1) $A = \begin{pmatrix} 2 & 1 \\ 3 & 3 \\ 1 & 5 \\ 3 & 0 \end{pmatrix}$ $\quad \begin{vmatrix} 2 & 1 \\ 3 & 3 \end{vmatrix} = 6-3 = 3 \neq 0 \Rightarrow \boxed{r=2}$

$A^+ = \begin{pmatrix} 2 & 1 & -1 \\ 3 & 3 & 2 \\ 1 & 5 & 10 \\ 3 & 0 & -5 \end{pmatrix}$ $\quad \begin{vmatrix} 2 & 1 & -1 \\ 3 & 3 & 2 \\ 1 & 5 & 10 \end{vmatrix} = 60-15+2+3-30-20 = 0$

$\begin{vmatrix} 2 & 1 & -1 \\ 3 & 3 & 2 \\ 3 & 0 & -5 \end{vmatrix} = -30+6+9+15 = 0$ $\Rightarrow \boxed{r^+=2}$ } S.C.D.

$\left.\begin{matrix} 2x+y=-1 \\ 3x+3y=2 \end{matrix}\right\}$ $\quad x = \dfrac{\begin{vmatrix} -1 & 1 \\ 2 & 3 \end{vmatrix}}{\begin{vmatrix} 2 & 1 \\ 3 & 3 \end{vmatrix}} = \boxed{\dfrac{-5}{3}}$ $\quad y = \dfrac{\begin{vmatrix} 2 & -1 \\ 3 & 2 \end{vmatrix}}{3} = \boxed{\dfrac{7}{3}}$

2) $A = \begin{pmatrix} 3 & -1 \\ -6 & 2 \\ 2 & 3 \\ 1 & 7 \end{pmatrix}$ $\quad \begin{vmatrix} 3 & -1 \\ -6 & 2 \end{vmatrix} = 6-6 = 0$

$\begin{vmatrix} 3 & -1 \\ 2 & 3 \end{vmatrix} = 9+2 = 11 \Rightarrow \boxed{r=2}$

$A^+ = \begin{pmatrix} 3 & -1 & 4 \\ -6 & 2 & -8 \\ 2 & 3 & 0 \\ 1 & 7 & -4 \end{pmatrix}$ $\quad \begin{vmatrix} 3 & -1 & 4 \\ -6 & 2 & -8 \\ 2 & 3 & 0 \end{vmatrix} = 0-72+16-16+72 = 0$

$\begin{vmatrix} 3 & -1 & 4 \\ 2 & 3 & 0 \\ 1 & 7 & -4 \end{vmatrix} = -36+56-12-8 = 0$ $\Rightarrow \boxed{r^+=2}$ } S.C.D.

$\left.\begin{matrix} 3x-y=4 \\ 2x+3y=0 \end{matrix}\right\}$ $\quad x = \dfrac{\begin{vmatrix} 4 & -1 \\ 0 & 3 \end{vmatrix}}{\begin{vmatrix} 3 & -1 \\ 2 & 3 \end{vmatrix}} = \boxed{\dfrac{12}{11}}$ $\quad y = \dfrac{\begin{vmatrix} 3 & 4 \\ 2 & 0 \end{vmatrix}}{11} = \boxed{\dfrac{-8}{11}}$

3) $A = \begin{pmatrix} 4 & 3 \\ 1 & 2 \\ 3 & -4 \\ 2 & -5 \end{pmatrix}$ $\quad \begin{vmatrix} 4 & 3 \\ 1 & 2 \end{vmatrix} = 8-3 = 5 \Rightarrow \boxed{r=2}$

$A^+ = \begin{pmatrix} 4 & 3 & 2 \\ 1 & 2 & -1 \\ 3 & -4 & 9 \\ 2 & -5 & 0 \end{pmatrix}$ $\quad \begin{vmatrix} 4 & 3 & 2 \\ 1 & 2 & -1 \\ 3 & -4 & 9 \end{vmatrix} = 72-8-9-12-27-16 = 0$

$\begin{vmatrix} 4 & 3 & 2 \\ 1 & 2 & -1 \\ 2 & -5 & 0 \end{vmatrix} = -10-6-8-20 = -44$ $\Rightarrow \boxed{r^+=3}$ } S. Incomp.

4) $A = \begin{pmatrix} 2 & -1 & 3 \\ 3 & -2 & 1 \\ 2 & 0 & 10 \\ 1 & 0 & 2 \end{pmatrix}$ $\quad \begin{vmatrix} 2 & -1 \\ 3 & -2 \end{vmatrix} = -4+3 = -1$

$\begin{vmatrix} 2 & -1 & 3 \\ 3 & -2 & 1 \\ 2 & 0 & 10 \end{vmatrix} = -40-2+12+30 = 0$

$\begin{vmatrix} 2 & -1 & 3 \\ 3 & -2 & 1 \\ 1 & 0 & 2 \end{vmatrix} = -8-1+6+6 = 3 \Rightarrow \boxed{r=3}$

$A^+ = \begin{pmatrix} 2 & -1 & 3 & 3 \\ 3 & -2 & 1 & -1 \\ 2 & 0 & 10 & 2 \\ 1 & 0 & 2 & 0 \end{pmatrix}$ $\quad \begin{vmatrix} 2 & -1 & 3 & 3 \\ 3 & -2 & 1 & -1 \\ 2 & 0 & 10 & 2 \\ 1 & 0 & 2 & 0 \end{vmatrix} = \begin{vmatrix} 2 & -1 & -1 & 3 \\ 3 & -2 & -5 & -1 \\ 2 & 0 & 6 & 2 \\ 1 & 0 & 0 & 0 \end{vmatrix} = -1 \cdot \begin{vmatrix} -1 & -1 & 3 \\ -2 & -5 & -1 \\ 0 & 6 & 2 \end{vmatrix} =$

$= -(10-36-4-6) = 36 \Rightarrow \boxed{r^+=4}$

S. Incompatible.

5] $A = \begin{pmatrix} 0 & 2 & -3 \\ 1 & -2 & 1 \\ 3 & -1 & 1 \end{pmatrix}$ $\quad \begin{vmatrix} 0 & 2 \\ 1 & -2 \end{vmatrix} = -2$

$\begin{vmatrix} 0 & 2 & -3 \\ 1 & -2 & 1 \\ 3 & -1 & 1 \end{vmatrix} = 3 + 6 - 18 - 2 = -11 \Rightarrow \boxed{r = 3}$

$A^+ = \begin{pmatrix} 0 & 2 & -3 & 4 \\ 1 & -2 & 1 & -1 \\ 3 & -1 & 1 & 2 \\ 1 & 1 & 2 & 0 \end{pmatrix}$ $\quad \begin{vmatrix} 0 & 2 & -3 & 4 \\ 1 & -2 & 1 & -1 \\ 3 & -1 & 1 & 2 \\ 1 & 1 & 2 & 0 \end{vmatrix} = \begin{vmatrix} 0 & 2 & -3 & 4 \\ 1 & -2 & 1 & -1 \\ 0 & 5 & -2 & 5 \\ 0 & 3 & 1 & 1 \end{vmatrix} = -1 \cdot \begin{vmatrix} 2 & -3 & 4 \\ 5 & -2 & 5 \\ 3 & 1 & 1 \end{vmatrix} = 0 \Rightarrow \boxed{r^+ = 3}$

$\Big\} \boxed{\text{S.C.D.}}$

$\left.\begin{aligned} 2y - 3z &= 4 \\ x - 2y + z &= -1 \\ 3x - y + z &= 2 \end{aligned}\right\} \rightarrow x = \dfrac{\begin{vmatrix} 4 & 2 & -3 \\ -1 & -2 & 1 \\ 2 & -1 & 1 \end{vmatrix}}{-11} = \dfrac{-13}{-11} = \boxed{\dfrac{13}{11}}$ $\qquad \boxed{y = \dfrac{7}{11}}$ $\qquad \boxed{z = -\dfrac{10}{11}}$

6] $A = \begin{pmatrix} -1 & 1 & -5 \\ 2 & -3 & 1 \\ 1 & 0 & 14 \\ 6 & -10 & -6 \end{pmatrix}$ $\quad \begin{vmatrix} -1 & 1 \\ 2 & -3 \end{vmatrix} = 3 - 2 = 1$

$\begin{vmatrix} -1 & 1 & -5 \\ 2 & -3 & 1 \\ 1 & 0 & 14 \end{vmatrix} = 42 + 1 - 15 - 28 = 0$

$\begin{vmatrix} -1 & 1 & -5 \\ 2 & -3 & 1 \\ 6 & -10 & -6 \end{vmatrix} = -18 + 100 + 6 - 90 + 12 - 10 = 0$

$\Big\} \Rightarrow \boxed{r = 2}$

$A^+ = \begin{pmatrix} -1 & 1 & -5 & 3 \\ 2 & -3 & 1 & 1 \\ 1 & 0 & 14 & -10 \\ 6 & -10 & -6 & 10 \end{pmatrix}$ $\quad \begin{vmatrix} -1 & 1 & 3 \\ 2 & -3 & 1 \\ 1 & 0 & -10 \end{vmatrix} = -30 + 1 + 9 + 20 = 0$

$\begin{vmatrix} -1 & 1 & 3 \\ 2 & -3 & 1 \\ 6 & -10 & 10 \end{vmatrix} = 30 - 60 + 6 + 54 - 20 - 10 = 0$

$\Big\} \Rightarrow \boxed{r^+ = 2}$

S. Compatible Indeterminado.

$\left.\begin{aligned} -x + y &= 3 + 5z \\ 2 - 3y &= 1 - z \end{aligned}\right\}$ $\quad x = \dfrac{\begin{vmatrix} 3+5z & 1 \\ 1-z & -3 \end{vmatrix}}{\begin{vmatrix} -1 & 1 \\ 2 & -3 \end{vmatrix}} = \dfrac{-9 - 15z - 1 + z}{1} = \boxed{-10 - 14z}$

$y = \dfrac{\begin{vmatrix} -1 & 3+5z \\ 2 & 1-z \end{vmatrix}}{1} = \dfrac{-1 + z - 6 - 10z}{1} = \boxed{-7 - 9z}$

$z = \boxed{\text{Cualquier nº real}}$

7] $A = \begin{pmatrix} 6 & 2 & -1 \\ 3 & 1 & -3 \\ 6 & 2 & -11 \end{pmatrix}$ $\quad \begin{vmatrix} 6 & 2 \\ 3 & 1 \end{vmatrix} = 6 - 6 = 0$

$\begin{vmatrix} 6 & -1 \\ 3 & -3 \end{vmatrix} = -18 + 3 = -15$

$\begin{vmatrix} 6 & 2 & -1 \\ 3 & 1 & -3 \\ 6 & 2 & -11 \end{vmatrix} = -66 - 6 - 36 + 6 + 66 + 36 = 0$

$\Big\} \Rightarrow \boxed{r = 2}$

$A^+ = \begin{pmatrix} 6 & 2 & -1 & 1 \\ 3 & 1 & -3 & -2 \\ 6 & 2 & -11 & -9 \end{pmatrix}$ $\quad \begin{vmatrix} 6 & -1 & 1 \\ 3 & -3 & -2 \\ 6 & -11 & -9 \end{vmatrix} = 162 - 33 + 12 + 18 - 27 - 132 = 0 \Rightarrow \boxed{r^+ = 2}$

Sistema Compatible Indeterminado

$$\begin{cases} 6x - z = 1 - 2y \\ 3x - 3z = -2 - y \end{cases}$$

$$x = \frac{\begin{vmatrix} 1-2y & -1 \\ -2-y & -3 \end{vmatrix}}{\begin{vmatrix} 6 & -1 \\ 3 & -3 \end{vmatrix}} = \frac{-3+6y-2-y}{-15} = \frac{-5+5y}{-15} = \boxed{\frac{-y+1}{3}}$$

$y = \boxed{\text{Cualquier nº real}}$

$$z = \frac{\begin{vmatrix} 6 & 1-2y \\ 3 & -2-y \end{vmatrix}}{-15} = \frac{-12-6y-3+6y}{-15} = \boxed{1}$$

8) $A = \left(\begin{array}{cccc} 1 & -2 & 3 & -1 \\ 3 & -1 & -1 & 2 \end{array}\right)$; $\begin{vmatrix} 1 & -2 \\ 3 & -1 \end{vmatrix} = -1+6 = 5 \Rightarrow \boxed{r=2}$

$A^+ = \left(\begin{array}{cccc|c} 1 & -2 & 3 & -1 & 1 \\ 3 & -1 & -1 & 2 & 4 \end{array}\right)$ $\boxed{r^+ = 2}$

$$\begin{cases} x - 2y = 1 - 3z + t \\ 3x - y = 4 + z - 2t \end{cases}$$

$$x = \frac{\begin{vmatrix} 1-3z+t & -2 \\ 4+z-2t & -1 \end{vmatrix}}{\begin{vmatrix} 1 & -2 \\ 3 & -1 \end{vmatrix}} = \frac{-1+3z-t+8+2z-4t}{5} = \boxed{\frac{7+5z-5t}{5}}$$

$$y = \frac{\begin{vmatrix} 1 & 1-3z+t \\ 3 & 4+z-2t \end{vmatrix}}{5} = \frac{4+z-2t-3+9z-3t}{5} = \boxed{\frac{1+10z-5t}{5}}$$

$z = \boxed{\text{Cualquier nº real}}$

$t = \boxed{\text{Cualquier nº real}}$

9) $A = \left(\begin{array}{cc|cc} 4 & -2 & -1 & -1 \\ -2 & 1 & 3 & 1 \end{array}\right)$

$\begin{vmatrix} 4 & -2 \\ -2 & 1 \end{vmatrix} = 4-4 = 0$

$\begin{vmatrix} 4 & -1 \\ -2 & 3 \end{vmatrix} = 12-2 = 10 \Rightarrow \boxed{r=2}$

$A^+ = \left(\begin{array}{cccc|c} 4 & -2 & -1 & -1 & 2 \\ -2 & 1 & 3 & 1 & 3 \end{array}\right)$ $\boxed{r^+ = 2}$

$$\begin{cases} 4x - z = z + 2y + t \\ -2x + 3z = 3 - y - t \end{cases}$$

$$x = \frac{\begin{vmatrix} z+2y+t & -1 \\ 3-y-t & 3 \end{vmatrix}}{\begin{vmatrix} 4 & -1 \\ -2 & 3 \end{vmatrix}} = \frac{6+6y+3t+3-y-t}{10} = \boxed{\frac{9+5y+2t}{10}}$$

$y = \boxed{\text{Cualquier nº real}}$

$$z = \frac{\begin{vmatrix} 4 & z+2y+t \\ -2 & 3-y-t \end{vmatrix}}{10} = \frac{12-4y-4t+4+4y+2t}{10} = \boxed{\frac{16-2t}{10}}$$

$t = \boxed{\text{Cualquier nº real}}$

10

$$A = \begin{pmatrix} 0 & 1 & -2 \\ 0 & 4 & 1 \\ 1 & -1 & 2 \end{pmatrix}$$

$$\begin{vmatrix} 0 & 1 \\ 0 & 4 \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 & -2 \\ 4 & 1 \end{vmatrix} = 1 + 8 = 9$$

$$\begin{vmatrix} 0 & 1 & -2 \\ 0 & 4 & 1 \\ 1 & -1 & 2 \end{vmatrix} = 1 + 8 = 9$$

$\Rightarrow \boxed{r = 3}$

$$A^+ = \left(\begin{array}{ccc|c} 0 & 1 & -2 & 3 \\ 0 & 4 & 1 & 1 \\ 1 & -1 & 2 & -1 \end{array}\right) \quad \boxed{r^+ = 3}$$

S.C.D.

$$x = \frac{\begin{vmatrix} 3 & 1 & -2 \\ 1 & 4 & 1 \\ -1 & -1 & 2 \end{vmatrix}}{\begin{vmatrix} 0 & 1 & -2 \\ 0 & 4 & 1 \\ 1 & -1 & 2 \end{vmatrix}} = \frac{24 + 2 - 1 - 8 - 2 + 3}{9} = \frac{18}{9} = \boxed{2}$$

$$y = \frac{\begin{vmatrix} 0 & 3 & -2 \\ 0 & 1 & 1 \\ 1 & -1 & 2 \end{vmatrix}}{10} = \frac{3 + 2}{9} = \boxed{\frac{5}{9}}$$

$$z = \frac{\begin{vmatrix} 0 & 1 & 3 \\ 0 & 4 & 1 \\ 1 & -1 & -1 \end{vmatrix}}{10} = \frac{1 - 12}{9} = \boxed{\frac{-11}{9}}$$

11

$$A = \begin{pmatrix} 3 & -1 & 2 \\ -1 & 1 & -3 \\ 2 & 0 & -1 \end{pmatrix}$$

$$\begin{vmatrix} 3 & -1 \\ -1 & 1 \end{vmatrix} = 3 - 1 = 2$$

$$\begin{vmatrix} 3 & -1 & 2 \\ -1 & 1 & -3 \\ 2 & 0 & -1 \end{vmatrix} = -3 + 6 - 4 + 1 = 0$$

$\Rightarrow \boxed{r = r^+ = 2}$

$$\begin{cases} 3x - y = -2z \\ -x + y = 3z \end{cases}$$

$$x = \frac{\begin{vmatrix} -2z & -1 \\ 3z & 1 \end{vmatrix}}{\begin{vmatrix} 3 & -1 \\ -1 & 1 \end{vmatrix}} = \frac{-2z + 3z}{2} = \boxed{\frac{z}{2}}$$

$$y = \frac{\begin{vmatrix} 3 & -2z \\ -1 & 3z \end{vmatrix}}{2} = \frac{9z - 2z}{2} = \boxed{\frac{7z}{2}}$$

$z = \boxed{\text{Cualquier nº real}}$

12

$$A = \begin{pmatrix} 5 & -2 & 3 \\ 1 & 0 & -1 \\ 2 & 1 & 0 \end{pmatrix}$$

$$\begin{vmatrix} 5 & -2 \\ 1 & 0 \end{vmatrix} = 2$$

$$\begin{vmatrix} 5 & -2 & 3 \\ 1 & 0 & -1 \\ 2 & 1 & 0 \end{vmatrix} = 3 + 4 + 5 = 12$$

$\Rightarrow \boxed{r = r^+ = 3}$ S.C.D.

$$\boxed{\begin{array}{l} x = 0 \\ y = 0 \\ z = 0 \end{array}}$$ Solución trivial.