (a+1)^3+1+1-(a+1)-(a+1)-(a+1) = (a+1)^3+2-3(a+1) =$$

$$= a^3+3a^2+\cancel{3a}+1+2-\cancel{3a}-3 = a^3+3a^2 = a^2(a+3)$$

$a^2(a-3)=0 \Rightarrow a = \begin{cases} 0 \\ -3 \end{cases}$

• Si $\left.\begin{array}{l} a\neq 0 \\ a\neq 3 \end{array}\right| \Rightarrow \left.\begin{array}{l} r(A)=3 \\ r(A^*)=3 \\ n=3 \end{array}\right\} \Rightarrow$ S. Compatible Determinado. Solución única

• Si $a=0 \rightarrow A=\begin{pmatrix} \boxed{1} & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ $r(A)=1$ por tener 3 filas iguales.

$A^*=\begin{pmatrix} \boxed{1} & 1 & 1 & 1 \\ 1 & 1 & 1 & 3 \\ 1 & 1 & 2 & -4 \end{pmatrix}$ $\begin{vmatrix} 1 & 2 \\ 1 & 4 \end{vmatrix} = 4-2=2\neq 0 \Rightarrow r(A^*)=2$ $\Big\} \Rightarrow$

$\Rightarrow$ S. Incompatible. Sin solución

• Si $a=-3 \rightarrow A=\begin{pmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{pmatrix}$ $\begin{vmatrix} -2 & 1 \\ 1 & -2 \end{vmatrix} = 3\neq 0 \Rightarrow r(A)=2$

$A^*=\begin{pmatrix} \boxed{\begin{matrix} -2 & 1 \\ 1 & -2 \end{matrix}} & 1 & -2 \\ 1 \quad 1 & -2 & 2 \end{pmatrix}$ $\begin{vmatrix} -2 & 1 & -2 \\ 1 & -2 & 0 \\ 1 & 1 & 2 \end{vmatrix} = 8-2-4-2=0$

$\Rightarrow r(A^*)=2$

$\Rightarrow$ Sist. Compatible Indeterminado. Infinitas soluciones

b) $\underline{a=-3}$:
$$\left.\begin{array}{l} -2x+y+z=-2 \\ x-2y+z=0 \\ \cancel{x/+y-2z\neq 2} \end{array}\right\} \qquad \left.\begin{array}{l} -2x+y=-2-z \\ x-2y=-z \end{array}\right\} \quad \boxed{z\in\mathbb{R}}$$

$$x=\frac{\begin{vmatrix} -2-z & 1 \\ -z & -2 \end{vmatrix}}{\begin{vmatrix} -2 & 1 \\ 1 & -2 \end{vmatrix}}=\frac{4+2z+z}{4-1}=\boxed{\frac{3z+4}{3}}$$

$$y=\frac{\begin{vmatrix} -2 & -2-z \\ 1 & -z \end{vmatrix}}{3}=\frac{2z+2+z}{3}=\boxed{\frac{3z+2}{3}}$$

Junio 16 fase General

Sea el número $\boxed{x|y|z}$. Es decir: x centenas, y decenas, z unidades

a)
$$\left.\begin{array}{l} y+z=5 \\ (100x+10y+z)-(100z+10y+x)=792 \end{array}\right\} \rightarrow \left.\begin{array}{l} y+z=5 \\ 99x-99z=792 \end{array}\right\} \rightarrow$$

$$\rightarrow \left.\begin{array}{l} y+z=5 \\ x-z=8 \end{array}\right\}$$

b) $A=\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & -1 \end{pmatrix}$ $\quad A^*=\begin{pmatrix} 0 & 1 & 1 & 5 \\ 1 & 0 & -1 & 8 \end{pmatrix}$ máximo rango $A^*=2$

c) $\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}=-1\neq 0 \Rightarrow \left.\begin{array}{l} r(A)=2 \\ r(A^*)=2 \\ n=3 \end{array}\right| \Rightarrow$ S. Compatible Indeterminado
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$$\left.\begin{array}{l} y+z=5 \\ x-z=8 \end{array}\right\} \rightarrow \boxed{\begin{array}{l} y=5-z \\ x=8+z \\ z\in\mathbb{R} \end{array}}$$

Como x, y, z deben ser enteros positivos menores que 10:

$z=0 \rightarrow x=8,\ y=5$
$z=1 \rightarrow x=9 \quad y=4$

$\boxed{\begin{array}{c} 850 \\ 941 \end{array}}$ son los números solución.

Junio 16 fase Específica

$$\left.\begin{array}{l} ax+z=2 \\ ax+ay+4z=8 \\ ay+2z=4 \end{array}\right\}$$

a) $A=\begin{pmatrix} a & 0 & 1 \\ a & a & 4 \\ 0 & a & 2 \end{pmatrix}$ $\quad A^*=\begin{pmatrix} a & 0 & 1 & 2 \\ a & a & 4 & 8 \\ 0 & a & 2 & 4 \end{pmatrix}$ máximo rango $A^*=3$

$$|A|=\begin{vmatrix} a & 0 & 1 \\ a & a & 4 \\ 0 & a & 2 \end{vmatrix}=2a^2+a^2-4a^2=-a^2$$

$|A|=0 \Rightarrow -a^2=0\ ;\ a=0$

- Si $a\neq 0 \Rightarrow \left.\begin{array}{l} r(A)=3 \\ r(A^*)=3 \\ n=3 \end{array}\right| \Rightarrow$ S. Compatible Determinado. Solución única

- Si $a=0 \rightarrow A=\begin{pmatrix} 0 & 0 & \boxed{1} \\ 0 & 0 & 4 \\ 0 & 0 & 2 \end{pmatrix}$ $\quad |1|\neq 0 \Rightarrow r(A)=1$

  $A^*=\begin{pmatrix} 0 & 0 & \boxed{1} & 2 \\ 0 & 0 & 4 & 8 \\ 0 & 0 & 2 & 4 \end{pmatrix}$ $\quad \begin{vmatrix} 1 & 2 \\ 4 & 8 \end{vmatrix}=0 \Rightarrow r(A^*)=2$??

  $n=3$ $\Bigg\} \Rightarrow$

$\Rightarrow$ S. Compatible Indeterminado. Infinitas Soluciones

b) $\underline{a=0}$:

$$\left.\begin{array}{r} z=2 \\ \cancel{4z\neq 8} \\ \cancel{2z\neq 4} \end{array}\right\} \longrightarrow \boxed{z=2}$$

Soluciones: $\boxed{\begin{array}{l} x\in\mathbb{R} \\ y\in\mathbb{R} \\ z=2 \end{array}}$

Julio 16 fase General

$$\left.\begin{array}{r} x+y+z=2 \\ ay+z=1 \\ x+2y+2z=3 \end{array}\right\}$$

a) $A=\begin{pmatrix} 1 & 1 & 1 \\ 0 & a & 1 \\ 1 & 2 & 2 \end{pmatrix}$ $\quad A^*=\begin{pmatrix} 1 & 1 & 1 & 2 \\ 0 & a & 1 & 1 \\ 1 & 2 & 2 & 3 \end{pmatrix}$ máximo rango $A^*=3$

$|A|=\begin{vmatrix} 1 & 1 & 1 \\ 0 & a & 1 \\ 1 & 2 & 2 \end{vmatrix} = 2a+1-a-2 = a-1$

$|A|=0 \Rightarrow a=1$

- Si $a\neq 1 \Rightarrow \left.\begin{array}{l} r(A)=3 \\ r(A^*)=3 \\ n=3 \end{array}\right\} \Rightarrow$ <u>S. Compatible Determinado</u>. Solución única

- Si $a=1 \longrightarrow A=\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 2 & 2 \end{pmatrix}$ $\quad \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix}=1\neq 0 \Rightarrow r(A)=2$

  $A^*=\begin{pmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 1 & 1 \\ 1 & 2 & 2 & 3 \end{pmatrix}$ $\quad \begin{vmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix}=3+1-2-2=0 \Rightarrow r(A^*)=2$, $n=3$ $\Rightarrow$

  $\Rightarrow$ <u>S. Compatible Indeterminado</u>. Infinitas Soluciones

b) $\underline{a=1}$

$$\left.\begin{array}{r} \boxed{x+y}+z=2 \\ \boxed{y}+z=1 \\ \cancel{x+2y+2z=3} \end{array}\right\} \longrightarrow \left.\begin{array}{r} x+y=2-z \\ y=1-z \\ z\in\mathbb{R} \end{array}\right\}$$

$x=\dfrac{\begin{vmatrix} 2-z & 1 \\ 1-z & 1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}} = \dfrac{2-z-1+z}{1} = \boxed{1}$

$y=\dfrac{\begin{vmatrix} 1 & 2-z \\ 0 & 1-z \end{vmatrix}}{1} = \boxed{1-z}$

Soluciones: $\boxed{\begin{array}{l} x=1 \\ y=1-z \\ z\in\mathbb{R} \end{array}}$

Julio 16 fase Específica

$$\left.\begin{array}{r} x+3y+az=3 \\ x+5y+5z=a \\ 2x+6y+3z=8 \end{array}\right\}$$

a) $A=\begin{pmatrix} 1 & 3 & a \\ 1 & 5 & 5 \\ 2 & 6 & 3 \end{pmatrix}$ $\quad A^*=\begin{pmatrix} 1 & 3 & a & 3 \\ 1 & 5 & 5 & a \\ 2 & 6 & 3 & 8 \end{pmatrix}$ máximo rango $A^*=3$

$|A|=\begin{vmatrix} 1 & 3 & a \\ 1 & 5 & 5 \\ 2 & 6 & 3 \end{vmatrix} = 15+6a+\cancel{30}-10a-9-\cancel{30} = 6-4a$

$|A|=0 \rightarrow 6-4a=0$; $a=3/2$

- Si $a\neq 3/2 \Rightarrow \left.\begin{array}{l} r(A)=3 \\ r(A^*)=3 \\ n=3 \end{array}\right\} \Rightarrow$ <u>S. Compatible Determinado</u>. Solución única

- Si $a=3/2 \longrightarrow A=\begin{pmatrix} 1 & 3 & 3/2 \\ 1 & 5 & 5 \\ 2 & 6 & 3 \end{pmatrix}$ $\quad \begin{vmatrix} 1 & 3 \\ 1 & 5 \end{vmatrix}=2\neq 0 \Rightarrow r(A)=2$

  $A^*=\begin{pmatrix} 1 & 3 & 3/2 & 3 \\ 1 & 5 & 5 & 3/2 \\ 2 & 6 & 3 & 8 \end{pmatrix}$ $\quad \begin{vmatrix} 1 & 3 & 3 \\ 1 & 5 & 3/2 \\ 2 & 6 & 8 \end{vmatrix} = 40+18+9-30-24-9=4\neq 0 \Rightarrow r(A^*)=3$

  $\Rightarrow$ <u>S. Incompatible</u>. Sin Solución

b) $\underline{\underline{a=1}}$

$$\left.\begin{array}{l} x+3y+z=3 \\ x+5y+5z=1 \\ 2x+6y+3z=8 \end{array}\right\}$$

$$x=\frac{\begin{vmatrix} 3 & 3 & 1 \\ 1 & 5 & 5 \\ 8 & 6 & 3 \end{vmatrix}}{\begin{vmatrix} 1 & 3 & 1 \\ 1 & 5 & 5 \\ 2 & 6 & 3 \end{vmatrix}}=\frac{45+6+120-40-9-90}{15+6+30-10-9-30}=\frac{32}{2}=\boxed{16}$$

$$y=\frac{\begin{vmatrix} 1 & 3 & 1 \\ 1 & 1 & 5 \\ 2 & 8 & 3 \end{vmatrix}}{2}=\frac{3+8+30-2-9-40}{2}=\frac{-10}{2}=\boxed{-5}$$

$$z=\frac{\begin{vmatrix} 1 & 3 & 3 \\ 1 & 5 & 1 \\ 2 & 6 & 8 \end{vmatrix}}{2}=\frac{40+18+6-30-24-6}{2}=\frac{4}{2}=\boxed{2}$$